ABSTRACT

CHATTERJEE, PAYEL. Development of a Risk Consistent Framework for Seismic Qualification of Equipment. (Under the direction of Dr. Abhinav Gupta).

IEEE 693 (2005) and IEEE 344 (2004) recommend qualification of equipment either by testing or by analysis depending on the voltage classification in order to ensure a safe shut down of power plants during or after an earthquake. The in-situ conditions at the time of installation such as structural properties and boundary conditions of the mounting arrangements can have a significant effect on the behavior and performance of such equipment. In majority of cases when heavy equipment is mounted on support structures, amplification can be either much less or much high than the recommended value of 2.5, depending on the rigidity of the support structures. The current practice for estimation of In-Structure Response Spectrum (ISRS) uses a decoupled analysis. Such an analysis neglects the effect of dynamic interaction between the equipment and the structure that can lead to significant reduction in the equipment response. For sensitive equipment such as relays and switches, the required response spectra (RRS) are often narrow banded due to filtering of ground motion through the structures on which the equipment may be mounted. Consequently, narrowband RRS are clipped to equivalent broadband spectra for fragility estimation. However, the recommended expressions for the clipping factors are based on excitations that are represented in terms of sine beat type motions and not actual earthquake motions

In this thesis, closed-form solutions for seismic amplifications of equipment are derived as functions of modal properties. Wherever possible, simplifications are made to these formulations depending on the combinations of mass and frequency ratios. The reduction in the equipment response due to interaction with the supporting structure is also studied for many different multi degree of freedom systems and the uncertainties in the reduction factor are investigated for a suite of mass ratios and damping ratios of the equipment. An attempt is made to estimate statistical bounds for the reduction factor. The effect of non-classical damping is also investigated. In case of equipment qualification based on equivalent broadband spectra, this study attempts to calculate the clipping factors for real earthquakes that are consistent with the site specific Probabilistic Seismic Hazard Analysis (PSHA) requirements. The purpose is to recommend a framework for seismic qualification of equipment that is consistent with Seismic Probabilistic Risk Assessment (SPRA) requirements.

Development of a Risk Consistent Framework for Seismic Qualification of Equipment

by
Payel Chatterjee

A thesis submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the degree of
Doctor of Philosophy

Civil Engineering

Raleigh, North Carolina

2017

APPROVED BY:

Dr. Abhinav Gupta
Chair of Advisory Committee

Dr. Murthy Guddati

Dr. John Baugh

Dr. Justin Post

DEDICATION

To *Ma*

BIOGRAPHY

Payel Chatterjee was born and brought up in the city of Kolkata in India. She joined the undergraduate program in Civil Engineering at Bengal Engineering and Science University, Shibpur, West Bengal, India in 2005 and received the Bachelors of Engineering (with honors) degree in February 2010. She has been awarded the merit scholarship twice during her undergraduate studies. Upon completion of her undergraduate studies, she joined Indian Oil Corporation Limited, a public sector unit in India, as an Engineering Officer. She worked there till June 2011 and joined North Carolina State University in August 2011 to pursue her Master of Science in Civil Engineering. She completed her M.S. in December 2013 and continued to pursue her doctoral degree in Structural Engineering and Mechanics in the CCEE department at North Carolina State University from January 2014.

Payel is a member of two honor societies, Phi Kappa Phi and Tau Beta Pi at NC State University. She has received the Zia graduate fellowship award for Structural Engineering from the department of CCEE at NC State University in 2016. She also got an internship opportunity at Idaho National Laboratory in Spring 2016.

Apart from doing her research work, she is also working as a Teaching Assistant for both undergraduate and graduate courses in Structural Engineering and Mechanics in NC State University. She is also actively involved with the Office of International Services at North Carolina State University as an Orientation Team member and is a member of the We are WE (Women in Engineering).

ACKNOWLEDGMENTS

A lot of people were instrumental in the completion of my research work. The first person to whom I would like to extend my sincere gratitude would be my advisor, Dr. Abhinav Gupta. His constant guidance and help both during my research work and also at the time of compiling this manuscript cannot be merely acknowledged in a few words. I would also like to thank Dr. Murthy Guddati, Dr. John Baugh and Dr. Justin Post for agreeing to be a part of my advisory committee and having faith in my research work.

My sincerest gratitude to my colleagues and friends Abhilash, Ankit, Saran, Sugandha and Shinyoung. A very special mention to Dr. Vivek Agarwal, my mentor at Idaho National Laboratory where I got the opportunity to intern.

Some of the people that helped me by their presence and influence in my life would be my parents and my brother who constantly motivated me not just verbally but also by placing their confidence in my ability and letting me pursue my dreams. I am also deeply grateful to Ayan who has been a rock throughout and supported me through thick and thin. There are definitely so many more people I would like to acknowledge, friends I have left behind and friends I have made here, without whom I would not have been able to make it so far.

Finally, I would also like to thank the Center for Nuclear Energy Facilities and Structures (CNEFS) at North Carolina State University and all the staff and faculty members of the Civil Engineering Department for providing me all the facilities and support in course of my graduate studies.

TABLE OF CONTENTS

LIST OF TABLES

PART II: CLOSED FORM SOLUTION FOR SEISMIC AMPLIFICATIONS IN ELECTRICAL EQUIPMENT

PART III: A PROBABILISTIC STUDY OF REDUCTION IN SEISMIC RESPONSE OF EQUIPMENT DUE TO COUPLED ANALYSIS

PART IV: A PRA-CONSISTENT FRAMEWORK FOR SEISMIC QUALIFICATION OF EQUIPMENT

LIST OF FIGURES

PART II: CLOSED FORM SOLUTION FOR SEISMIC AMPLIFICATIONS IN ELECTRICAL EQUIPMENT

PART III: A PROBABILISTIC STUDY OF REDUCTION IN SEISMIC RESPONSE OF EQUIPMENT DUE TO COUPLED ANALYSIS

PART IV: A PRA-CONSISTENT FRAMEWORK FOR SEISMIC QUALIFICATION OF EQUIPMENT

PART I: INTRODUCTION

1. General

The reliability of a power plant in case of an earthquake depends on the performance of the individual equipment as well as the effect of interaction between the different components during and after the event. To ensure the safe shut down of a plant, the equipment needs to be qualified either by testing or dynamic analysis as per the requirements of IEEE 344 [1], IEEE 693 [2], and EPRI [3]. The observations from testing are used to characterize the dynamic behavior of the system. The seismic probabilistic risk assessment (PRA) involves the estimation of fragility curves for plant equipment that has been qualified by testing or analysis. EPRI [3] recommends different factors for equipment response and capacity needed in the fragility computations.

Typically, substation equipment are large and bulky systems mounted on various types of support structures. The in-situ conditions at the time of installation such as structural properties and boundary conditions of the mounting arrangements can have a significant effect on the behavior and performance of such equipment. Therefore, it is important to consider the effects of the actual mounting conditions on the equipment qualification. When the design details of the support conditions are not known, IEEE 693 [2] recommends the use of an amplification factor equal to 2.5 on the ground motion to allow for possible amplifications by the support structure. In majority of cases when heavy equipment are mounted on support structures, the amplifications are either much less or greater than the recommended value of 2.5 [4, 5]. For practical purposes, a closed form equation may be useful in estimating the amplification that can be directly utilized for qualification of the equipment. Studies [6, 7] have been conducted to estimate the dynamic response of coupled primary-secondary systems in case of lightly damped structures with very low mass ratios. The effect of mounting condition on the amplification of heavier equipment needs to be studied.

In case of lighter equipment (secondary systems), for classically damped primary-secondary systems, the equations of motion can be uncoupled into independent modal equations. In case of systems that have different damping ratios for the primary and secondary systems, the coupled system becomes non-classically damped. For such systems, the equations of motion can no longer be uncoupled and the off-diagonal terms in the transformed damping matrix cannot be ignored. The reduction in the floor and incabinet response spectra peaks due to mass interaction can be calculated for multiple ground motions that are consistent with the site specific Probabilistic Seismic Hazard Analysis (PSHA) requirements. Subsequently, a probabilistic model for such reduction factors can be useful in estimating the In-structure Response Spectra (ISRS) for the equipment in case of equipment mounted on MDOF primary systems. The effect of mass ratios and frequency ratios on the response spectra can also be assessed for such systems.

The seismic qualification of an equipment based on testing requires the equipment to continue to function when subjected to a specific test response spectrum (TRS). Broad banded ground motions are found to cause more damage to equipment than the filtered narrow banded excitations [8, 9, 10, 11, 12, 13]. As a result, the amplified response of the equipment is clipped to transform the narrow banded spectra to an equivalent broadband response. The broadband clipping factor (C_B) and the modal interaction clipping factor (C_{MI}) together contribute to the definition of the overall clipping factor (C_C). The recommendations given in the current codes of practice are based on analyses of artificially generated input motions that are combinations of narrow banded sine beat type excitations.

It is proposed to study the existing methodologies for estimating the clipping factors (C_B, C_{MI} and C_C) using harmonic input motions as well as actual ground motions. A probabilistic framework needs to be developed based on actual ground motions that satisfy the requirements of the PSHA for specific locations. Wherever necessary, the models are validated and compared with respect to the existing recommendations to evaluate their performance relative to the current methodologies.

2. Background

Depending on the voltage classification and their relative importance in a substation, the equipment can be inherently acceptable, or requires qualification by testing, finite element analysis, and static coefficient analysis [1, 2]. For qualification of equipment based on testing, EPRI [3] recommends different factors for equipment response and capacity for fragility computations of equipment. Depending on the size of the equipment, IEEE 693 [2] recommends a part or the whole equipment to be mounted as per in-situ conditions for testing. The response from the testing is used to characterize the dynamic behavior of the system. The equipment under consideration needs to be qualified with respect to a broad band response spectrum. Experimental observations by Merz [8,9] indicates that broad band spectra produce greater relay chatter and structural damage as compared to narrowband spectra. Studies [3, 10, 11, 12, 13] suggest that narrow frequency high spectral peaks of the required response spectra need to be scaled down to produce a damage effective clipped required response spectra (RRS). Therefore, the amplified response at the degree of freedom of the equipment is the point of interest for qualification based on dynamic testing.

IEEE 693 [2] recommends the consideration of different installation parameters on the qualification of the equipment under in-situ conditions. Some of the important installation parameters as per IEEE 693 [2] are: equipment assembly, site response characteristics, soil-structure interaction (SSI), support structures, base isolation, suspended equipment, anchorage, and conductor loading from the conductor dynamic and adjacent equipment interaction. IEEE 693 [2] recommends the following methods when the equipment cannot be mounted for testing or modeled for analysis in its in-service configuration: a) modifying existing qualified support, b) qualification on multiple supports, c) qualification without support when support parameters are known, and d) qualification without support when support parameters are not known. Often times the design details of the support structure may not be known to the equipment manufacturer and IEEE 693 [2] recommends the use of an amplification of 2.5 times the ground motion for the equipment mounted on such supports. In case of qualification of equipment for nuclear power generating station, IEEE 344 [1] requires consideration of seismic adequacy of the supporting structure and the potential for adverse seismic interaction.

A recent study [4, 5] has compared results from simplified 2 degree-of-freedom (DOF) models (representing the support and the equipment) and corresponding single degree-of-freedom (SDOF) models representing the stand-alone equipment. The mass and stiffness of the equipment is typically higher than that of the supporting structures. Hence the analysis is limited to high mass ratio and frequency ratios between the equipment-support systems. The results of the study suggest that in general the amplification factor of 2.5 as specified by the IEEE 693 [2] standard is a conservative estimate in absence of support dynamic data.

For all practical purposes, it is always beneficial if a closed form equation is available to calculate the amplification ratio for a system. Previous studies [6, 7, 14] propose closed form solutions for dynamic response of coupled primary-secondary systems. However, these formulations are for lightly damped systems (classically damped) with low mass ratios (lighter secondary systems). A hybrid methodology is proposed [15] for the seismic qualification of heavier equipment such as transformers and gas insulated switchgears (GIS) which uses experimental data from shake table tests of an equipment unit as well as the analytical results obtained from a detailed finite element analysis. In this study, a closed form equation is developed to model the dynamic behavior of multi degree of freedom systems starting from the equation of motion of the system. Such formulations are useful to estimate the amplified response of equipment with high mass ratios.

In case of classically damped systems, the off-diagonal terms in the transformed damping matrix are ignored and the equation of motion can be uncoupled into independent modal equations by pre-multiplying the classical damping matrix by the transpose of the mode shape matrix and post-multiplying it by the mode shape matrix. However, in case of non-classical damping such an approximation is not accurate.

Studies [6, 7, 14, 16, 17, 18, 19] have been conducted over the years to develop simplified closed-form solutions as a reasonable approximation to the complete coupled problem. These studies are mainly conducted on classically damped secondary systems in which lighter secondary systems (such as piping in a nuclear power plant) are mounted on relatively heavier supporting structures . In this thesis, these developments are summarized and simplified closed-form solutions are presented to account for coupling between primary and secondary systems.

Other studies have also been conducted over the last two to three decades to develop different methods that can overcome the limitations of uncoupled analysis and consider non-classical damping and dynamic interaction between equipment and structures while estimating the ISRS accurately. Lin and Liu [20], USNRC [21] and RDT Standard [22] study the effect of coupling on the secondary system response as functions of mass and frequency ratios of the primary-secondary systems to identify regions in which decoupling cannot be permitted. Several semi-empirical as well as heuristic methods have been developed to perform coupled analysis of secondary systems [23, 24, 25, 26]. Singh [27, 28, 29] uses the random vibration approach to develop a response spectrum method for non-classically damped systems. Gupta [30] and Gupta and Jaw [31] have developed approximate methods for evaluating the complex eigenvalues and eigenvectors of non-classically damped primary-secondary systems. The floor response spectrum method or ISRS method can be used to calculate the response of the secondary system [6, 7, 30, 32, 33, 34, 35, 36].

Zacharia [14] has proposed closed-form equations to calculate the in-structure response of single degree of freedom (SDOF) secondary systems connected to multi-degree of freedom (MDOF) primary systems accounting for dynamic mass interaction and non-classical damping. The formulations are developed for tuned or nearly tuned and detuned systems for lightly damped structures and for very small mass ratios. It is shown that in tuned or nearly tuned cases, due to interaction between the primary and secondary systems, the spectral response of the secondary system may be excessively overestimated if the effect of mass interaction is not considered. The effect of non-classical damping also needs to be incorporated when calculating the reduction factor for equipment response due to mass interaction.

In case of lighter equipment such as relays and switches, the input motion gets filtered through the building and supporting structures such that the resulting excitation is narrow banded in nature. However, broad banded ground motions are found to cause more damage to such equipment than the filtered narrow banded excitations. As a result, the definition of acceleration capacity used in the fragility models involve clipped response spectra for both TRS and RRS. Kana [12] shows that the development of appropriate dynamic amplification for devices mounted in equipment is based on the root-mean-square (RMS) severity changes with bandwidth, multimode interaction and multi-axis excitation. Due to the absence of multi-mode response, variable RMS severity over the bandwidth of the spectra and the lack of interaction of nonlinear responses, a narrow banded input is judged to be less severe from a fragility point of view.

The current recommendation for clipping factor as per EPRI [3] considers a combined effect of broadband clipping factor (C_B) and modal interaction clipping factor (C_{MI}) that incorporates the high RMS severity ratio and modal interaction in case of broadband spectrum. Currently, the site-specific PSHA of nuclear power plants is conducted as per these EPRI guidelines [37]. The recommended clipping factors are developed for artificially generated input motions that are combinations of one or more narrow banded harmonic type excitations.

Majority of the ground motion studies are directed towards the sites in Western United States that are characterized by low frequency ground motions. However, the seismic hazard-consistent ground motion for sites in the Central and Eastern United states (CEUS) often contains significant amounts of high-frequency motion [38, 39]. The performance of power plant components during these high-frequency motions has not been studied in detail. EPRI [38, 39] provides guidelines for the effects of high-frequency ground motion on NPP components based on shake table test results. According to this report, for SPRAs, the fragility methodology in EPRI [3] is directly applicable for both low and high-frequency input motions. The median horizontal cabinet amplification factors presented in EPRI [3] are applicable in the high-frequency case as well. The High Frequency Program conducted by EPRI [38] shows that the high-frequency test capacity is greater than the low-frequency test capacity. Therefore, in absence of high-frequency test capacity, at a minimum the low-frequency spectral acceleration may be extended as a surrogate capacity for a given component subjected to high-frequency range of ground motions. In case of narrow band RRS, the clipping procedure as recommended in EPRI [3] is validated for the high frequency range in the Phase 1 portion of the EPRI Program [38]. There is ongoing research into high frequency fragilities that can be instrumental in the evolution of the current recommendations as additional SPRAs are performed for sites with significant high frequency ground motions.

3. Objective

A study of the existing literature illustrates the limitations in the existing methodologies for estimating the correct dynamic response for the seismic qualification of equipment. The effect of mounting arrangement in case of lighter equipment is studied in detail [6, 7, 14, 16, 17, 18, 19, 30, 31, 32, 33, 34, 35]. The amplification in equipment response due to interaction with the supporting structures needs to be studied for heavier equipment mounted on lighter supporting structures.

In case of lighter equipment, the reduction in the peak spectral response due to dynamic interaction with the primary supporting structure can be probabilistically quantified in terms of the mass and frequency ratios of the system. Also, in case of sensitive equipment such as relays and switches, the filtered narrowband amplified responses need to be clipped to corresponding broad banded spectra for the fragility estimation of the equipment. Since earthquake motions are random in nature, an attempt is made to propose a risk consistent framework for the clipping of the narrowband spectra by using real earthquakes as input motions.

The objectives of the present study can be classified into three broad categories. The key tasks of the proposed research for each of these categories are outlined as follows.

i. Development of a closed form equation for the amplification at the degree of freedom of the equipment for moderately heavier equipment and lighter supporting structure systems: The steps required to achieve the objectives of this research are summarized below.

- Identify system configurations representative of the behavior of equipment-support system at the actual site of installation;
- Validate existing recommendations for the amplification value requirements of equipment-support systems;
- Develop a closed-form solution for the amplification at the degree of freedom of the equipment;
- Validate the closed-form formulation with response spectrum analysis results from single degree of freedom primary and single degree of freedom secondary systems (SDOF-SDOF) to verify its accuracy;
- Propose simplifications to the closed-form formulations, if any;

ii. Estimation of the reduction in the peak spectral response due to coupling effect in an MDOF equipment-support structure and development of a probabilistic model for the reduction factor in case of tuned or nearly tuned primary-secondary systems: The steps required to achieve the objectives of this research are summarized below.

  - Calculate peak spectral responses for different configurations of SDOF secondary systems mounted on MDOF primary systems considering coupling effect as well as different ground motions that are consistent with the PSHA requirements for the locations of interest;
  - Calculate peak spectral response for uncoupled SDOF secondary systems for different configurations of the MDOF primary systems subjected to multiple PSHA consistent ground motions;

- Compare the coupled and uncoupled spectral responses and estimate the reduction factors for transforming uncoupled in-structure spectra to reduced and more accurate coupled spectra;
- Develop a probabilistic model for the reduction factors and study the effect of mass ratio and frequency ratio on the reduction factors.

iii. Validation of the existing methodology for clipping of narrowband response spectra for equipment such as relays and switches and development of a risk consistent framework as per the site specific requirements of Probabilistic Seismic Hazard Analysis (PSHA): The steps required to achieve the objectives of this research are summarized below.

- Investigate the factors such as variable RMS severity and modal interaction that contribute to increased relay chatter and structural damage caused by broadband spectra in comparison to amplified narrowband response spectra for the equipment;
- Calculate the extent of clipping necessary for narrowband response spectra to transform into an equivalent broadband spectra when the equipment is subjected to harmonic excitations as well as combinations of multiple sine beat type motions;
- Develop clipping factors for equipment subjected to real earthquake motions and compare them with the recommended values in the existing codes of practice;

- Develop a probabilistic model for the estimation of clipping factors based on calculations on real earthquake type input motions.

.

4. Organization

The thesis is divided into five main parts. The first part gives an introduction to the problem being considered followed by a discussion of the main objectives of the research. The next three parts are manuscripts which will be submitted for possible publication in peer reviewed journals.

The effect of mass interaction between the equipment and the supporting structures has a significant contribution in estimating the dynamic response of the equipment for qualification purposes. Extensive studies [6, 7, 14, 16, 17, 18, 19, 30, 31, 32, 33, 34, 35] are conducted to estimate the coupled dynamic response of secondary systems (equipment) in case of lightly damped systems with low mass ratios. The seismic amplifications for heavier equipment such as transformers mounted on lighter, more flexible supporting structures have not been studied in detail. The second part of the thesis studies the effect of mass and frequency ratios on the seismic amplifications for such systems. Using the equations of motion for multi degree of freedom systems, closed form equations are derived as functions of modal properties. Wherever possible, simplifications are made to these formulations depending on the combinations of mass and frequency ratios.

In case of lighter sensitive equipment such as relays and switches mounted inside cabinets, the current practice for estimation of In-Structure Response Spectrum (ISRS) neglects the effect of dynamic interaction between the equipment and the structure. In case of high frequency ground motions, such uncoupled analysis not only results in spectra with high peaks but may also result in peaks in high frequency regions. The third part of the thesis compares the coupled equipment response with respect to decoupled response for high frequency and low frequency input motions. A parametric study is conducted to estimate statistical bounds for the reduction factor that can be directly applied to the uncoupled peak responses to give more accurate spectral responses. The effect of non-classical damping of the system is also investigated.

Due to filtering of input motions through the different supporting structures, the response spectra for equipment become more and more narrow banded in nature. However, the equipment needs to be qualified with respect to a wide frequency range. EPRI [3] recommends expressions for clipping factors to estimate equivalent broadband spectral response of the equipment for qualification purposes. However, the studies undertaken to establish these expressions are based on narrowband excitations that are typically combinations of sine beat type motions. The fourth part of the thesis attempts to calculate the clipping factors for real earthquakes that are consistent with the PSHA requirements for a location of interest and develop a risk consistent framework from equipment qualification point of view.

The last part presents a summary and conclusion of the work that are discussed in detail in the second, third and fourth parts.

REFERENCES

[1] IEEE 344, 2004. “IEEE recommended practice for seismic qualification of class 1E equipment for nuclear power generating stations”.

[2] IEEE 693, 2005. “IEEE recommended practice for seismic design of substations”.

[3] EPR1, 1994. “Methodology for developing seismic fragilities.” TR-103959.

[4] Esfandiari, S., Fujisaki, E., Schiff, A., 2013. “Study of support amplification factors for varying frequency & mass ratios of equipment mounted on support.” Trans. 22nd Conference on Structural Mechanics in Reactor Technology, San Francisco, CA, USA, Division IV.

[5] EPRI, 2013. “Study of support amplification factors for various frequency and mass ratios of idealized equipment mounted on a support structure.” Palo Alto, CA, 3002000767.

[6] Gupta A., Gupta, A. K., 1994a. “New developments in coupled seismic analysis of equipment and piping.” Proceedings of the 13th International Conference on Structural Mechanics in Reactor Technology, Porto Alegre, Brazil.

[7] Gupta A., Gupta, A. K., 1994b. “Applications of new developments in coupled seismic analysis of piping systems.” Proceedings of the 13th International Conference on Structural Mechanics in Reactor Technology, Porto Alegre, Brazil.

[8] Merz, K. L., 1991. “Generic seismic ruggedness of power plant equipment, Rev. 1.” Palo Alto, CA, Electric Power Research Institute, NP-5223.

[9] Merz, K. L., 1991. “Seismic ruggedness of relays.” Palo Alto, CA, Electric Power Research Institute, NP-7147.

[10] Kana, D. D., Pomerening, D. J., 1983. “Correlation of methodologies for seismic qualification test of nuclear plant equipment.” SWRI-6582-002, prepared for U.S. Nuclear Regulatory Commission by Southwest Research Institute, San Antonio, Texas.

[11] Kana, D. D., 1989. “Use of amplification factors for developing equipment base seismic capacity for relays.” Letter report to P. Y. Chen, U. S. Nuclear Regulatory Commission.

[12] Kana, D. D., Chen, P. Y., 1991. “Dynamic amplification factors for seismic evaluation of devices mounted in equipment.” Trans. 11th International Conference on Structural Mechanics in Reactor Technology, K28/5, 337-342.

[13] Kana, D. D., Pomerening, D. J., 1987. “Determination of Modal Interaction Correction for Narrowband Fragility Data.” Transactions of the 9th International Conference on Structural Mechanics in Reactor Technology, JK Panel Session (NUREG/CR-0088), 181-205.

[14] George, Z., 1996. “Development and Evaluation of Methods for Coupled Seismic Analysis of Piping Systems.” Doctoral Thesis, Department of Civil Engineering, North Carolina State University, Raleigh, NC.

[15] Chatterjee, P., Gupta, A., 2014. “Reconciliation of experimental and analytical behavior of gas-insulated switchgear.” Technical Report No.-CNEFS-05/2014, Center for Nuclear Energy Facilities and Structures, NC State University, Raleigh, NC.

[16] Gupta, A. K., 1990. “Response Spectrum Method in Seismic Analysis and Design of Structures.” Blackwell Scientific Publications, Inc., ISBN 0-8493-8628-4.

[17] Tadinada, S. K., Gupta, A., 2011. “Consideration of uncertainties in seismic analysis of non-classically damped coupled systems.” Nuclear Engineering and Design, Vol. 241, Issue. 6, pp. 2034-2044.

[18] Gupta, A., Choi, B., 2005. “Consideration of uncertainties in seismic analysis of coupled building-piping systems.” Nuclear Engineering and Design, Vol. 235, pp. 2071-2086.

[19] Gupta, A, Gupta, A. K., 1998. “Missing mass effect in coupled analysis, I: complex modal properties.” J. Struct. Eng. ASCE, 124 (5), pp. 490-495.

[20] Lin, C. W., Liu, T. H., 1975. “A discussion of coupling and resonant effects for integrated systems consisting of subsystems.” Proceedings, Extreme Load Conditions and Limit Analysis Procedures for Structural Reactor Safeguards and Containment Structures, U3/3, Berlin.

[21] U. S. Nuclear Regulatory Commission, 1975. “Standard Review Plan, Section 3.7.2, Seismic System Analysis.”

[22] RDT Standard F9-2T, 1974. “Seismic Requirements for Design of Nuclear Power Plants and Test Facilities.”

[23] Penizen, J., Chopra, A. K., 1965. “Earthquake Response of Appendage on a Multi-Story Building.” Proceedings, 3rd World Conference on Earthquake Engineering, Vol. II, New Zealand.

[24] Biggs, J. M., Roesset, J. M., 1970. “Seismic Analysis of Equipment Mounted on a Massive Structure.” Seismic Design for Nuclear Power Plants, Ed. By Hansen, R. J., MIT press, Cambridge, Massachusetts.

[25] Kapur, K. K., Shao, L. C., 1973. “Generation of Seismic Floor Response Spectra for Equipment Design.” Specialty Conference on Structural Design of Nuclear Power Plant Facilities, ASCE, Chicago, Illinois.

[26] Hadjian, A. H., 1977. “On the decoupling of secondary systems for seismic analysis.” Proceedings, Sixth World Conference on Earthquake Engineering, New Delhi, India.

[27] Singh, M. P., Suarez, L. E., 1986. “A Perturbation Analysis of the Eigenproperties of Equipment-Structure Systems.” Nuclear Engineering and Design, Vol. 97, No. 2, pp. 167-185.

[28] Singh, M. P., Suarez, L. E., 1988. “A Method for Dynamic Coupling with Non-classical Damping Effects.” Journal of Sound and Vibration, Vol. 120, No. 3, pp. 539-555.

[29] Singh, M. P., Burdisso, R. A., 1987. “Multiply Supported Secondary Systems II: Seismic Inputs.” Earthquake Engineering and Structural Dynamics, Vol. 15, No. 1, pp. 73-90.

[30] Gupta, A. K., 1984. “Seismic Response of Multiply Connected MDOF Primary and MDOF Secondary Systems.” Nuclear Engineering and Design, Vol. 81, No. 3, pp. 385-394.

[31] Gupta, A. K., Jaw, J. W., 1986. “Complex Modal Properties of Coupled Moderately Light Equipment-Structure Systems.” Nuclear Engineering and Design, Vol. 91, No. 2, pp. 171-178.

[32] Gupta, A. K., Jaw, J. W., 1986. “Seismic Response of Non-classically Damped Systems.” Nuclear Engineering and Design, Vol. 91, No. 2, pp. 153-159.

[33] Gupta, A. K., Jaw, J. W., 1986. “Response Spectrum Method for Non-classically Damped Systems.” Nuclear Engineering and Design, Vol. 91, No. 2, pp. 161-169.

[34] Gupta, A. K., Jaw, J. W., 1986. “Coupled Response Spectrum Analysis of Secondary Systems Using Uncoupled Modal Properties.” Nuclear Engineering and Design, Vol. 92, No. 1, pp. 61-68.

[35] Gupta, A. K., Jaw, J. W., 1985. “CREST, a Computer Program for Coupled Response Spectrum Analysis of Secondary Systems.” User’s Manual, Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University, Raleigh, North Carolina.

[36] Megahed, H. A., Gupta, A. K., 1992. “Topics in Seismic Response of Non-classically Damped Systems.” Report, Research Program on Nuclear Power Plant Structures, Equipment and Piping, Department of Civil Engineering, North Carolina State University, Raleigh, NC.

[37] Hwang, K., Tong, W. H., Dar, A., 2013. “Bruce B seismic probabilistic risk assessment.” Trans. 22nd Conference on Structural Mechanics in Reactor Technology, San Francisco, CA, USA, Division IX.

[38] EPRI, 2015. “High Frequency Program: Application Guidelines for Functional Confirmation and Fragility Evaluation.” 3002004396, Draft Report.

[39] EPRI, 2007. “Program on Technology Innovation: The Effects of High Frequency Ground Motion on Structures, Components, and Equipment in Nuclear Power Plants.” 1015108.

PART II: CLOSED-FORM SOLUTION FOR SEISMIC AMPLIFICATIONS IN ELECTRICAL EQUIPMENT

1. Introduction

The reliability of a power plant in case of an earthquake depends on the performance of the individual equipment as well as the effect of interaction between the different components during and after the event. Such equipment can be part of an electrical substation or nuclear power plant and needs to be seismically qualified to ensure the safe shutdown of the power plant in case of a seismic event.

Depending on the voltage classification and their relative importance in a substation, the equipment may be considered inherently acceptable, or may require qualification by testing, finite element analysis, static coefficient analysis, etc. [1, 2]. For qualification of equipment based on testing, EPRI [3] recommends different factors for response and capacity to compute fragility of equipment. The response from the testing is used to characterize the dynamic behavior of the system. The equipment under consideration needs to be qualified with respect to a broad band response spectrum.

Typically, substation equipment are large and bulky systems mounted on various types of support structures (transformers, circuit breakers etc.). These support structures may vary in complexity from a single steel pole to complex steel frame structures. Fig. 1 shows a typical 242 kV dead tank breaker mounted on supporting structures [4]. The installation conditions, such as the mounting arrangement can have a significant effect on the behavior and performance of such equipment. Quite often, the vendor may need to qualify the equipment without knowing the design details of the supporting structures. IEEE 693 [2] recommends several methods for equipment qualification as per the requirements of the actual installation site. Depending on the in-situ condition, the equipment can be qualified by modifying existing qualified support or with respect to different possible support configurations. The code also recommends equipment qualification without support in case the support conditions are “not known” or as per site requirements. Under such situations, IEEE 693 [2] requires a uniform amplification factor of 2.5 on the ground motion to allow for possible amplifications by the support structure. Furthermore, for bushings mounted on a transformer, IEEE 693 [2] recommends an amplification factor of 2.0 to account for possible amplification effects of the transformer. In this case, it is assumed that the dynamic properties of the transformer are “not known”. Interestingly, the recommended values of 2.5 and 2.0 are based on professional expertise. The two values can appear to be inconsistent and are in most cases considered excessively conservative. Yet, there is no fundamental basis for such conclusions.

In case of qualification of equipment for nuclear power generating stations (Class 1E equipment), IEEE 344 [1] requires consideration of seismic adequacy of the supporting structure and the potential for adverse seismic interaction.

The actual amplification of the equipment, however, depends on the dynamic characteristics of both the equipment and the support. The equipment to support mass and stiffness ratios therefore contribute to the overall dynamic behavior of the system and in turn affect the amplified response of the equipment. Electric Power Research Institute has supported a recent study [5, 6] on evaluation of the amplifications using simplified two degree of freedom (DOF) models (representing the support and the equipment) and on corresponding single degree of freedom (SDOF) models representing the stand-alone equipment. The system configuration is defined in terms of the ratios of masses and frequencies between the primary (support) and secondary (equipment) systems. The mass and stiffness of the equipment in such systems are typically higher than that of the supporting structures. Hence the analysis is limited to high mass and frequency ratios between the equipment-support systems. The results of the study suggest that in general the amplification factor of 2.5 as specified by the IEEE 693 [2] standard is a good conservative estimate.

The work presented in the exploratory study by EPRI [5, 6] can be advanced further by developing a closed form equation to calculate the amplification ratio for a system. Such a formulation can provide the much needed fundamental basis for selecting appropriate amplification factor and also help in evaluation of actual amplification factors for specific cases of a given equipment and support configuration.

The closed form solution to estimate the amplification of input motion is developed using response spectrum analysis on a 2 DOF coupled system. The formulation is validated with respect to the corresponding time history analysis results. The closed form equations can be used to give an estimate of the amplification for a given frequency ratio and mass ratio of the equipment-support system. These closed form equations are further simplified depending on the ranges of the mass ratio and frequency ratio for the system under consideration. The accuracy of the simplified formulations is also evaluated with respect to the complete formulation.

2. Requirements of IEEE-693 and IEEE-344 for Equipment Mounted on Supports

IEEE 693 [2] gives the minimum requirements for amplification ratios when the equipment cannot be mounted for testing or modeled for analysis in its in-situ configuration.

When the equipment is mounted on supports different from those used in an existing qualification, the equipment-support system is acceptable if the modified support is dynamically equivalent or the amplification of the equipment response due to the support is lower than that due to the existing qualified support. In case, it is necessary to have multiple supports, the complete model needs to be analyzed to identify the system configuration that would reach the limit state first. IEEE 693 [2] requires the qualification of such equipment when it is mounted or modeled on the most seismically vulnerable configuration of the equipment/structures to be used. When the support conditions are known, the equipment can be tested without supports. The code requires the equipment to be qualified with respect to an amplified input ground motion which is 1.1 times the amplification value for the known support system. However, the most critical case for equipment qualification is when the support conditions are not known. Under such circumstances the equipment is mounted directly on the shake table and tested by an input ground motion which is amplified by a factor of 2.5. The qualification will be acceptable when the in-situ demand (required response spectrum) is enveloped by the response spectra from the amplified ground motion (test response spectrum).

In case of nuclear power generating stations (Class 1E equipment), IEEE 344 [1] has identified that the dynamic response of equipment on structures may be amplified or attenuated depending on the damping and natural frequency of the system. There is no direct specification for the amplification factor. The code recommends to consider the installation features while qualifying the equipment both by testing and/or analysis.

The following section reviews the results of the research undertaken as a part of an EPRI project [5] to study the variability of amplification factor based on variations of frequency and mass ratio of the standalone equipment to the support.

3. Existing Study on Amplification Factor for Equipment Mounted on Support

This section follows the procedure outlined in EPRI [5] to calculate the amplification ratio for different system configurations. Similar to the above mentioned study [5], the equipment mounted on a support structure is modeled as a 2-DOF system with dynamic properties in one direction as shown in Fig. 2. The system has two lumped masses: m_p for the primary system (support structure) and m_s for the secondary system (or the equipment). The corresponding uncoupled frequencies are f_p and f_s respectively. The response spectrum for the system can be generated by conducting a coupled time history analysis on the system using an input time history compatible with the IEEE 693 [2] specified input ground motion spectra. Figure 3 shows the IEEE 693 [2] recommended required response spectrum (RRS) anchored to 1.0g for high performance level qualification. A 2% damping ratio is considered for both the primary and the secondary systems. The analysis can be broadly classified into two parts:

- Dynamic analysis of the combined 2 DOF system;
- Study of the equipment stand-alone response subjected to 2.5 times the ground motion (or IEEE- 693 spectral shape anchored to 2.5g).

These two sets of analyses are repeated for a combination of 5 mass ratios (m_s/m_p = 0.1, 0.5, 1.0, 2.0, 4.0), 4 frequency ratios (f_s/f_p) and 4 support stand-alone frequencies (fp = 5, 10, 20, 33 Hz). For each of these two sets of analyses, the following response parameters are studied:

- Plots of response spectra at the base of the equipment vs the IEEE-693 input spectra anchored to 1.0g;
- Peak acceleration and displacement response at the equipment mass point for coupled analysis as well as stand-alone analysis;
- Peak force response at the equipment mass point.

The primary objective of the study [5] is to estimate the variation of amplification ratio for different combinations of mass and frequency ratios of the 2 DOF systems. The results from the study are validated in this section. Fig. 4 shows the variation of normalized amplification ratio vs. frequency ratio for support frequency of 5Hz. Fig. 5 shows the variation of normalized amplification ratio vs. mass ratio for support frequency of 5Hz. In addition, Fig. 6 shows the plot of amplified response spectrum at the top of the support structure superimposed on the ground spectrum for the analysis case corresponding to a support frequency of 10 Hz with frequency ratio of 2.0 and mass ratio of 0.5. The amplification factors are plotted as a function of frequency. The observations from this study [5] is summarized as follows:

- 95% of the analysis cases show that an amplification factor of 2.5 is conservative;
- Only 4% of the cases show an amplification factor greater than 2.5

The observations described above show that assuming an amplification factor equal to 2.5 may lead to conservative design. There is also a possibility that this factor may be greater than 2.5. There is no closed form solution available to estimate the amplification. The formulation presented in this paper is an attempt to develop a closed-form solution that allows accurate estimation of the amplification of equipment response due to the interaction between the support and the equipment.

4. Development of Closed-Form Formulation

4.1 Effect of mass ratio on the Response Spectrum Analysis

The response spectra at the secondary system is plotted for a range of mass ratios and frequency ratios of the system. Figs 7-12 show the plots of the response spectra for coupled as well as uncoupled analyses for various combinations of mass and frequency ratios of the primary and secondary systems. In response spectrum method, the modal responses are combined using different mode combination techniques [7]. Depending on whether the modal frequencies are closely spaced or not, the responses from different modes are combined using Double Sum, SRSS (square root of sum of squares) etc. [7].

In most cases, the substation equipment is heavier and less flexible than the mounting arrangement. As can be observed from the plots, the response at the secondary degree of freedom is primarily dominated by one mode for cases with relatively larger values of frequency ratio. Also, in case of higher masses of the secondary system, the modal frequencies are not closely spaced. So the modal responses in case of the detuned systems can be combined using SRSS. A single mode approximation can provide quite a good estimation in such systems.

4.2 Amplification ratio for multi degree of freedom system

The equation of motion for a multi degree of freedom system can be written as follows:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{u_b\}\ddot{u}_{g_0}\mathrm{e}^{\mathrm{i}\Omega\mathrm{t}} \tag{1}$$

where *[M]*, *[C]* and *[K]* are the mass, damping and stiffness matrices, $\{u_b\}$ is the influence force vector at the base of the system,$\ddot{u}_g(t)$ is the ground acceleration function at the base with frequency *Ω*, and {u} is the relative displacement response vector.

For the 2-DOF system, as shown in Fig. 2,

$$[M] = \begin{bmatrix} m_p & 0 \\ 0 & m_s \end{bmatrix}, [K] = \begin{bmatrix} k_p + k_s & -k_s \\ -k_s & k_s \end{bmatrix},$$

$$[C] = \begin{bmatrix} c_p + c_s & -c_s \\ -c_s & c_s \end{bmatrix} \tag{2}$$

where, k_p and k_s are the stiffness for the primary system (support) and secondary system (equipment) respectively and m_p and m_s are the corresponding masses.

An eigenvalue analysis for the 2-DOF system gives the following expression for the circular frequencies:

$$\omega^2 = \frac{{w_p}^2}{2}\left[(1 + rf^2 + f^2) \pm (1 - f^2)\sqrt{1 + \frac{2rf^2(1+f^2+0.5rf^2)}{(1-f^2)^2}}\right] \tag{3}$$

where, $\omega_p = \sqrt{\frac{k_p}{m_p}}$ is the natural frequency of the primary system and ω_s is the corresponding natural frequency of the secondary system, *r* and *f* are the mass ratio and frequency ratio of the system, as defined in section 3.

The eigenvectors normalized to unity can be expressed as :

$$\{\varphi_1\} = \begin{Bmatrix} \dfrac{2rf^2}{[1 + rf^2 - f^2 - (1 - f^2)\sqrt{1 + \dfrac{2rf^2(1 + f^2 + 0.5rf^2)}{(1 - f^2)^2}}} \\ 1.0 \end{Bmatrix}$$

$$\{\varphi_2\} = \begin{Bmatrix} \dfrac{2rf^2}{[1 + rf^2 - f^2 + (1 - f^2)\sqrt{1 + \dfrac{2rf^2(1 + f^2 + 0.5rf^2)}{(1 - f^2)^2}}} \\ 1.0 \end{Bmatrix}$$

(4)

where, $r = m_s/m_p$, $f = f_s/f_p$

The eigenvectors or the mode shapes can be mass-normalized as:

$$\{\Phi_i\} = \frac{\{\varphi_i\}}{\sqrt{D_i}} \tag{5}$$

where,

$$D_i = \{\varphi_i\}^T[M]\{\varphi_i\}$$

The participation factors can be calculated as:

$$\gamma_i = \{\Phi_i\}^T[M]\{u_b\} \tag{6}$$

If S_{a1} and S_{a2} are the spectral accelerations at mode frequencies ω_1 and ω_2 from ground response spectrum, the total acceleration can be calculated as

$$\{\ddot{u}_{i\,max}\} = \gamma_i\{\Phi_i\}S_{a\,i} \tag{7}$$

The total maximum acceleration ($\ddot{U}_{p\,max}^{\;\;T}$) and ($\ddot{U}_{s\,max}^{\;\;T}$) can be obtained by modal combination. In case of detuned systems the modal responses can be combined using SRSS rule [9]. The amplification ratio is calculated by dividing the total maximum acceleration for the secondary system (equipment) by the ordinates of the ground response spectrum corresponding to the frequency of the secondary system, $S_{a,ground}$.

Mathematically,

$$Amplification\,ratio = \frac{\ddot{U}_{s\,max}^{\;T}}{S_{a,ground}} \tag{8}$$

In the following section, the formulation of amplification at the equipment degree of freedom using response spectrum analysis is validated with respect to exact time history analysis results.

5. Validation of Closed Form Equation for Amplification of Equipment

The natural frequency of the primary system is considered to be equal to 4 Hz. The frequencies, mode shapes and modal participation factors as estimated using equations (3), (4) and (6) match with the results from eigenvalue analysis of the 2 DOF system. The formulation is validated further by comparing the values of the maximum spectral responses at the two degrees of freedom from the analytical formulations and time history analyses. Tables 1 through 5 shows the comparison for the closed-form formulation and time history results.

The validated closed form equations can be used to estimate the amplification ratio at the secondary system. This degree of freedom corresponds to the equipment which typically is heavier and more rigid than the supporting structure. So essentially only one mode contributes to the estimation of the amplification ratio. The accuracy of such estimation is shown in Table 6. As can be observed from the results, the one mode approximation works well for the type of system considered.

The complete form of the closed-form solution for amplification ratio is a polynomial function of mass and frequency ratios. For the convenience of design and analysis, the complete formulations can be simplified further. Such simplifications are based on the assumptions of the range of mass ratio and frequency ratio of the equipment-support system. The following sections discuss the development of these simplified formulations. The performance of these equations are also compared with respect to the complete solution for amplification.

6. Simplified Analytical Models for Different System Configurations

The dynamic response of equipment is dependent on the mass ratio and frequency ratio of the system. The system configuration can be broadly classified into four categories:

Case 1: $r>1$, $f>1$: These are the type of systems where heavy bulky equipment is mounted on light, flexible supporting structures.

Case 2: $r>1$, $f<1$: These are systems where the equipment is heavier than the supporting structures but the support is more rigid than the equipment.

Case 3: $r<1$, $f<1$: This case represents equipment supported on structures such as building which are heavier and more rigid than the equipment.

Case 4: *r*<1, *f*>1: In such a case, the equipment is lighter than the support but more rigid.

The amplification ratio using response spectrum analysis is given by:

$$Amplification\, ratio = \frac{\gamma_1 \varphi_{12} S_{a,\omega 1}}{S_{a,ground}}$$

The simplified analytical models for amplification ratio for the above four cases are discussed below.

6.1 Case 1: *r*>1, *f*>1: One Mode Approximation

In case of heavy, stiff equipment mounted on light flexible supports, the first eigenvalue can be given by:

$$\lambda_1^2 \approx \frac{k_p}{m_p + m_s} = \frac{\omega_p^2}{1 + r}$$

The frequency for the first mode is approximated by:

$$\omega_1 \approx \frac{\omega_p}{\sqrt{1 + r}}$$

The mode shape (normalized to unity) for the first mode is:

$$\varphi_{11} = \frac{k_s}{k_p + k_s + \lambda_1^2 m_p} = \frac{f^2 r(1 + r)}{(1 + rf^2)(1 + r) + 1}$$

$$\varphi_{12} = 1$$

As both *r* and *f* are greater than 1, we can neglect any first order term in the denominator. Simplifying, the first mode shape normalized to unity is:

$$\varphi_1 \approx \begin{Bmatrix} \dfrac{\theta_1}{(2+\theta_1)} \\ 1 \end{Bmatrix}$$

where,

$$\theta_1 = rf^2(1+r)$$

The mass normalized Eigen vector for the first mode shape is:

$$\Phi_1 \approx \begin{Bmatrix} \sqrt{\dfrac{\theta_1}{(2+\theta_1)(1+r)}} \\ \sqrt{\dfrac{(2+\theta_1)}{\theta_1(1+r)}} \end{Bmatrix}$$

The modal participation factor for the first mode is given by:

$$\gamma_1 = (\Phi_{11} + \Phi_{12}r)$$

Substituting the values we get,

$$\gamma_1 \approx \sqrt{\frac{\theta_1}{(2+\theta_1)(1+r)}} + r\sqrt{\frac{(2+\theta_1)}{\theta_1(1+r)}}$$

Therefore, at the degree of freedom of the equipment,

$$\gamma_1\Phi_{12} = 1 + \frac{2r}{\theta_1(1+r)},$$

where,

$$\theta_1 = rf^2(1+r) \tag{9}$$

The simplified formulation for the amplification ratio is compared with the corresponding one mode approximation for the complete closed form equation developed using response spectrum analysis. Table 7 and Fig. 13 compare the performance of such simplified formulations. The figure shows that as the frequency ratio increases the simplified form of the closed form solution converges to the exact results. In case of frequency ratios close to 1, the simplified form always gives the higher values for the amplification ratios.

6.2 Case 2: *r*>1, *f*<1: One Mode Approximation

The analytical formulation for the modal frequency given by equation (3) can be simplified by neglecting higher order terms in *f* as *f*<1.

For the first mode, equation (3) simplifies as:

$$\omega_1 \approx \frac{\omega_p^2}{2}(1 + rf^2 - \sqrt{2rf^2})$$

Using similar simplifications by neglecting higher order terms in *f*, the expression for the unity normalized eigenvector in the first mode as given by equation (4) reduces to:

$$\varphi_{11} \approx \frac{rf^2}{\sqrt{1+4rf^2}}$$

The mass normalized mode shape for the first mode is given by:

$$\Phi_{11} = \frac{rf^2}{(r + 4r^2f^2 + 4r^2f^4)^{0.5}}$$

A plot of the denominator for different values of *r* and *f* (*r*>1, *f*<1) shows that it converges to $\sqrt{r}$ as frequency ratio gets smaller. Fig. 14 shows the plot of the denominator with respect to different frequency and mass ratios in the range of the current case study.

The observations from this figure can be summarized as follows:

- The denominator in the equation for the modal participation factor, γ_1, is plotted as a function of the frequency ratio, *f*.
- As the frequency ratio becomes smaller (*f*<1), for all mass ratios, the denominator converges to the value of $\sqrt{r}$.
- However, for very large mass ratios, the convergence is not truly satisfied. But in such cases the exact value of the denominator will always be higher than $\sqrt{r}$ and consequently the exact amplification ratio will be lower than the simplified value.

As will be discussed later, the simplification in the closed form solution using this approximation gives a good estimate for the modal properties and the amplification ratio.

Therefore, the mass normalized model shapes can be approximated as follows:

$$\Phi_{11} \approx \sqrt{r}f^2$$

$$\Phi_{12} \approx \sqrt{\frac{1 + 4rf^2}{r}}$$

Using, $\gamma_1 = (\Phi_{11} + \Phi_{12}r)$, the modal participation factor for the first mode simplifies to:

$$\gamma_1 = \frac{rf^2}{\sqrt{r + 4r^2f^2 + 4r^2f^4}} + \frac{r\sqrt{1 + 4rf^2}}{\sqrt{r + 4r^2f^2 + 4r^2f^4}} \approx \sqrt{r}f^2 + \sqrt{r}\sqrt{1 + 4rf^2}$$

Therefore, at the degree of freedom of the equipment, for f<1 neglecting fourth order terms in the denominator,

$$\gamma_1\Phi_{12} \approx \frac{f^2}{(\theta_2)^{0.5}} + 1$$

where,

$$\theta_2 = (1 + 4rf^2)$$

(10)

The performance of this simplified formulation with respect to the complete model is shown in Fig. 15. As mentioned before, the simplification as given in equation (10) always calculates the higher values of the amplification ratio and can be used to estimate the amplified response conservatively. As the frequency ratio becomes smaller the simplified solution converges to the exact results.

6.3 Case 3: *r*<1, *f*<1: One Mode Approximation

When both the mass ratio and frequency ratio are less than 1, the fourth order terms in f can be neglected.

Therefore, $\sqrt{r + 4r^2f^2 + 4r^2f^4} \approx \sqrt{r + 4r^2f^2}$.

The eigenvector and the participation factor for the first mode at the degree of freedom of the equipment can be approximated by:

$$\gamma_1 \approx \frac{rf^2}{\sqrt{r + 4r^2f^2 + 4r^2f^4}} + \frac{r\sqrt{1 + 4rf^2}}{\sqrt{r + 4r^2f^2 + 4r^2f^4}} \approx \frac{rf^2 + r\sqrt{1 + 4rf^2}}{\sqrt{r + 4r^2f^2}}$$

$$\Phi_{12} \approx \sqrt{1 + 4rf^2}$$

Therefore, the simplification becomes,

$$\gamma_1\varphi_{12} = \frac{\theta_3}{r(1 + 4\theta_3)^{0.5}} + 1$$

where,

$$\theta_3 = \mathrm{r}f^2$$

(11)

6.4 Case 4: *r*<1, *f*>1: One Mode Approximation

This type of system configuration has a simplified formulation similar to case 1. As the equipment has higher stiffness than the supporting structures (*f*>1), the first eigenvalue can be given by:

$$\lambda_1^2 \approx \frac{k_p}{m_p + m_s} = \frac{\omega_p^2}{1 + r}$$

The frequency for the first mode is approximated by:

$$\omega_1 \approx \frac{\omega_p}{\sqrt{1 + r}}$$

Following similar procedure as Case 1, at the degree of freedom of the equipment,

$$\gamma_1\Phi_{12} = 1 + \frac{2r}{\theta_1(1 + r)}$$

where,

$$\theta_1 = rf^2(1+r) \tag{12}$$

Figs. 16 and 17 compare the results of the simplified formulations for cases 3 and 4 with respect to the one mode approximation of the complete formulation. As can be observed from Fig. 16, for very small mass ratios, the simplified closed form solution results in underestimation of the amplification ratio. However, the simplification works well for moderately light systems. In case of lighter but stiffer equipment, as in Case 4, the simplified closed form solution converges with the exact results.

The next section provides a discussion of the observations from the different case studies for both the complete closed form solution and the simplified forms for the different system configurations presented in the current section.

7. Discussion of Results

The observations from the results can be summarized as follows:

- The spectral response as well as the amplification ratio for the secondary system matches well with exact time history analysis results.
- Often only one mode approximation can be used for such heavy, rigid equipment mounted on lighter and flexible support structures.
- The closed form equations can be further simplified based on the range of mass ratios and frequency ratios of the system.
- In case of systems with mass ratios and frequency ratios greater than 1 (r>1, f>1), the simplified formulation with one mode approximation gives good estimates for

the amplification ratio and the estimated values are higher than the corresponding values from the complete formulation.

- For $r = 1$ and $f = 1$, the amplification ratio is exactly equal to 1.16;
- In case of mass ratio greater than 1 and frequency ratio less than 1 ($r > 1$, $f < 1$), for smaller frequency ratios ($f < 0.5$), the simplified formulation converges with the complete closed form solution. However, as the frequency ratio approaches 1, the simplification always estimates higher amplification values than the complete formulation.
- When both the mass ratios and frequency ratios are small ($r < 1$, $f < 1$), the simplified formulation converges with the exact results for small frequency ratios. However, the convergence is not good as the frequency ratio approaches 1 in case of very light equipment ($r < 0.25$). For such systems, since all the modes of the system participates in the dynamic response of the system, one mode approximation does not give accurate results. In such cases both the modes need to be used to estimate the equipment response [8, 9, 10].
- The simplified one mode approximation gives good results for the case when the equipment is mounted on structures that are heavier but less rigid ($r < 1$, $f > 1$).

The results indicate that the simplified formulation can be used to give conservative estimates of the amplification in the equipment. For the necessary combinations of the mass ratio and frequency ratio, Equations 9 through 12 can be used to estimate the amplification ratio for equipment mounted on structures.

8. Summary and Conclusions

The existing code recommendations require the use of an amplification factor of 2.5 on the ground motion to allow for possible amplifications by the support structure when the design details of the support conditions are not known. In case of bushings on transformer, IEEE 693 [2] recommends an amplification factor of 2.0. Therefore, there is an inconsistency in the existing recommendations (2.5 vs 2.0). EPRI [5] has studied the effect of mass and frequency ratios on the amplification of the equipment response and makes the following observations:

- For small mass ratios and frequency ratios, the amplification ratio is typically lesser than 2.
- For stiff supporting structures, the amplification factor for lighter and flexible equipment usually ranges between 1 and 2.
- When rigid equipment is mounted on flexible supports, the incabinet motion may be amplified much higher than 2.5 times the input ground motion.

When the mass and stiffness ratios of the equipment-support system are known, a closed-form solution in terms of these parameters can enable the estimation of the amplification ratio directly. The study presented in this paper makes an attempt to develop a closed form equation for amplification ratio in terms of the mass and frequency ratios of the system. The response at the point of interest is calculated using response spectrum analysis. The equipment-support system can be typically modeled as a two degree-of-freedom system with high mass and frequency ratios. For such systems, one mode approximations in the response spectrum analysis to estimate the amplification ratio for the equipment give values that match the results of time history analyses. Wherever possible, depending on the combination of the mass ratio and frequency ratio, simplifications are made to the closed-form formulations. The simplified models are validated with respect to the complete formulations. These formulations can be directly utilized to compute the amplified responses for seismic qualification of equipment.

REFERENCES

[1] IEEE 344, 2004. “IEEE recommended practice for seismic qualification of class 1E equipment for nuclear power generating stations”.

[2] IEEE 693, 2005. “IEEE recommended practice for seismic design of substations”.

[3] EPR1, 1994. “Methodology for developing seismic fragilities.” TR-103959.

[4] Freeman, W., 2008. “Seismic considerations of circuit breakers.” IEEE Tutorial, ABB Inc., Mt Pleasant PA, USA.

[5] Esfandiari, S., Fujisaki, E., Schiff, A. 2013. “Study of support amplification factors for varying frequency & mass ratios of equipment mounted on support.” Trans. 22nd Conference on Structural Mechanics in Reactor Technology, San Francisco, CA, USA, Division IV.

[6] EPRI, 2013. “Study of support amplification factors for various frequency and mass ratios of idealized equipment mounted on a support structure.” Palo Alto, CA, 3002000767.

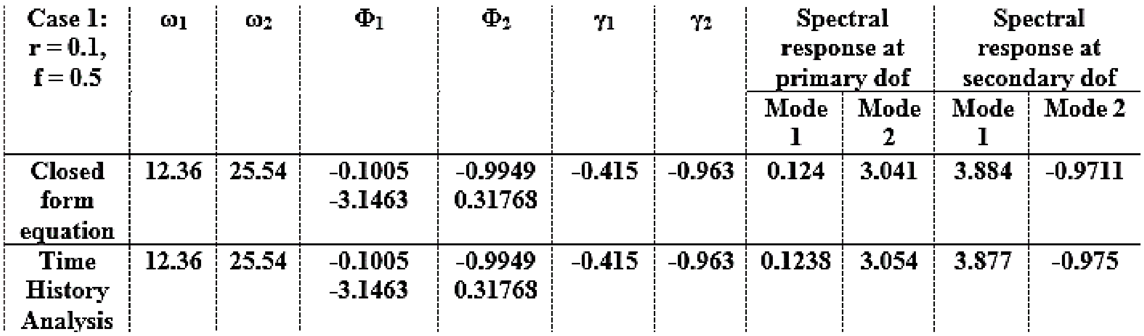
[7] Gupta, A. K., 1990. “Response spectrum method in seismic analysis and design of structures.” Blackwell Scientific Publications.

[8] Gupta A., Gupta, A. K., 1994a. “New developments in coupled seismic analysis of equipment and piping.” Proceedings of the 13th International Conference on Structural Mechanics in Reactor Technology, Porto Alegre, Brazil.

[9] Gupta A., Gupta, A. K., 1994b. “Applications of new developments in coupled seismic analysis of piping systems.” Proceedings of the 13th International Conference on Structural Mechanics in Reactor Technology, Porto Alegre, Brazil.

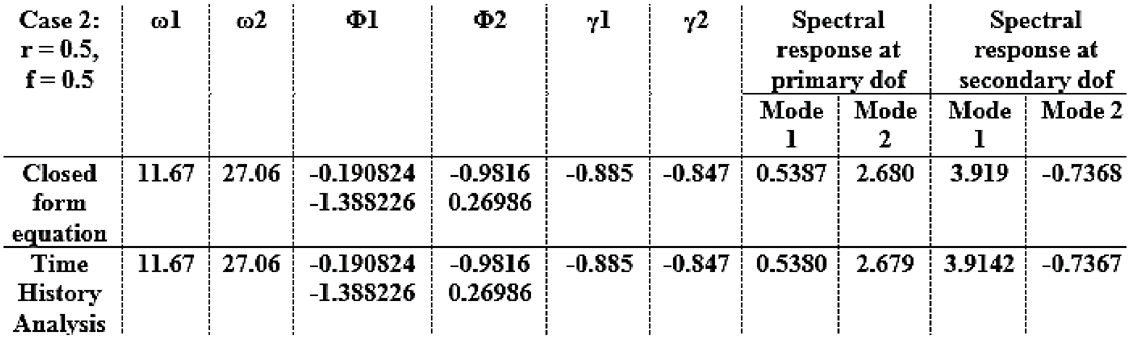
[10] Zacharia, G. 1996. “Development and evaluation of methods for coupled seismic analysis of piping systems.”

Table 1: Validation of closed form equation with time history analysis results for case 1



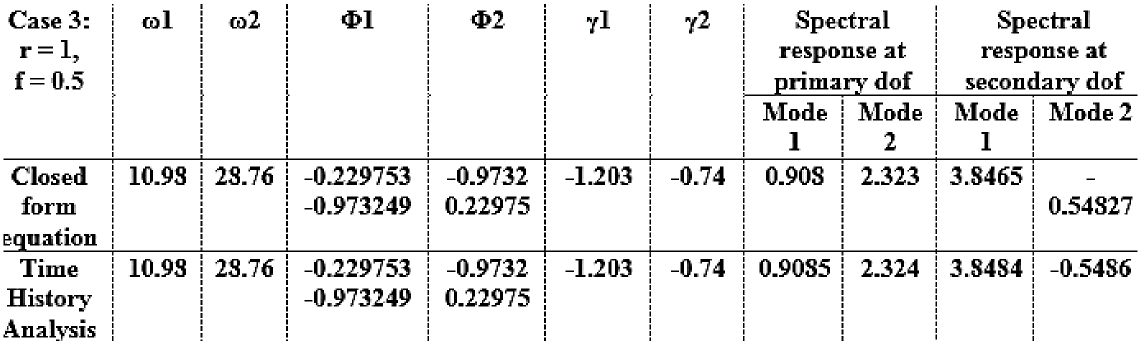
Case 1: r = 0.1, f = 0.5	ω_1	ω_2	Φ_1	Φ_2	γ_1	γ_2	Spectral response at primary dof		Spectral response at secondary dof	
							Mode 1	Mode 2	Mode 1	Mode 2
Closed form equation	**12.36**	**25.54**	**-0.1005 -3.1463**	**-0.9949 0.31768**	**-0.415**	**-0.963**	**0.124**	**3.041**	**3.884**	**-0.9711**
Time History Analysis	**12.36**	**25.54**	**-0.1005 -3.1463**	**-0.9949 0.31768**	**-0.415**	**-0.963**	**0.1238**	**3.054**	**3.877**	**-0.975**

Table 2: Validation of closed form equation with time history analysis results for case 2



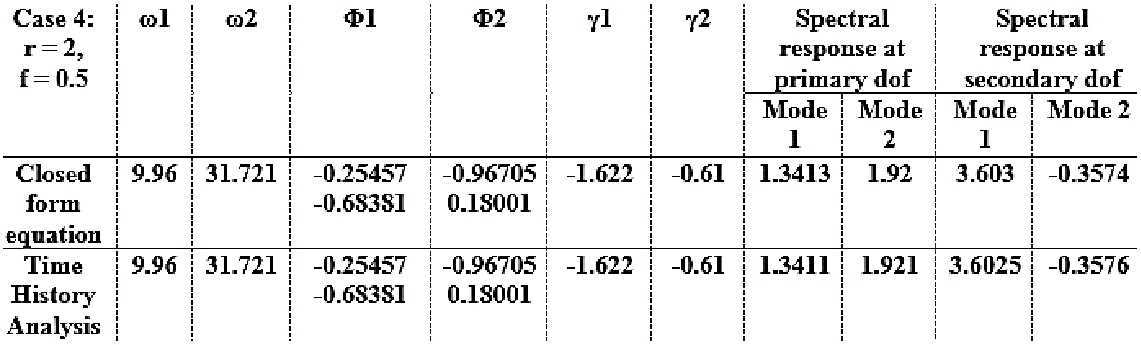
Case 2: r = 0.5, f = 0.5	ω1	ω2	Φ1	Φ2	γ1	γ2	Spectral response at primary dof		Spectral response at secondary dof	
							Mode 1	Mode 2	Mode 1	Mode 2
Closed form equation	**11.67**	**27.06**	**-0.190824 -1.388226**	**-0.9816 0.26986**	**-0.885**	**-0.847**	**0.5387**	**2.680**	**3.919**	**-0.7368**
Time History Analysis	**11.67**	**27.06**	**-0.190824 -1.388226**	**-0.9816 0.26986**	**-0.885**	**-0.847**	**0.5380**	**2.679**	**3.9142**	**-0.7367**

Table 3: Validation of closed form equation with time history analysis results for case 3



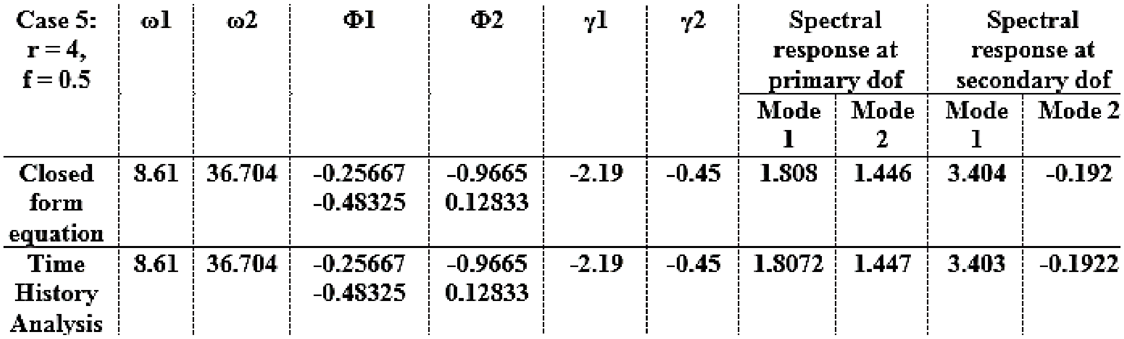
Case 3: r = 1, f = 0.5	ω1	ω2	Φ1	Φ2	γ1	γ2	Spectral response at primary dof		Spectral response at secondary dof	
							Mode 1	Mode 2	Mode 1	Mode 2
Closed form equation	10.98	28.76	-0.229753 -0.973249	-0.9732 0.22975	-1.203	-0.74	0.908	2.323	3.8465	-0.54827
Time History Analysis	10.98	28.76	-0.229753 -0.973249	-0.9732 0.22975	-1.203	-0.74	0.9085	2.324	3.8484	-0.5486

Table 4: Validation of closed form equation with time history analysis results for case 4



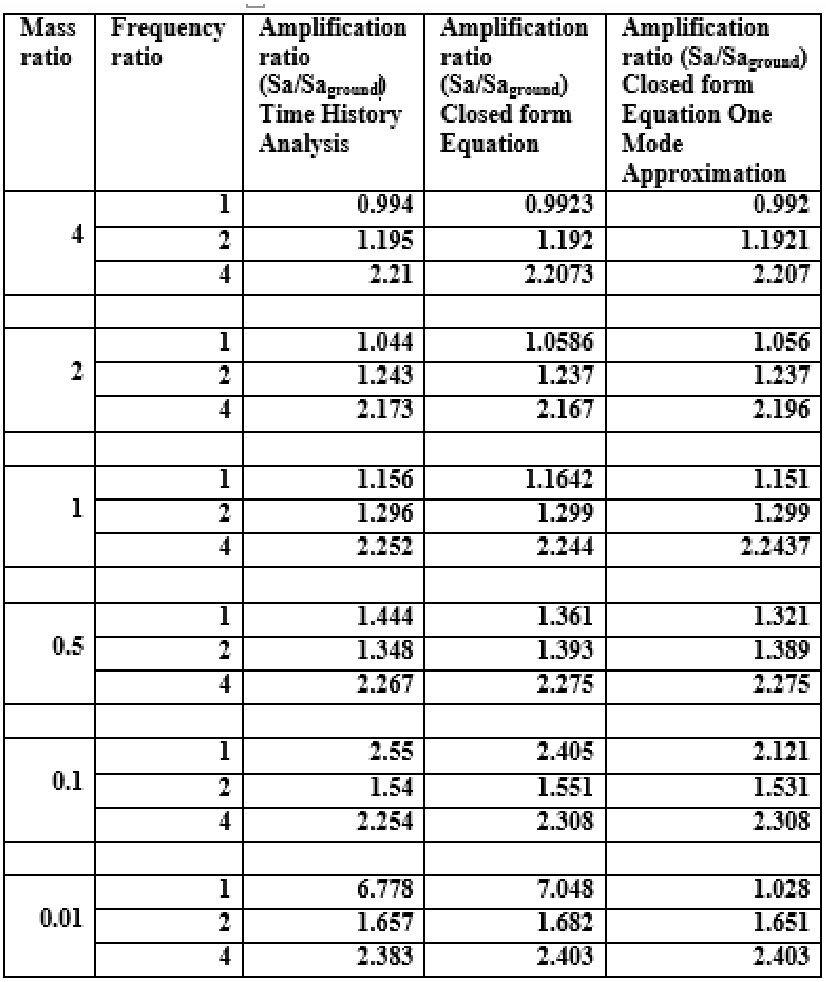
Case 4: r = 2, f = 0.5	ω1	ω2	Φ1	Φ2	γ1	γ2	Spectral response at primary dof		Spectral response at secondary dof	
							Mode 1	Mode 2	Mode 1	Mode 2
Closed form equation	9.96	31.721	-0.25457 -0.68381	-0.96705 0.18001	-1.622	-0.61	1.3413	1.92	3.603	-0.3574
Time History Analysis	9.96	31.721	-0.25457 -0.68381	-0.96705 0.18001	-1.622	-0.61	1.3411	1.921	3.6025	-0.3576

Table 5: Validation of closed form equation with time history analysis results for case 5



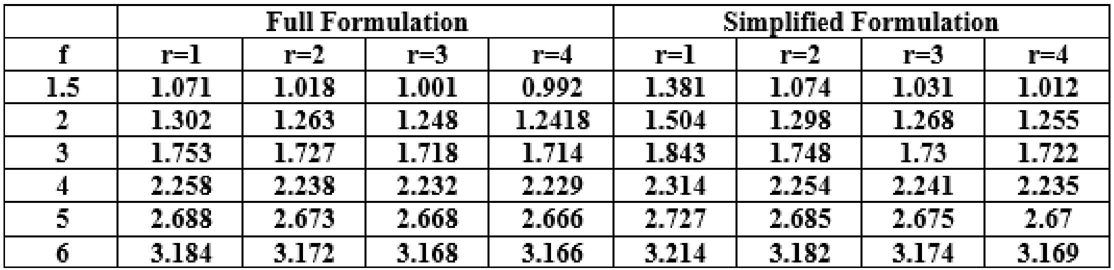
Case 5: r = 4, f = 0.5	ω1	ω2	Φ1	Φ2	γ1	γ2	Spectral response at primary dof		Spectral response at secondary dof	
							Mode 1	Mode 2	Mode 1	Mode 2
Closed form equation	8.61	36.704	-0.25667 -0.48325	-0.9665 0.12833	-2.19	-0.45	1.808	1.446	3.404	-0.192
Time History Analysis	8.61	36.704	-0.25667 -0.48325	-0.9665 0.12833	-2.19	-0.45	1.8072	1.447	3.403	-0.1922

Table 6. Comparison of amplification ratio between time history analyses and closed form equation



Mass ratio	Frequency ratio	Amplification ratio (Sa/Sa_{ground}) Time History Analysis	Amplification ratio (Sa/Sa_{ground}) Closed form Equation	Amplification ratio (Sa/Sa_{ground}) Closed form Equation One Mode Approximation
4	1	0.994	0.9923	0.992
	2	1.195	1.192	1.1921
	4	2.21	2.2073	2.207
2	1	1.044	1.0586	1.056
	2	1.243	1.237	1.237
	4	2.173	2.167	2.196
1	1	1.156	1.1642	1.151
	2	1.296	1.299	1.299
	4	2.252	2.244	2.2437
0.5	1	1.444	1.361	1.321
	2	1.348	1.393	1.389
	4	2.267	2.275	2.275
0.1	1	2.55	2.405	2.121
	2	1.54	1.551	1.531
	4	2.254	2.308	2.308
0.01	1	6.778	7.048	1.028
	2	1.657	1.682	1.651
	4	2.383	2.403	2.403

Table 7: Comparison of amplification ratio from simplified formulation with the complete analytical formulation for *r>1, f>1*



	Full Formulation				Simplified Formulation			
f	**r=1**	**r=2**	**r=3**	**r=4**	**r=1**	**r=2**	**r=3**	**r=4**
1.5	**1.071**	**1.018**	**1.001**	**0.992**	**1.381**	**1.074**	**1.031**	**1.012**
2	**1.302**	**1.263**	**1.248**	**1.2418**	**1.504**	**1.298**	**1.268**	**1.255**
3	**1.753**	**1.727**	**1.718**	**1.714**	**1.843**	**1.748**	**1.73**	**1.722**
4	**2.258**	**2.238**	**2.232**	**2.229**	**2.314**	**2.254**	**2.241**	**2.235**
5	**2.688**	**2.673**	**2.668**	**2.666**	**2.727**	**2.685**	**2.675**	**2.67**
6	**3.184**	**3.172**	**3.168**	**3.166**	**3.214**	**3.182**	**3.174**	**3.169**

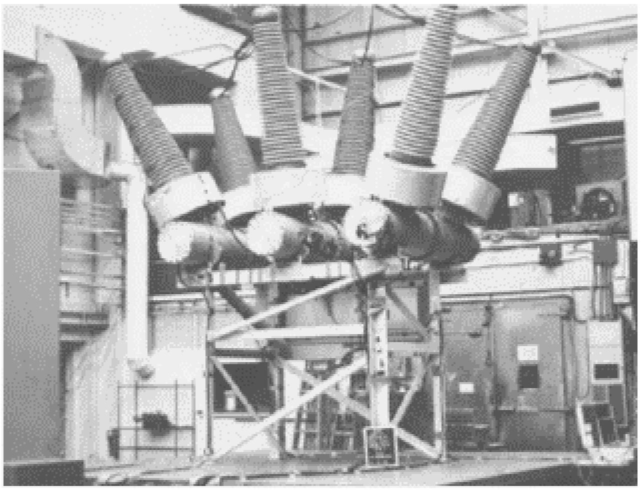


Fig. 1. 242 kV Dead Tank Breaker

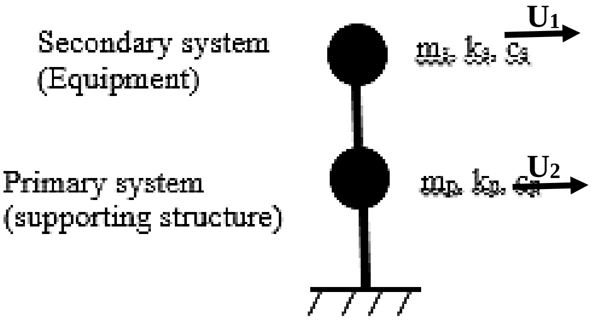


Fig. 2. Lumped stiffness-mass model for equipment-support system

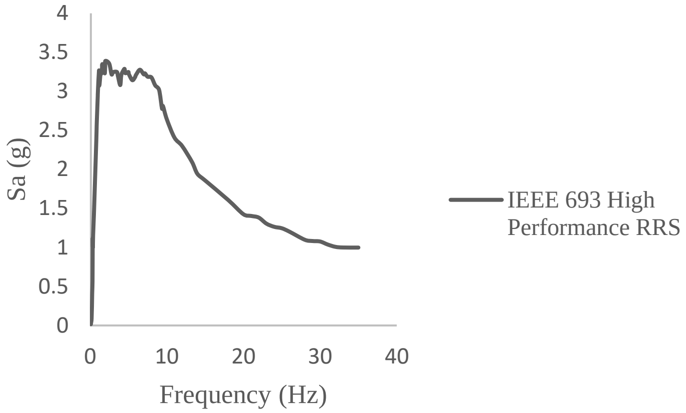


Fig. 3. IEEE-693 (2005) recommended RRS for high performance qualification, anchored to 1.0g

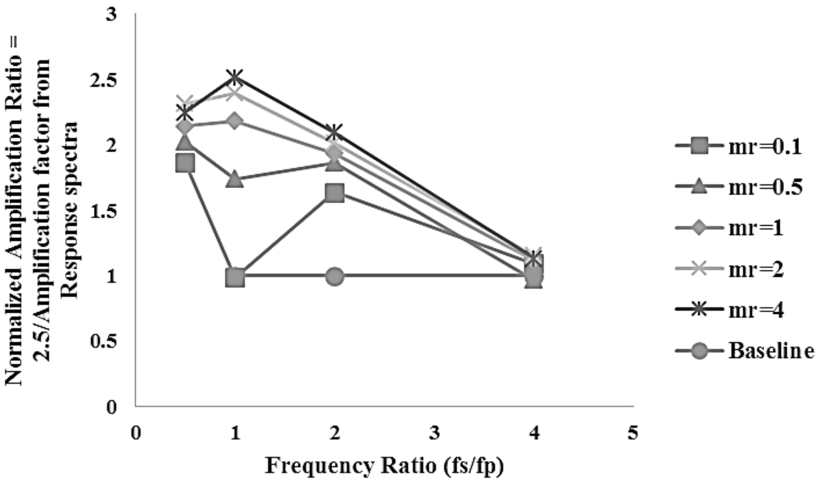


Fig. 4. Normalized amplification ratio vs. frequency ratio for support frequency = 5Hz

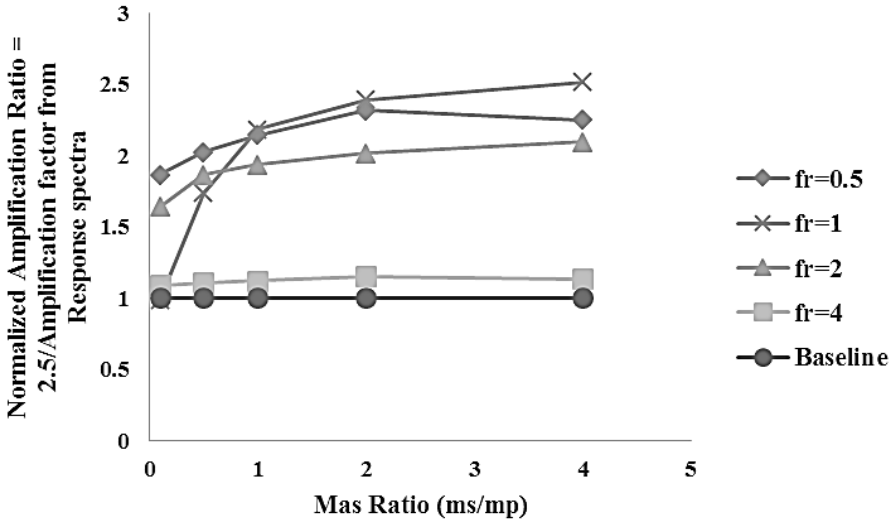


Fig. 5. Normalized amplification ratio vs. mass ratio for support frequency = 5Hz

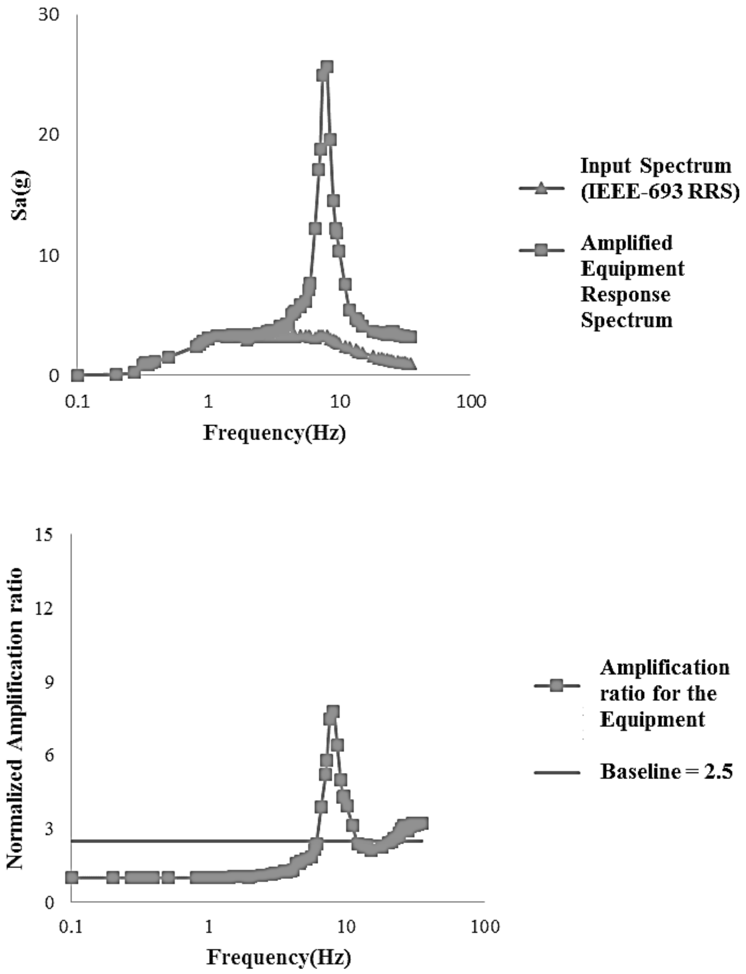


Fig. 6. Amplification ratio for a support frequency = 10 Hz, mass ratio = 2 and frequency Ratio = 0.5

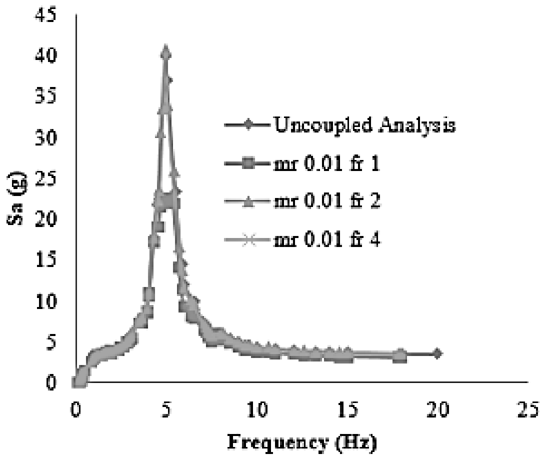


Fig. 7. Response spectra for mass ratio = 0.01

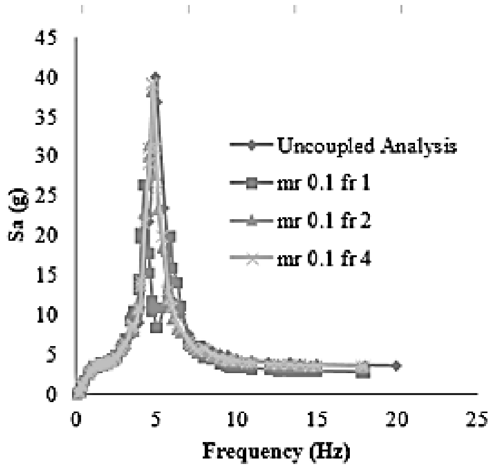
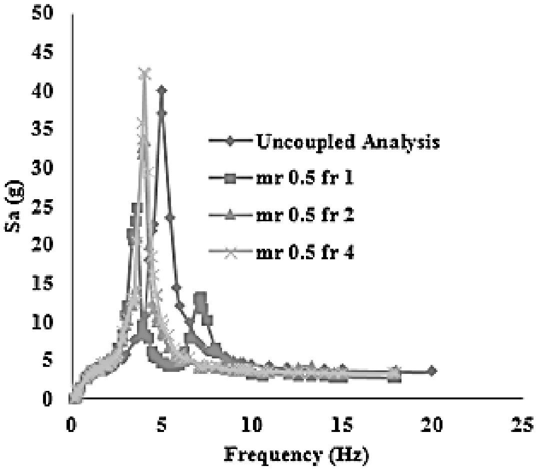


Fig. 8. Response spectra for mass ratio = 0.1



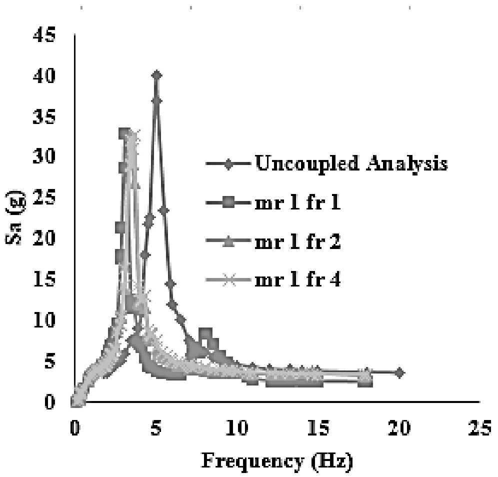


Fig. 9. Response spectra for mass ratio = 0.5

Fig. 10. Response spectra for mass ratio = 1

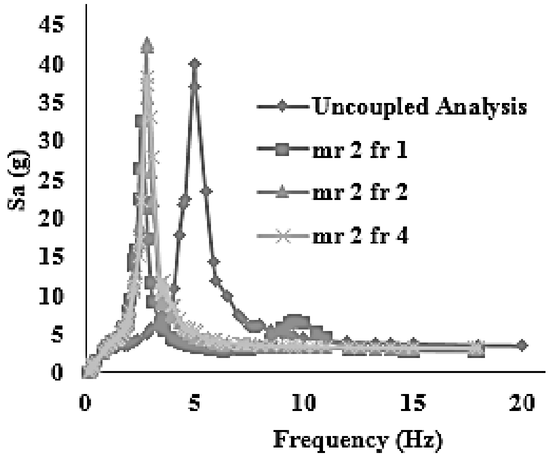


Fig. 11. Response spectra for mass ratio = 2

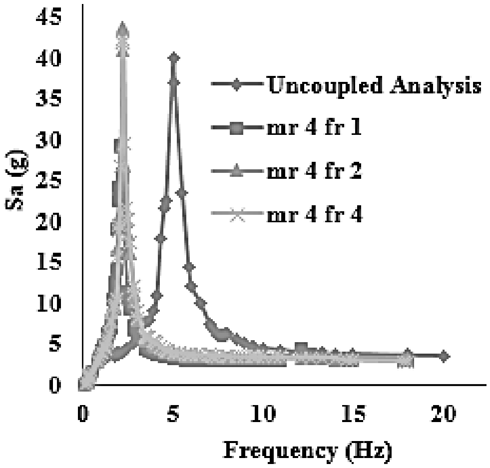


Fig. 12. Response spectra for mass ratio = 4

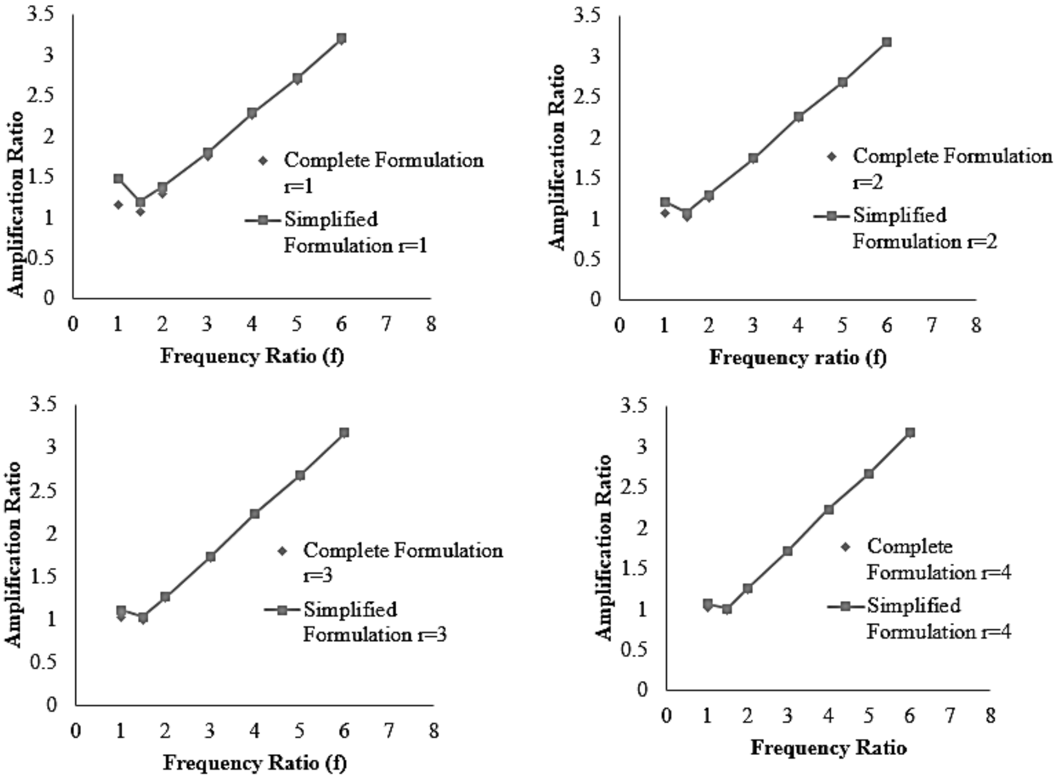


Fig. 13: Amplification ratio for mass ratio>1, frequency ratio>1

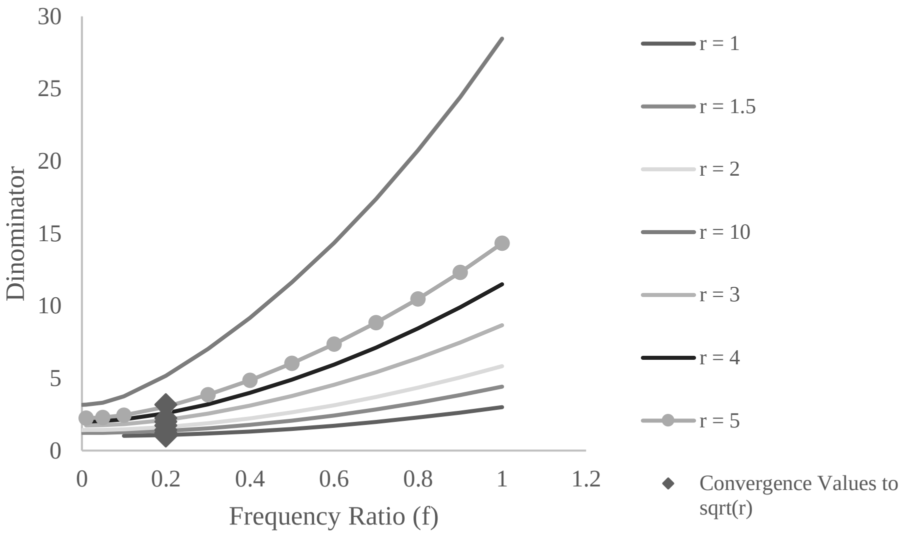


Fig. 14: Convergence Study for denominator for Case 2

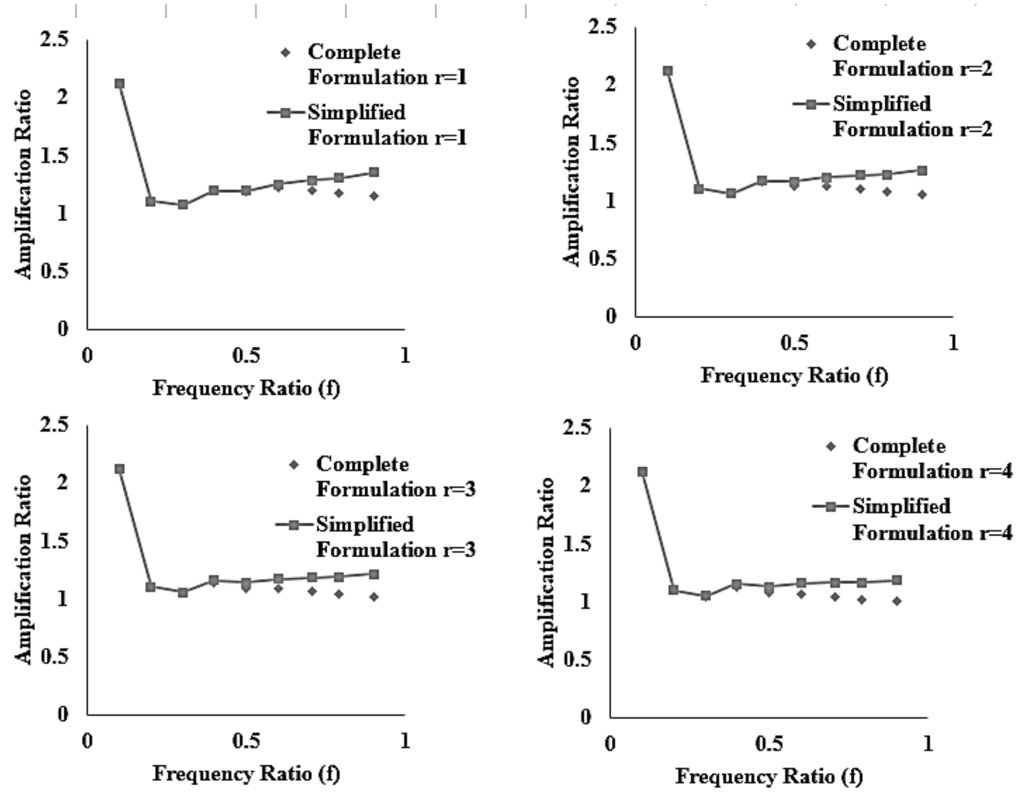


Fig. 15: Amplification ratio for mass ratio>1, frequency ratio<1

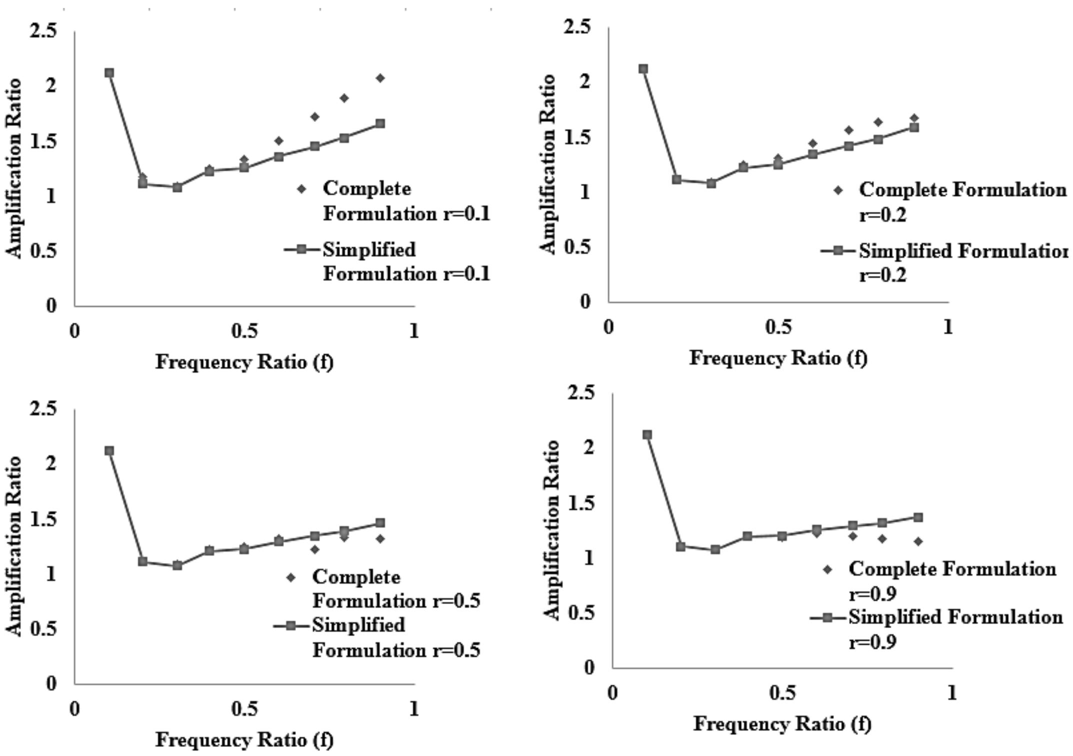


Fig. 16: Amplification ratio for mass ratio<1, frequency ratio<1

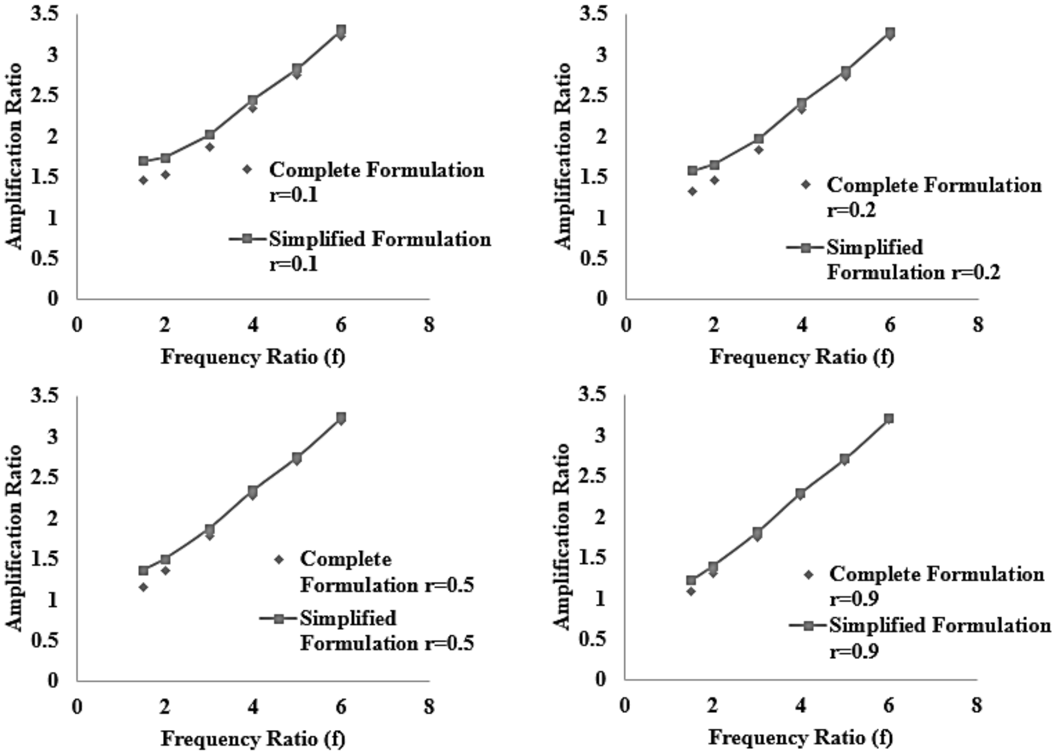


Fig. 17: Amplification ratio for mass ratio<1, frequency ratio>1

PART III: A PROBABILISTIC STUDY OF REDUCTION IN SEISMIC RESPONSE OF EQUIPMENT DUE TO COUPLED ANALYSIS

1. Introduction

The functional failure of sensitive equipment such as relays and switches mounted inside the cabinets is a concern for nuclear power plants performing seismic probabilistic risk assessments. A number of nuclear power plants in the Central and Eastern US are estimating high In-Structure Response Spectra (ISRS) particularly in the high frequency (HF) range. The common practice to estimate ISRS involves the seismic analysis of a secondary system using a decoupled model of the oscillator excited by the floor response spectrum. Such an approach uses significantly less computational resources compared to coupled time history analyses. However, a considerable reduction of ISRS amplitude can be achieved when the interaction between the primary system and secondary system is considered.

Studies [1, 2] show that a decoupled analysis of the secondary system, that ignores the interaction between the structure (primary system) and the mounted equipment (secondary system), may lead to conservative results when the equipment is tuned to one of the modes of the structure. In addition, the interaction reduces the calculated response of the equipment by a considerable amount.

For classically damped multi-degree-of-freedom (MDOF) systems, the equations of motion can be uncoupled into independent modal equations which further simplifies the calculation of equipment response. However, in case of systems that have different damping ratios for the primary and secondary systems, the coupled system becomes non-classically damped. For such systems the equations of motion can no longer be uncoupled using the conventional modal analysis and the off-diagonal terms in the transformed damping matrix cannot be ignored.

Studies [1, 2, 3, 4, 5, 6, 7] have been conducted over the years to develop simplified closed-form solutions as a reasonable approximation to the complete coupled problem. Other studies have also been conducted over the last two to three decades to develop different methods that can overcome the limitations of uncoupled analysis and consider non-classical damping and dynamic interaction between equipment and structures while estimating the ISRS accurately. Lin and Liu [8], USNRC [9] and RDT Standard [10] have studied the effect of coupling on the secondary system response as functions of mass and frequency ratios of the primary-secondary systems to identify regions in which decoupling cannot be permitted. Several semi-empirical as well as heuristic methods have been developed to perform coupled analysis of secondary systems [11, 12, 13, 14]. Alternatives to the use of time history analyses have been developed. Singh [15, 16, 17] has used the random vibration approach to develop a response spectrum method for non-classically damped systems. Gupta [18] and Gupta and Jaw [19] have developed approximate methods for evaluating the complex eigenvalues and eigenvectors of non-classically damped primary-secondary systems. The floor response spectrum method or ISRS method can be used to calculate the response of the secondary system [1, 2, 18, 20, 21, 22, 23, 24].

Zacharia [6] has proposed closed-form equations to calculate the in-structure response of single degree of freedom (SDOF) secondary systems connected to multi-degree of freedom (MDOF) primary systems accounting for dynamic mass interaction and non-classical damping. The formulations are developed for tuned or nearly tuned and detuned systems for lightly damped systems and for very small mass ratios. It is shown that in tuned or nearly tuned cases, due to interaction between the primary and secondary systems, the spectral response of the secondary system may be excessively overestimated if the effect of mass interaction is not considered. The effect of non-classical damping also needs to be incorporated when calculating the reduction factor for equipment response due to coupling effect.

Majority of the ground motion studies are directed towards the sites in Western United States that are characterized by low frequency ground motions. However, the seismic hazard-consistent ground motion for sites in the Central and Eastern United states (CEUS) often contains significant amounts of high-frequency vibratory motion [26, 27]. This paper attempts to explore the potential use of the simplified closed form solutions in estimating a more realistic ISRS and to implement the solutions in a simplified form for future use. Emphasis is laid on non-classically damped systems subjected to high frequency ground motions. The study further investigates the reduction in equipment response due to interaction between the equipment and structure and proposes probabilistic models for the reduction factors using results from simple systems representative of realistic structural systems and a suite of equipment mass, damping values and primary structure frequencies. The reduction factors are evaluated from a statistical study by varying the key parameters with an intent to recommend a statistical definition of reduction factor. Such a definition is likely to facilitate a incorporation of reduction factors in seismic probabilistic risk assessment (SPRA) studies. The next section of this paper discusses the analytical formulations used to estimate the equipment response spectra for non-classically damped systems.

2. Non-classically Damped Systems

The conventional method for calculating the response of secondary systems is by using the in-structure response spectrum (ISRS) which ignores the interaction between the primary and secondary systems. The effect of interaction is most significant when the secondary system is tuned or nearly tuned to the primary system, that is, in the resonant frequency region. Several studies have been conducted to evaluate the coupled response of the secondary system [1, 2, 6, 18, 20, 21, 22, 23, 24]. A complete modal solution of a coupled structure and equipment system that are non-classically damped gives complex eigenvalues and eigenvectors. For the purpose of simplification and practical use, several approximations have been suggested. The complex eigenvalues can be calculated directly using closed form equations [6]. In this section, the validity of these simplified formulations is evaluated with respect to complete modal solution.

The complexity of the problem in case of non-classically damped systems can be further simplified by using transformed modal solution. The resulting eigenvalue analysis is simple and involves calculation of coupled response of the equipment by using uncoupled modal properties of the systems. The following sections discuss the different methodologies for calculation of coupled equipment response.

2.1 Mode Superposition Based Time History Analysis

The equation of motion of for an N-DOF system is given by:

$$M\ddot{U} + C\dot{U} + KU = -MU_b\ddot{u}_g(t)$$

(1)

where, *M*, *C* and *K* are the mass, damping and stiffness matrices respectively; *U* is the relative displacement vector; U_b is the static displacement vector for a unit displacement at the base in the direction of input motion; $\ddot{u}_g(t)$ is the ground acceleration time history.

In case of non-classical damping, the eigenvalue analysis of the equation of motion can be conducted by transforming the above equation of motion into a 2*N*-dimensional matrix equation as follows:

$$A\dot{Y} + BY = -Q\ddot{u}_g$$

where,

$$Y = \begin{Bmatrix} \dot{U} \\ U \end{Bmatrix}, Q = \begin{Bmatrix} 0 \\ MU_b \end{Bmatrix}, A = \begin{bmatrix} 0 & -M \\ -M & -C \end{bmatrix}, B = \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}$$

The eigenvalue analysis of the above transformed equation yields *N* complex eigenvectors and eigenvalues along with their conjugates. The complex eigenvectors can be expressed as: $\begin{Bmatrix} \lambda_i \Psi_i \\ \Psi_i \end{Bmatrix}$ where, λ_i is the complex eigenvalue for mode *i*.

The complex eigenvector, Ψ_i can be split into two real mode shapes, Ψ_i^d, Ψ_i^v that correspond to the displacement and velocity components of the complex mode shape. The real mode shapes are given as:

$$\Psi_i^d = -2Re\bar{\lambda}_i F_i \Psi_i$$

$$\Psi_i^v = -2ReF_i\Psi_i$$

(2)

where, *Re* stands for the real part of the product of complex numbers;

$$F_i = \frac{1}{a_i}\Psi_i^T MU_b$$

where,

$$a_i = 2\lambda_i \Psi_i^T M \Psi_i + \Psi_i^T C \Psi_i$$

(3)

Let ω_i and ξ_i be the circular frequency and damping ratio in mode *I* for the *N*-DOF system. The eigenvalue can be written as:

$$\lambda_i = -\xi_i \omega_i + i\omega_i \sqrt{1 - \xi_i^2}$$

(4)

Using the above equation (4), along with the eigenvalue analysis results, the circular frequency and damping ratios for the *N* modes can be calculated.

If x_i and $\dot{x}_i$ are the displacement and velocity time histories for a single degree of freedom system for mode *i*, corresponding to circular frequency ω_i and damping ratio, ξ_i, then,

$$U_i^d = \Psi_i^d x_i$$

$$U_i^v = \Psi_i^v \dot{x}_i$$

Then the displacement in mode *i* is given by

$$U_i = U_i^d - U_i^v$$

Combining all the modes,

$$U = \sum_{i=1}^{N} U_i$$

(5)

In order to arrive at the equipment response spectra, the relative displacement time history with respect to the connecting degree of freedom is calculated. For each equipment frequency, the maximum value of this relative displacement is calculated, u_{rel}^{max} and for each equipment frequency, the spectral response is calculated by $\omega^2 u_{rel}^{max}$.

2.2 Mode Superposition Based Time History Analysis Using Closed Form Solutions

Zacharia [6] has derived the closed form equations to calculate the coupled response of equipment connected to multi degree of freedom structures using response spectrum analysis. These closed form solutions have been developed for lightly damped systems with low mass ratios. The effect of non-classical damping can be neglected for such systems. This section uses similar closed form equations to calculate the eigenvalue properties of the system using time history analyses as discussed in section 2.1. However, the effect of non-classical damping is also considered.

Let $\omega_{pi}, \xi_{pi}, \gamma_{pi}, \Phi_{ci}, \omega_s, \xi_s$ represent the primary and secondary system properties. $\omega_{pi}, \xi_{pi}, \gamma_{pi}, \Phi_{ci}$ are the primary system mode frequency, damping ratio, modal participation factor and mode shape at the connecting DOF for mode *i*. ω_s, ξ_s are the secondary system frequency and damping ratio.

The primary mode *I* is identified whose mode will interact with the secondary system. This is the mode whose uncoupled circular frequency is closest to that of the secondary system. For mode *I*, the eigenvalues are calculated by the following closed form equation:

$$\lambda_I^* = -\left(\xi + \frac{\overline{\xi_d}}{2}\right)\omega + i\omega\sqrt{1+\bar{\beta}}$$

$$\lambda_s^* = -\left(\xi - \frac{\overline{\xi_d}}{2}\right)\omega + i\omega\sqrt{1-\bar{\beta}}$$

(6)

The coupled angular frequencies and damping ratios are approximated as:

$$\omega_I^* = sqrt\left(\omega^2\left(1+\bar{\beta}+\frac{\bar{\beta}^2}{2}-\frac{\beta^2}{2}\right)\right)$$

$$\omega_s^* = sqrt\left(\omega^2\left(1-\bar{\beta}+\frac{\bar{\beta}^2}{2}-\frac{\beta^2}{2}\right)\right)$$

$$\xi_I^* = \frac{\omega}{\omega_I^*}(\xi + \frac{\bar{\xi}_d}{2})$$

$$\xi_s^* = \frac{\omega}{\omega_s^*}(\xi - \frac{\bar{\xi}_d}{2})$$

(7)

where,

$$\beta = \frac{\omega_{pI}^2 - \omega_s^2}{\omega_{pI}^2 + \omega_s^2}$$

$$\omega^2 = \frac{\omega_{pI}^2 + \omega_s^2}{2}$$

$$\xi = \frac{\omega_p\xi_p + \omega_s\xi_s}{2\omega}$$

$$\xi_d = \frac{\omega_p\xi_p - \omega_s\xi_s}{\omega}$$

$$\bar{\beta}, \bar{\xi}_d = sgn(\beta, \xi_d)\sqrt{\frac{\pm(\beta^2 - \xi_d^2 + r) + \sqrt{(\beta^2 - \xi_d^2 + r)^2 + 4\beta^2\xi_d^2}}{2}}$$

Similar to the exact solution, the response spectrum for the secondary system can be developed by plotting $\omega^2 u_{rel}^{max}$ as a function of the secondary system frequency. u_{rel}^{max} is the maximum relative displacement of the secondary system with respect to the primary system connecting degree of freedom. The relative displacement is a combination of the displacement and velocity part of the complex eigenvectors as discussed in the exact solution section. For a coupled 2 DOF system, the displacement and velocity components of the relative displacement for a particular frequency of the secondary system is calculated as:

$$\Delta u_i^d = (\Psi_{si}^d - \Psi_{pi}^d)x_i$$

$$\Delta u_i^v = (\Psi_{si}^v - \Psi_{pi}^v)\dot{x}_i$$

where, $x_i,\ \dot{x}_i$ are the displacement and velocity time histories for a SDOF oscillator with circular frequency same as in mode i.

So, for tuned case ($\beta = 0\ or\ 0 < \beta < 0.5, or - 0.5 < \beta < 0$), the closed form solutions for the displacement and velocity components of the eigenvectors in mode I^* are given by:

$$\Psi_{sI^*}^d - \Psi_{pI^*}^d \approx \Psi_{sI^*}^d = \gamma_{pI} * \Phi_{cI}\left(-\frac{X}{Den}(X^2 + Y^2 + \bar{r}_I) + \frac{1}{2}\right) * \left(\frac{\omega_I^*}{\omega_s}\right)^2$$

$$\Psi_{sI^*}^v - \Psi_{pI^*}^v \approx \Psi_{sI^*}^v = -\gamma_{pI} * \Phi_{cI} * \frac{Y}{Den}(X^2 + Y^2 - \bar{r}_I)(1 - |\beta|) * \left(\frac{\omega_I^*}{\omega_s^2}\right)$$

(8)

When the mode I^* is detuned with mode s^*, the velocity component of the mode shape can be neglected. The closed form solution for the detuned case is:

$$\Psi_{sI^*}^d - \Psi_{pI^*}^d = -\gamma_{pI} * \Phi_{cI} * \frac{{\omega_{pI}}^2}{{\omega_{pI}}^2 - {\omega_s}^2}$$

$$\Psi^{v}_{sI^*} - \Psi^{v}_{pI^*} \approx 0$$

(9)

For all other modes it is assumed that they are detuned and the mode frequencies and damping ratios for the coupled MDOF primary-SDOF secondary system are equal to the primary system modal values. The corresponding closed form solution for the factor $(\Psi^{d}_{si} - \Psi^{d}_{pi})$ is given by:

$$\Psi^{d}_{si} - \Psi^{d}_{pi} = -\gamma_{pi} * \Phi_{ci} * \frac{{\omega_{pi}}^{2}}{{\omega_{pi}}^{2} - {\omega_{s}}^{2}}$$

The velocity part of the mode shape is neglected in this case as well.

$$\Psi^{v}_{si} - \Psi^{v}_{pi} \approx 0$$

The closed form solution for the coupled secondary system mode is given by:

$$\Psi^{d}_{ss^*} - \Psi^{d}_{ps^*} = -\sum_{j \neq s^*} \Psi^{d}_{sj} - \Psi^{d}_{pj}$$

(10)

The sum of $\Psi^{d}_{si}, \Psi^{d}_{pi}$ for all the modes for each DOF is equal to 1. Therefore, the sum of their difference over all the modes should be zero. The above closed form solution is based on that assumption.

The velocity component of the coupled secondary system is given by:

$$\Psi^{v}_{ss^*} - \Psi^{v}_{ps^*} \approx \Psi^{v}_{ss^*} = \gamma_{pI} * \Phi_{cI} * \frac{Y}{Den}(X^2 + Y^2 - \bar{r}_I)(1 - |\beta|) * \left(\frac{\omega_s^*}{\omega_s^2}\right)$$

(11)

where

$$X = \beta + \bar{\beta}$$

$$Y = \xi_d + \bar{\xi}_d$$

$$Den = 4X^2Y^2 + (X^2 - Y^2 + \bar{r}_I)^2$$

where,

$$\bar{r}_I = r_I(1 - abs(\beta))$$

and,

$$r_I = m_s * {\Phi_{cI}}^2$$

where, m_s is the mass of the secondary system.

Similar to exact mode superposition, the displacement and velocity time histories for the SDOF oscillator with frequencies equal to the modal frequencies for the coupled system can be calculated using piecewise exact integration. Then, the corresponding maximum relative displacement at the secondary system is calculated as follows for the MDOF primary- SDOF secondary system:

$$U_{rel} = \left[\left((\Psi^d_{sI^*} - \Psi^d_{pI^*}\right))x^*_I - \left(\Psi^v_{sI^*} - \Psi^v_{pI^*}\right))\dot{x}^*_I\right] + \left[\left(\Psi^d_{ss^*} - \Psi^d_{ps^*}\right)x^*_s - (\Psi^v_{ss^*} - \Psi^v_{ps^*})\dot{x}^*_s\right] + \sum_{i \neq I^*, s^*} \left(\Psi^d_{si} - \Psi^d_{pi}\right)x_i \tag{12}$$

The maximum value of this displacement is then used to calculate the spectral response corresponding to the secondary system frequency. A plot of these spectral responses for different secondary system frequencies results in the response spectrum plot.

The formulation presented above is a complete formulation that requires eigenvalue analysis of the coupled system model which in turn requires the mass, stiffness, and damping matrices of the coupled system. Since recreating the complete model of the coupled system can be a formidable task, the coupled equipment response can also be obtained by using the uncoupled modal properties of the primary and secondary systems using transformed modal solution which is discussed in section 2.3. Existing models of the primary system (building) which are used to evaluate the ISRS in the conventional way give the modal properties of the primary system and the secondary system is a SDOF oscillator. Therefore, the transformation presented in section 2.3 makes the use of already available information on modal properties from existing models to facilitate the calculation of ISRS using coupled analysis by accounting for the interaction between the primary and the secondary system modes.

2.3 Transformed Modal Solution: Coupled Equipment Response from Uncoupled Modal Properties

As discussed earlier, this section presents the modified formulation for the coupled problem in which the required matrices for the coupled system are formulated using the modal properties of the uncoupled systems. Such a formulation simplifies the evaluation of ISRS using coupled analysis as the modal properties of the uncoupled systems are typically available from existing models. The main assumption behind transformed modal solution is that the uncoupled primary and secondary systems are classically damped by themselves. In terms of the uncoupled modal properties, the response can be expressed as:

$$U = \Phi X, \qquad \Phi = \begin{bmatrix} \Phi_p & 0 \\ 0 & \Phi_s \end{bmatrix}, \qquad X^T = \begin{bmatrix} X_p^T & X_s^T \end{bmatrix}$$

(15)

where, the subscripts *p* and *s* denotes the primary and secondary systems respectively.

For a single dof secondary system with mass m_s, $\Phi_s = 1/\sqrt{m_s}$.

The transformed modal solution involves transforming the *M*, *C* and *K* matrices such that the free vibration equation becomes,

$$[M^*]\{\ddot{X}\} + [C^*]\{\dot{X}\} + [K^*]\{X\} = \{0\}$$

(16)

For an *m* degree of freedom primary system,

$$[M^*] = \begin{bmatrix} [1]_{mxm} & 0 \\ 0 & 1 \end{bmatrix}_{(m+1)x(m+1)} = [I]$$

The mass ratio of the system in each mode corresponding to α^{th} mode of the secondary system is given by,

$$\sqrt{r_{i\alpha}} = \Phi_{ci}\gamma_s$$

$$or, \sqrt{r_i} = \Phi_{ci}\sqrt{m_s}, \quad i = 1, \ldots\ldots, m$$

where, Φ_{ci} is the mode shape of the primary system for mode i, at the connecting degree of freedom with the oscillator and $\gamma_s = \sqrt{m_s}$ is the modal participation factor for the single degree of freedom oscillator (α=1).

For the transformed equation of motion, the elements of $[K^*]$ and $[C^*]$ matrices are:

$$k_{ij} = \omega_{pi}^2 + r_i\omega_s^2, \qquad i = j$$

$$k_{ij} = \sqrt{r_i r_j}\omega_s^2, \qquad i \neq j$$

$$k_{\alpha\alpha} = \omega_s^2$$

and,

$$c_{ij} = 2\omega_{pi}\xi_{pi} + 2r_i\omega_s\xi_s, \qquad i = j$$

$$c_{ij} = 2\sqrt{r_i r_j}\omega_s\xi_s, \qquad i \neq j$$

$$c_{\alpha\alpha} = 2\omega_s\xi_s$$

The corresponding eigenvalue analysis does not involve complex numbers. Similar to complete modal solution, parameters $a_i, F_i, \psi_i^d, \psi_i^s$ are calculated as follows:

$$a_i = 2\lambda_i\{X_i\}^T\{X_i\} + \{X_i\}^T[C^*]\{X_i\}$$

$$F_i = \frac{1}{a_i}\{X_i\}^T\left[\gamma_{p1}\,\gamma_{p2} \dots\dots \gamma_{pm}\ \sqrt{m_s}\right]$$

$$\psi_i^d = -2Re\lambda_i F_i[\Phi]\{X_i\}$$

$$\psi_i^v = -2ReF_i[\Phi]\{X_i\}$$

(17)

For each value of equipment frequency, the maximum value of relative displacement is calculated similar to complete modal solution and for each equipment frequency, the spectral response is calculated by $\omega^2 u_{rel}^{max}$.

The transformed modal solution can be further simplified based on the assumption that the effect of coupling between the primary system modes and secondary system is most significant when the oscillator is tuned or nearly tuned to the frequency of the primary system. The next section discusses the simplified two mode approximation of the methodology to calculate the peak values of the ISRS. The comparison of the different methodologies for calculating equipment response spectra is also illustrated in later sections.

3. Simplified Transformed Modal Solution: Two-mode Approximation

The transformed modal solution can be further simplified to approximate ISRS by considering only the mode *I* that is closest to the equipment frequency. The coupled equipment response is then calculated using modal properties for the primary system for mode *I* only. This section gives the formulation of this two-mode approximation for estimating coupled ISRS using uncoupled primary and secondary system properties.

Let,

ω_{pI} = the mode frequency of the primary system closest to the oscillator frequency,

ω_s= the oscillator frequency,

ξ_{pI} = the damping ratio of the primary system for mode *I*,

ξ_s = the damping ratio of the secondary system,

r = mass ratio = $\frac{m_s}{m_p}$,

Φ_{cI} = eigenvalue for the primary system mode *I* at the connecting degree of freedom of the secondary system with the primary system,

γ_{pI} = mass participation factor for the primary system for mode *I*.

The transformed modal solution becomes simplified further to solving the eigenvalue problem for *2X2* mass and stiffness matrices as given below:

$$M^* = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad K^* = \begin{bmatrix} \omega_{pI}^2 + r_I\omega_s^2 & -\sqrt{r_I}\omega_s^2 \\ -\sqrt{r_I}\omega_s^2 & \omega_s^2 \end{bmatrix} \tag{18}$$

where,

$$r_I = m_s \Phi_{cI}^2$$

The eigenvalue solution solving the following equation:

$$\begin{vmatrix} \omega_{pI}^2 + r_I\omega_s^2 + \lambda^2 & -\sqrt{r_I}\omega_s^2 \\ -\sqrt{r_I}\omega_s^2 & \omega_s^2 + \lambda^2 \end{vmatrix} = 0$$

Let $a_{11} = \omega_{pI}^2 + r_I\omega_s^2, \quad a_{12} = a_{21} = -\sqrt{r_I}\omega_s^2, \quad a_{22} = \omega_s^2$

The eigenvalues are given by:

$$\lambda^2 = \frac{-(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2}$$

The corresponding angular frequencies are given by:

$$\omega_{1,2} = \sqrt{\frac{-(a_{11} + a_{22}) \pm \sqrt{(a_{11} + a_{22})^2 - 4(a_{11}a_{22} - a_{12}a_{21})}}{2}}$$

(19)

Solving for eigenvectors, unity normalized mode shapes are:

$$\psi_1' = \begin{Bmatrix} \dfrac{-(a_{22} + \omega_1^2)}{a_{21}} \\ 1 \end{Bmatrix}, \quad \psi_2' = \begin{Bmatrix} \dfrac{-(a_{22} + \omega_2^2)}{a_{21}} \\ 1 \end{Bmatrix}$$

The mass normalized mode shapes are given by:

$$\psi_i = \frac{\psi_i'}{\sqrt{D_i}}, \quad D_i = {\psi_{i1}'}^2 + 1$$

(20)

For mode *i*, where, *i*=1, 2, the displacement component of the eigenvectors are as follows:

$$\psi_d^i = F_i\{\Phi_i\}$$

$$F_i = \psi_i^T \gamma_c, \quad \Phi_i = \Phi_c \psi_i, \qquad \Phi_c = \begin{bmatrix} \Phi_{cI} & 0 \\ 0 & \Phi_S \end{bmatrix}, \qquad \gamma_c = \begin{bmatrix} \gamma_{pI} \\ \gamma_s \end{bmatrix}$$

(21)

Let S_A^1, S_A^2 be the spectral responses corresponding to ω_1, ξ_1 and ω_2, ξ_2 respectively. Then the instructure spectral response for the two modes are approximated by the following equations:

$$S_{Ai}^* = \psi_{dr}^i S_{Ai}, \;\; \psi_{dr}^i = \psi_{ds}^i - \psi_{dp}^i$$

(22)

Using double sum for combination of modal responses [25],

$$S_A^* = \sqrt{{S_{A1}^*}^2 + {S_{A2}^*}^2 + 2\bar{\epsilon}_{12} S_{A1}^* S_{A2}^*}$$

(23)

The ISRS for the equipment can be estimated using two mode approximation by identifying the primary system mode that is closest to the oscillator frequency and calculating the spectral response for the tuned system.

The following section considers a ten degree of freedom system and compares the different methodologies for estimating ISRS and the matching of the two mode approximation results to the complete solutions.

4. Example case studies and results

In this section, the spectral values as calculated from the various formulations presented above are compared. A ten floor building (primary structure) coupled with an equipment (secondary structure) with non-classical damping is analyzed. Two high frequency ground motions (HFEQ1 and HFEQ2) are used. The ground spectrum of the two input motions for damping ratio of 5% are shown in Figs. 1 and 2. The peak frequencies for the two motions are given in Table 1. Two primary systems of ten floors with stiffness of each floor equal to 75000N/m and 55000N/m are selected for analyzing using HFEQ1 and HFEQ2 respectively. Fig. 3 shows the two systems. These primary structures have frequencies that are tuned with the peak of the ground response spectra for the corresponding ground motions.

The mass ratios of secondary to primary structure considered for this example are 0.05, 0.1, 0.15, 0.2, 0.25. The secondary system is placed at different floors of primary structure and the response spectrum of the secondary structure is evaluated for each case. The equipment response spectra for different mass ratios of the oscillator connected to different levels for the two systems are shown below in Figs. 4 through 23. To incorporate the effect of non-classical damping, the primary system has a damping ratio of 5% and the single degree of freedom oscillator has a damping ratio of 2%. The coupled ISRS for each of these cases are compared to the corresponding uncoupled response spectra. In this example, the spectral values from the two mode approximation are also evaluated and compared to the peaks of ISRS for HFEQ1 and HFEQ2.

.

As can be observed from all the above plots, as the mass ratio of the system increases and the secondary system is connected to higher degrees of freedom of the primary system, the effect of interaction between the structure and equipment becomes more significant.

The conventional practice of estimating the ISRS using uncoupled analysis can give excessively conservative estimates. The level of reduction in the equipment response due to the coupling effect can be studied for a suite of systems and input ground motions. Subsequently, a probabilistic definition of such reduction factors can be useful in estimating the ISRS for the equipment in case of equipment mounted on MDOF primary systems.

The following section identifies different system configurations and calculates the reduction factors in these systems. Furthermore, a statistical study is conducted by considering the effect of uncertainty in the stiffness of the primary system, mass ratio of the system and the damping ratio of the secondary system on the equipment response.

5. Probabilistic distributions for reduction due to coupling

The reduction in the equipment response due to interaction between the structure and equipment is most prominent at the peaks of the ISRS when the equipment (secondary system) is tuned or nearly tuned to the primary system. For five high frequency ground motions (EQ1, EQ2, EQ3, EQ4, EQ5), suites of single, two and five degrees of freedom primary systems are identified such that the primary system frequencies lie in the region of peaks associated with the ground spectra for each of the input motions. The spectral response from uncoupled analysis, complete modal solution of the coupled system, and two mode approximation for a 2-DOF primary system – SDOF secondary system are given in tables 2 through 11. The corresponding reductions in equipment response are also given in the tables.

5.1 Effect of uncertainty in system properties

The effect of uncertainty in the equipment response due to variation in system properties can be studied by conducting a parametric study. For the purpose of the current study, a single degree of freedom equipment (secondary system) is connected to a single degree of freedom structure (primary system). Fig. 24 shows the single degree of freedom oscillator connected to a single degree of freedom primary system. The main assumptions for this study can be summarized as follows:

- Let m_p, k_p be the mass and stiffness of the single degree of freedom primary system (structure) and m_s, k_s be that for the single degree of freedom of oscillator (equipment).
- The parameters varied are the stiffness of the primary system (k_p), the mass ratio of the secondary system with respect to the primary system (r) and the damping ratio of the secondary system (ξ_s).
- Five high frequency ground motions and one low frequency ground motion (ElCentro) are used as input ground motions for the parametric study.
- A suite of single dof primary systems are identified such that their frequency coincides with the peaks of the input ground spectrum. The secondary system is tuned to the primary system, that is, the frequency ratio is 1.

- For the parametric study, k_p, r, and ξ_s are assumed to be normally distributed. The details of the parameter distributions are explained in the subsequent sections.

Selection of systems

The primary systems are selected such that the frequency of the system coincides with the peak spectral response for the input ground motions. The ground spectrum for the five high frequency ground motions and the low frequency ElCentro ground motion are shown in Figs. 1, 2, 25, 26, 27 and 28 respectively.

Statistical distribution for stiffness of the primary system is taken as:

$$k_p \sim N(\mu = System\ Stiffness, Coefficient\ of\ Varation = 0.15)$$

Statistical distribution for mass ratio is taken as:

Each of the time history analysis is analyzed for five mass ratios: 0.05, 0.1, 0.15, 0.2, 0.25. For each of these analyses, the distribution of mass ratio (r) is:

$$r \sim N(\mu = 0.5, 0.1, 0.15, 0.2, 0.25, Coefficient\ of\ Varation = 0.05)$$

Statistical distribution for damping ratio of the oscillator is taken as:

$$\xi_s \sim N(\mu = 0.05, Coefficient\ of\ Varation = 0.1)$$

The main assumptions of the study can be summarized as flows:

- Random samples are generated from each of these normally distributed parameters

- A set of 100 random combinations of the parameters are used to run the analyses.
- The coupled equipment response using complete mode superposition of the time history analysis results and the uncoupled equipment response by conducting time history analyses on a decoupled oscillator are generated for such 100 random simulations.
- The cumulative distribution function (cdf) for the raw data are plotted.
- The data is also fitted to a lognormal distribution and cdf for the estimated lognormal parameters are plotted along with the empirical cdf.

The uncoupled equipment response spectra for each of the case studies are given in Figs. 29, 31, 33, 35, 37 and 39. The corresponding empirical and fitted cumulative distribution function (cdf) plots from each of the analysis cases are shown in Figs. 30, 32, 34, 36, 38 and 40. The cdf of the reduction factor shows good fit with lognormal distribution.

From the distribution of the reduction factors, the median and other percentile values can be estimated to give point estimates as well as upper-bound values of the reduction in equipment response due to coupled analysis. In addition to studying the effect of mass ratios on the reduction factor, the effect of variation in damping ratios between the equipment and structure can also be investigated. The following section studies the variation of the percentile estimates of the reduction factor as a function of the mass ratios of the system and changing damping ratios of the equipment.

5.2 Variation of reduction factor with mass ratio and damping ratio

The effects of mass ratio of the system as well as change in damping ratios of the equipment on the estimated percentile values of the reduction factor are studied in the current section. For the suite of systems identified, the median values of the reduction factor can be calculated for different damping ratios of the oscillator. The damping ratio of the structure (primary system) is fixed at 5%. For the equipment (secondary system), the damping ratios are varied from 2%, 3%, 4%, 5%, 6% and 7%. For each of the above damping ratio combinations, the median of the reduction factor is calculated for the systems for mass ratios 0.05, 0.1, 0.15, 0.2 and 0.25. In addition to the median values, the median plus one log-normal standard deviation (Median + β) and median plus two log-normal standard deviations (Median + 2β) have also been investigated. These correspond to 87.5% Non-exceedance probability (NEP) and 95.3% NEP, respectively. As an illustrative example, the results for the five degree of freedom system subjected to HFEQ3 input motion and the oscillator connected to the top degree of freedom are given. Fig. 41 shows the system with the oscillator connected at the top degree of freedom. Figs. 42, 43 and 44 show the surface plots of the reduction factor with 50%, 87.5% and 95.3% NEP respectively as a joint function of mass ratios and damping ratios of the single degree of freedom oscillator.

6. Discussion of Results

The spectral response for single degree of freedom oscillators are estimated using two mode approximation for time history analysis when the oscillator is tuned to the modes of the primary system. The results from the two mode approximation as discussed in section 4 can be summarized as follows:

- The responses are good approximations of the exact results, especially around the peak regions.
- For a range of mass ratios of the system (5%, 10%, 15%, 20% and 25%) and oscillator locations (connecting degree of freedom with the primary system), the approximate response is found to be close to the exact solution.
- When the oscillator frequency is tuned to modes of the primary system that do not have a peak in spectra, the approximation tends to give higher values.
- For some of the higher values of mass ratios, the approximation may lead to under-prediction of equipment response.

The results from the parametric study of the reduction in equipment response due to coupled analysis as discussed in section 5 are as follows:

- The effect of dynamic interaction between the primary system (structure) and the secondary system (equipment) is shown to be increasingly significant as the mass ratio increases.
- The reduction in spectral response is also affected considerably in case of non-classically damped systems as shown in Tables 2 through 11.

Figs. 32, 34, 36, 38, 40 show the cumulative distribution plots for the reduction factor for the illustrated single degree of freedom primary system case when the structure and the equipment are both tuned to the peak frequency of the input ground motion. The analysis is extended to a suite of two and five degree of freedom systems. The results show that:

- The cumulative distribution function of the reduction factor can be approximated by lognormal distribution.
- As the mass ratio increases, the coupled response can be as low as 30% of the uncoupled response.

A further study on the effect of mass ratio of the system and non-classical damping on the estimated median and other statistically significant percentile values as shown in Figs. 42, 43 and 44 shows that:

- The median values of the reduction factor is around 0.25 for mass ratios of 25% and oscillator damping ratio of 2%.
- Even in case of lighter equipment (mass ratio = 5%), there is a 50% probability of around 60% reduction in equipment response using coupled analysis.
- Reduction in coupled response can be as low as 50% of the uncoupled response with a 95.3% (Median + 2β) Non-exceedance probability (NEP) for mass ratio = 25%.

The above results emphasize the significance of incorporating the dynamic interaction between the equipment and its supporting structure in equipment response. The effect of non-classical damping also needs to be considered. A knowledge of the distribution of the reduction factor can help to estimate the statistical limiting values that can be directly applied to uncoupled analysis results to calculate ISRS that are less conservative than the current recommendations.

7. Summary and Conclusions

The reduction in ISRS due to coupling of primary and secondary systems is most significant in the peak regions of the spectra (tuned or nearly tuned oscillators) and for non-classically damped systems. Also, in case of input motions that have frequency content in regions higher than 30 Hz or so, using uncoupled analysis may result in ISRS with peaks in high frequency region. The observations from the current study show that:

- As the mass ratio of the system increases, coupled response can be significantly lower than the corresponding uncoupled response in the peak regions.
- For high frequency ground motions, coupled analysis can lead to ISRS with significantly reduced peaks.

Therefore, including the effects of mass interaction and non-classical damping can be very useful in SPRA studies, especially for regions subjected to high frequency ground motions. The present study aims to quantify the reduction in response in terms of a factor which when applied to the peaks of uncoupled spectra gives more realistic ISRS for equipment qualification purposes.

A statistical study of the reduction factor using a suite of multi degree of freedom systems has been conducted. Using random sampling of the parameters, such as, stiffness of primary system, mass ratio of the system and damping ratio of the secondary system, the effect of uncertainty of these parameters on the equipment response is studied. Based on these samples, the distribution of the reduction factor is plotted for a range of systems. The study shows that:

- The cdf of the reduction factors has good fit with lognormal distribution.

- As the mass ratio increases, the reduced coupled response can be as low as 30% of the uncoupled response.
- The effect of reduction in equipment response is significant for non-classically damped systems.
- Reduction in coupled response can be as low as 50% of the uncoupled response with 95.3% (Median+2β) Non-exceedance probability (NEP) for a mass ratio of 0.25 in a non-classical damped system ($\xi_p = 5\%, \xi_s = 2\%$).

The results from this paper can be used to estimate reduction factors with high NEPs which when applied to the uncoupled analysis results may provide ISRS of equipment that are much lower than the current requirements. This may help to make the qualification requirements for equipment to be less conservative, especially when they need to be qualified with respect to high frequency ground motions.

REFERENCES

[1] Gupta A., Gupta, A. K., 1994a. "New developments in coupled seismic analysis of equipment and piping." Proceedings of the 13th International Conference on Structural Mechanics in Reactor Technology, Porto Alegre, Brazil.

[2] Gupta A., Gupta, A. K., 1994b. "Applications of new developments in coupled seismic analysis of piping systems." Proceedings of the 13th International Conference on Structural Mechanics in Reactor Technology, Porto Alegre, Brazil.

[3] Gupta, A. K., 1990. "Response Spectrum Method in Seismic Analysis and Design of Structures." Blackwell Scientific Publications, Inc., ISBN 0-8493-8628-4.

[4] Tadinada, S. K., Gupta, A., 2011. "Consideration of uncertainties in seismic analysis of non-classically damped coupled systems." Nuclear Engineering and Design, Vol. 241, Issue. 6, pp. 2034-2044.

[5] Gupta, A., Choi, B., 2005. "Consideration of uncertainties in seismic analysis of coupled building-piping systems." Nuclear Engineering and Design, Vol. 235, pp. 2071-2086.

[6] George, Z., 1996. "Development and Evaluation of Methods for Coupled Seismic Analysis of Piping Systems." Doctoral Thesis, Department of Civil Engineering, North Carolina State University, Raleigh, NC.

[7] Gupta, A, Gupta, A. K., 1998. "Missing mass effect in coupled analysis, I: complex modal properties." J. Struct. Eng. ASCE, 124 (5), pp. 490-495.

[8] Lin, C. W., Liu, T. H., 1975. "A discussion of coupling and resonant effects for integrated systems consisting of subsystems." Proceedings, Extreme Load Conditions and Limit

Analysis Procedures for Structural Reactor Safeguards and Containment Structures, U3/3, Berlin.

[9] U. S. Nuclear Regulatory Commission, 1975. "Standard Review Plan, Section 3.7.2, Seismic System Analysis."

[10] RDT Standard F9-2T, 1974. "Seismic Requirements for Design of Nuclear Power Plants and Test Facilities."

[11] Penizen, J., Chopra, A. K., 1965. "Earthquake Response of Appendage on a Multi-Story Building." Proceedings, 3rd World Conference on Earthquake Engineering, Vol. II, New Zealand.

[12] Biggs, J. M., Roesset, J. M., 1970. "Seismic Analysis of Equipment Mounted on a Massive Structure." Seismic Design for Nuclear Power Plants, Ed. By Hansen, R. J., MIT press, Cambridge, Massachusetts.

[13] Kapur, K. K., Shao, L. C., 1973. "Generation of Seismic Floor Response Spectra for Equipment Design." Specialty Conference on Structural Design of Nuclear Power Plant Facilities, ASCE, Chicago, Illinois.

[14] Hadjian, A. H., 1977. "On the decoupling of secondary systems for seismic analysis." Proceedings, Sixth World Conference on Earthquake Engineering, New Delhi, India.

[15] Singh, M. P., Suarez, L. E., 1986. "A Perturbation Analysis of the Eigenproperties of Equipment-Structure Systems." Nuclear Engineering and Design, Vol. 97, No. 2, pp. 167-185.

[16] Singh, M. P., Suarez, L. E., 1988. "A Method for Dynamic Coupling with Non-classical Damping Effects." Journal of Sound and Vibration, Vol. 120, No. 3, pp. 539-555.

[17] Singh, M. P., Burdisso, R. A., 1987. "Multiply Supported Secondary Systems II: Seismic Inputs." Earthquake Engineering and Structural Dynamics, Vol. 15, No. 1, pp. 73-90.

[18] Gupta, A. K., 1984. "Seismic Response of Multiply Connected MDOF Primary and MDOF Secondary Systems." Nuclear Engineering and Design, Vol. 81, No. 3, pp. 385-394.

[19] Gupta, A. K., Jaw, J. W., 1986. "Complex Modal Properties of Coupled Moderately Light Equipment-Structure Systems." Nuclear Engineering and Design, Vol. 91, No. 2, pp. 171-178.

[20] Gupta, A. K., Jaw, J. W., 1986. "Seismic Response of Non-classically Damped Systems." Nuclear Engineering and Design, Vol. 91, No. 2, pp. 153-159.

[21] Gupta, A. K., Jaw, J. W., 1986. "Response Spectrum Method for Non-classically Damped Systems." Nuclear Engineering and Design, Vol. 91, No. 2, pp. 161-169.

[22] Gupta, A. K., Jaw, J. W., 1986. "Coupled Response Spectrum Analysis of Secondary Systems Using Uncoupled Modal Properties." Nuclear Engineering and Design, Vol. 92, No. 1, pp. 61-68.

[23] Gupta, A. K., Jaw, J. W., 1985. "CREST, a Computer Program for Coupled Response Spectrum Analysis of Secondary Systems." User's Manual, Center for Nuclear Power Plant Structures, Equipment and Piping, North Carolina State University, Raleigh, North Carolina.

[24] Gupta A. K., 1990. "Response Spectrum Method in Seismic Analysis and Design of Structures." Blackwell Scientific Publications.

[25] Megahed, H. A., Gupta, A. K., 1992. “Topics in Seismic Response of Non-classically Damped Systems.” Report, Research Program on Nuclear Power Plant Structures, Equipment and Piping, Department of Civil Engineering, North Carolina State University, Raleigh, NC.

[26] EPRI, 2015. “High Frequency Program: Application Guidelines for Functional Confirmation and Fragility Evaluation.” 3002004396, Draft Report.

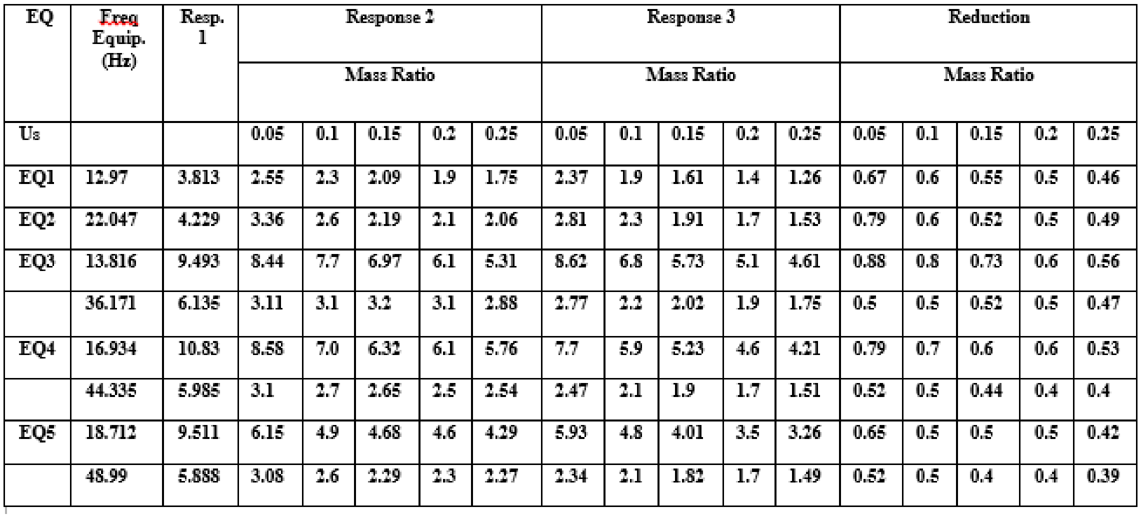
[27] EPRI, 2007. “Program on Technology Innovation: The Effects of High Frequency Ground Motion on Structures, Components, and Equipment in Nuclear Power Plants.” 1015108.

[28] Humar, J., 2012. “Dynamics of Structures.” Third Edition, CRC Press

Table 1: Selection of systems based on peak frequencies of ground spectra for HFEQ1 and HFEQ2

Ground Motion	Frequencies of peaks in ground spectra (Hz)	Primary system stiffness (N/m)	Frequencies of Primary system modes (Hz)
HFEQ1	16.47, 17.78, 19.53, 33.96, 45.15, 66.7	75000	6.5, 19.3978, 31.85, 43.58, 54.35, 63.9, 72.026, 78.54, 83.3, 86.2
HFEQ2	21.69, 24.59, 26.78, 29.32, 31.43, 36.53, 38.08, 38.41, 57.72	55000	5.58, 16.6, 27.3, 37.325, 46.54, 54.72, 61.68, 67.26, 71.3, 73.8

Table 2: Two degree of freedom primary system with oscillator connected at dof 1: $\xi_s = 5\%$

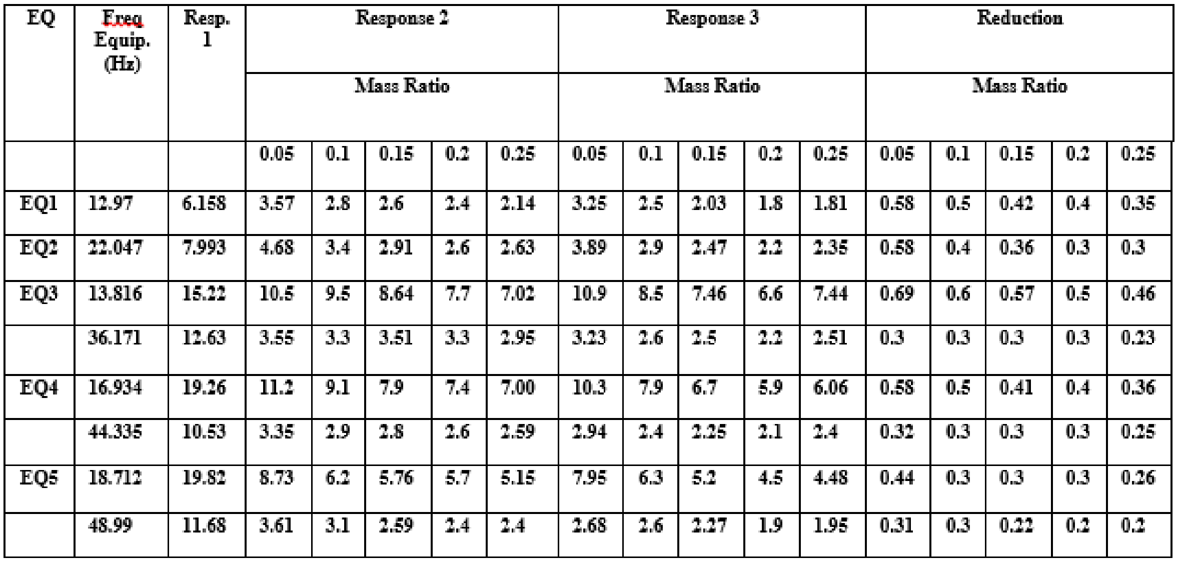


EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass Ratio					Mass Ratio					Mass Ratio				
Us			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	12.97	3.813	2.55	2.3	2.09	1.9	1.75	2.37	1.9	1.61	1.4	1.26	0.67	0.6	0.55	0.5	0.46
EQ2	22.047	4.229	3.36	2.6	2.19	2.1	2.06	2.81	2.3	1.91	1.7	1.53	0.79	0.6	0.52	0.5	0.49
EQ3	13.816	9.493	8.44	7.7	6.97	6.1	5.31	8.62	6.8	5.73	5.1	4.61	0.88	0.8	0.73	0.6	0.56
	36.171	6.135	3.11	3.1	3.2	3.1	2.88	2.77	2.2	2.02	1.9	1.75	0.5	0.5	0.52	0.5	0.47
EQ4	16.934	10.83	8.58	7.0	6.32	6.1	5.76	7.7	5.9	5.23	4.6	4.21	0.79	0.7	0.6	0.6	0.53
	44.335	5.985	3.1	2.7	2.65	2.5	2.54	2.47	2.1	1.9	1.7	1.51	0.52	0.5	0.44	0.4	0.4
EQ5	18.712	9.511	6.15	4.9	4.68	4.6	4.29	5.93	4.8	4.01	3.5	3.26	0.65	0.5	0.5	0.5	0.42
	48.99	5.888	3.08	2.6	2.29	2.3	2.27	2.34	2.1	1.82	1.7	1.49	0.52	0.5	0.4	0.4	0.39

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution
- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 3: Two degree of freedom primary system with oscillator connected at dof 1: $\xi_s = 2\%$

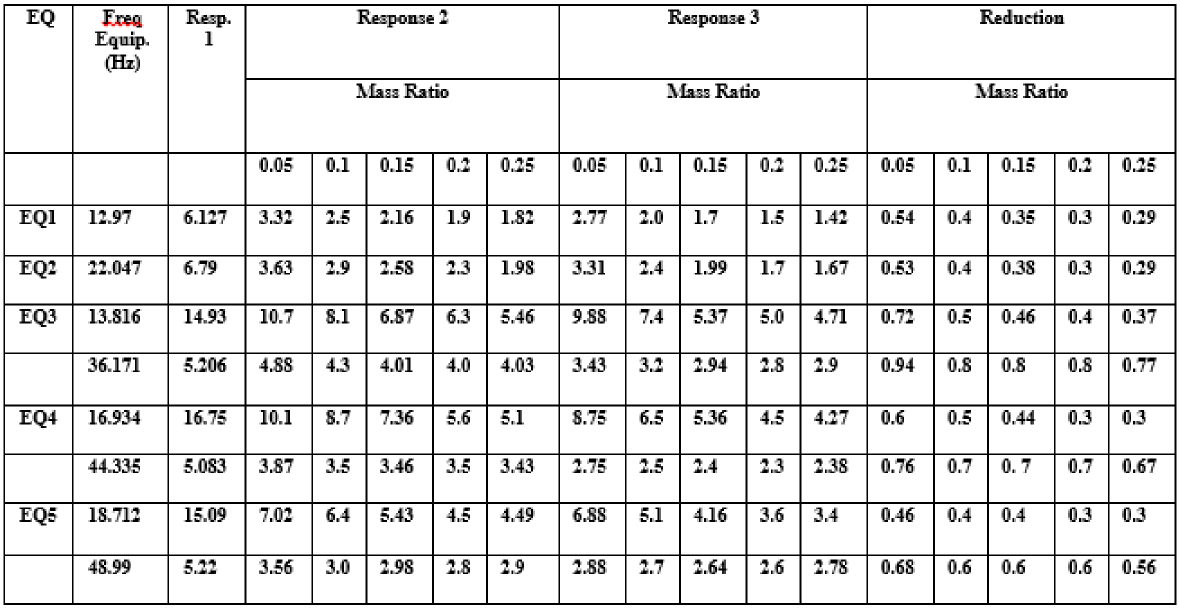


EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass Ratio					Mass Ratio					Mass Ratio				
			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	12.97	6.158	3.57	2.8	2.6	2.4	2.14	3.25	2.5	2.03	1.8	1.81	0.58	0.5	0.42	0.4	0.35
EQ2	22.047	7.993	4.68	3.4	2.91	2.6	2.63	3.89	2.9	2.47	2.2	2.35	0.58	0.4	0.36	0.3	0.3
EQ3	13.816	15.22	10.5	9.5	8.64	7.7	7.02	10.9	8.5	7.46	6.6	7.44	0.69	0.6	0.57	0.5	0.46
	36.171	12.63	3.55	3.3	3.51	3.3	2.95	3.23	2.6	2.5	2.2	2.51	0.3	0.3	0.3	0.3	0.23
EQ4	16.934	19.26	11.2	9.1	7.9	7.4	7.00	10.3	7.9	6.7	5.9	6.06	0.58	0.5	0.41	0.4	0.36
	44.335	10.53	3.35	2.9	2.8	2.6	2.59	2.94	2.4	2.25	2.1	2.4	0.32	0.3	0.3	0.3	0.25
EQ5	18.712	19.82	8.73	6.2	5.76	5.7	5.15	7.95	6.3	5.2	4.5	4.48	0.44	0.3	0.3	0.3	0.26
	48.99	11.68	3.61	3.1	2.59	2.4	2.4	2.68	2.6	2.27	1.9	1.95	0.31	0.3	0.22	0.2	0.2

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution
- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 4: Two degree of freedom primary system with oscillator connected at dof 2: $\xi_s = 5\%$



EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass Ratio					Mass Ratio					Mass Ratio				
			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	12.97	6.127	3.32	2.5	2.16	1.9	1.82	2.77	2.0	1.7	1.5	1.42	0.54	0.4	0.35	0.3	0.29
EQ2	22.047	6.79	3.63	2.9	2.58	2.3	1.98	3.31	2.4	1.99	1.7	1.67	0.53	0.4	0.38	0.3	0.29
EQ3	13.816	14.93	10.7	8.1	6.87	6.3	5.46	9.88	7.4	5.37	5.0	4.71	0.72	0.5	0.46	0.4	0.37
	36.171	5.206	4.88	4.3	4.01	4.0	4.03	3.43	3.2	2.94	2.8	2.9	0.94	0.8	0.8	0.8	0.77
EQ4	16.934	16.75	10.1	8.7	7.36	5.6	5.1	8.75	6.5	5.36	4.5	4.27	0.6	0.5	0.44	0.3	0.3
	44.335	5.083	3.87	3.5	3.46	3.5	3.43	2.75	2.5	2.4	2.3	2.38	0.76	0.7	0. 7	0.7	0.67
EQ5	18.712	15.09	7.02	6.4	5.43	4.5	4.49	6.88	5.1	4.16	3.6	3.4	0.46	0.4	0.4	0.3	0.3
	48.99	5.22	3.56	3.0	2.98	2.8	2.9	2.88	2.7	2.64	2.6	2.78	0.68	0.6	0.6	0.6	0.56

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution
- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

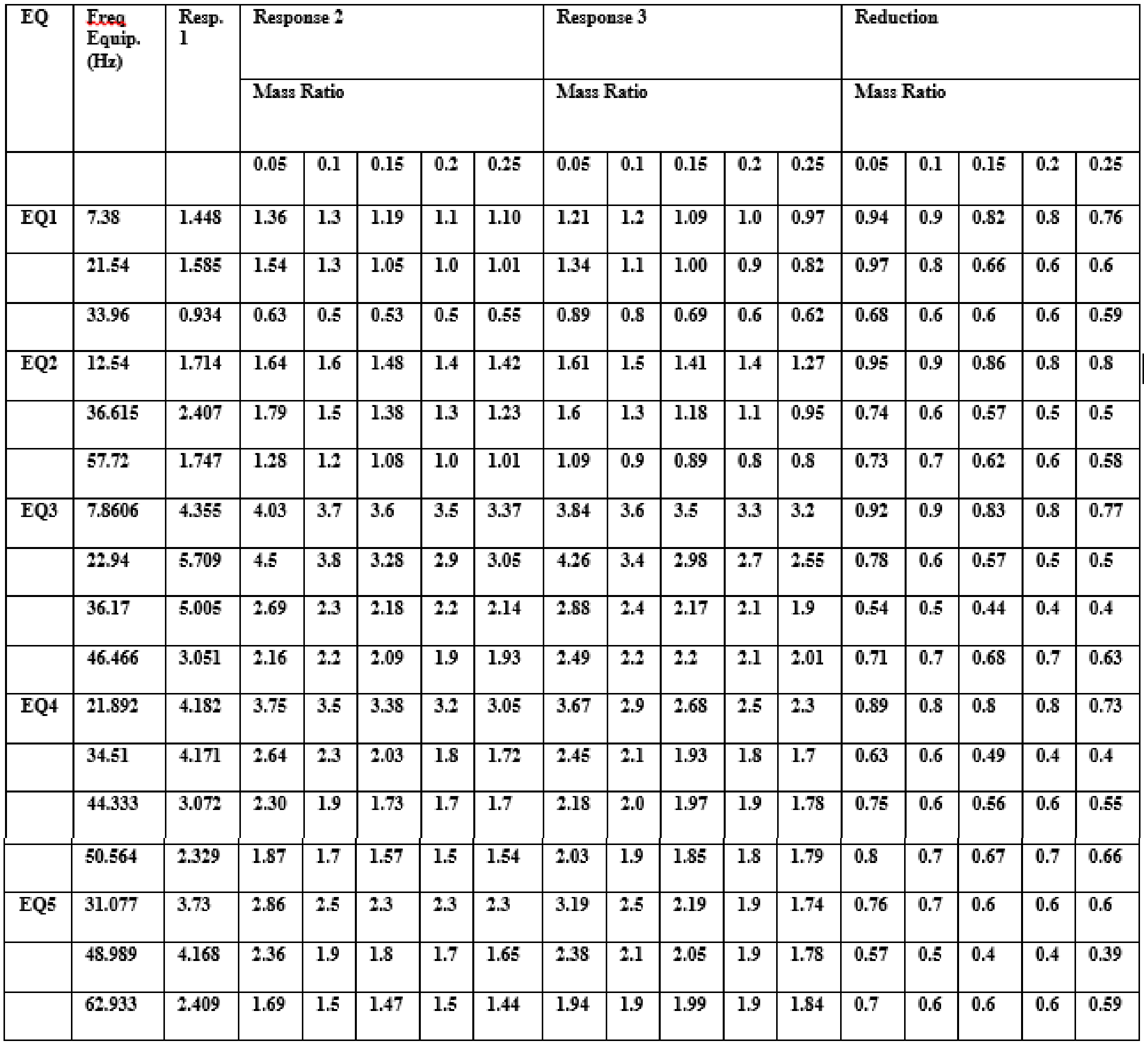
Table 5: Two degree of freedom primary system with oscillator connected at dof 2: $\xi_s = 2\%$

EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass Ratio					Mass Ratio					Mass Ratio				
			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	12.97	9.841	4.17	3.1	2.55	2.4	2.09	3.54	2.6	2.18	1.9	1.88	0.42	0.3	0.3	0.3	0.21
EQ2	22.047	12.69	4.86	3.7	3.11	2.8	2.4	4.28	3.1	2.46	2.2	2.02	0.38	0.3	0.25	0.2	0.19
EQ3	13.816	23.89	13.2	10	8.56	8.0	6.52	12.8	9.6	7.49	6.3	5.61	0.27	0.3	0.36	0.4	0.55
	36.171	8.266	5.64	4.5	4.03	4.0	4.11	4.42	3.7	3.51	3.2	2.99	0.49	0.8	0.48	0.5	0.68
EQ4	16.934	30.61	13.1	10	8.94	6.4	5.92	11.5	8.2	6.7	5.7	5.2	0.43	0.4	0.29	0.2	0.19
	44.335	7.368	4.24	3.6	3.59	3.7	3.43	3.05	2.7	2.63	2.6	2.49	0.57	0.5	0.49	0.5	0.47
EQ5	18.712	31.72	8.8	7.7	6.49	5.4	5.59	8.8	6.4	5.14	4.3	4.48	0.28	0.2	0.2	0.2	0.17
	48.99	8.701	3.9	2.9	3.02	2.9	2.95	3.4	3.2	3.13	3.1	2.94	0.45	0.3	0.4	0.3	0.34

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution
- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 6: Five degree of freedom primary system with oscillator connected at dof 1: $\xi_s = 5\%$

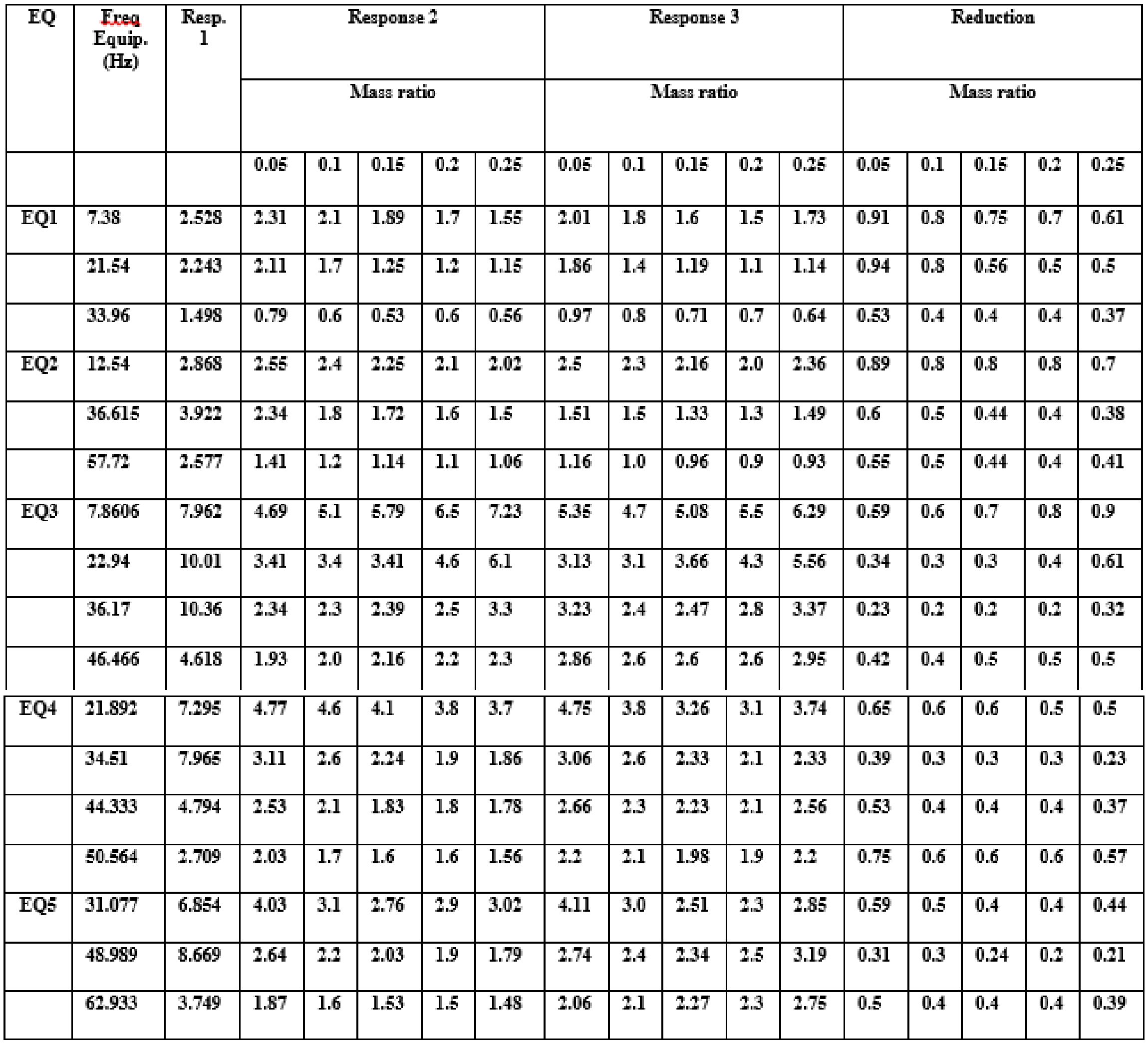


EQ	**Freq Equip. (Hz)**	**Resp. 1**	**Response 2**					**Response 3**					**Reduction**				
			Mass Ratio					**Mass Ratio**					**Mass Ratio**				
			0.05	**0.1**	**0.15**	**0.2**	**0.25**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**
EQ1	7.38	1.448	1.36	1.3	1.19	1.1	1.10	1.21	1.2	1.09	1.0	0.97	0.94	0.9	0.82	0.8	0.76
	21.54	1.585	1.54	1.3	1.05	1.0	1.01	1.34	1.1	1.00	0.9	0.82	0.97	0.8	0.66	0.6	0.6
	33.96	0.934	0.63	0.5	0.53	0.5	0.55	0.89	0.8	0.69	0.6	0.62	0.68	0.6	0.6	0.6	0.59
EQ2	12.54	1.714	1.64	1.6	1.48	1.4	1.42	1.61	1.5	1.41	1.4	1.27	0.95	0.9	0.86	0.8	0.8
	36.615	2.407	1.79	1.5	1.38	1.3	1.23	1.6	1.3	1.18	1.1	0.95	0.74	0.6	0.57	0.5	0.5
	57.72	1.747	1.28	1.2	1.08	1.0	1.01	1.09	0.9	0.89	0.8	0.8	0.73	0.7	0.62	0.6	0.58
EQ3	7.8606	4.355	4.03	3.7	3.6	3.5	3.37	3.84	3.6	3.5	3.3	3.2	0.92	0.9	0.83	0.8	0.77
	22.94	5.709	4.5	3.8	3.28	2.9	3.05	4.26	3.4	2.98	2.7	2.55	0.78	0.6	0.57	0.5	0.5
	36.17	5.005	2.69	2.3	2.18	2.2	2.14	2.88	2.4	2.17	2.1	1.9	0.54	0.5	0.44	0.4	0.4
	46.466	3.051	2.16	2.2	2.09	1.9	1.93	2.49	2.2	2.2	2.1	2.01	0.71	0.7	0.68	0.7	0.63
EQ4	21.892	4.182	3.75	3.5	3.38	3.2	3.05	3.67	2.9	2.68	2.5	2.3	0.89	0.8	0.8	0.8	0.73
	34.51	4.171	2.64	2.3	2.03	1.8	1.72	2.45	2.1	1.93	1.8	1.7	0.63	0.6	0.49	0.4	0.4
	44.333	3.072	2.30	1.9	1.73	1.7	1.7	2.18	2.0	1.97	1.9	1.78	0.75	0.6	0.56	0.6	0.55
	50.564	2.329	1.87	1.7	1.57	1.5	1.54	2.03	1.9	1.85	1.8	1.79	0.8	0.7	0.67	0.7	0.66
EQ5	31.077	3.73	2.86	2.5	2.3	2.3	2.3	3.19	2.5	2.19	1.9	1.74	0.76	0.7	0.6	0.6	0.6
	48.989	4.168	2.36	1.9	1.8	1.7	1.65	2.38	2.1	2.05	1.9	1.78	0.57	0.5	0.4	0.4	0.39
	62.933	2.409	1.69	1.5	1.47	1.5	1.44	1.94	1.9	1.99	1.9	1.84	0.7	0.6	0.6	0.6	0.59

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution
- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 7: Five degree of freedom primary system with oscillator connected at dof 1: $\xi_s = 2\%$

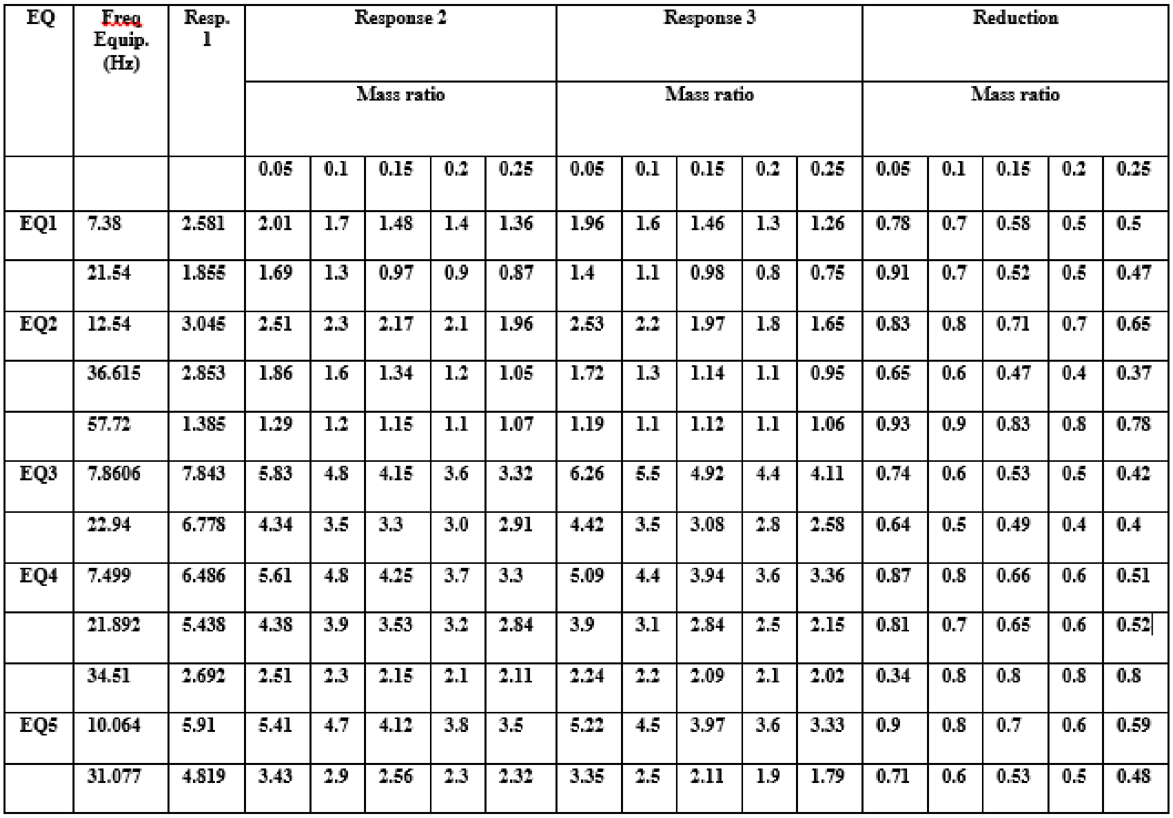


EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass ratio					Mass ratio					Mass ratio				
			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	7.38	2.528	2.31	2.1	1.89	1.7	1.55	2.01	1.8	1.6	1.5	1.73	0.91	0.8	0.75	0.7	0.61
	21.54	2.243	2.11	1.7	1.25	1.2	1.15	1.86	1.4	1.19	1.1	1.14	0.94	0.8	0.56	0.5	0.5
	33.96	1.498	0.79	0.6	0.53	0.6	0.56	0.97	0.8	0.71	0.7	0.64	0.53	0.4	0.4	0.4	0.37
EQ2	12.54	2.868	2.55	2.4	2.25	2.1	2.02	2.5	2.3	2.16	2.0	2.36	0.89	0.8	0.8	0.8	0.7
	36.615	3.922	2.34	1.8	1.72	1.6	1.5	1.51	1.5	1.33	1.3	1.49	0.6	0.5	0.44	0.4	0.38
	57.72	2.577	1.41	1.2	1.14	1.1	1.06	1.16	1.0	0.96	0.9	0.93	0.55	0.5	0.44	0.4	0.41
EQ3	7.8606	7.962	4.69	5.1	5.79	6.5	7.23	5.35	4.7	5.08	5.5	6.29	0.59	0.6	0.7	0.8	0.9
	22.94	10.01	3.41	3.4	3.41	4.6	6.1	3.13	3.1	3.66	4.3	5.56	0.34	0.3	0.3	0.4	0.61
	36.17	10.36	2.34	2.3	2.39	2.5	3.3	3.23	2.4	2.47	2.8	3.37	0.23	0.2	0.2	0.2	0.32
	46.466	4.618	1.93	2.0	2.16	2.2	2.3	2.86	2.6	2.6	2.6	2.95	0.42	0.4	0.5	0.5	0.5
EQ4	21.892	7.295	4.77	4.6	4.1	3.8	3.7	4.75	3.8	3.26	3.1	3.74	0.65	0.6	0.6	0.5	0.5
	34.51	7.965	3.11	2.6	2.24	1.9	1.86	3.06	2.6	2.33	2.1	2.33	0.39	0.3	0.3	0.3	0.23
	44.333	4.794	2.53	2.1	1.83	1.8	1.78	2.66	2.3	2.23	2.1	2.56	0.53	0.4	0.4	0.4	0.37
	50.564	2.709	2.03	1.7	1.6	1.6	1.56	2.2	2.1	1.98	1.9	2.2	0.75	0.6	0.6	0.6	0.57
EQ5	31.077	6.854	4.03	3.1	2.76	2.9	3.02	4.11	3.0	2.51	2.3	2.85	0.59	0.5	0.4	0.4	0.44
	48.989	8.669	2.64	2.2	2.03	1.9	1.79	2.74	2.4	2.34	2.5	3.19	0.31	0.3	0.24	0.2	0.21
	62.933	3.749	1.87	1.6	1.53	1.5	1.48	2.06	2.1	2.27	2.3	2.75	0.5	0.4	0.4	0.4	0.39

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution
- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 8: Five degree of freedom primary system with oscillator connected at dof 2: $\xi_s = 5\%$

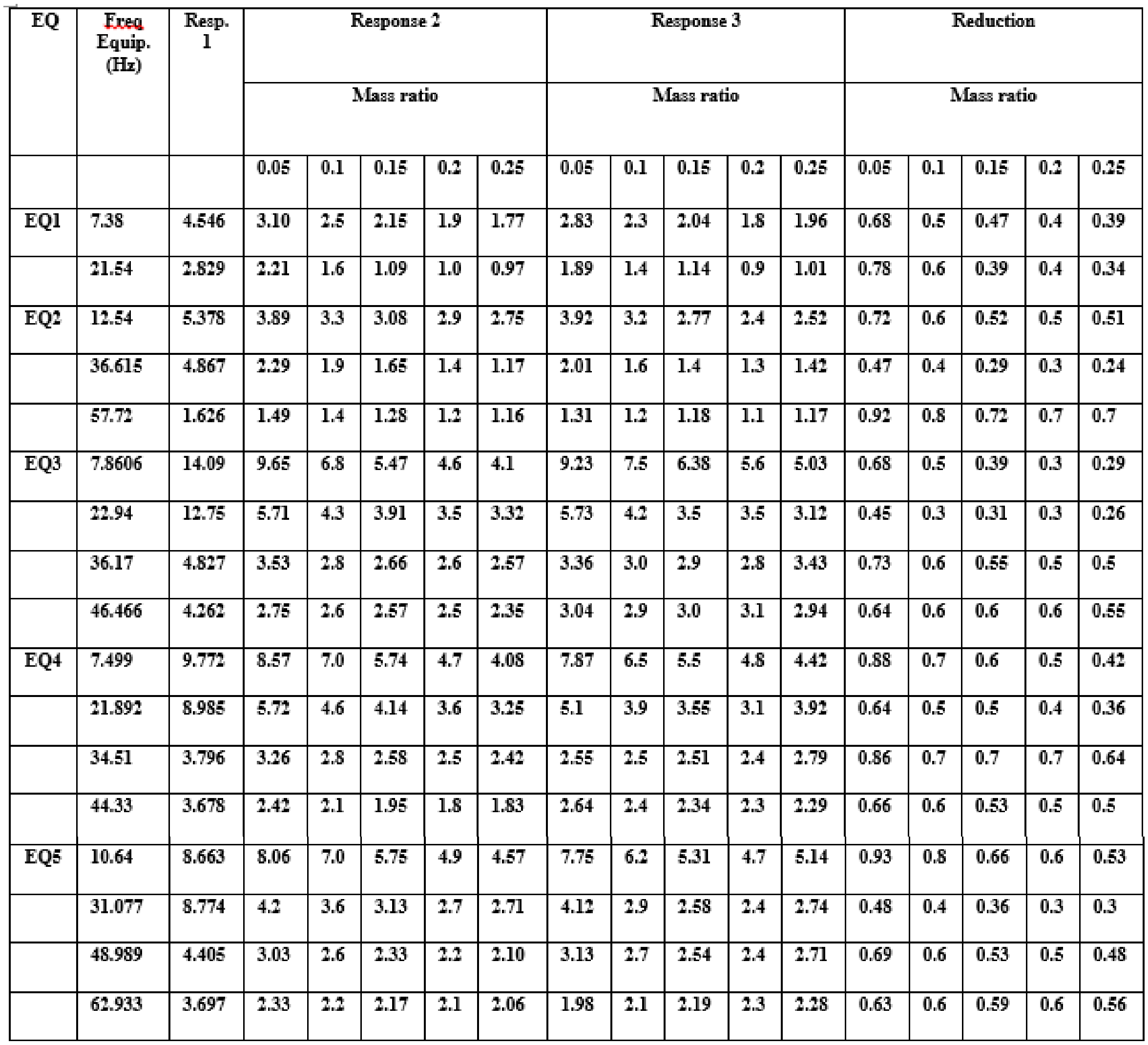


EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass ratio					Mass ratio					Mass ratio				
			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	7.38	2.581	2.01	1.7	1.48	1.4	1.36	1.96	1.6	1.46	1.3	1.26	0.78	0.7	0.58	0.5	0.5
	21.54	1.855	1.69	1.3	0.97	0.9	0.87	1.4	1.1	0.98	0.8	0.75	0.91	0.7	0.52	0.5	0.47
EQ2	12.54	3.045	2.51	2.3	2.17	2.1	1.96	2.53	2.2	1.97	1.8	1.65	0.83	0.8	0.71	0.7	0.65
	36.615	2.853	1.86	1.6	1.34	1.2	1.05	1.72	1.3	1.14	1.1	0.95	0.65	0.6	0.47	0.4	0.37
	57.72	1.385	1.29	1.2	1.15	1.1	1.07	1.19	1.1	1.12	1.1	1.06	0.93	0.9	0.83	0.8	0.78
EQ3	7.8606	7.843	5.83	4.8	4.15	3.6	3.32	6.26	5.5	4.92	4.4	4.11	0.74	0.6	0.53	0.5	0.42
	22.94	6.778	4.34	3.5	3.3	3.0	2.91	4.42	3.5	3.08	2.8	2.58	0.64	0.5	0.49	0.4	0.4
EQ4	7.499	6.486	5.61	4.8	4.25	3.7	3.3	5.09	4.4	3.94	3.6	3.36	0.87	0.8	0.66	0.6	0.51
	21.892	5.438	4.38	3.9	3.53	3.2	2.84	3.9	3.1	2.84	2.5	2.15	0.81	0.7	0.65	0.6	0.52
	34.51	2.692	2.51	2.3	2.15	2.1	2.11	2.24	2.2	2.09	2.1	2.02	0.34	0.8	0.8	0.8	0.8
EQ5	10.064	5.91	5.41	4.7	4.12	3.8	3.5	5.22	4.5	3.97	3.6	3.33	0.9	0.8	0.7	0.6	0.59
	31.077	4.819	3.43	2.9	2.56	2.3	2.32	3.35	2.5	2.11	1.9	1.79	0.71	0.6	0.53	0.5	0.48

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution
- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 9: Five degree of freedom primary system with oscillator connected at dof 2: $\xi_s = 2\%$



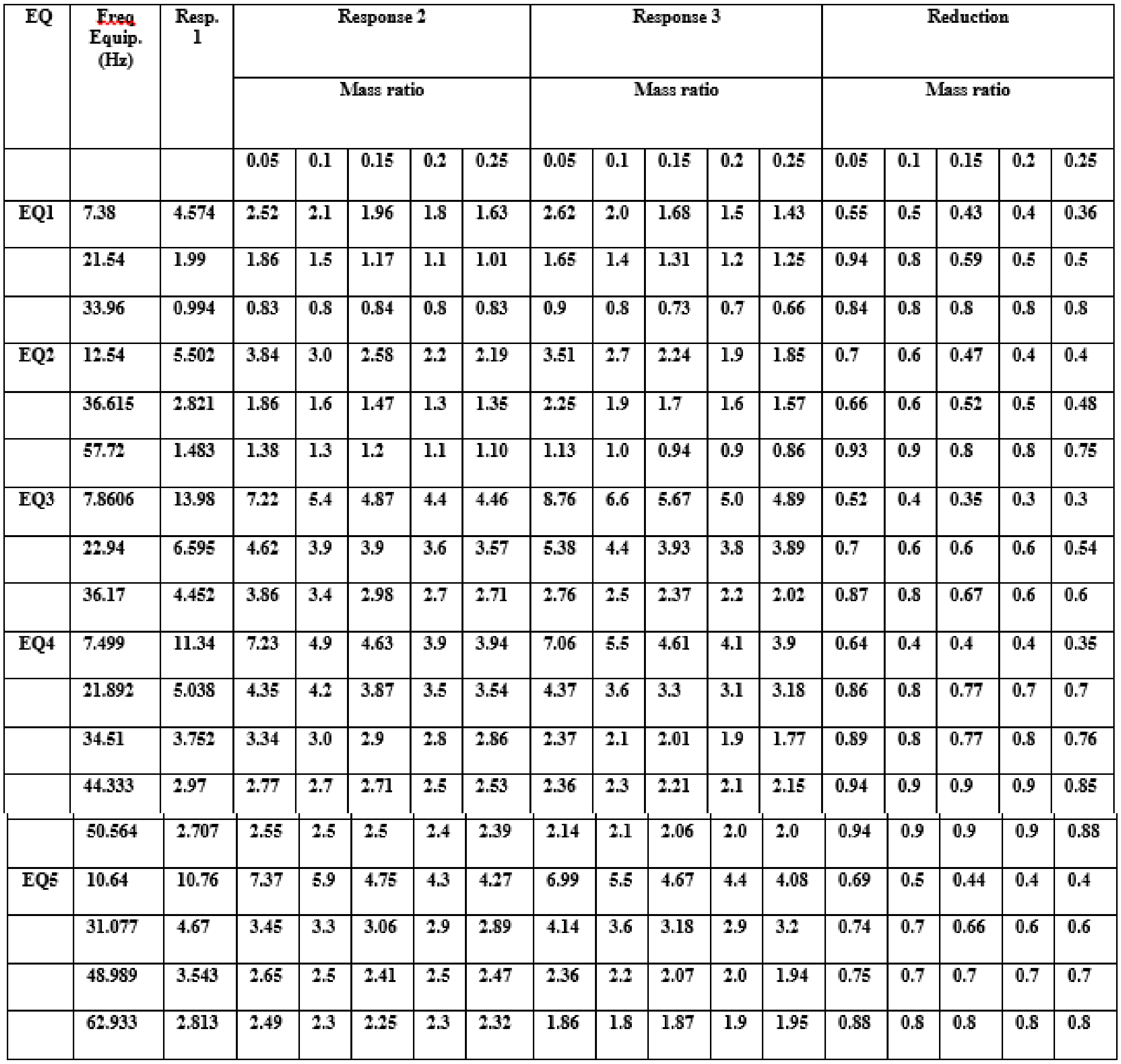
EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass ratio					Mass ratio					Mass ratio				
			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	7.38	4.546	3.10	2.5	2.15	1.9	1.77	2.83	2.3	2.04	1.8	1.96	0.68	0.5	0.47	0.4	0.39
	21.54	2.829	2.21	1.6	1.09	1.0	0.97	1.89	1.4	1.14	0.9	1.01	0.78	0.6	0.39	0.4	0.34
EQ2	12.54	5.378	3.89	3.3	3.08	2.9	2.75	3.92	3.2	2.77	2.4	2.52	0.72	0.6	0.52	0.5	0.51
	36.615	4.867	2.29	1.9	1.65	1.4	1.17	2.01	1.6	1.4	1.3	1.42	0.47	0.4	0.29	0.3	0.24
	57.72	1.626	1.49	1.4	1.28	1.2	1.16	1.31	1.2	1.18	1.1	1.17	0.92	0.8	0.72	0.7	0.7
EQ3	7.8606	14.09	9.65	6.8	5.47	4.6	4.1	9.23	7.5	6.38	5.6	5.03	0.68	0.5	0.39	0.3	0.29
	22.94	12.75	5.71	4.3	3.91	3.5	3.32	5.73	4.2	3.5	3.5	3.12	0.45	0.3	0.31	0.3	0.26
	36.17	4.827	3.53	2.8	2.66	2.6	2.57	3.36	3.0	2.9	2.8	3.43	0.73	0.6	0.55	0.5	0.5
	46.466	4.262	2.75	2.6	2.57	2.5	2.35	3.04	2.9	3.0	3.1	2.94	0.64	0.6	0.6	0.6	0.55
EQ4	7.499	9.772	8.57	7.0	5.74	4.7	4.08	7.87	6.5	5.5	4.8	4.42	0.88	0.7	0.6	0.5	0.42
	21.892	8.985	5.72	4.6	4.14	3.6	3.25	5.1	3.9	3.55	3.1	3.92	0.64	0.5	0.5	0.4	0.36
	34.51	3.796	3.26	2.8	2.58	2.5	2.42	2.55	2.5	2.51	2.4	2.79	0.86	0.7	0.7	0.7	0.64
	44.33	3.678	2.42	2.1	1.95	1.8	1.83	2.64	2.4	2.34	2.3	2.29	0.66	0.6	0.53	0.5	0.5
EQ5	10.64	8.663	8.06	7.0	5.75	4.9	4.57	7.75	6.2	5.31	4.7	5.14	0.93	0.8	0.66	0.6	0.53
	31.077	8.774	4.2	3.6	3.13	2.7	2.71	4.12	2.9	2.58	2.4	2.74	0.48	0.4	0.36	0.3	0.3
	48.989	4.405	3.03	2.6	2.33	2.2	2.10	3.13	2.7	2.54	2.4	2.71	0.69	0.6	0.53	0.5	0.48
	62.933	3.697	2.33	2.2	2.17	2.1	2.06	1.98	2.1	2.19	2.3	2.28	0.63	0.6	0.59	0.6	0.56

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution

- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 10: Five degree of freedom primary system with oscillator connected at dof 5: $\xi_s = 5\%$



EQ	Freq Equip. (Hz)	Resp. 1	Response 2					Response 3					Reduction				
			Mass ratio					Mass ratio					Mass ratio				
			0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25	0.05	0.1	0.15	0.2	0.25
EQ1	7.38	4.574	2.52	2.1	1.96	1.8	1.63	2.62	2.0	1.68	1.5	1.43	0.55	0.5	0.43	0.4	0.36
	21.54	1.99	1.86	1.5	1.17	1.1	1.01	1.65	1.4	1.31	1.2	1.25	0.94	0.8	0.59	0.5	0.5
	33.96	0.994	0.83	0.8	0.84	0.8	0.83	0.9	0.8	0.73	0.7	0.66	0.84	0.8	0.8	0.8	0.8
EQ2	12.54	5.502	3.84	3.0	2.58	2.2	2.19	3.51	2.7	2.24	1.9	1.85	0.7	0.6	0.47	0.4	0.4
	36.615	2.821	1.86	1.6	1.47	1.3	1.35	2.25	1.9	1.7	1.6	1.57	0.66	0.6	0.52	0.5	0.48
	57.72	1.483	1.38	1.3	1.2	1.1	1.10	1.13	1.0	0.94	0.9	0.86	0.93	0.9	0.8	0.8	0.75
EQ3	7.8606	13.98	7.22	5.4	4.87	4.4	4.46	8.76	6.6	5.67	5.0	4.89	0.52	0.4	0.35	0.3	0.3
	22.94	6.595	4.62	3.9	3.9	3.6	3.57	5.38	4.4	3.93	3.8	3.89	0.7	0.6	0.6	0.6	0.54
	36.17	4.452	3.86	3.4	2.98	2.7	2.71	2.76	2.5	2.37	2.2	2.02	0.87	0.8	0.67	0.6	0.6
EQ4	7.499	11.34	7.23	4.9	4.63	3.9	3.94	7.06	5.5	4.61	4.1	3.9	0.64	0.4	0.4	0.4	0.35
	21.892	5.038	4.35	4.2	3.87	3.5	3.54	4.37	3.6	3.3	3.1	3.18	0.86	0.8	0.77	0.7	0.7
	34.51	3.752	3.34	3.0	2.9	2.8	2.86	2.37	2.1	2.01	1.9	1.77	0.89	0.8	0.77	0.8	0.76
	44.333	2.97	2.77	2.7	2.71	2.5	2.53	2.36	2.3	2.21	2.1	2.15	0.94	0.9	0.9	0.9	0.85
	50.564	2.707	2.55	2.5	2.5	2.4	2.39	2.14	2.1	2.06	2.0	2.0	0.94	0.9	0.9	0.9	0.88
EQ5	10.64	10.76	7.37	5.9	4.75	4.3	4.27	6.99	5.5	4.67	4.4	4.08	0.69	0.5	0.44	0.4	0.4
	31.077	4.67	3.45	3.3	3.06	2.9	2.89	4.14	3.6	3.18	2.9	3.2	0.74	0.7	0.66	0.6	0.6
	48.989	3.543	2.65	2.5	2.41	2.5	2.47	2.36	2.2	2.07	2.0	1.94	0.75	0.7	0.7	0.7	0.7
	62.933	2.813	2.49	2.3	2.25	2.3	2.32	1.86	1.8	1.87	1.9	1.95	0.88	0.8	0.8	0.8	0.8

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution

- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

Table 11: Five degree of freedom primary system with oscillator connected at dof 5: $\xi_s = 2\%$

EQ	**Freq Equip. (Hz)**	**Resp. 1**	**Response 2**					**Response 3**					**Reduction**				
			Mass ratio					**Mass ratio**					**Mass ratio**				
			0.05	**0.1**	**0.15**	**0.2**	**0.25**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**	**0.05**	**0.1**	**0.15**	**0.2**	**0.25**
EQ1	**7.38**	**8.068**	**3.56**	**2.6**	**2.4**	**2.3**	**2.02**	**3.6**	**2.7**	**2.13**	**1.9**	**1.79**	**0.44**	**0.3**	**0.3**	**0.3**	**0.25**
	21.54	**2.638**	**2.37**	**1.9**	**1.4**	**1.1**	**1.07**	**2.05**	**1.7**	**1.49**	**1.4**	**1.25**	**0.9**	**0.7**	**0.53**	**0.4**	**0.4**
	33.96	**1.384**	**0.97**	**0.8**	**0.85**	**0.8**	**0.84**	**0.98**	**0.8**	**0.75**	**0.7**	**0.7**	**0.7**	**0.6**	**0.6**	**0.6**	**0.6**
EQ2	**12.54**	**9.831**	**5.41**	**4.1**	**3.36**	**3.1**	**2.75**	**4.88**	**3.6**	**2.91**	**2.5**	**2.23**	**0.55**	**0.4**	**0.34**	**0.3**	**0.28**
	36.615	**4.585**	**2.31**	**2.0**	**1.74**	**1.6**	**1.51**	**2.56**	**2.1**	**1.78**	**1.7**	**1.57**	**0.5**	**0.4**	**0.38**	**0.3**	**0.3**
	57.72	**1.995**	**1.53**	**1.4**	**1.26**	**1.2**	**1.1**	**1.2**	**1.0**	**0.97**	**0.9**	**0.99**	**0.77**	**0.7**	**0.6**	**0.6**	**0.56**
EQ3	**7.8606**	**24.97**	**9.31**	**6.6**	**5.6**	**5.2**	**5.6**	**11.3**	**8.1**	**6.8**	**5.9**	**5.65**	**0.39**	**0.3**	**0.22**	**0.2**	**0.2**
	22.94	**11.73**	**6.01**	**4.6**	**4.5**	**4.2**	**3.9**	**6.52**	**5.2**	**4.54**	**4.4**	**3.9**	**0.51**	**0.4**	**0.38**	**0.3**	**0.3**
	36.17	**7.815**	**4.64**	**3.8**	**3.28**	**2.9**	**2.78**	**3.2**	**2.9**	**2.69**	**2.5**	**2.55**	**0.59**	**0.5**	**0.42**	**0.4**	**0.36**
EQ4	**7.499**	**16.88**	**9.53**	**6.6**	**6.16**	**5.6**	**5.04**	**9.79**	**6.8**	**5.7**	**5.1**	**4.82**	**0.56**	**0.4**	**0.37**	**0.3**	**0.3**
	21.892	**8.545**	**5.26**	**4.9**	**4.52**	**4.2**	**3.99**	**5.33**	**4.4**	**4.17**	**4.1**	**3.19**	**0.62**	**0.6**	**0.53**	**0.5**	**0.47**
	34.51	**6.539**	**3.72**	**3.4**	**3.04**	**3.1**	**3.04**	**2.96**	**2.6**	**2.46**	**2.4**	**2.9**	**0.57**	**0.5**	**0.46**	**0.5**	**0.46**
	44.333	**3.297**	**3.05**	**2.8**	**2.75**	**2.7**	**2.55**	**2.79**	**2.5**	**2.41**	**2.3**	**2.36**	**0.92**	**0.9**	**0.84**	**0.8**	**0.77**
	50.564	**2.786**	**2.58**	**2.5**	**2.5**	**2.5**	**2.39**	**2.39**	**2.3**	**2.27**	**2.2**	**2.13**	**0.93**	**0.9**	**0.9**	**0.9**	**0.86**
EQ5	**10.64**	**15.81**	**9.9**	**7.4**	**5.8**	**5.5**	**5.4**	**9.28**	**7.0**	**5.95**	**5.1**	**4.96**	**0.63**	**0.5**	**0.37**	**0.3**	**0.3**
	31.077	**8.497**	**4.16**	**3.6**	**3.38**	**3.2**	**3.29**	**5.00**	**4.4**	**3.72**	**3.5**	**3.2**	**0.49**	**0.4**	**0.4**	**0.4**	**0.38**
	48.989	**6.481**	**3.05**	**2.7**	**2.53**	**2.5**	**2.56**	**2.77**	**2.4**	**2.31**	**2.2**	**2.82**	**0.47**	**0.4**	**0.4**	**0.4**	**0.39**
	62.933	**3.477**	**2.56**	**2.4**	**2.26**	**2.3**	**2.33**	**2.11**	**1.9**	**1.96**	**2.0**	**2.02**	**0.74**	**0.7**	**0.65**	**0.7**	**0.67**

Note:

- Response 1: Spectral response of the oscillator for an uncoupled analysis
- Response 2: Spectral response of the oscillator for coupled analysis using complete transformed mode solution

- Response 3: Spectral response of the oscillator for coupled analysis using two mode approximation in response spectrum method
- Damping ratio of primary system = 5%

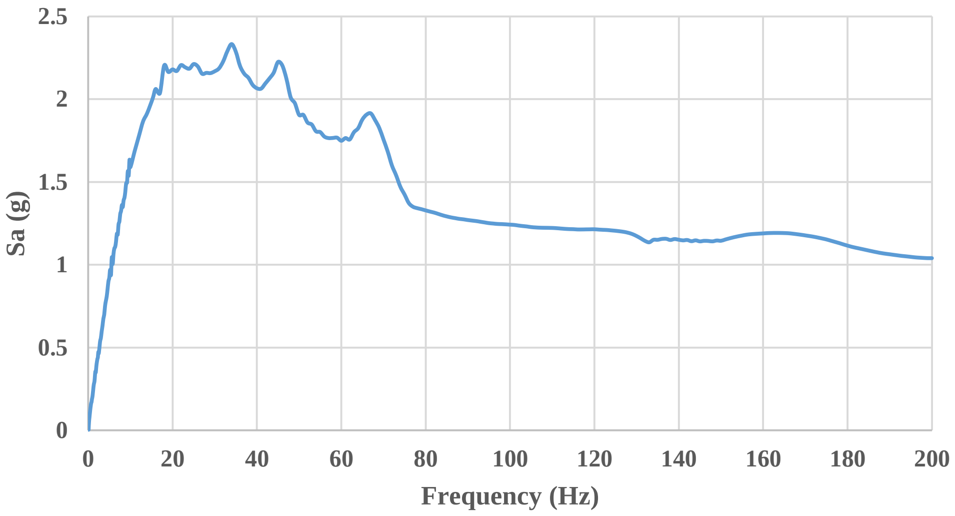


Fig. 1. Ground spectrum for high frequency earthquake 1 (HFEQ1)

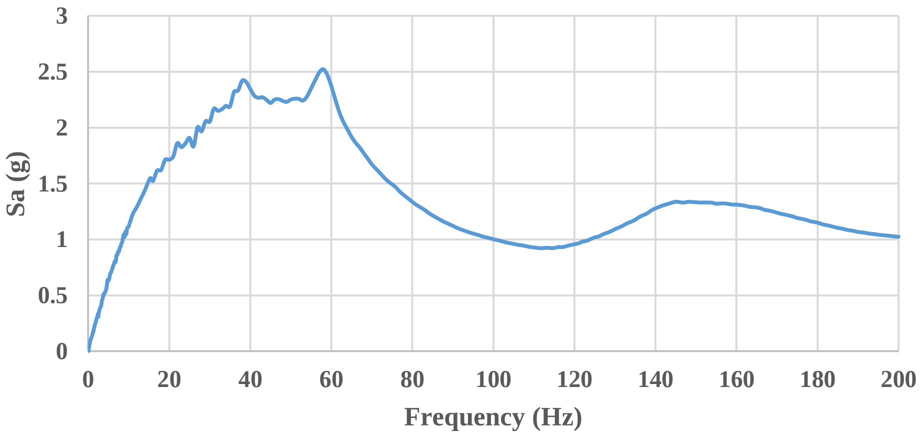


Fig. 2. Ground spectrum for high frequency earthquake 2 (HFEQ2)

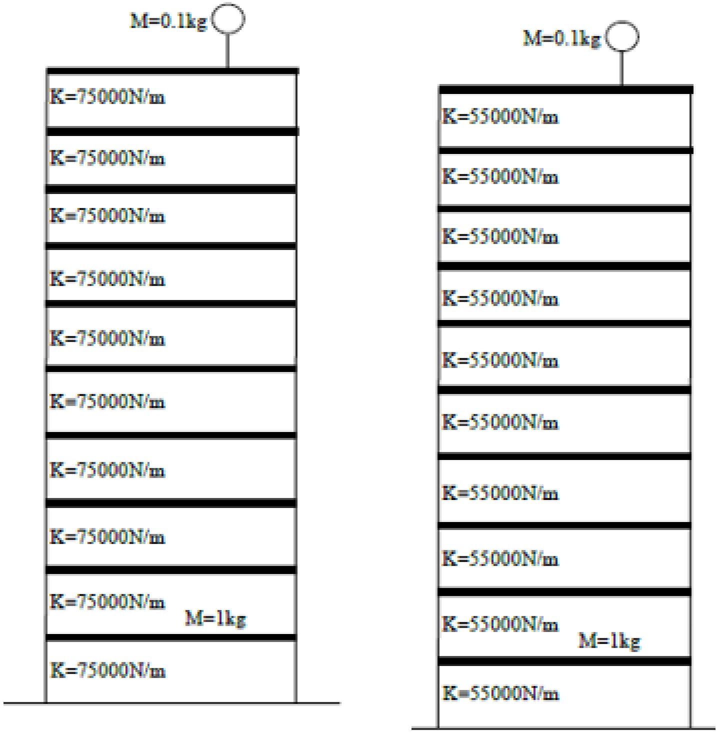


Fig. 3. Representative ten degree of freedom primary system with oscillator connected to top:

System 1 (HFEQ1) & System II (HFEQ2)

Case A: System I

Equipment mounted on 10th floor

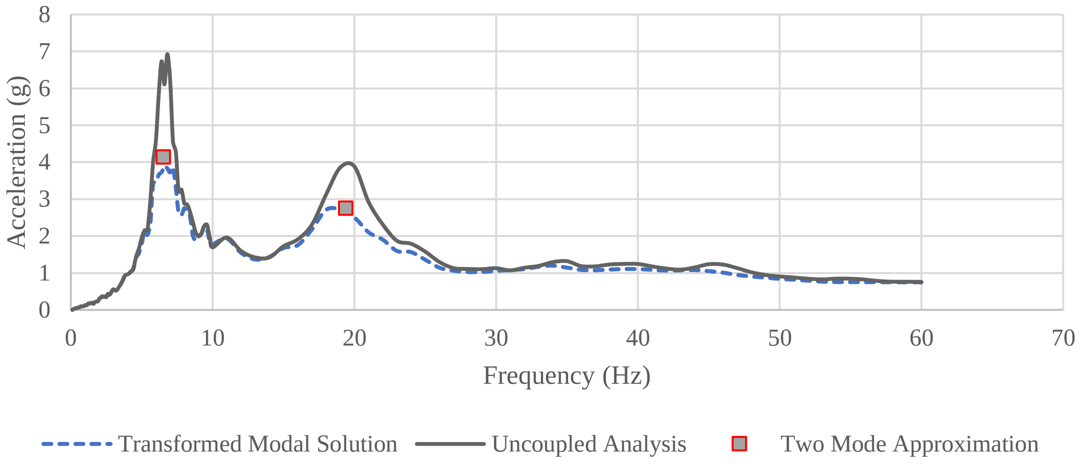


Fig. 4. ISRS for equipment mounted on 10th floor, Mass ratio: 0.05

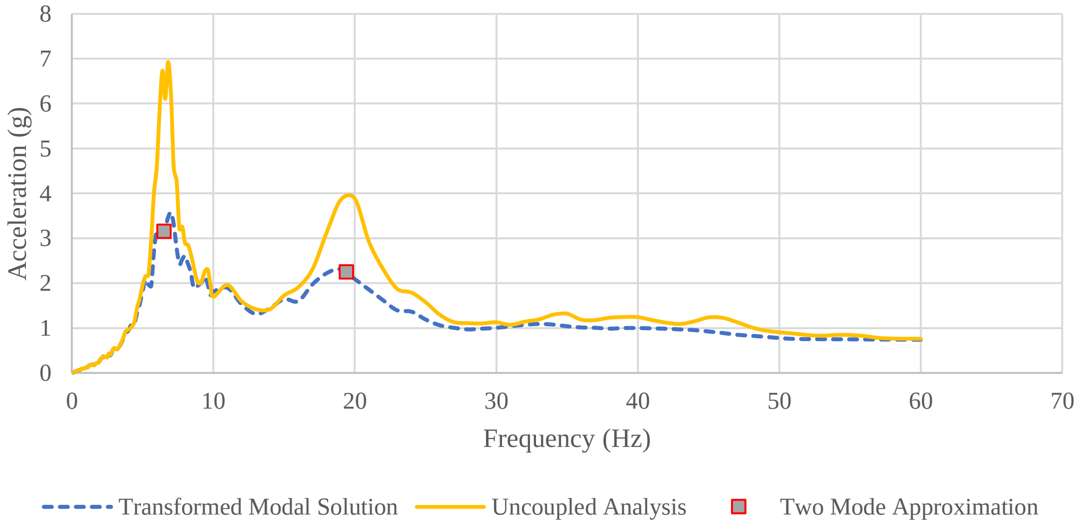


Fig. 5. ISRS for equipment mounted on 10th floor, Mass ratio: 0.1

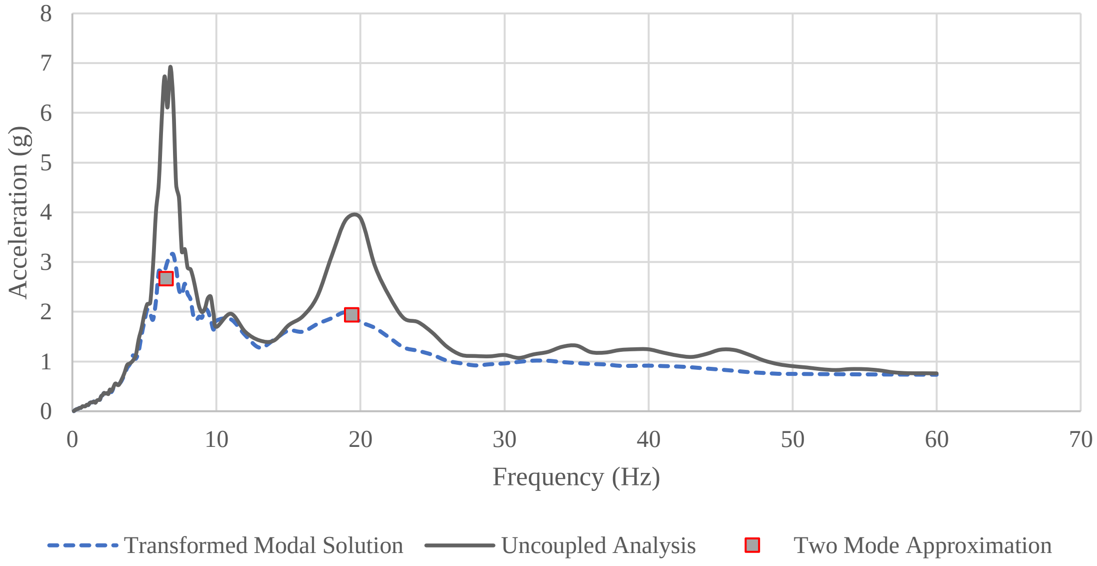


Fig. 6. ISRS for equipment mounted on 10th floor, Mass ratio: 0.15

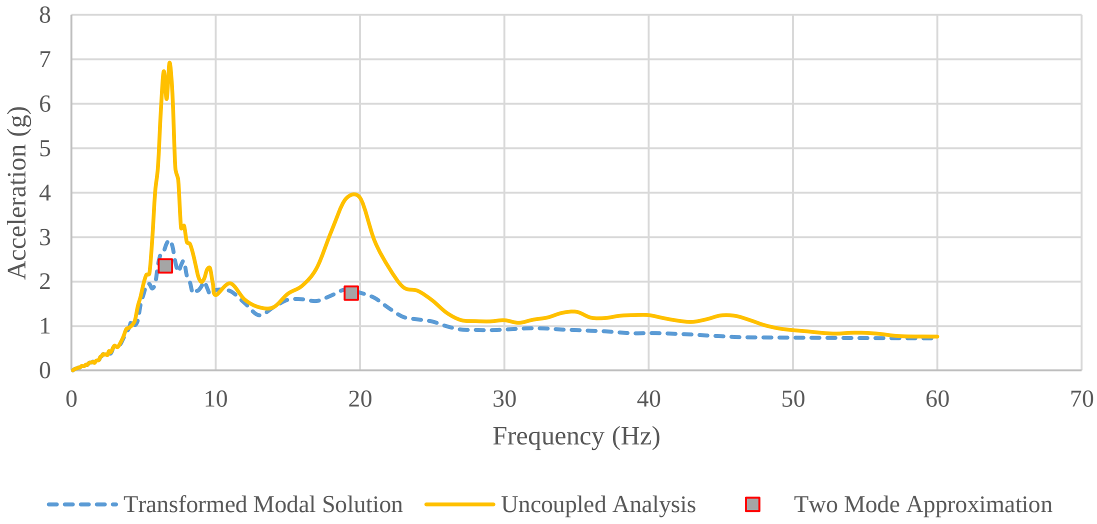


Fig. 7. ISRS for equipment mounted on 10th floor, Mass ratio: 0.2

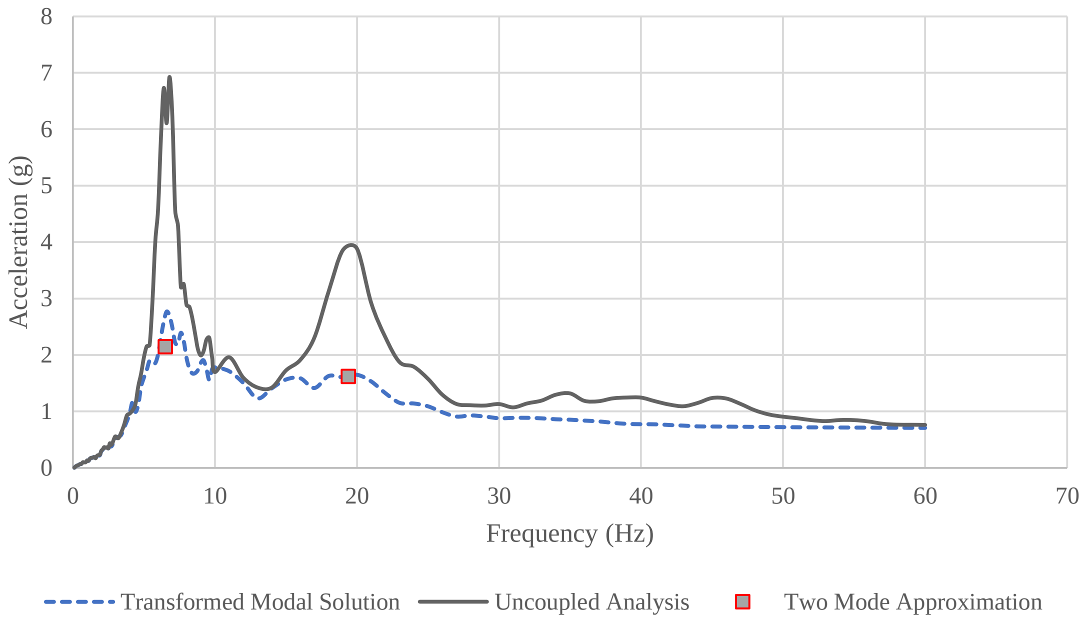


Fig. 8. ISRS for equipment mounted on 10th floor, Mass ratio: 0.25

Equipment mounted on 5th floor

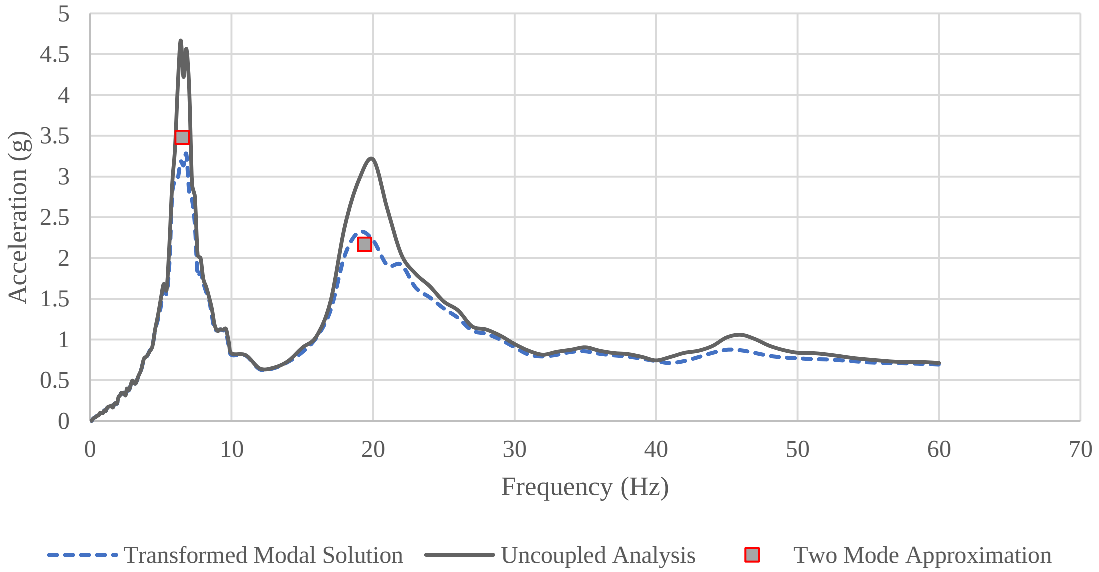


Fig. 9. ISRS for equipment mounted on 5th floor, Mass ratio: 0.05

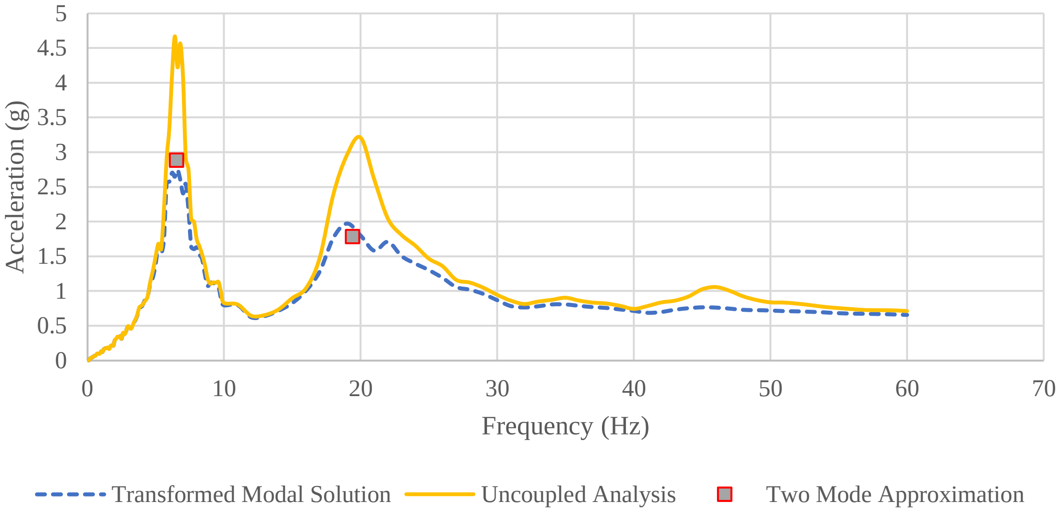


Fig. 10. ISRS for equipment mounted on 5th floor, Mass ratio: 0.1

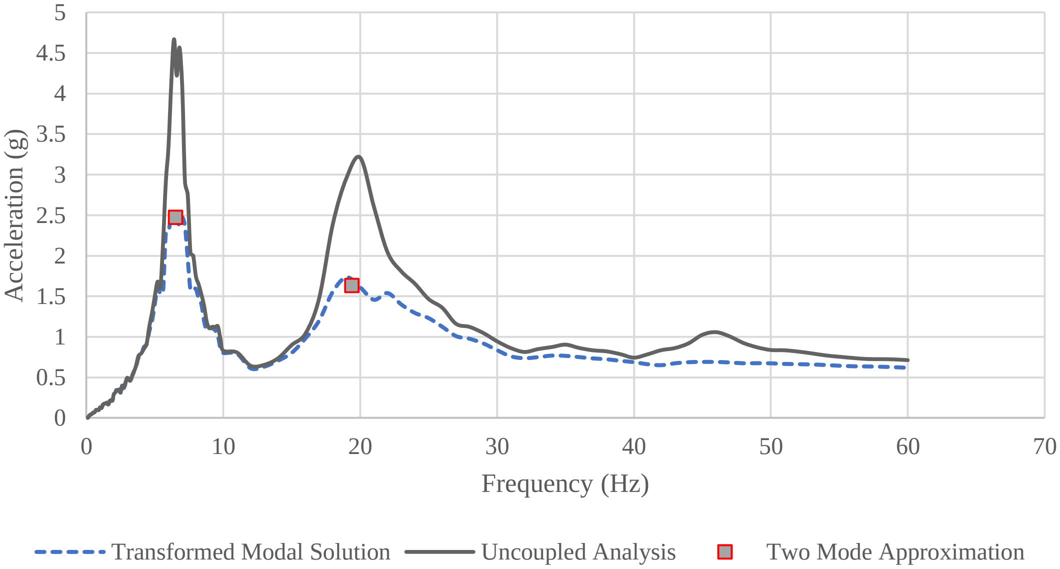


Fig. 11. ISRS for equipment mounted on 5th floor, Mass ratio: 0.15

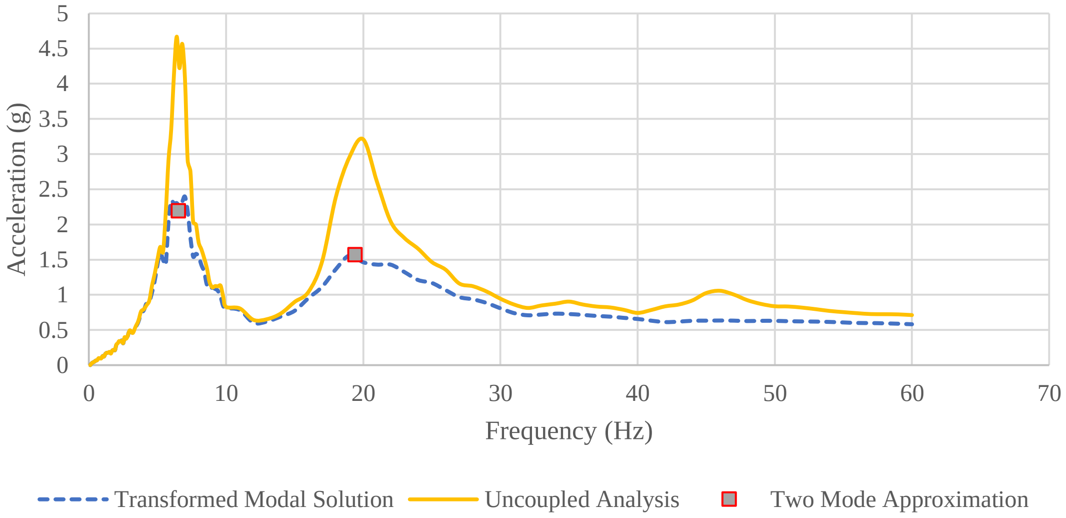


Fig. 12. ISRS for equipment mounted on 5th floor, Mass ratio: 0.2

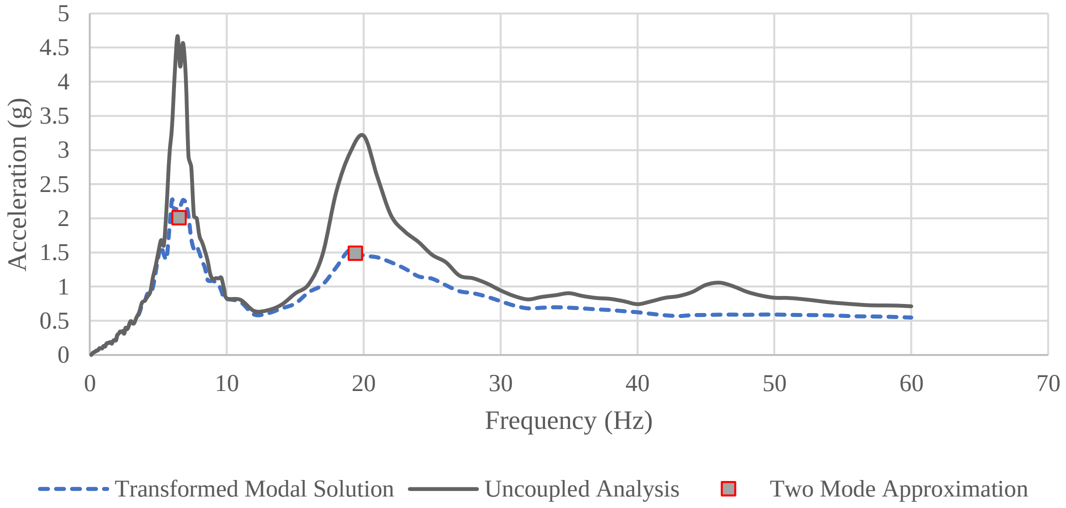


Fig. 13. ISRS for equipment mounted on 5th floor, Mass ratio: 0.25

Case B: System II

Equipment mounted on 10th floor

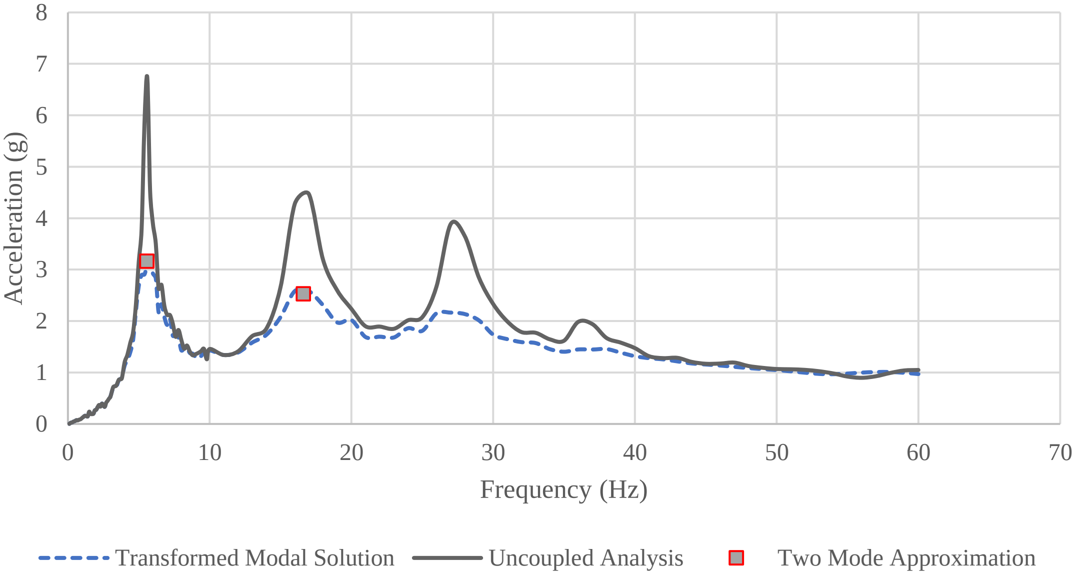


Fig. 14. ISRS for equipment mounted on 10th floor, Mass ratio: 0.05

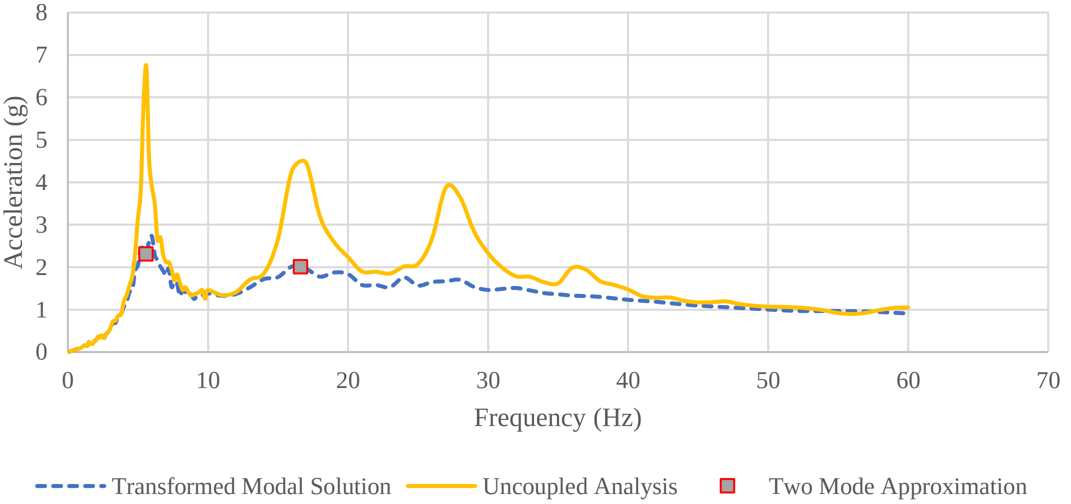


Fig. 15. ISRS for equipment mounted on 10th floor, Mass ratio: 0.1

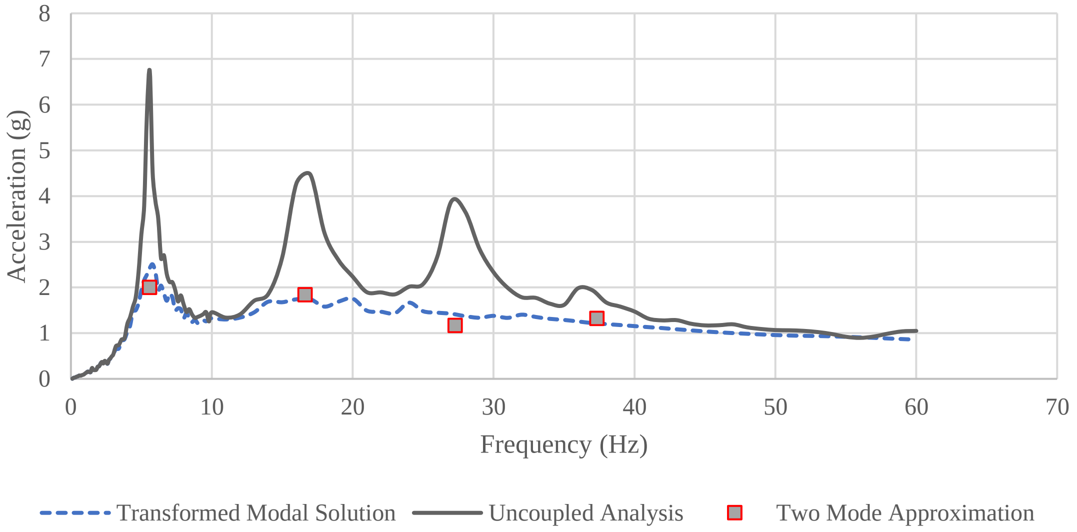


Fig. 16. ISRS for equipment mounted on 10th floor, Mass ratio: 0.15

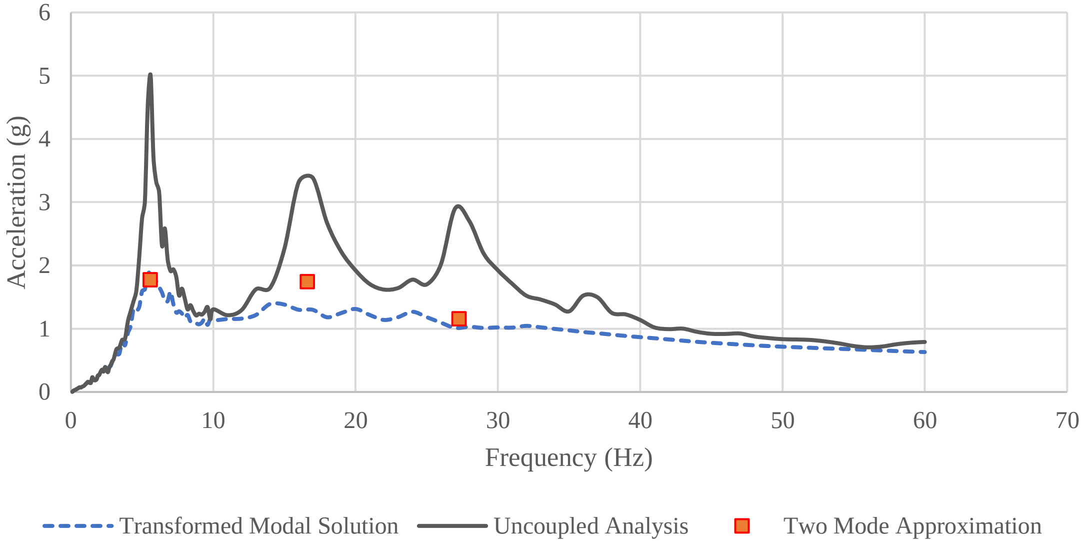


Fig. 17. ISRS for equipment mounted on 10th floor, Mass ratio: 0.2

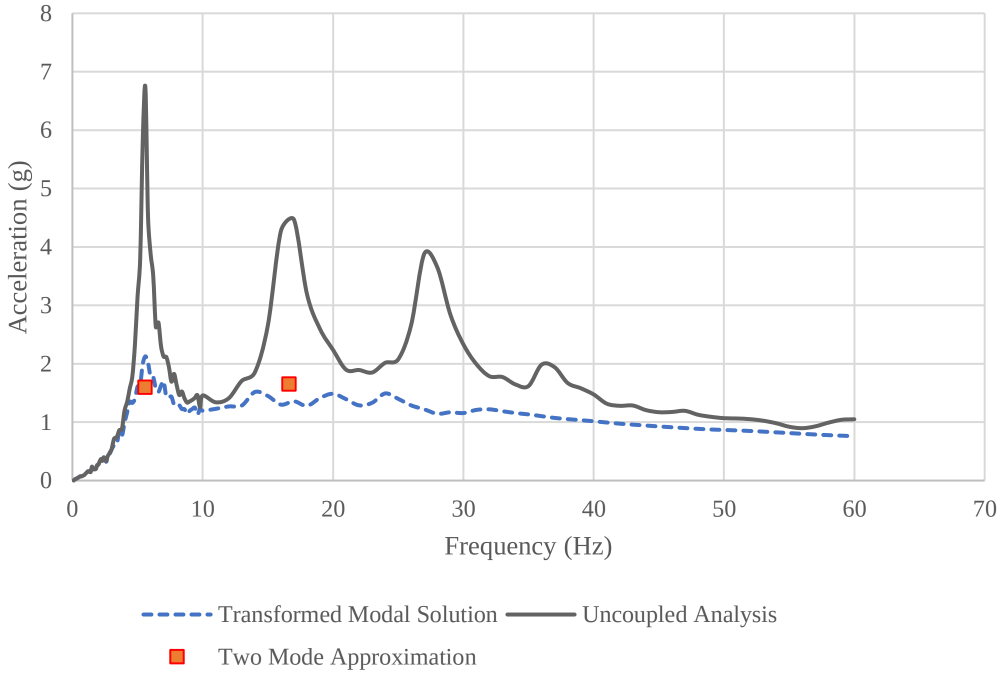


Fig. 18. ISRS for equipment mounted on 10th floor, Mass ratio: 0.25

Equipment mounted on 5th floor

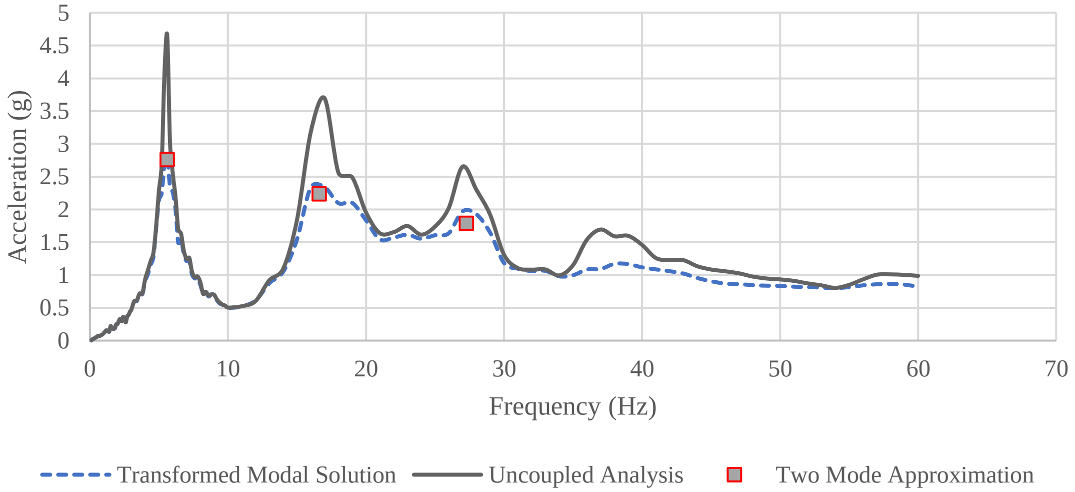


Fig. 19. ISRS for equipment mounted on 5th floor, Mass ratio: 0.05

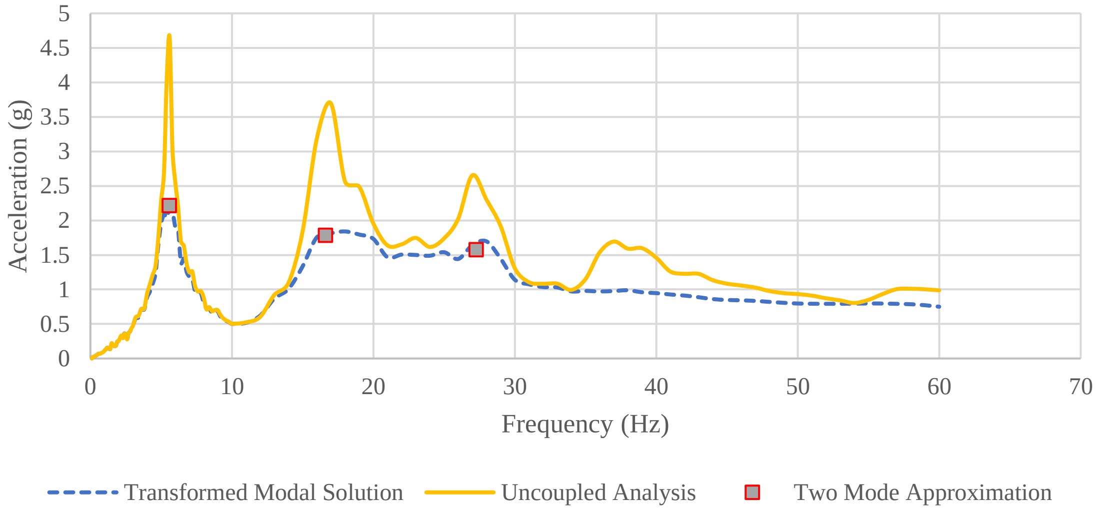


Fig. 20. ISRS for equipment mounted on 5th floor, Mass ratio: 0.1

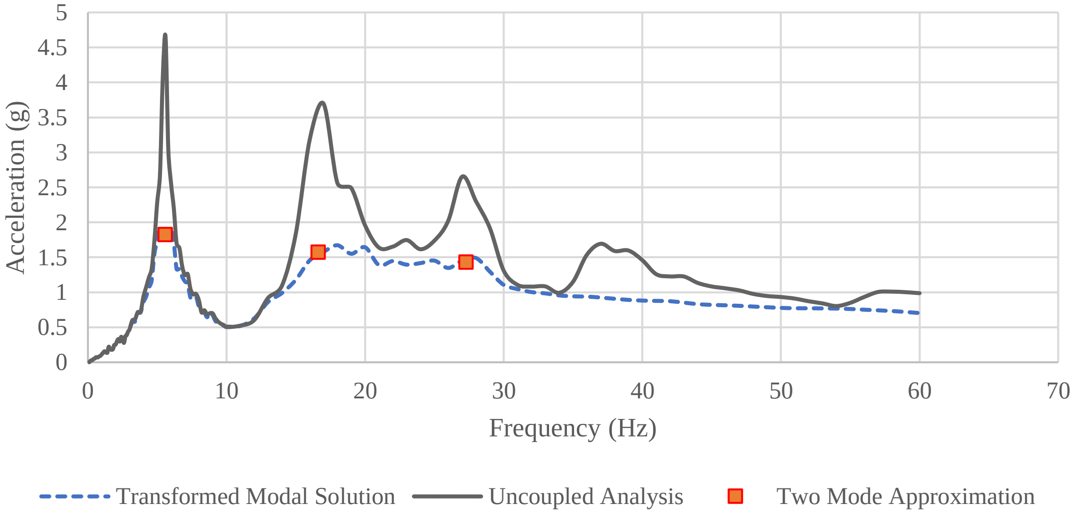


Fig. 21. ISRS for equipment mounted on 5th floor, Mass ratio: 0.15

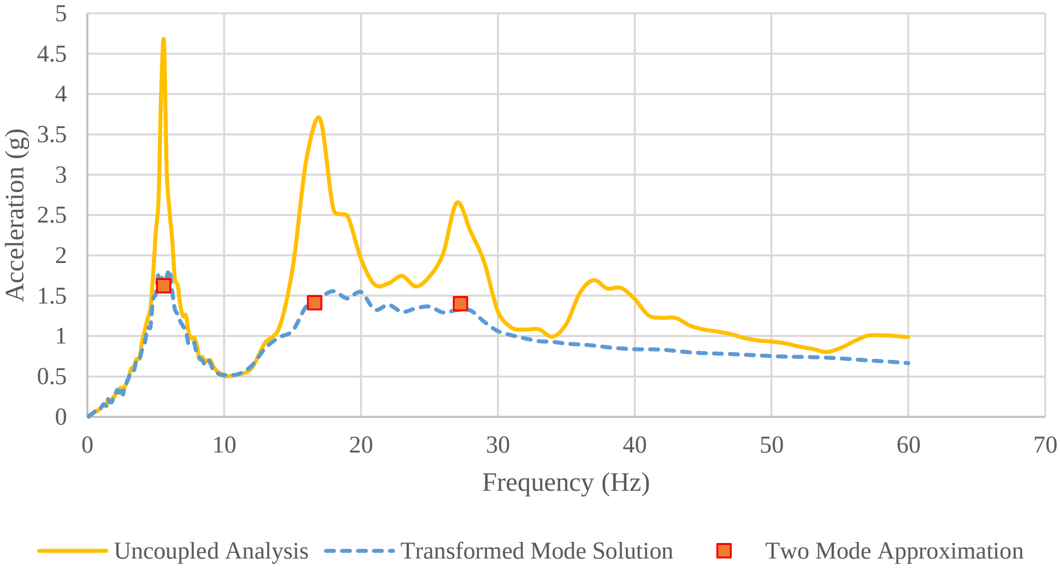


Fig. 22. ISRS for equipment mounted on 5th floor, Mass ratio: 0.2

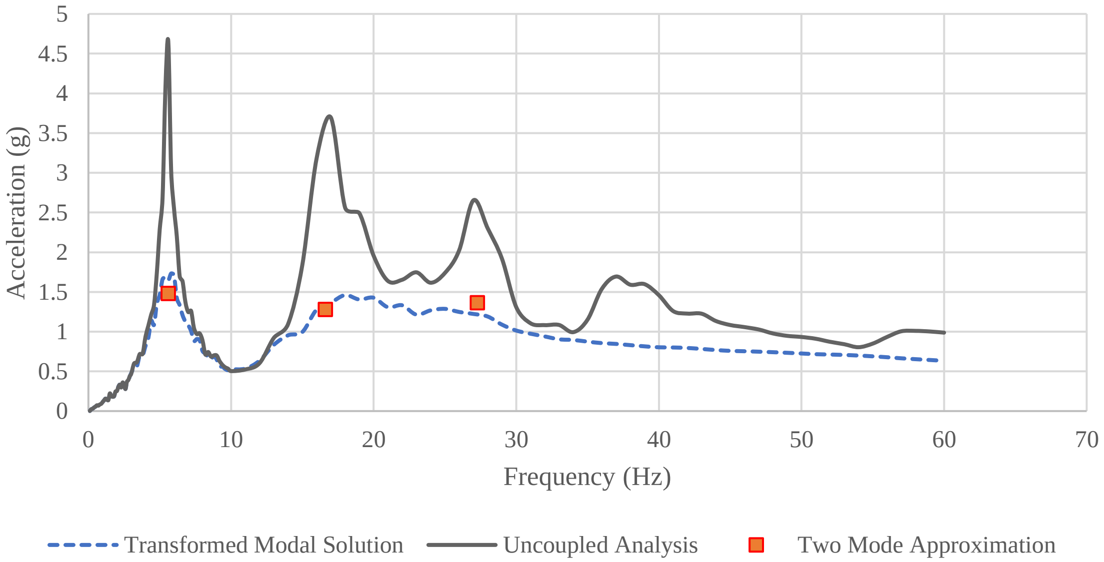


Fig. 23. ISRS for equipment mounted on 5th floor, Mass ratio: 0.25

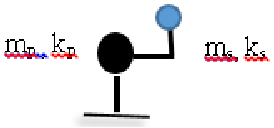


Fig. 24. Schematic representation of a single degree of freedom equipment connected to single degree of freedom structure

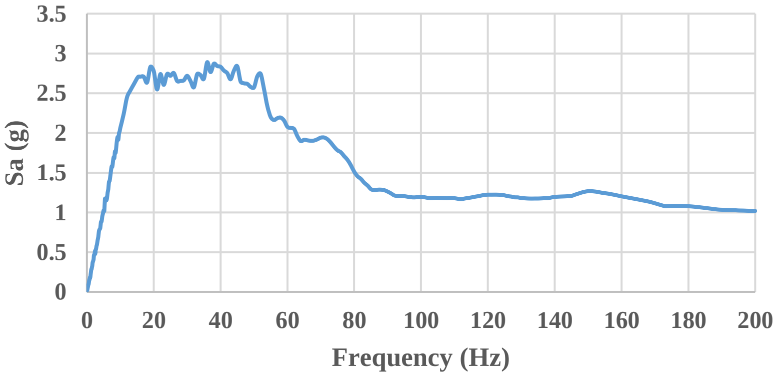


Fig. 25. Ground spectrum for high frequency earthquake 3 (HFEQ3)

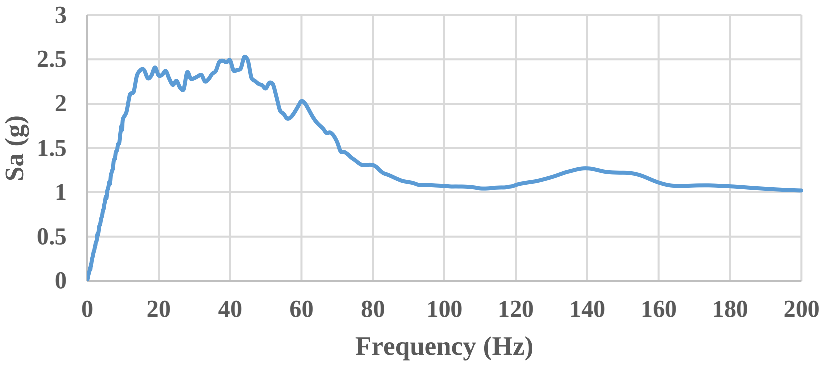


Fig. 26. Ground spectrum for high frequency earthquake 4 (HFEQ4)

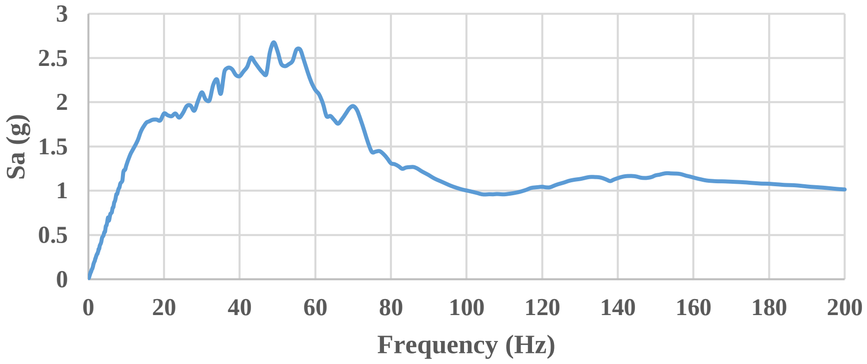


Fig. 27. Ground spectrum for high frequency earthquake 5 (HFEQ5)

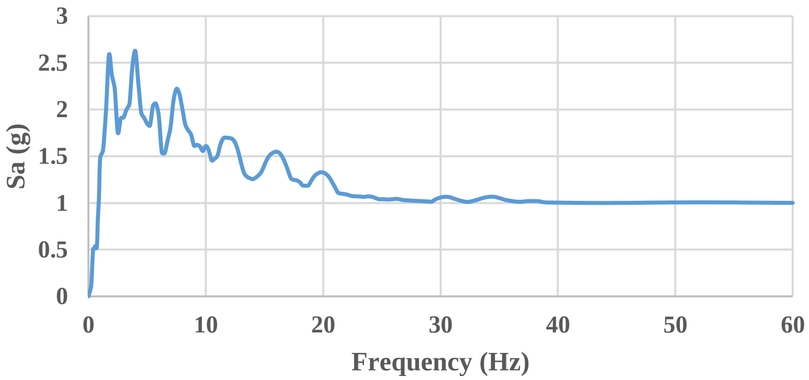


Fig. 28. Ground spectrum for low frequency earthquake – ElCentro (LFEQ)

<u>Case 1:</u> Ground motion: HFEQ1

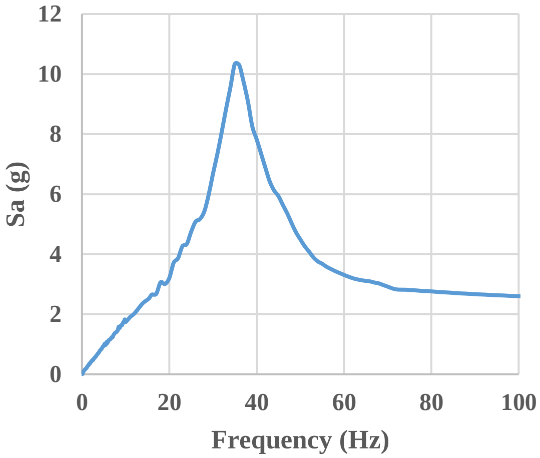
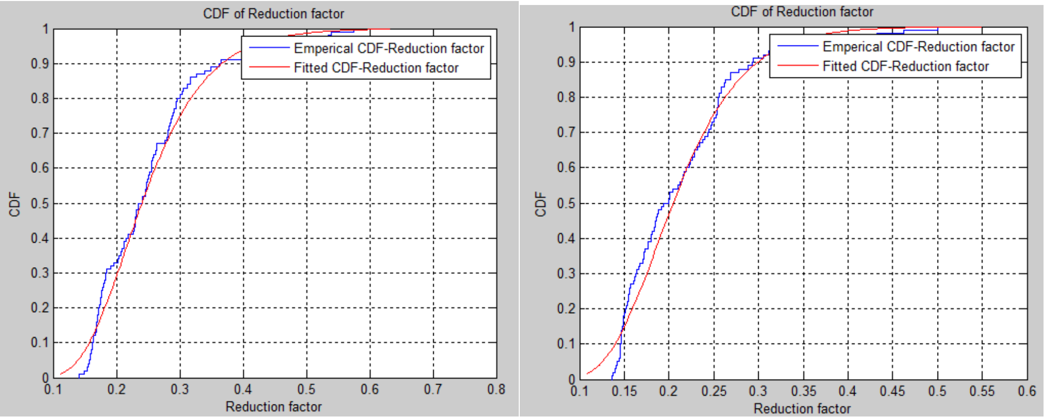
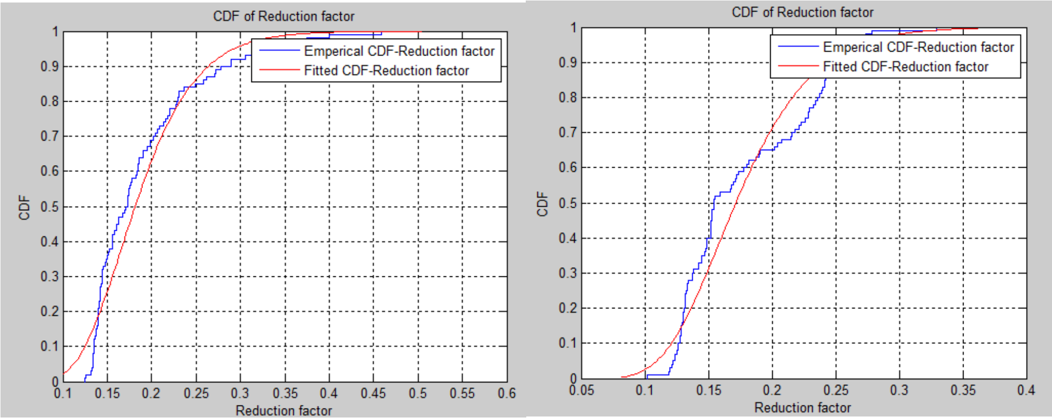


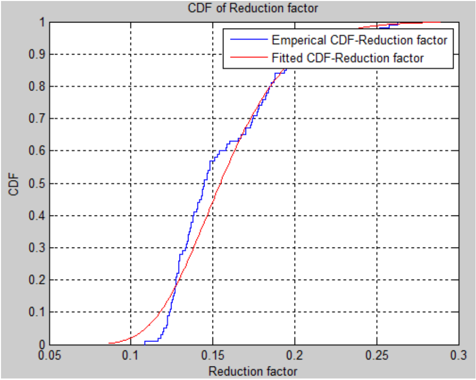
Fig. 29. Uncoupled response spectrum for equipment: HFEQ1



(a) Mass ratio: 0.05 (b) Mass ratio: 0.1



(c) Mass ratio: 0.15 (d) Mass ratio: 0.2



(e) Mass ratio: 0.25

Fig. 30. Plots of cumulative distribution functions for reduction factor: HFEQ1

<u>Case 2:</u> Ground motion: HFEQ2

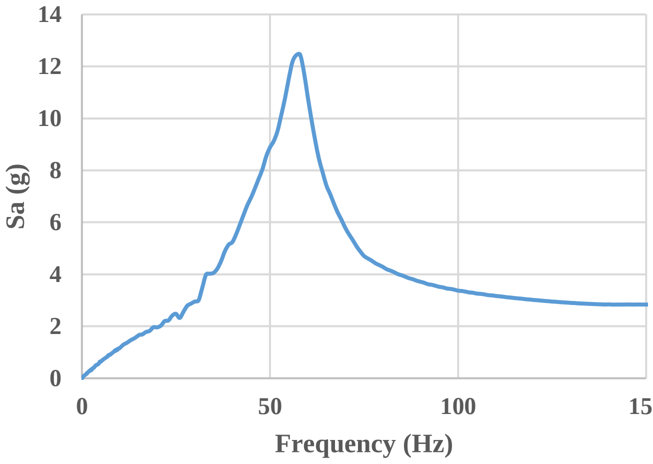
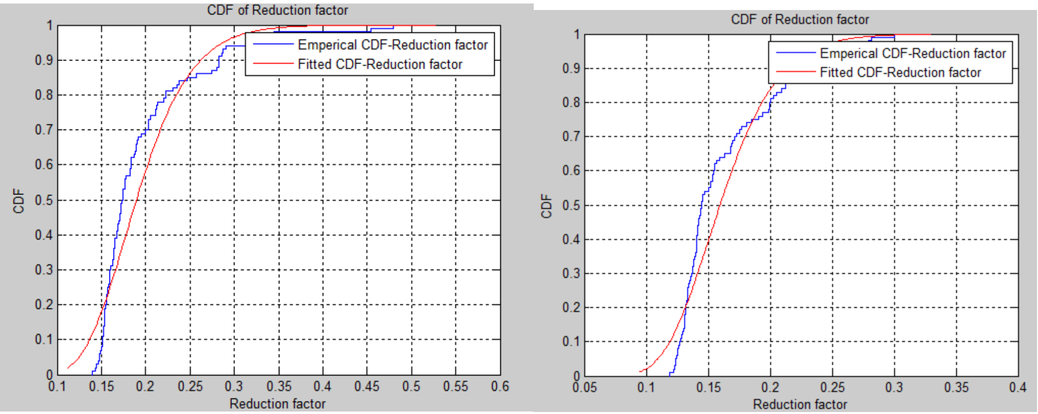
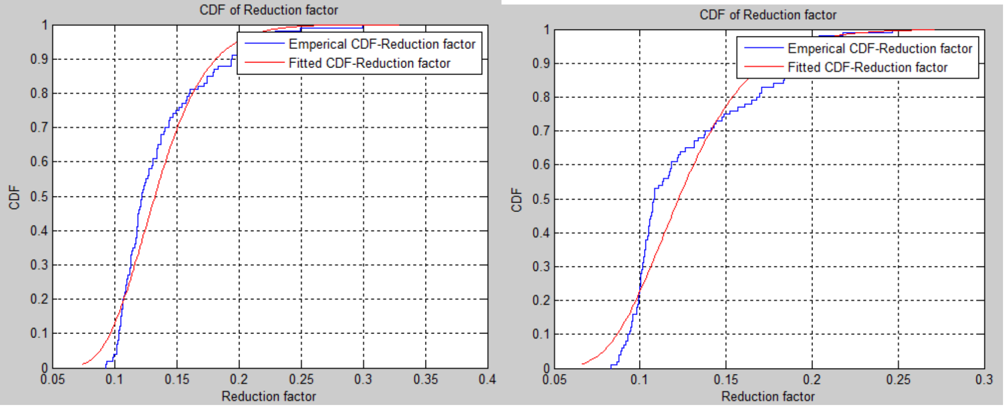


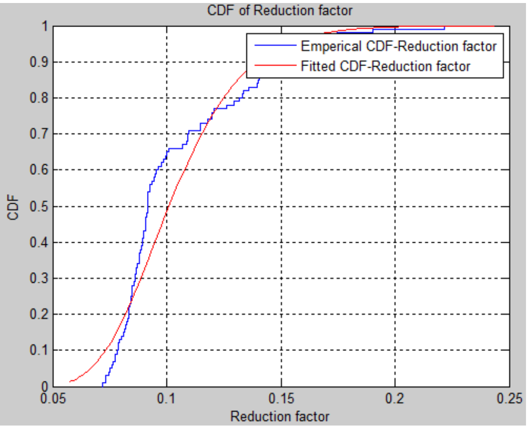
Fig. 31. Uncoupled response spectrum for equipment: HFEQ2



(a) Mass ratio: 0.05 (b) Mass ratio: 0.1



(c) Mass ratio: 0.15 (d) Mass ratio: 0.2



(e) Mass ratio: 0.25

Fig. 32. Plots of cumulative distribution functions for reduction factor: HFEQ2

Case 3: Ground motion: HFEQ3

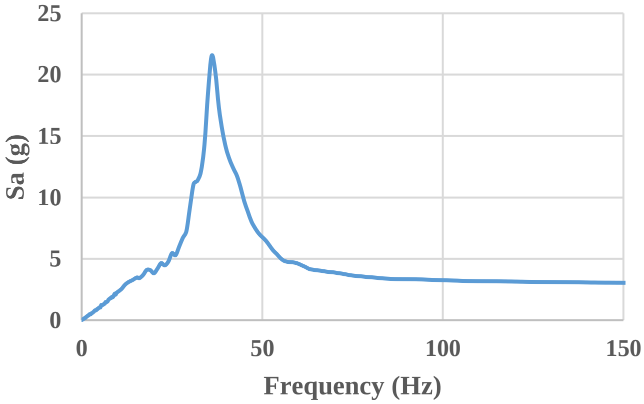
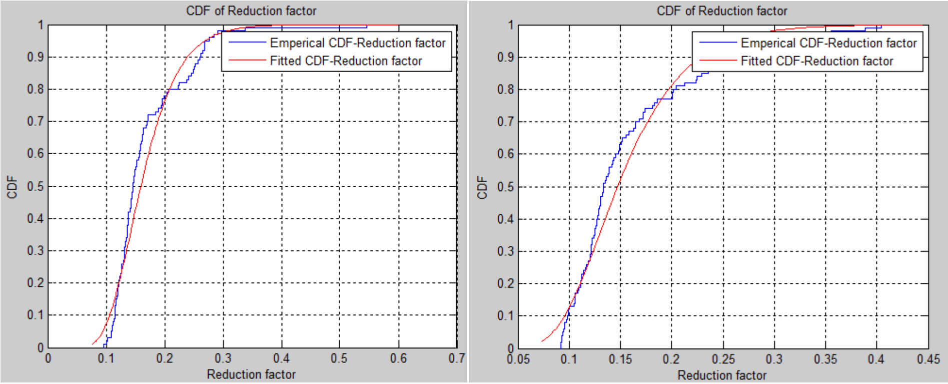
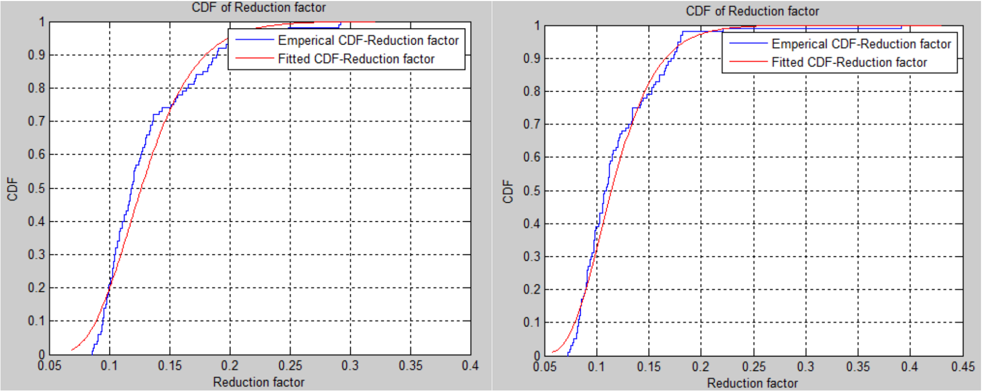


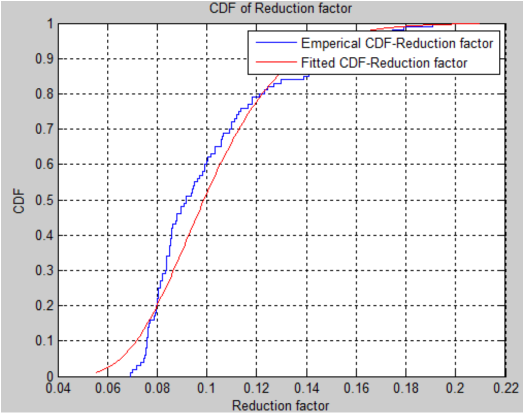
Fig. 33. Uncoupled response spectrum for equipment: HFEQ3



(a) Mass ratio: 0.05 (b) Mass ratio: 0.1



(c) Mass ratio: 0.15 (d) Mass ratio: 0.2



(e) Mass ratio: 0.25

Fig. 34. Plots of cumulative distribution functions for reduction factor: HFEQ3

Case 4: Ground motion: HFEQ4

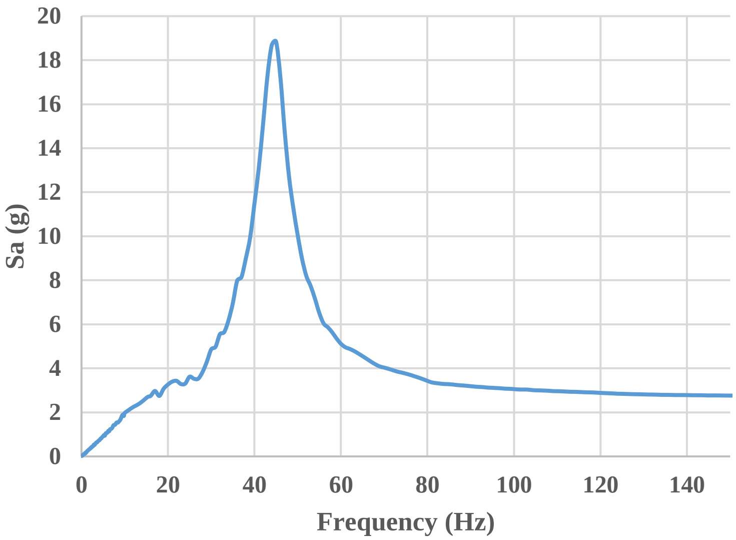
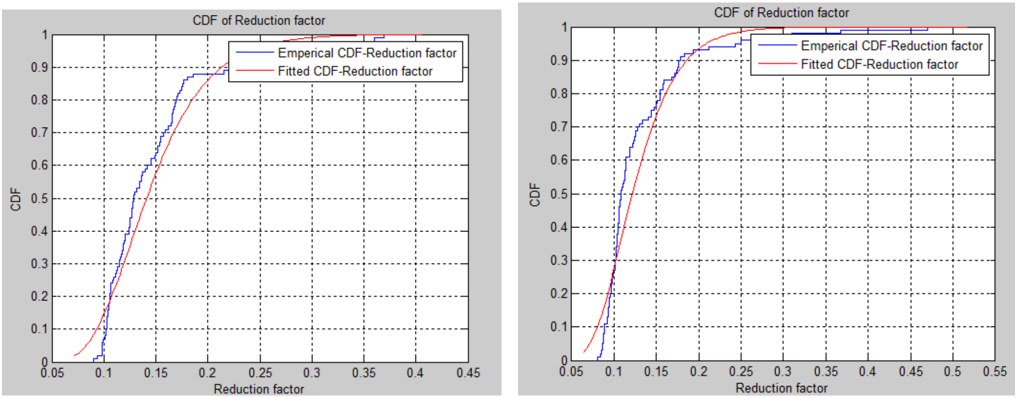
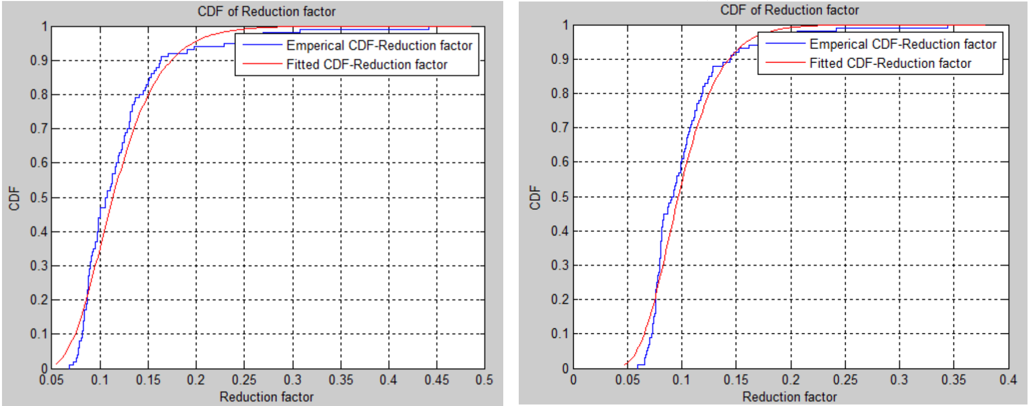


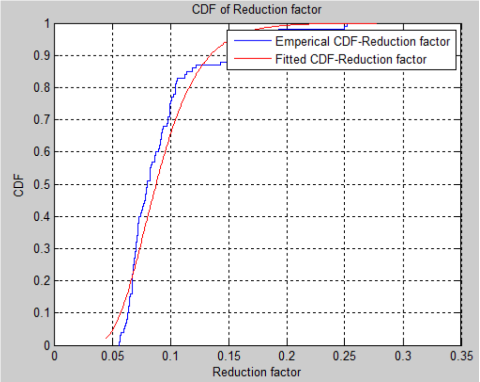
Fig. 35. Uncoupled response spectrum for equipment: HFEQ4



(a) Mass ratio: 0.05 (b) Mass ratio: 0.1



(c) Mass ratio: 0.15 (d) Mass ratio: 0.2



(e) Mass ratio: 0.25

Fig. 36. Plots of cumulative distribution functions for reduction factor: HFEQ4

Case 5: Ground motion: HFEQ5

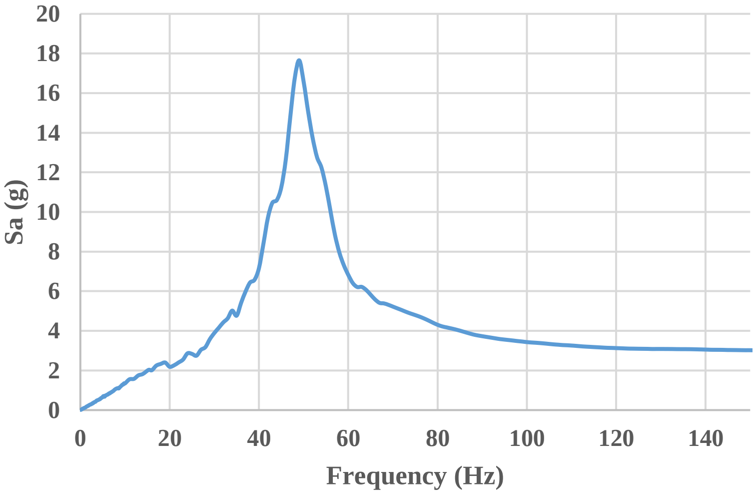
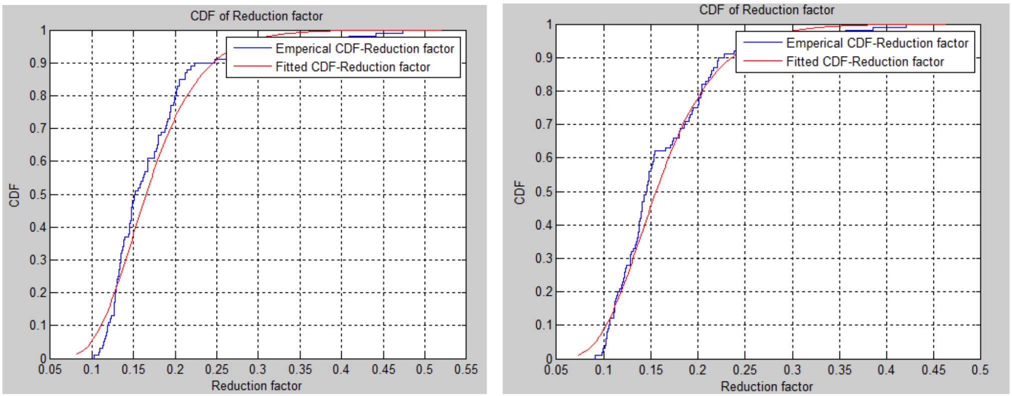
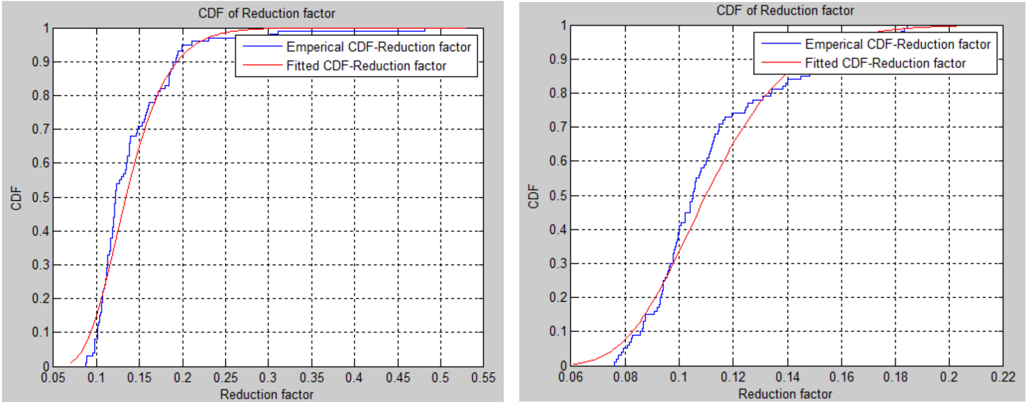


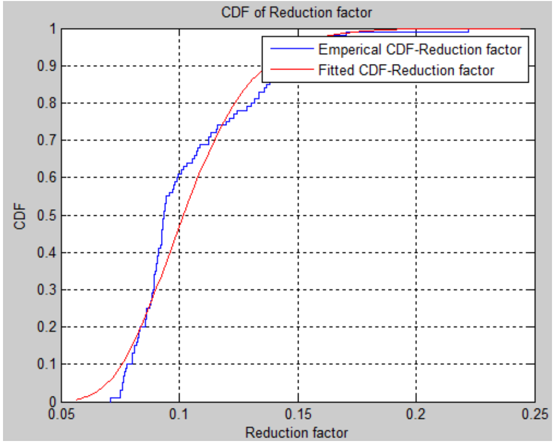
Fig. 37. Uncoupled response spectrum for equipment: HFEQ5



(a) Mass ratio: 0.05 (b) Mass ratio: 0.1



(c) Mass ratio: 0.15 (d) Mass ratio: 0.2



(e) Mass ratio: 0.25

Fig. 38. Plots of cumulative distribution functions for reduction factor: HFEQ5

Case 6: Ground motion: LFEQ

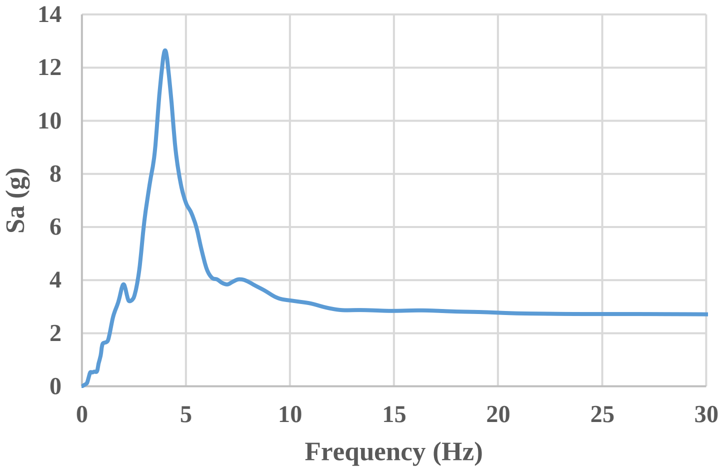
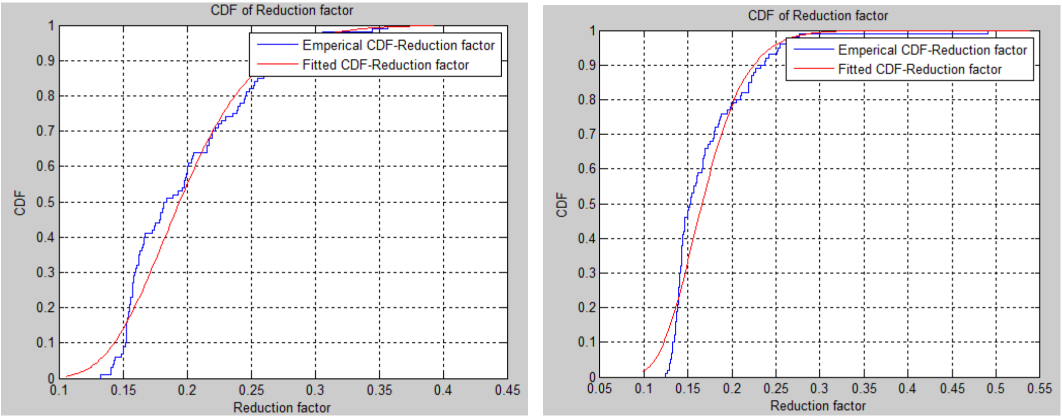
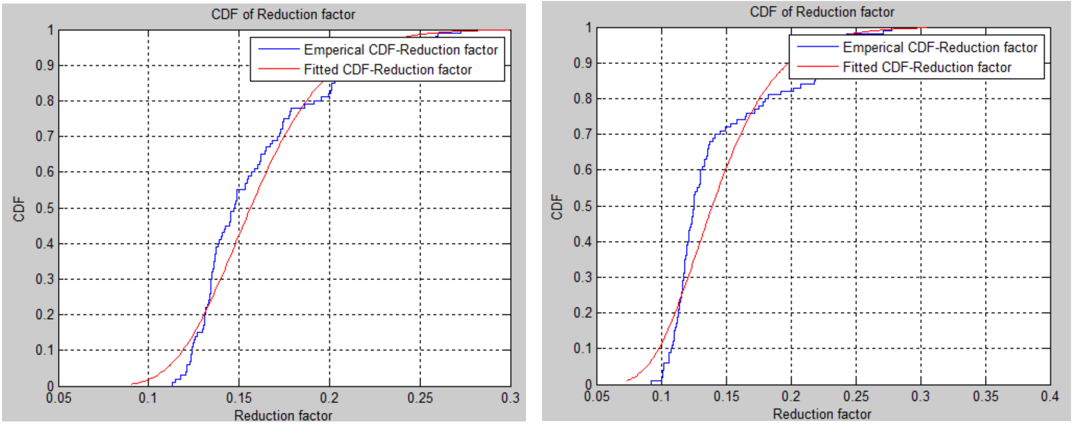


Fig. 39. Uncoupled response spectrum for equipment: LFEQ



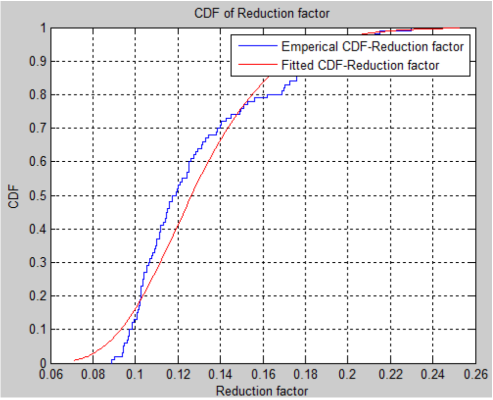
(a) Mass ratio: 0.05

(b) Mass ratio: 0.1



(c) Mass ratio: 0.15

(d) Mass ratio: 0.2



(e) Mass ratio: 0.25

Fig. 40. Plots of cumulative distribution functions for reduction factor: LFEQ

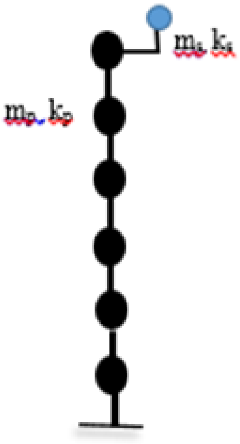


Fig. 41. Schematic representation of five degree of freedom structure (primary system), oscillator connected at top

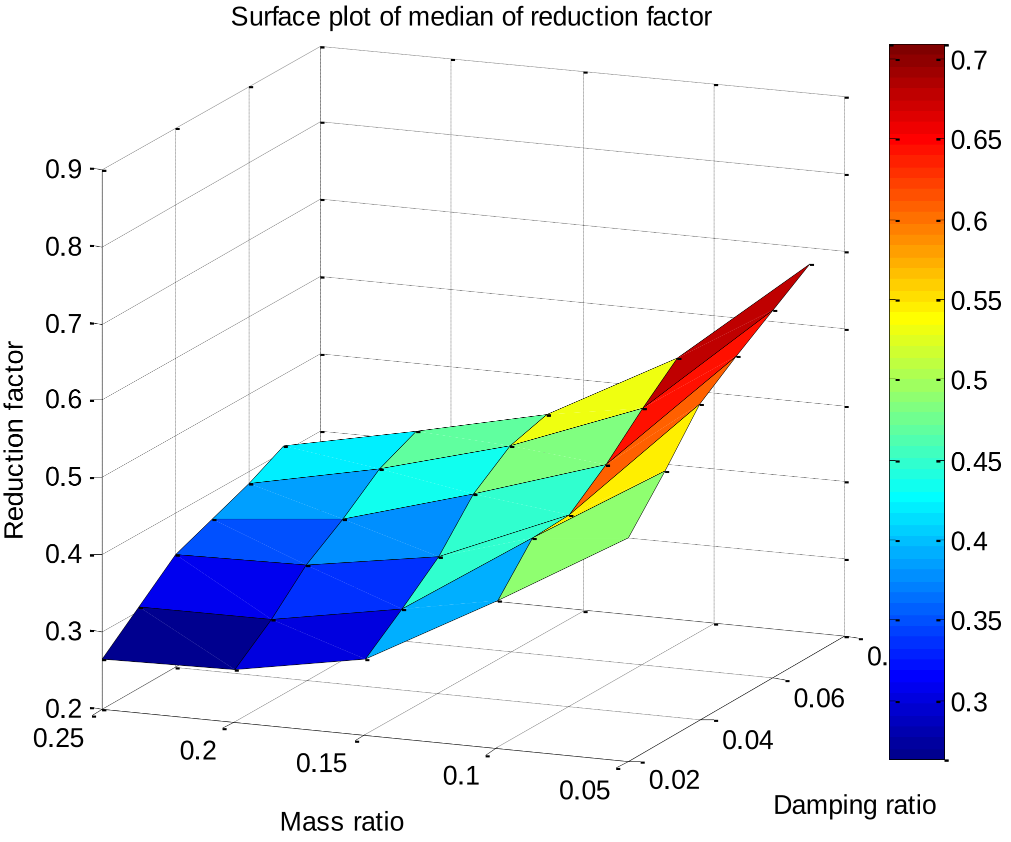


Fig. 42. Variation of Median of Reduction factor with mass ratio and damping ratio for a five degree of freedom primary system, oscillator connected at top

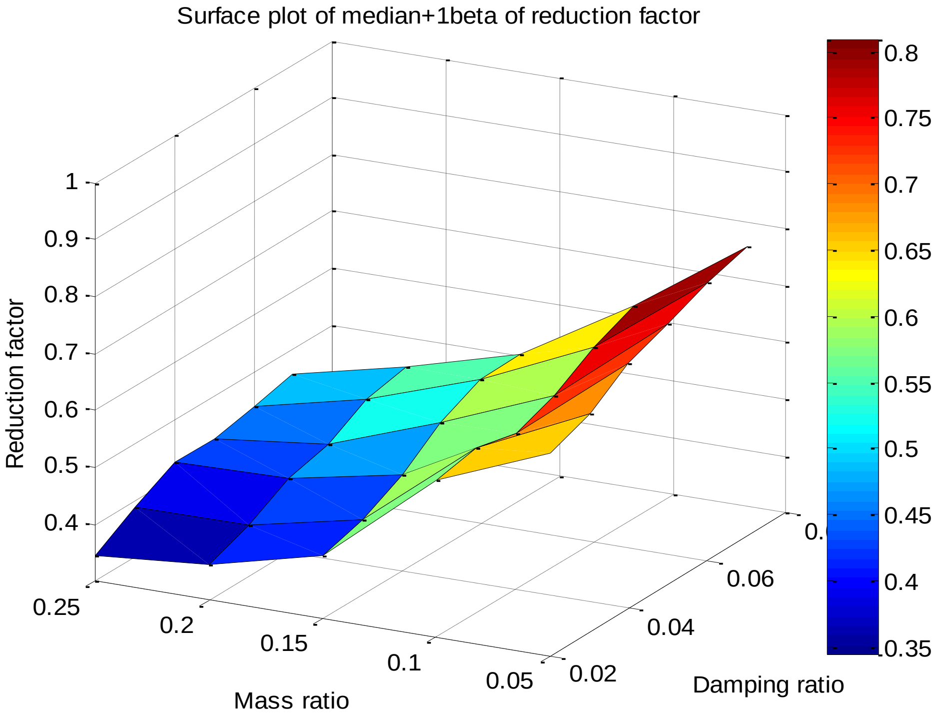


Fig. 43. Variation of Median + 1β Reduction factor with mass ratio and damping ratio for a five degree of freedom primary system, oscillator connected at top

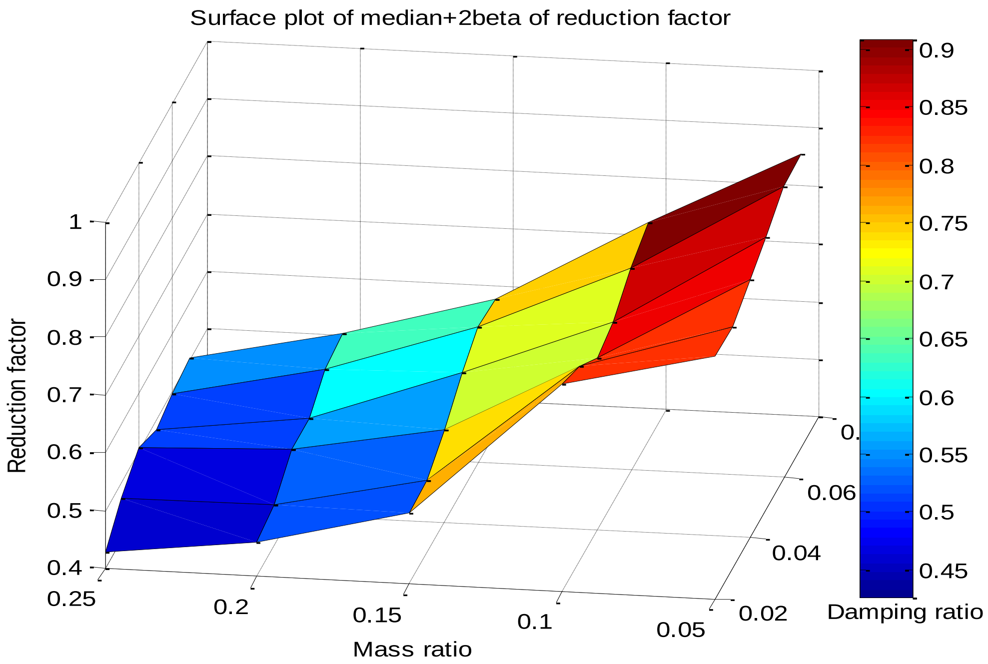


Fig. 44. Variation of Median + 2β Reduction factor with mass ratio and damping ratio for a five degree of freedom primary system, oscillator connected at top

PART IV: A PRA-CONSISTENT FRAMEWORK FOR SEIMIC QUALIFICATION OF EQUIPMENT

1. Introduction

The seismic performance of an equipment in a power plant, such as nuclear and electrical, is governed by the qualification requirements of IEEE standards [1, 2]. To ensure the safe shut down of the plant during or after an earthquake, the equipment needs to be qualified either by testing or dynamic analysis [1, 2, 3].

The seismic qualification of an equipment based on testing requires the equipment to continue to function when subjected to a specific test response spectrum (TRS). Broad banded ground motions are found to cause more damage to equipment than the filtered narrow banded excitations. Experimental observations by Merz [4, 5] indicate that broad band spectra produce greater relay chatter and structural damage as compared to narrowband spectra. Studies [4, 6, 7] suggest that narrow frequency high spectral peaks of the required response spectrum (RRS) need to be scaled down to produce a clipped RRS. As a result, the definition of acceleration capacity used in the fragility models use clipped response spectra for both test response spectrum (TRS) and required response spectrum (RRS). EPRI [3] recommends different factors for equipment response and capacity that are used to compute fragility of equipment. The clipping factor is one such factor that is applied to narrowband response spectra to transform to equivalent broadband spectra. The transformed broadband spectra are used for equipment qualification purposes.

Kana [8] shows that the amplification for specific equipment can be quantified by comparing the base excitation spectra with the response spectra at elevated locations that have been generated analytically or measured under test conditions. The study discusses the development of appropriate dynamic amplification for devices mounted in equipment based on the root-mean-square (RMS) severity changes with bandwidth, multimode interaction and multi-axis excitation. A narrow banded input is judged to be less severe from a fragility point of view due to the absence of multi-mode response, variable RMS severity over the bandwidth of the spectra, and the lack of interaction of nonlinear responses. The effect of modal interaction in case of narrowband excitation can be incorporated by a study of the fragility response of the equipment at critical frequency for broadband and narrowband input motions [9].

As per EPRI [3], the clipping factor for narrowband response spectra is a combined effect of broadband correction factor (C_B) and modal interaction correction factor (C_{MI}) that incorporates the high RMS severity ratio and modal interaction respectively. Currently, the site-specific probabilistic seismic hazard analysis (PSHA) of nuclear power plants are conducted as per these EPRI guidelines [10]. However, these factors are developed for input motions that are combinations of one or more harmonic type excitations. The present paper reviews the development of the clipping factor used in current practice. The underlying principles are then applied for the case of real earthquakes to calculate the corresponding correction factors for actual ground motions and evaluate how they compare with the values recommended by EPRI [3]. Such a comparison forms a basis in understanding the need for improving the current recommendations and develop a risk-consistent framework for evaluating the clipped response spectra needed in fragility calculations of the equipment being qualified.

2. Requirement for Clipping of Response Spectra for Equipment Qualification

Typically, power plant equipment are mounted on supporting structures. In case of heavier equipment such as cabinets and transformers, they are located directly on a floor level. The lighter sensitive devices such as relays and switches are mounted inside the electrical cabinets. Due to filtering of input motions through the different structures and systems, broadband excitations become more and more narrow banded in nature. Therefore, the RRS for an equipment is usually narrow banded. Fig. 1 shows the filtering of ground motion through structures at different levels in a typical nuclear power plant. As mentioned before, studies show that broadband spectra are more critical in estimating the performance of equipment [4, 5, 6, 7].

The equipment demand and capacity for the fragility analysis are computed in terms of RRS and TRS respectively. During testing or analysis, equipment are qualified with respect to broadband test response spectra (TRS). This ensures that the equipment is qualified for a wide range of excitation frequencies. So, the TRS is typically broad banded and does not need to be clipped for qualification purpose. The filtered RRS for the equipment needs to be clipped by applying appropriate clipping factors. The determination of the degree of clipping depends on the center frequency as well as the bandwidth of the response spectrum of the equipment being qualified.

Mathematically,

$$S_{a,\ broadband} = C_c * S_{a,\ narrowband} \tag{1}$$

where, Cc is the desired clipping factor.

The clipping factor is a probabilistic estimate for a combination of two factors: broadband correction factor (C_B) and modal interaction correction factor (C_{MI}). The broadband correction factor, C_B, accounts for the effect of high RMS value in case of broadband ground motion. The modal interaction correction factor, C_{MI}, considers the effect of interaction between different modes on the response of the equipment when it is subjected to broadband excitation.

An understanding of the overall clipping factor therefore requires an in depth discussion on the development of C_B and C_{MI}. The following section provides a detailed discussion on broadband correction factor.

3. Development of Broadband Correction Factor, C_B, for Equipment

Kana [8] defines C_B as a comparative estimate between the RMS severity ratio of a broadband and a narrowband spectra. The RMS severity ratio for a response spectra is defined as:

$$Severity\ Ratio = \frac{A_{RMS}}{S_{a,peak}} = \frac{A_{RMS}}{PGA} * \frac{PGA}{S_{a,peak}}$$

$$= \frac{A_{RMS}}{PGA} * Maximum\ Response\ Factor \tag{2}$$

where, A_{RMS} is the RMS value for the input motion, PGA is the peak ground acceleration for the input motion and $S_{a,peak}$ is the peak value of the corresponding response spectrum for the input motion. The RMS value of any time history, $f(t)$, is calculated as follows:

$$f_{\mathrm{rms}} = \lim_{T\to\infty} \sqrt{\frac{1}{T}\int_0^T [f(t)]^2\, dt}. \tag{3}$$

The severity ratio is a function of the maximum response factor for the ground motion. The latter can be calculated for a variety of excitations as discussed below:

3.1 Maximum Response Factor for Harmonic Ground Excitation

The maximum response factor in case of harmonic ground acceleration is given by the peak value of the transmissibility ratio. At resonance (frequency ratio = 1), the transmissibility ratio = $1/2\xi$ where ξ is the damping ratio for the single degree of freedom under consideration. Fig. 2 shows the plot of the transmissibility ratio for harmonic excitations as functions of different damping ratios.

3.2 Maximum Response Factor for Sine Beat Type Excitation

Kana [8] characterizes the input motions by a combination of sine beat type motions. For such motions, the response can be calculated analytically from the equation of motion:

$$m\ddot{u} + c\dot{u} + ku = -m * [cos2\pi f_1 t + cos2\pi f_2 t]$$

$or,$

$$\ddot{u} + {}^{c}/_{m}\,\dot{u} + {}^{k}/_{m}\,u = -[cos2\pi f_1 t + cos2\pi f_2 t]$$

where, f_1 and f_2 are the excitation frequencies of the harmonic motions that comprise the sine beat motion.

The displacement time history at the degree of freedom is:

$$u(t) = -\frac{m}{\omega^2}\left[\begin{array}{l}\frac{1}{\sqrt{((1-\beta_1^2)^2+(2\xi\beta_1)^2)}}\cos\left(2\pi f_1 t - tan^{-1}\left(\frac{2\xi\beta_1}{1-\beta_1^2}\right)\right)\\ +\frac{1}{\sqrt{((1-\beta_2^2)^2+(2\xi\beta_2)^2)}}\cos\left(2\pi f_2 t - tan^{-1}\left(\frac{2\xi\beta_2}{1-\beta_2^2}\right)\right)\end{array}\right] \tag{4}$$

where, $\beta_1 = f_1/f$ and $\beta_2 = f_2/f$; $\omega = 2\pi f$ is the natural frequency of the oscillator.

The peak spectral response is given by peak value of $\omega^2 * \mathrm{u}_{\mathrm{max}}$. The maximum response factor is plotted for different types of excitations including actual earthquakes and is shown in Fig. 3. As can be observed from the figure, the maximum response factors in case of actual earthquake time histories are much lower than that for sine-beat type harmonic excitation. This is because the maximum response factor in case of sine beat type motions corresponds to the steady state response for tuned or nearly tuned oscillators. In case of actual earthquakes, the input motions comprise of a wide range of frequencies which are not exactly tuned to the frequencies of the oscillators. Even if the frequencies are tuned or closely tuned, the duration of the pulse is very small. As a result the complete steady state response does not develop in case of real earthquakes.

The bandwidth and center frequency of the input motion play a pivotal role in the dynamic response of the structure. The plots of maximum response factor as a function of bandwidth and *B* (*B* = ratio of bandwidth to center frequency of the input motion) are shown in Figs. 4 and 5.

The input ground motions for calculating the maximum response factors for single degree of freedom systems are generated using combinations of multiple harmonic excitations such that the generated input motions have specific center frequencies (5Hz, 10Hz etc.).

After the maximum response factors are calculated for different excitations, the severity ratio is calculated as discussed in the next section.

3.3 Severity Ratios for Different Ground Motions

The RMS severity ratio is calculated as a product of the maximum response factor for the input motion and the ratio of RMS to peak ground acceleration for the motion. In case of harmonic excitation with 1g amplitude, the ratio of the peak to RMS value is equal to $\sqrt{2}$. In case of Gaussian random motion, this value is nearly equal to 3. For the calculation of RMS severity ratio that defines the broadband correction factor, artificial Gaussian random motions are used for analysis. These motions are artificially generated using combinations of multiple sine beat type motions.

The severity ratios are calculated for the excitations with center frequencies 5Hz and 10Hz by dividing the mean value of the maximum response factor by $\sqrt{2}$ for harmonic excitations and a value equal to 3 for artificially generated random motions. Fig. 6 shows the plot of severity ratio as a function of *B*. EPRI [3] defines the bandwidth of a response spectrum as the range of frequency that contains 80% of the peak spectral response. Fig. 7 shows a typical response spectrum along with its bandwidth and center frequency as per this definition.

The procedure discussed in detail above can be used to evaluate C_B for real earthquakes. The next section discusses the result obtained by doing so and compares the C_B values, evaluated for real earthquakes, to those evaluated above using harmonic excitations.

3.4 Broadband Correction for Real Earthquakes

The primary objective of this research is to develop a PRA (Probabilistic Risk Assessment) consistent framework that incorporates the requirements of PSHA for particular sites of interest. For exploration and illustration purposes, a representative location of interest is chosen as the City of Oaks, Los Angeles, CA (longitude: 117.856W, latitude: 34.102). The corresponding Uniform Hazard Spectrum (UHS) with 2% probability of exceedance (P.E.) in 50 years is selected from the USGS database (http://geohazards.usgs.gov/hazardtool/application.php).

Twenty one earthquakes are chosen for the location, which are consistent with the PSHA requirements for the site. These earthquakes are chosen such that the median response spectrum for the unscaled ground motions is similar to the corresponding predicted UHS. Fig. 8 shows the UHS as per USGS database and unscaled median spectrum for the 21 earthquakes on the same plot.

The maximum response factor and the corresponding RMS severity ratio are calculated for these 21 earthquakes and their mean and median design spectra. Figs. 9, 10 and 11 show plots for these earthquakes with PGA equal to 1g that correspond to Figs. 4 through 6.

The severity ratio for the different motions as per EPRI [3] recommendations can be plotted on the same plot as the actual earthquake results. Figs 12 and 13 show the comparison of the values between harmonic type excitations and actual earthquake motions.

Kana [8] defies for the bandwidth correction factor C_B as:

$$C_B = \frac{Severity\ Ratio_{narrowband\ spectrum}}{Severity\ Ratio_{broadband\ spectrum}} \tag{5}$$

When the equipment is qualified with respect to a RG 1.60 spectrum excitation, the RMS severity ratio for such broadband excitation is estimated, from the EPRI recommended plot for a B_{RG} equal to 1.5, as 0.135. However, in case of real earthquakes that are consistent with requirements of the PSHA for the location of interest, the equipment has to be qualified w.r.t. to the corresponding UHS. Fig. 14 compares the estimated C_B values for the real earthquakes consistent with the representative location of interest and the recommended lower bound (unconservative) estimate, C_B' [3]:

$$C_B' = 0.32 + 0.4B \qquad 0.15 \leq B \leq 0.9 \tag{6}$$

where, B = Bandwidth/Center Frequency of the spectrum.

4. Development of Modal Interaction Correction Factor, C_{MI}, for Equipment

Studies suggest that broadband excitations are usually applicable when the performance of the equipment is affected by interacting modes [3, 9]. Kana [9] recommends a modal interaction correction factor to transform a narrowband response to an equivalent broadband spectrum. The narrowband fragility function envelope, for which no modal interaction has occurred, needs to be clipped to transform into an equivalent broadband input for which modal interaction takes place. The fragility response of a simple two degree-of-freedom oscillator is studied for representative narrowband and broadband excitations. This model is representative of different practical physical systems, depending on the values of stiffness, mass and damping involved. Such model can be used to represent electrical cabinet mounted on heavier supporting structures or a relay mounted on cabinet.

4.1 Effect of Modal Interaction for Narrowband Harmonic Excitation

The two degree-of-freedom system is subjected to broadband and narrowband excitations and the corresponding Fragility Response Spectrum (FRS) is developed. The failure levels are defined in terms of the peak acceleration response of each of the two masses. This type of failure is relevant to acceleration sensitive devices located at some elevated position of the equipment structure.

In case of broadband excitations, the input motion is scaled to 1g PGA and the peak responses at the two degrees of freedom are calculated. The FRS is obtained by determining the appropriate PGA level that produces a 1g peak response at each of the masses. For the purpose of the study, Kana [9] uses R.G. 1.60 as the broadband spectrum. Artificial time history is generated that has a response spectrum that matches the R.G. 1.60 horizontal spectrum for 5% damping and 1g PGA.

In case of narrowband input motion, a harmonic excitation is used that has maximum amplitude equal to 1. The magnitude of the transfer function for each of the masses is inverted, which gives the input level required to produce 1g output at the masses. This input level is converted to a spectral value and the resulting narrowband FRS is equivalent to the response spectrum obtained from the envelope of peak spectral values from a slowly-swept sine-wave or independently-applied discrete sine-wave excitation, whose amplitude produces 1g peak response at the degrees of freedom [9].

The values from the FRS are used to calculate the modal interaction correction factor at each mass point. For the narrowband calculations, the minimum value of the FRS is noted along with the corresponding critical frequency. Then the PGA level of an equivalent broadband spectrum which has identical spectral value at the critical frequency is calculated. Finally, a numerical ratio is determined between the PGA level of the actual broadband FRS and the equivalent broadband FRS based on narrowband results. Figs. 15 and 16 show the C_{MI} values for different frequency ratios of the two degree-of-freedom oscillator at the two mass points for different mass ratios. The primary supporting structure has a weight of 1000 lbs and the damping ratio of the system is 5%.

The results from the study suggest that the use of a universal correction factor of 0.7 can be considered to be conservative for most cases studied and the minimum value occurs when the two masses are exactly tuned and there is failure at the primary degree-of-freedom.

Based on the above study, the current recommendations [3] suggest that the C_{MI} lies within the range of 0.85 to 1.0. The following equation is suggested by EPRI [3] standards as a lower bound for C_{MI}:

$$C_{MI} = 0.39 + 1.4B \leq 1.0 \tag{7}$$

where, B = Bandwidth/Center Frequency of the spectrum.

4.2 Effect of Modal Interaction for Real Earthquakes

The modal interaction correction factor is calculated for the 21 real earthquakes that are consistent with the PSHA requirements for the location of interest. The corresponding median spectrum is used to generate the corresponding broadband excitation. The main purpose is to identify the absence or presence of modal interaction for the narrowband floor spectra and incabinet spectra due to filtering of ground motion through the system. Fig. 17 shows the C_{MI} for the narrowband excitations and compares them to the code recommended lower bound values of C_{MI} as a function of B, as given by equation (7).

As can be observed from the figure, most of the values are greater than 1. This indicates that in case of real earthquakes there is interaction of modes even in case of filtered narrowband spectra at the floor and equipment levels. Under such circumstances, in case of real earthquakes the modal interaction correction factor may not be applicable. The broadband correction factor can be representative of the clipping required to transform a narrowband spectrum to a more damage causing broadband spectrum. The next section of the paper discusses about the overall clipping factor, C_C, as a combination of C_B and C_{MI}, in case of real earthquakes and compares them with the code recommended values.

5. Clipping Factor for Narrowband Response Spectra

The narrowband RRS of an equipment is clipped to an effective broad-frequency RRS_C by applying the broadband correction and the modal interaction correction factors, as described in sections 3 and 4 respectively.

Mathematically,

$$C_C = C_B * C_{MI} \tag{8}$$

However, the calculation of both the factors consists of responses at the two mass points for a coupled two degree-of-freedom system. This may result in double-counting of the benefit of a narrow bandwidth [3]. In order to avoid this, EPRI [3] gives the following conservative estimate of C_C. This definition is based on the Conservative Deterministic Failure Margin (CDFM) approach [11] that considers High Confidence Low Probability of Failure (HCLPF) analysis for the estimation of C_B and C_{MI}. This philosophy defines the demand at about the 84% Non-exceedance Probability (NEP) (i.e. about $+0.5\beta$ level). The CDFM clipping factors are as follows:

$$\begin{aligned} C_C &= 0.55 && B \leq 0.2 \\ C_C &= 0.4 + 0.75B && 0.2 \leq B \leq 0.8 \\ C_C &= 1.0 && B \geq 0.8 \end{aligned} \tag{9}$$

where, B = Bandwidth/Center Frequency of the spectrum.

For the purpose of fragility analysis the EPRI recommends median values for C_C and the logarithmic standard deviations for uncertainty. Assuming a 99% NEP (i.e. β=0.01), these equations are developed assuming the following constraints:

- CDFM values of C_C are conservative and should be at or above the 0.5β level.
- $C_C=C_B*C_{MI}$ gives an unconservative estimate of the clipping factor and should be approximately at the -2β level.
- At +2β level the clipping factor should be less than or equal to 1.0.

The median values and logarithmic standard deviation for the clipping factor as recommended by EPRI [3] are given below.

$$Median\,Value: \hat{C}_c = 0.30 + 0.86B \qquad B \le 0.4$$

$$\hat{C}_c = 0.50 + 0.36B \qquad B > 0.4$$

$$Logarithmic\,Standard\,Deviation: \beta_u = 0.37 - 0.5B \qquad B \le 0.4$$

$$\beta_u = 0.24 - 0.17B \qquad B > 0.4 \tag{10}$$

where, B = Bandwidth/Center Frequency of the spectrum.

The following section considers three case studies for a tuned two degree-of-freedom system, representing the supporting structure and the equipment, to calculate the clipping factor from C_B and C_{MI} for real earthquakes. The mass ratio of the system is considered as 0.01 and the frequency ratio is 1. In the case studies the frequency of the primary system is considered as 10Hz, 5Hz and 2Hz respectively. Finally the clipping factors are compared to the existing code recommended values.

6. Comparative Study of the Combined Clipping Factor for Structures Subjected to Real Earthquakes

6.1 Case 1: Supporting Structure Frequency = 10 Hz

The broadband correction and the modal interaction correction factors are calculated following the EPRI [3] definitions for a tuned system. The weight of the primary system (for example, the supporting structure), W_1 = 1000 lbs and that of the secondary system (for example, the equipment), W_2 = 10 lbs. The system is subjected to the 21 earthquakes that are consistent with the PSHA requirements for the location of interest and C_B and C_{MI} are estimated with respect to the broad banded median design spectrum. Tables 1 and 2 show the values of C_B, C_{MI} for this case study for masses 1 and 2 respectively. As can be observed from the tables, some of the C_{MI} values are greater than 1. For those cases, C_{MI} is taken as 1. The corresponding C_B*C_{MI} and recommended CDFM and median C_C values are also shown in the tables.

The C_B*C_{MI} values for the floor and incabinet spectra are plotted in Fig.18 and compared with the corresponding code recommended median values of C_C.

6.2 Case 2: Supporting Structure Frequency = 5 Hz

The mass ratio and the frequency ratio of the two degree-of-freedom are considered the same as in case 1, i.e. 0.01 and 1 respectively. However, the frequency of the supporting structure is changed to 5 Hz. Tables 3 and 4 show the values of C_B, C_{MI} for this case study. As can be observed from the table the majority of the values of $\mathrm{C_{MI}}$ are greater than 1. This indicates that in case of real earthquakes, the input motion is not as narrow banded as in case of harmonic excitations. Therefore, there is substantial modal interaction for the RRS as well. The C_{MI} values greater than 1 are assumed to be 1 for the calculation of C_C. The corresponding C_B*C_{MI} and recommended CDFM and median C_C values are also shown in the tables.

The C_B*C_{MI} values for the floor and incabinet spectra are plotted in Fig.19 and compared with the corresponding code recommended median values of C_C.

6.3 Case 3: Supporting Structure Frequency = 2 Hz

The frequency of the supporting structure is changed to 2 Hz. Table 5 and 6 show the values of C_B, C_{MI} for this case study. Similar to the previous case, the majority of the values of C_{MI} are greater than 1. The C_{MI} values greater than 1 are assumed to be 1 for the calculation of C_C. The corresponding C_B*C_{MI} and recommended CDFM and median C_C values are also shown in the tables.

The C_B*C_{MI} values for the floor and incabinet spectra are plotted in Fig. 20 and compared with the corresponding code recommended median values of C_C.

7. Discussion of Results

The observations from the results can be summarized as follows:

A comparison of Figs. 4, 5 and 6 with Figs. 9, 10 and 11 can be summarized as follows:

- No distinct pattern similar to Gaussian random motions is visible in case of actual earthquake time histories.
- The results from artificial excitations generated from a combination of harmonic type excitations are not representative of the dynamic behavior of systems subjected to real earthquakes.

The observations from Figs. 12, 13 and 14 are:

- Real earthquakes have much lower severity ratios.
- As mentioned in section 3.2, the EPRI [3] reference values for maximum response factors corresponding to sine beat type motions are conservative compared to actual earthquakes.
- The recommended lower bound value of C_B, as given by equation (6), are calculated with respect to R.G. 1.60 spectra which has a severity ratio equal to 0.135. However, in case of real earthquakes, the severity ratio and bandwidth of the corresponding broadband spectra would be different from R.G. 1.60.
- The broadband correction factor for actual ground motion as observed in case of a seismic event is lower;

Fig. 17 shows that:

- EPRI [3] recommended values of C_{MI} as a function of B, as given by equation (7), increases linearly from around 0.4 and should be less than or equal to 1 for tuned two degree-of-freedom systems.
- In case of real earthquakes, this value is usually greater than 1.
- There is multi-mode interaction for the filtered narrow banded response spectra in case of actual earthquake time histories as discussed in section 4.2.
- The assumption of absence of modal interaction for narrowband response spectra may be relaxed in case of real earthquakes.

The results from Figs. 18, 19, 20 and tables 1 through 6 can be summarized as follows:

- Clipping factors for the floor and incabinet spectra for real earthquakes as calculated by C_B*C_{MI} are lower than the code recommended median C_C.
- In case of real earthquakes modal interaction is present. So C_{MI} in most cases is equal to 1.
- The broadband correction factor plays the major role in clipping of narrow band filtered response spectra for real earthquake time histories.
- For the case of tuned system with natural frequency equal to 2 Hz, some of the calculated values of clipping factor are higher than the recommended values given by equation (10). But most of the values are still lower than the median C_C values. Therefore, the above observations recommends that:

- For real earthquakes, the narrowband response spectra needs to be clipped to an equivalent broadband spectra due to the variable RMS severity ratio only.
- Multi-mode interaction is usually present in case of equipment responses when the equipment is subjected to actual earthquakes.

The overall clipping as recommended in current practice using Fragility Analysis can be conservative. This necessitates the development of a probabilistic framework after considering real earthquakes, as is discussed in the present work.

8. Summary and Conclusions

The present work makes an attempt to study the existing recommendations for the qualification of equipment under seismic loading based on their acceleration capacity. EPRI [3] recommends the median acceleration capacity of equipment and structures in terms of the TRS and RRS, based on testing and analysis results on such systems. In case of narrowband spectra, these response spectra need to be clipped to equivalent broadband spectra that would incorporate a wider frequency range. Such broadband spectra cause more relay chatter and damage in equipment. The clipping of the spectra considers the effect of variable RMS severity and interaction between multiple modes in case of broad frequency bandwidth of the corresponding ground motion. However, the estimation of the clipping factors based on the bandwidth and center frequency of the spectra, as recommended [3], correspond to harmonic type input motions which are extremely narrow banded in frequency content. In case of actual earthquake ground motion there is considerable inter-mode interactions and the RMS severity of such input motions is lower than sine beat type motion.

21 earthquakes that are consistent with site specific PSHA requirements are used to study the effect of broadband correction and modal interaction correction in case of floor and incabinet spectra due to filtering of ground motion at different levels of the structure and equipment. The observations from this study can be summarized as follows:

- Real earthquakes show random scattering of points unlike harmonic motions.
- Results from harmonic type motions are not representative of the behavior of actual earthquakes.
- Severity ratio for real earthquakes are over estimated if we use the EPRI [3] recommendations directly. Therefore, broadband correction factors are much lower as compared to harmonic type excitations.
- In case of real earthquakes, the equipment response includes interaction between different modes. Therefore, for all practical purposes, modal interaction correction may not be necessarily applicable to clipping of narrowband response spectra for seismic qualification of equipment.
- The combined effect of broadband and modal interaction correction for narrowband spectra gives lower overall value for the clipping factor in case of real earthquakes than the code recommended corresponding median values based on 1% NEP.
- The code recommends that the full coupling of C_B and C_{MI} gives an unconservative estimate of clipping factor. However, the present study reveals that in case of calculations based on actual earthquakes, the estimated clipping based on full coupling are lower than the median clipping factor values for a particular B.

- This necessitates the use of real earthquakes to develop a more probabilistic framework for seismic qualification of equipment.

The present paper, therefore, discusses the excessive conservatism in the clipping of amplified responses for equipment as per current recommendations which results in high seismic risk assessments. The observations from the study demonstrate the necessity of assessing the risk for an equipment/structure based on actual ground motions and development of a PRA consistent framework for seismic qualification of equipment.

REFERENCES

[1] IEEE 693, 2005. “IEEE recommended practice for seismic design of substations”.

[2] IEEE 344, 2004. “IEEE recommended practice for seismic qualification of class 1E equipment for nuclear power generating stations”.

[3] EPR1, 1994. “Methodology for developing seismic fragilities.” TR-103959.

[4] Merz, K. L. 1991. “Generic seismic ruggedness of power plant equipment, Rev. 1.” Palo Alto, CA, Electric Power Research Institute, NP-5223.

[5] Merz, K. L. 1991. “Seismic ruggedness of relays.” Palo Alto, CA, Electric Power Research Institute, NP-7147.

[6] Kana, D. D., Pomerening, D. J. 1983. “Correlation of methodologies for seismic qualification test of nuclear plant equipment.” SWRI-6582-002, prepared for U.S. Nuclear Regulatory Commission by Southwest Research Institute, San Antonio, Texas.

[7] Kana, D. D. 1989. “Use of amplification factors for developing equipment base seismic capacity for relays.” Letter report to P. Y. Chen, U. S. Nuclear Regulatory Commission.

[8] Kana, D. D., Chen, P. Y. 1991. “Dynamic amplification factors for seismic evaluation of devices mounted in equipment.” Trans. 11th International Conference on Structural Mechanics in Reactor Technology, K28/5, 337-342.

[9] Kana, D. D., Pomerening, D. J. 1987. “Determination of Modal Interaction Correction for Narrowband Fragility Data.” Transactions of the 9th International Conference on Structural Mechanics in Reactor Technology, JK Panel Session (NUREG/CR-0088), 181-205.

[10] Hwang, K., Tong, W. H., Dar, A. 2013. “Bruce B seismic probabilistic risk assessment.” Trans. 22nd Conference on Structural Mechanics in Reactor Technology, San Francisco, CA, USA, Division IX.

[11] EPRI, 1991. “A Methodology for Assessment of Nuclear Power Plant Seismic Margin.” Palo Alto, California, prepared by Jack R. Benjamin and Associates, Inc., et al., NP-6041, Revision 1.

[12] Kaplan, S., Garrick, B. J., 1981, “On the Quantitative Definition of Risk”, Risk Analysis, 1, 11-27.

[13] Field, E. H., “Probabilistic Seismic Hazard Analysis (PSHA) A Primer.”

[14] Tadinada, S. K., 2012, “A Bayesian Framework for Probabilistic Seismic Fragility Assessment of Structures”, Doctoral Thesis, North Carolina State University.

[15] Kennedy, R. P., 2007, “Performance-Goal Based (Risk Informed) Approach for establishing the SSE Site Specific Response Spectrum for Future Nuclear Power Plants”, Jaeger Lecture, SMiRT-19, Toronto, Canada.

[16] http://geohazards.usgs.gov/hazardtool/application.php

Table 1: Comparison of Correction Factors for 2 dof system Subjected to Real Earthquakes and EPRI [3] Recommended Values for Mass 1, Primary system frequency = 10Hz

B	CB	Recommended CB	CMI	Modified CMI	Recommended CMI	CB*CMI	Recommended Cc (CDFM)	Recommended Median Cc
0.028	0.12		0.76	0.76	0.43	0.09	0.55	0.32
0.057	0.18		0.81	0.81	0.47	0.14	0.55	0.35
0.070	0.16		0.58	0.58	0.49	0.09	0.55	0.36
0.124	0.28		1.19	1.00	0.56	0.28	0.55	0.41
0.124	0.12		0.73	0.73	0.56	0.09	0.55	0.41
0.130	0.17		1.06	1.00	0.57	0.17	0.55	0.41
0.197	0.14	0.40	1.13	1.00	0.67	0.14	0.55	0.47
0.160	0.23	0.38	1.26	1.00	0.61	0.23	0.55	0.44
0.162	0.20	0.38	0.77	0.77	0.62	0.15	0.55	0.44
0.195	0.32	0.40	0.84	0.84	0.66	0.27	0.55	0.47
0.221	0.37	0.41	0.85	0.85	0.70	0.31	0.57	0.49
0.204	0.34	0.40	0.68	0.68	0.68	0.23	0.55	0.48
0.206	0.23	0.40	0.94	0.94	0.68	0.22	0.55	0.48
0.200	0.20	0.40	0.64	0.64	0.67	0.13	0.55	0.47
0.204	0.22	0.40	1.27	1.00	0.68	0.22	0.55	0.48
0.199	0.25	0.40	0.70	0.70	0.67	0.18	0.55	0.47
0.200	0.39	0.40	0.80	0.80	0.67	0.31	0.55	0.47
0.206	0.33	0.40	0.93	0.93	0.68	0.31	0.55	0.48
0.251	0.22	0.42	0.72	0.72	0.74	0.16	0.59	0.52
0.263	0.12	0.43	0.77	0.77	0.76	0.09	0.60	0.59

Table 2: Comparison of Correction Factors for 2 dof system Subjected to Real Earthquakes and EPRI [3] Recommended Values for Mass 2, Primary system frequency = 10Hz

B	**CB**	**Recommended CB**	**CMI**	**Modified CMI**	**Recommended CMI**	**CB*CMI**	**Recommended Cc (CDFM)**	**Recommended Median Cc**
0.016	0.134	NA	1.268	1.000	0.413	0.134	0.550	0.314
0.028	0.118	NA	0.680	0.680	0.430	0.080	0.550	0.325
0.029	0.140	NA	0.623	0.623	0.431	0.087	0.550	0.325
0.040	0.228	NA	0.708	0.708	0.447	0.161	0.550	0.335
0.056	0.104	NA	0.972	0.972	0.469	0.101	0.550	0.348
0.065	0.181	NA	0.871	0.871	0.481	0.157	0.550	0.356
0.070	0.105	NA	0.960	0.960	0.488	0.100	0.550	0.360
0.070	0.276	NA	0.552	0.552	0.488	0.153	0.550	0.360
0.070	0.217	NA	1.009	1.000	0.488	0.217	0.550	0.360
0.071	0.148	NA	0.873	0.873	0.489	0.129	0.550	0.361
0.071	0.139	NA	0.913	0.913	0.489	0.127	0.550	0.361
0.072	0.149	NA	0.697	0.697	0.491	0.104	0.550	0.362
0.096	0.146	NA	0.830	0.830	0.525	0.121	0.550	0.383
0.112	0.278	NA	0.767	0.767	0.546	0.213	0.550	0.396
0.114	0.084	NA	0.603	0.603	0.550	0.051	0.550	0.398
0.116	0.111	NA	1.092	1.000	0.553	0.111	0.550	0.400
0.123	0.146	NA	0.699	0.699	0.563	0.102	0.550	0.406
0.126	0.211	NA	0.985	0.985	0.566	0.208	0.550	0.408
0.131	0.198	NA	0.508	0.508	0.573	0.101	0.550	0.412
0.143	0.120	NA	0.857	0.857	0.590	0.103	0.550	0.423
0.167	0.327	0.387	0.745	0.745	0.624	0.244	0.550	0.444

Table 3: Comparison of Correction Factors for 2 dof system Subjected to Real Earthquakes and EPRI [3] Recommended Values for Mass 1, Primary system frequency = 5Hz

B	CB	Recommended CB	CMI	Modified CMI	Recommended CMI	CB*CMI	Recommended Cc (CDFM)	Recommended Median Cc
0.0267	0.2252	NA	1.3431	1.0000	0.4273	0.2252	0.5500	0.3229
0.0455	0.1828	NA	1.5765	1.0000	0.4536	0.1828	0.5500	0.3391
0.0556	0.1935	NA	1.4407	1.0000	0.4678	0.1935	0.5500	0.3478
0.0727	0.2783	NA	1.5623	1.0000	0.4918	0.2783	0.5500	0.3625
0.0889	0.1569	NA	1.6073	1.0000	0.5144	0.1569	0.5500	0.3764
0.0909	0.4403	NA	1.7471	1.0000	0.5173	0.4403	0.5500	0.3782
0.0909	0.2047	NA	2.0727	1.0000	0.5173	0.2047	0.5500	0.3782
0.1000	0.3400	NA	2.1565	1.0000	0.5300	0.3400	0.5500	0.3860
0.1000	0.2296	NA	1.9394	1.0000	0.5300	0.2296	0.5500	0.3860
0.1111	0.2600	NA	1.6587	1.0000	0.5456	0.2600	0.5500	0.3956
0.1111	0.2506	NA	1.6492	1.0000	0.5456	0.2506	0.5500	0.3956
0.2000	0.4383	0.4000	2.2359	1.0000	0.6700	0.4383	0.5500	0.4720
0.2000	0.2062	0.4000	2.0856	1.0000	0.6700	0.2062	0.5500	0.4720
0.2000	0.1968	0.4000	1.9916	1.0000	0.6700	0.1968	0.5500	0.4720
0.2000	0.2711	0.4000	2.1468	1.0000	0.6700	0.2711	0.5500	0.4720
0.2791	0.2855	0.4316	1.3269	1.0000	0.7807	0.2855	0.6093	0.5400
0.2857	0.5044	0.4343	1.4401	1.0000	0.7900	0.5044	0.6143	0.5457
0.2963	0.1877	0.4385	1.4939	1.0000	0.8048	0.1877	0.6222	0.5548
0.5000	0.2177	0.5200	1.7820	1.0000	1.0000	0.2177	0.7750	0.6800
0.5017	0.1517	0.5207	1.8733	1.0000	1.0000	0.1517	0.7763	0.6806
0.6250	0.2403	0.5700	1.3984	1.0000	1.0000	0.2403	0.8688	0.7250

Table 4: Comparison of Correction Factors for 2 dof system Subjected to Real Earthquakes and EPRI [3] Recommended Values for Mass 2, Primary system frequency = 5Hz

B	**CB**	**Recommended CB**	**CMI**	**Modified CMI**	**Recommended CMI**	**CB*CMI**	**Recommended Cc (CDFM)**	**Recommended Median Cc**
0.095	1.327	1.000	0.523	0.163		0.163	0.550	0.382
0.098	1.992	1.000	0.527	0.172		0.172	0.550	0.384
0.100	1.659	1.000	0.530	0.218		0.218	0.550	0.386
0.100	1.343	1.000	0.530	0.237		0.237	0.550	0.386
0.167	2.236	1.000	0.623	0.196	0.387	0.196	0.550	0.443
0.167	1.441	1.000	0.623	0.269	0.387	0.269	0.550	0.443
0.182	1.494	1.000	0.645	0.179	0.393	0.179	0.550	0.456
0.205	1.607	1.000	0.677	0.205	0.402	0.205	0.554	0.476
0.211	1.577	1.000	0.685	0.190	0.404	0.190	0.558	0.481
0.286	1.398	1.000	0.790	0.282	0.434	0.282	0.614	0.546
0.316	1.939	1.000	0.832	0.252	0.446	0.252	0.637	0.572
0.346	1.562	1.000	0.875	0.201	0.458	0.201	0.660	0.598
0.556	1.747	1.000	1.000	0.172	0.542	0.172	0.817	0.700
0.643	1.649	1.000	1.000	0.220	0.577	0.220	0.882	0.731
0.667	2.156	1.000	1.000	0.320	0.587	0.320	0.900	0.740
0.813	2.086	1.000	1.000	0.227	0.645	0.227	1.000	0.793
0.837	2.073	1.000	1.000	0.309	0.655	0.309	1.000	0.801
0.846	1.440	1.000	1.000	0.251	0.658	0.251	1.000	0.805
0.914	1.873	1.000	1.000	0.344		0.344	1.000	0.829
1.080	2.147	1.000	1.000	0.445		0.445	1.000	0.889
2.083	1.782	1.000	1.000	0.553		0.553	1.000	1.250

Table 5: Comparison of Correction Factors for 2 dof system Subjected to Real Earthquakes and EPRI [3] Recommended Values for Mass 1, Primary system frequency = 2Hz

B	**CB**	**Recommended CB**	**CMI**	**Modified CMI**	**Recommended CMI**	**CB*CMI**	**Recommended Cc (CDFM)**	**Recommended Median Cc**
0.100	0.243		1.464	1.000	0.530	0.243	0.550	0.386
0.100	0.226		1.183	1.000	0.530	0.226	0.550	0.386
0.100	0.439		1.981	1.000	0.530	0.439	0.550	0.386
0.100	0.440		1.531	1.000	0.530	0.440	0.550	0.386
0.100	0.300		1.384	1.000	0.530	0.300	0.550	0.386
0.100	0.367		1.649	1.000	0.530	0.367	0.550	0.386
0.111	0.309		0.831	0.831	0.546	0.257	0.550	0.396
0.125	0.461		1.508	1.000	0.565	0.461	0.550	0.408
0.125	0.277		1.342	1.000	0.565	0.277	0.550	0.408
0.125	0.283		1.412	1.000	0.565	0.283	0.550	0.408
0.125	0.263		1.492	1.000	0.565	0.263	0.550	0.408
0.125	0.238		1.531	1.000	0.565	0.238	0.550	0.408
0.125	0.408		1.675	1.000	0.565	0.408	0.550	0.408
0.143	0.294		0.657	0.657	0.590	0.193	0.550	0.423
0.222	0.157	0.520	1.363	1.000	0.701	0.157	0.567	0.491
0.250	0.559	0.520	1.633	1.000	0.740	0.559	0.588	0.515
0.250	0.303	0.520	1.269	1.000	0.740	0.303	0.588	0.515
0.286	0.555	0.520	0.846	0.846	0.790	0.470	0.614	0.546
0.330	0.300	0.616	1.488	1.000	0.852	0.300	0.647	0.584
0.429	0.531	0.620	0.683	0.683	0.990	0.363	0.721	0.654

Table 6. Comparison of Correction Factors for 2 dof system Subjected to Real Earthquakes and EPRI [3] Recommended Values for Mass 2, Primary system frequency = 2Hz

B	**CB**	**Recommended CB**	**CMI**	**Modified CMI**	**Recommended CMI**	**CB*CMI**	**Recommended Cc (CDFM)**	**Recommended Median Cc**
0.050	0.205	NA	1.231	1.000	0.460	0.205	0.550	0.343
0.050	0.249	NA	1.716	1.000	0.460	0.249	0.550	0.343
0.056	0.381	NA	0.341	0.341	0.469	0.130	0.550	0.348
0.100	0.202	NA	1.274	1.000	0.530	0.202	0.550	0.386
0.100	0.190	NA	1.640	1.000	0.530	0.190	0.550	0.386
0.100	0.250	NA	1.446	1.000	0.530	0.250	0.550	0.386
0.100	0.232	NA	1.578	1.000	0.530	0.232	0.550	0.386
0.100	0.188	NA	1.784	1.000	0.530	0.188	0.550	0.386
0.100	0.373	NA	1.778	1.000	0.530	0.373	0.550	0.386
0.100	0.313	NA	0.820	0.820	0.530	0.257	0.550	0.386
0.100	0.370	NA	1.699	1.000	0.530	0.370	0.550	0.386
0.100	0.194	NA	1.075	1.000	0.530	0.194	0.550	0.386
0.100	0.125	NA	1.160	1.000	0.530	0.125	0.550	0.386
0.125	0.426	NA	1.013	1.000	0.565	0.426	0.550	0.408
0.125	0.369	NA	1.533	1.000	0.565	0.369	0.550	0.408
0.125	0.617	NA	1.443	1.000	0.565	0.617	0.550	0.408
0.125	0.229	NA	1.324	1.000	0.565	0.229	0.550	0.408
0.125	0.189	NA	1.306	1.000	0.565	0.189	0.550	0.408
0.125	0.227	NA	1.210	1.000	0.565	0.227	0.550	0.408
0.125	0.211	NA	1.116	1.000	0.565	0.211	0.550	0.408
0.125	0.198	NA	1.550	1.000	0.565	0.198	0.550	0.408

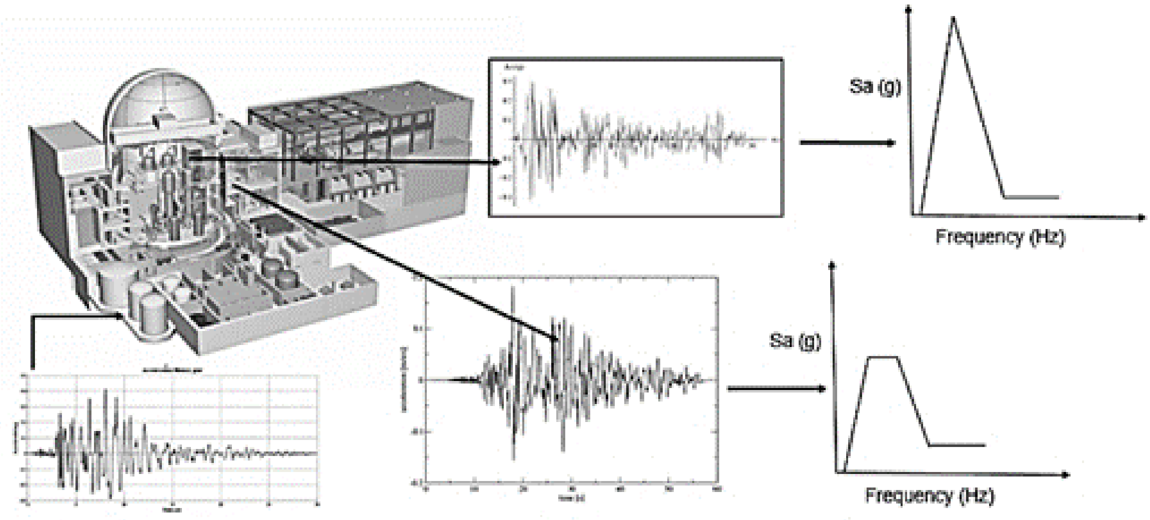


Fig. 1. Filtering of ground motion through structures

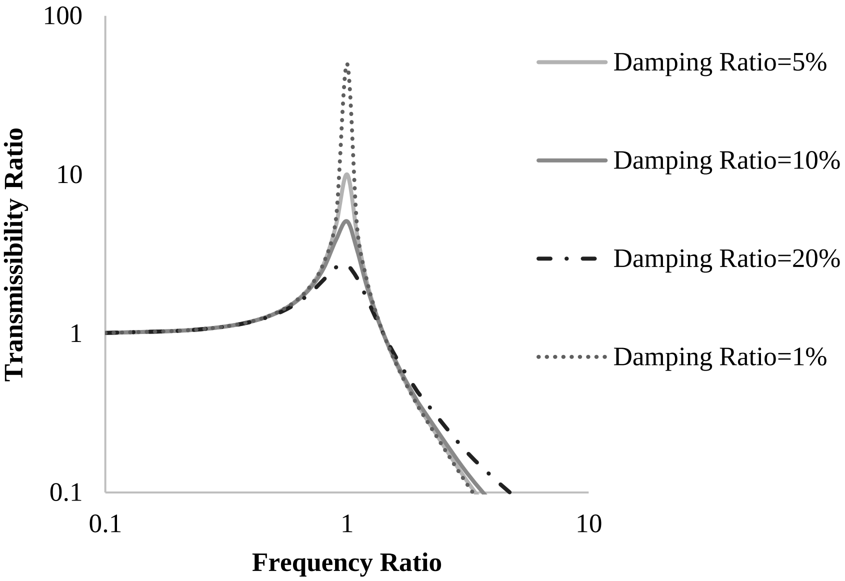


Fig. 2. Transmissibility Ratio for Harmonic Excitations

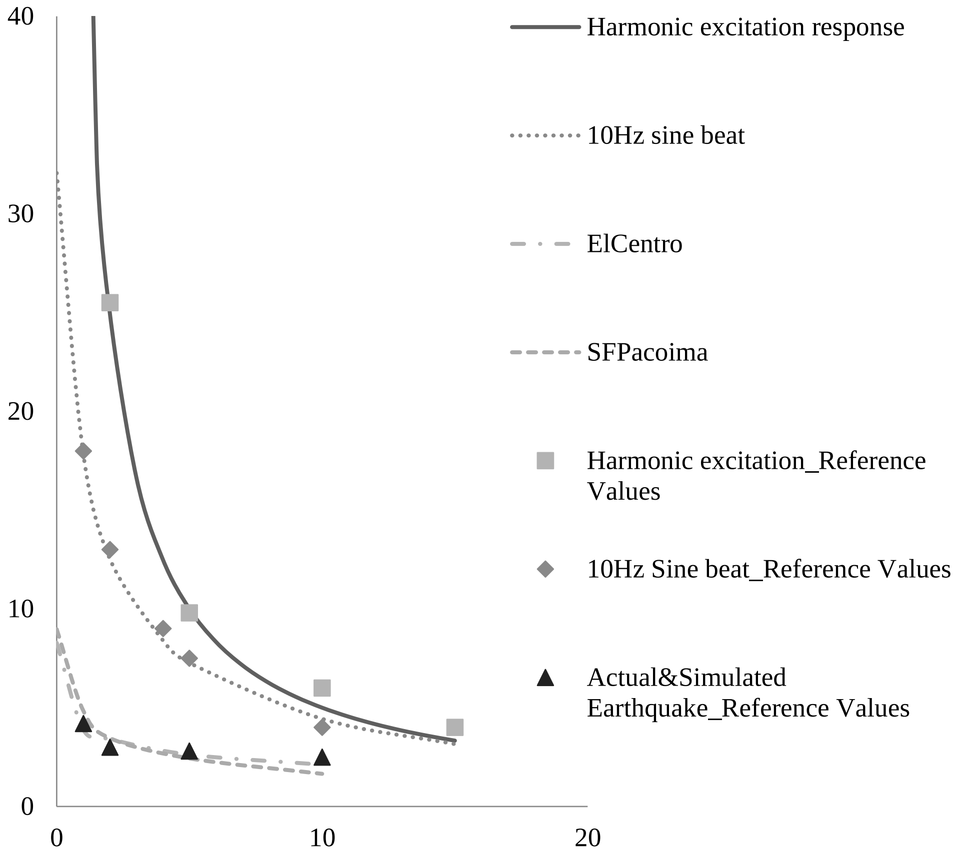


Fig. 3. Maximum Response Factor for Different Excitations

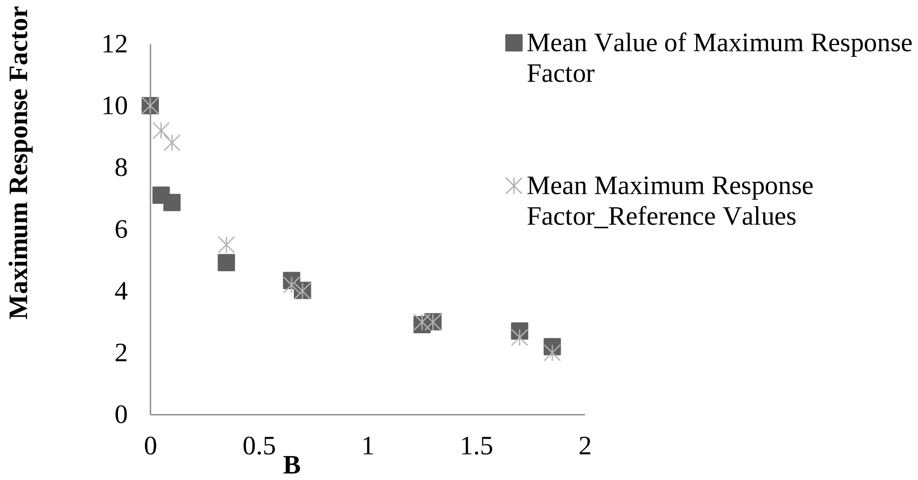


Fig. 4. Maximum Response Factor for Different Bandwidth of Input Motion

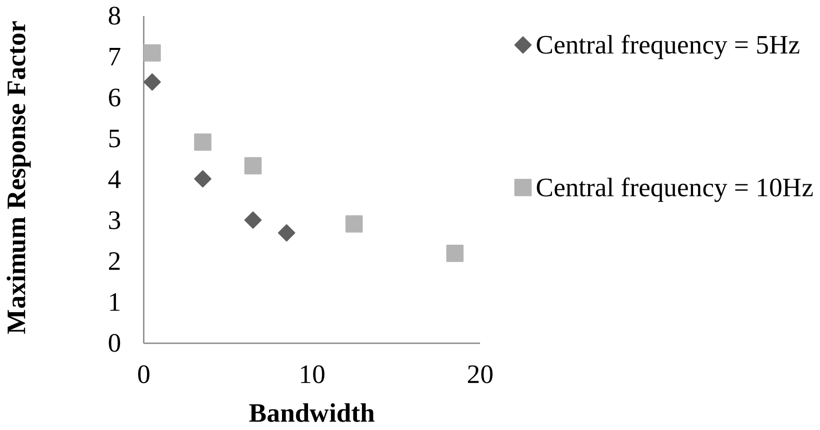


Fig. 5. Mean Values of Maximum Response Factor

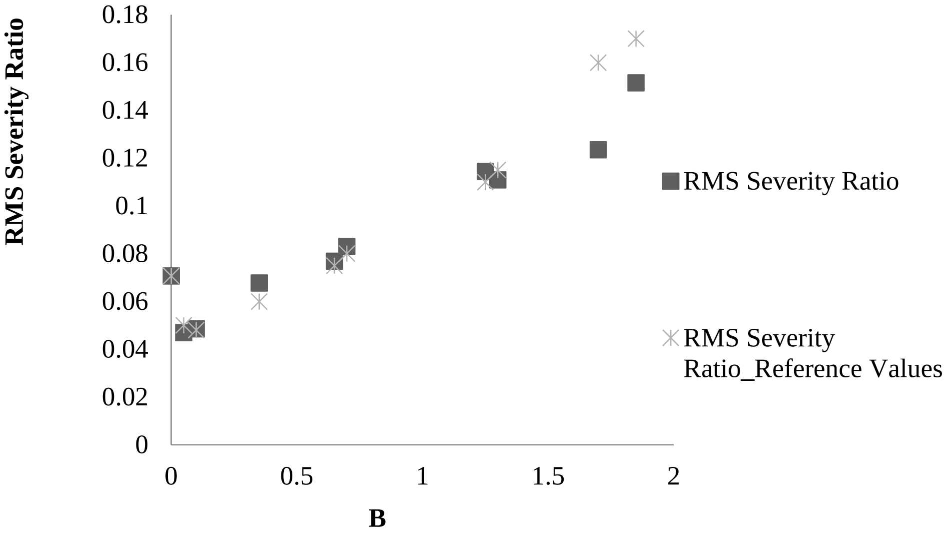


Fig. 6. Severity Ratio for different ground motions

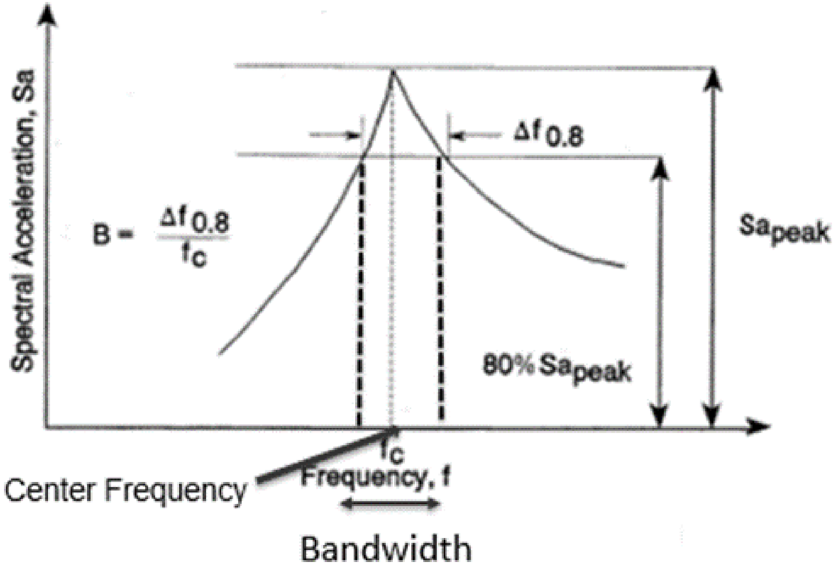


Fig. 7. Bandwidth and Center frequency, EPRI [3]

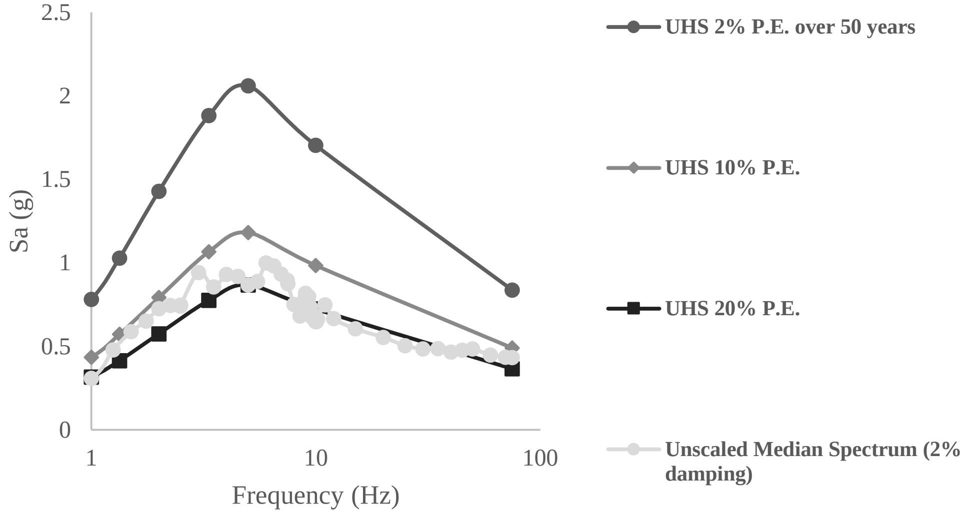


Fig. 8. Uniform Hazard Spectra and Unscaled Median Spectra for 21 earthquakes

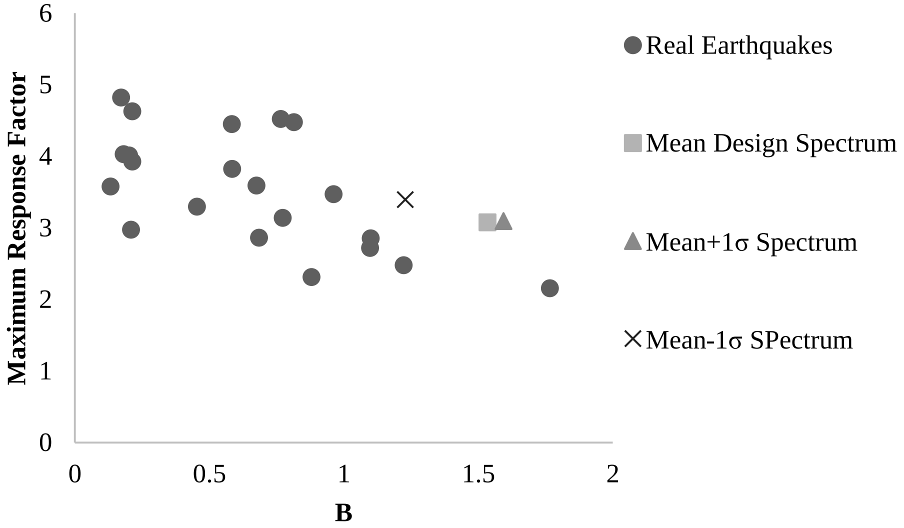


Fig. 9. Maximum Response Factor for Real Earthquakes

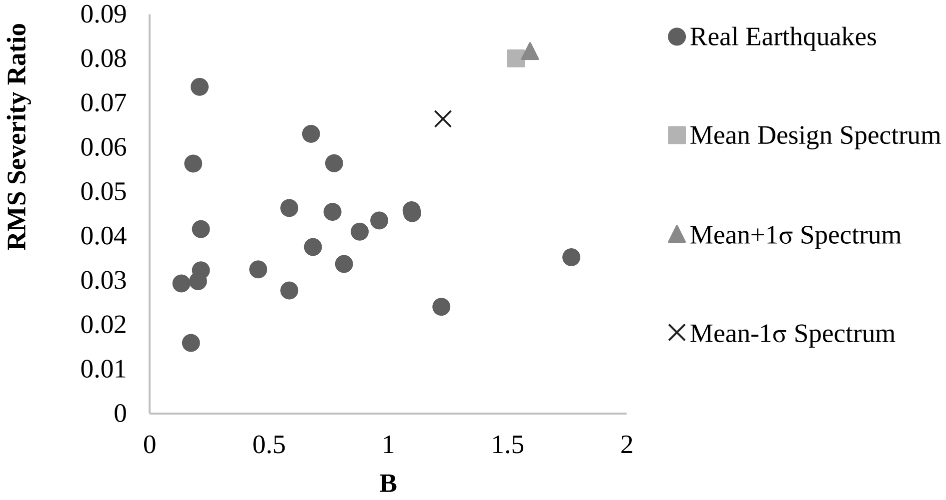


Fig. 10. Maximum Response Factors as a function of B

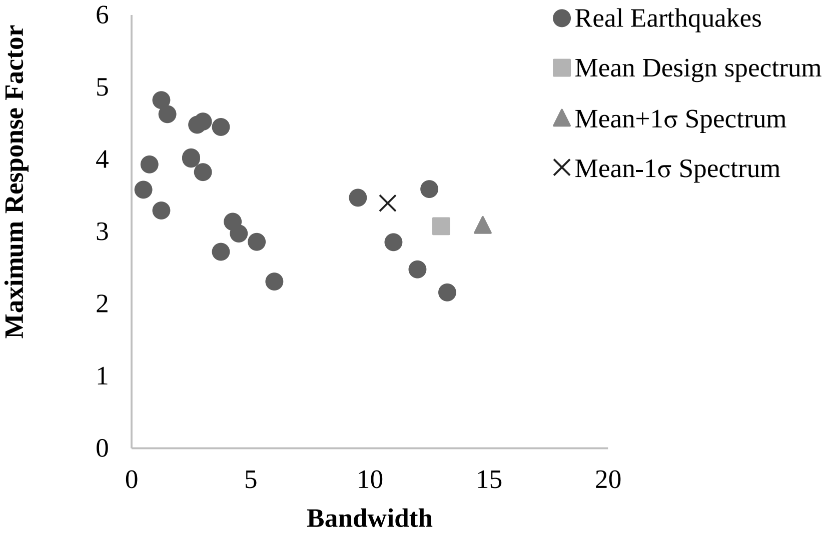


Fig. 11. RMS Severity Ratio for Real Earthquakes

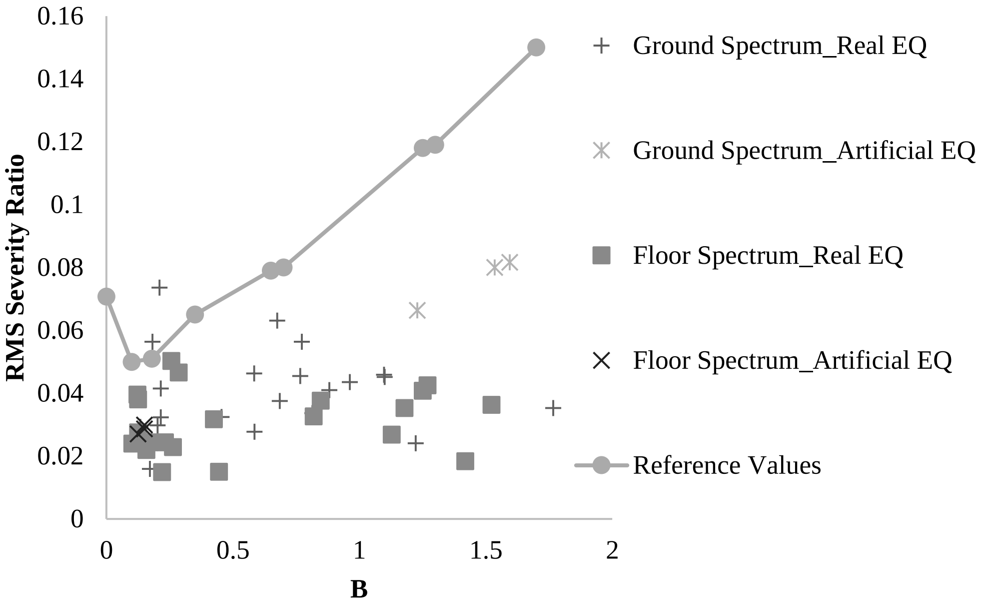


Fig. 12. Severity Ratios for Ground and Floor Spectra for Real Earthquakes w.r.t. EPRI Reference Values

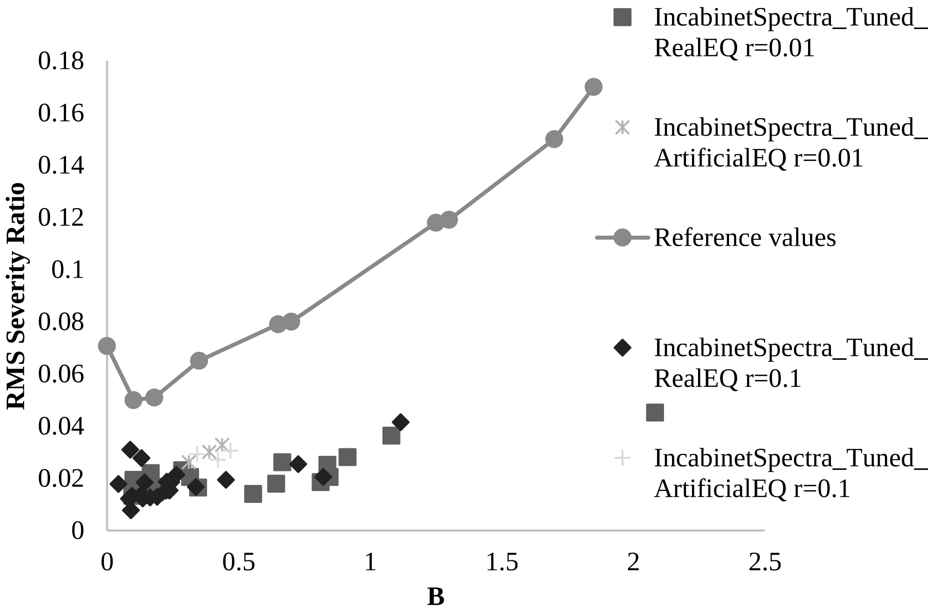


Fig. 13. Severity Ratios for In-cabinet Spectra for Real Earthquakes w.r.t. EPRI Reference Values

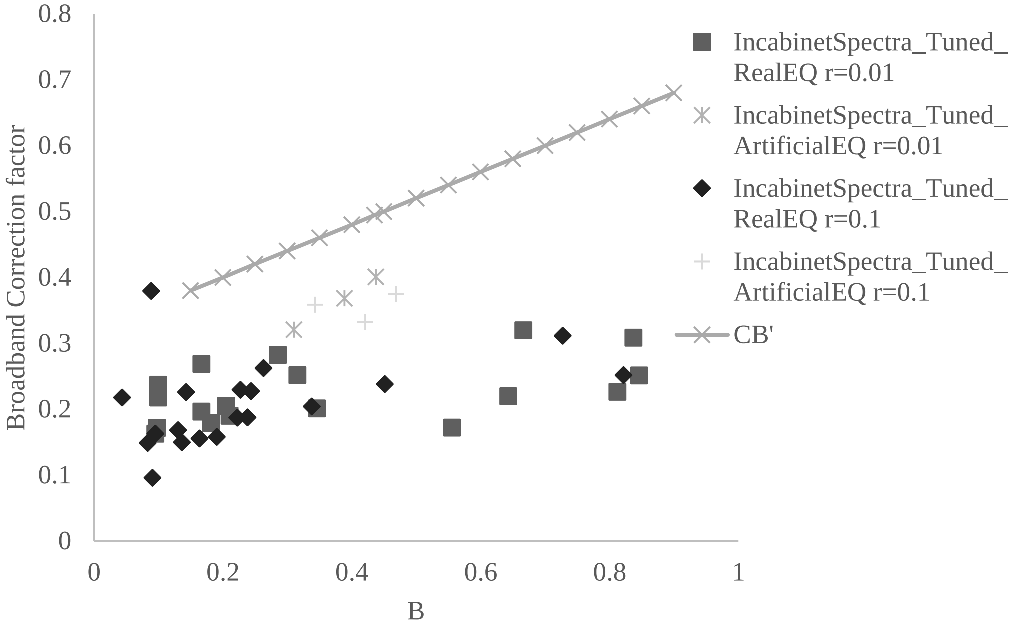


Fig. 14: Comparison of C_B for Actual Earthquakes with the EPRI [3] Recommended Values

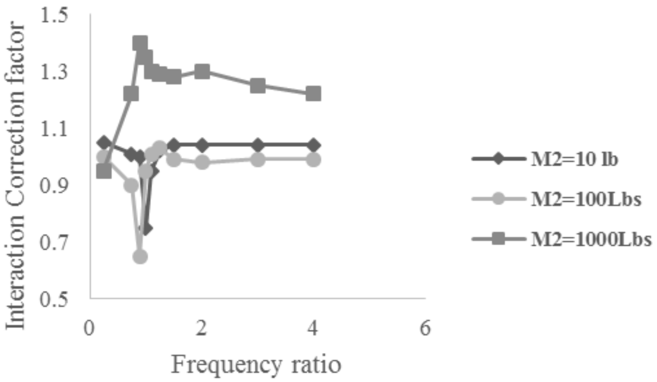


Fig. 15: Interaction Correction Factor for Mass 1 for Narrowband Harmonic Excitation

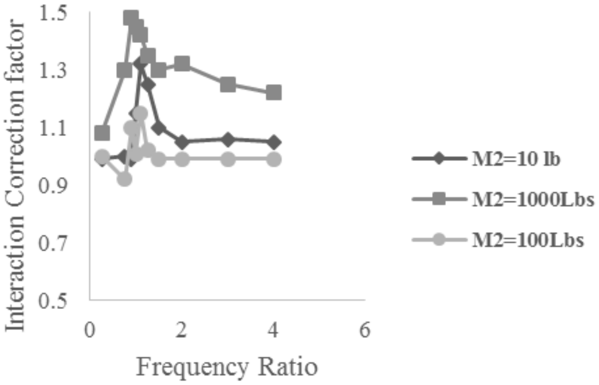


Fig. 16: Interaction Correction Factor for Mass 2 for Narrowband Harmonic Excitation

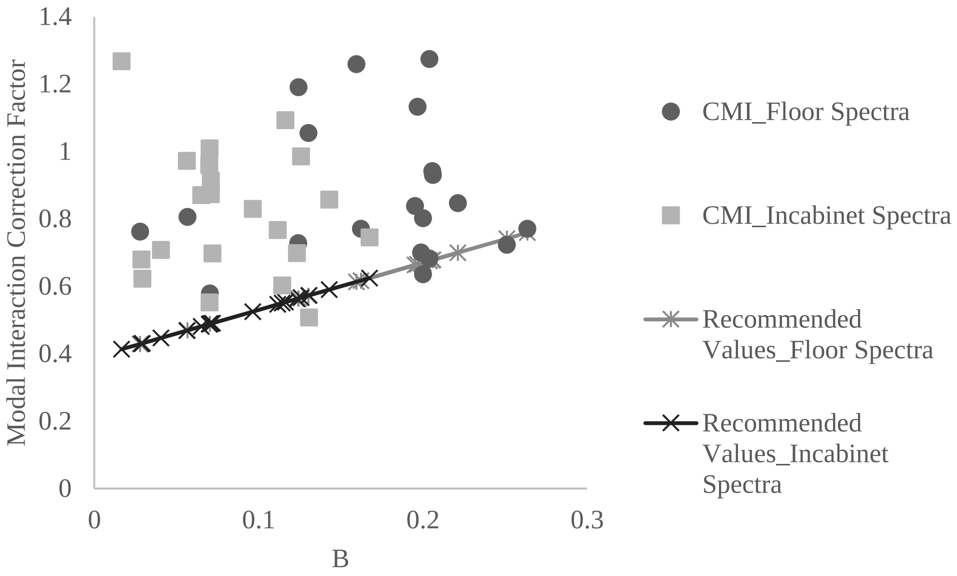


Fig. 17: Comparison of C_{MI} for Actual Earthquakes with EPRI [3] Recommended Values

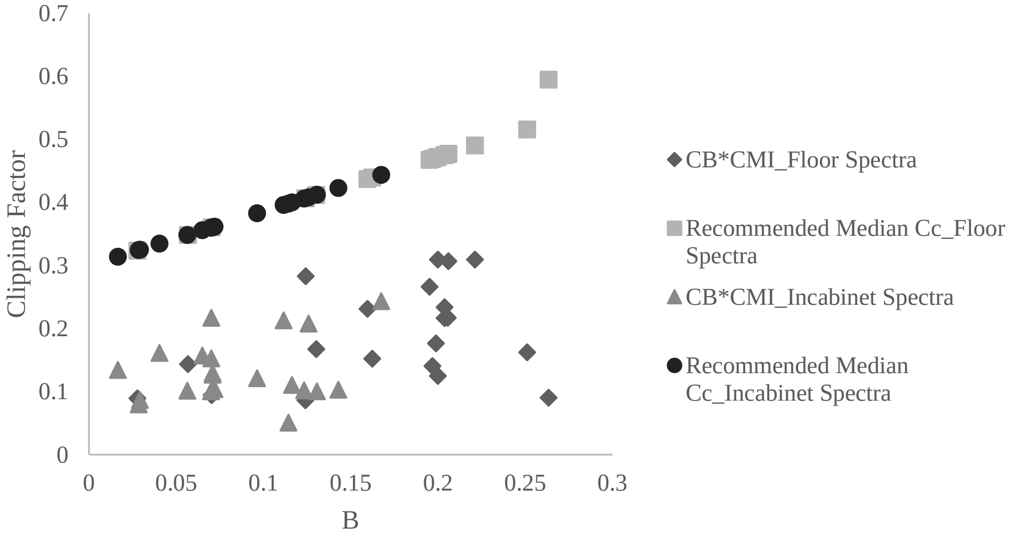


Fig. 18: Comparison of Clipping Factor for Actual Earthquakes with EPRI [3]

Recommended Values, for Coupled Tuned System, Primary System Frequency = 10 Hz

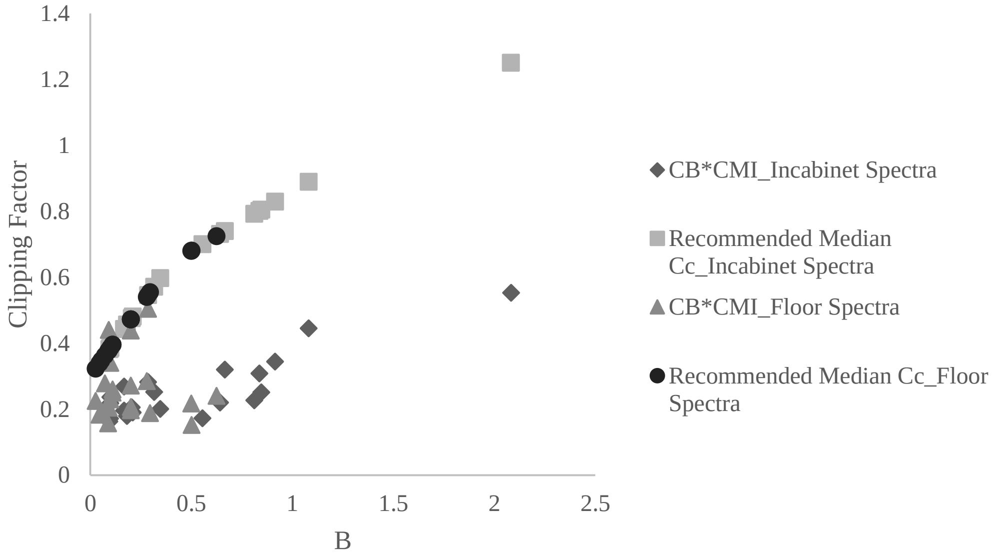


Fig. 19: Comparison of Clipping Factor for Actual Earthquakes with EPRI [3]

Recommended Values, for Coupled Tuned System, Primary System Frequency = 5 Hz

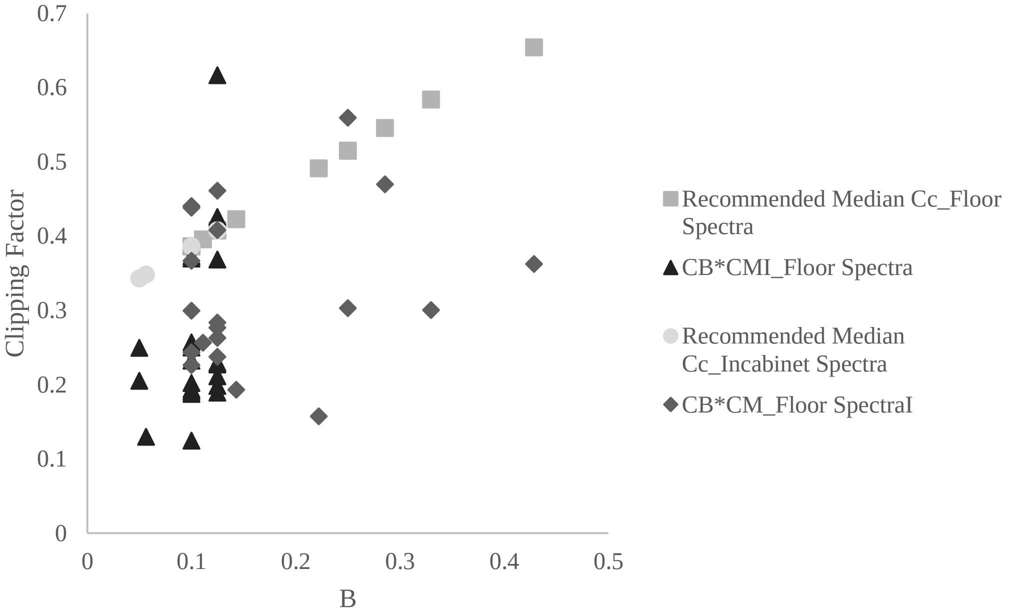


Fig. 20: Comparison of Clipping Factor for Actual Earthquakes with EPRI [3]

Recommended Values, for Coupled Tuned System, Primary System Frequency = 2 Hz

PART V: SUMMARY AND CONCLUSIONS

1. General

The seismic qualification of electrical instruments is a key aspect of SPRA studies being conducted by many nuclear plants. To ensure a safe shut down of the plant during and after an earthquake, the equipment needs to be qualified either by testing or dynamic analysis as per the requirements of IEEE 344 [1], IEEE 693 [2] and EPRI [3]. The response from the testing or analysis is used to characterize the dynamic behavior of the system. The key purpose of this research is to study the dynamic response of equipment for qualification under seismic loading. The observations from this study are summarized as follows.

The existing code recommendations require the use of an amplification factor of 2.5 on the ground motion to allow for possible amplifications by the support structure when the design details of the support conditions are not known. In case of bushings on transformer, IEEE 693 [2] recommends an amplification factor of 2.0. Therefore, there is an inconsistency in the existing recommendations (2.5 vs 2.0). Also, the current practice for seismic qualification of sensitive equipment such as relays and switches in nuclear power plants involves estimation of In-structure Response Spectra (ISRS) using uncoupled analysis on a secondary system subjected to a floor motion. The ISRS are commonly generated by neglecting the interaction between structure (primary system) and the equipment (secondary system). However, studies have shown that this simplification may lead to conservative results when equipment is tuned to one of the modes of the structure. An appreciable reduction of ISRS amplitude can be achieved when considering the interaction between the primary and the secondary systems. The reduction in ISRS due to coupling of primary and secondary systems is most significant in the peak regions of the spectra (tuned or nearly tuned oscillators) and for non-classically damped systems. Also, in case of input motions that have frequency content in regions higher than 30 Hz or so, using uncoupled analysis may result in ISRS with peaks in high frequency regions.

The seismic qualification of systems, structures and components in a nuclear power plant necessitates the use of broadband spectra when equipment is qualified by dynamic testing. Broadband spectra are found to cause more damage to equipment due to higher RMS severity and multi-mode interaction over a broad frequency range. In case of narrowband spectra, as is common in case of sensitive equipment such as relays and switches, the response spectra need to be clipped to obtain an equivalent broadband spectra that would incorporate a wider frequency range. The estimation of the clipping factors based on the bandwidth and central frequency of the spectra, as recommended by EPRI [3], correspond to harmonic type input motions which are extremely narrow banded in frequency content. In case of actual earthquake ground motions that the equipment are subjected to in real life, there is considerable inter-mode interaction and the RMS severity of such input motions are lower than sine beat type motions.

The second, third and fourth chapters in this thesis make an attempt to recommend risk consistent frameworks that can address the above mentioned limitations in the current practice. The observations from each of these parts are discussed below.

2. Development of a closed form solution for amplification of equipment response

Simple closed form solutions are proposed for estimating the amplification in equipment response due to interaction with supporting structures on which the equipment is mounted. These formulations are validated with respect to those obtained from the complete solution for a MDOF system. The observations from the studies conducted by EPRI [4] show that for small mass ratios and frequency ratios the amplification is typically lesser than 2. In case of stiff supporting structures, the amplification factor for lighter and flexible equipment usually ranges between 1 and 2. Also, when rigid equipment is mounted on flexible supports, the incabinet motion may be amplified much higher than 2.5 times the input ground motion.

This necessitates the development of a closed form solution that will enable the estimation of the amplification ratio directly when the mass and stiffness ratios of the equipment-support system are known. The objective and outcome of the current study are summarized as follows:

- The study makes an attempt to develop a closed form equation for amplification ratio in terms of the mass and frequency ratios of the system.
- The response at the point of interest are calculated using response spectrum analysis.
- The equipment-support system are modeled as a two degree-of-freedom system with high mass and frequency ratios.
- For tuned or nearly tuned system, only one mode can be used to approximate the amplification ratio for the equipment using response spectrum analysis.
- The one mode approximations are found to give values that are close to the results from actual time history analyses.
- Depending on the combination of the mass ratio and frequency ratio, simplifications are made to the analytical formulations. The simplified models are validated with respect to the complete formulations.

- These formulations can be directly utilized to compute the amplified responses for seismic qualification of equipment.

3. Reduction in equipment response due to coupled analysis

The dynamic response of equipment mounted on supporting structures can be significantly lower when the interaction between the equipment and support is considered. As the mass ratio of the system increases, coupled response can be significantly lower than the corresponding uncoupled response at the peak regions. For high frequency ground motions, coupled analysis can lead to In-structure Response Spectra (ISRS) with no peaks in the rigid regions. This study makes an attempt to quantify the reduction in response as a factor which when applied to uncoupled equipment response can give more realistic ISRS for equipment qualification purposes. The key steps in the estimation of this reduction factor are discussed below.

- A parametric study of the reduction in equipment response due to coupling effect is conducted for a suite of multi degree of freedom systems.
- Using random sampling of the parameters, such as, stiffness of primary system, mass ratio of the system and damping ratio of the secondary system, the effect of uncertainty in these parameters on the equipment response is studied.
- The distribution of the reduction factor is plotted for a range of systems based on these samples.

The results from the analysis have shown that:

- The cumulative distribution functions for the reduction factors have good fit with lognormal distribution.
- As the mass ratio increases, the reduced coupled response can be as low as 30% of the uncoupled response.
- The effect of reduction in equipment response is significant for non-classically damped systems.
- Reduction in coupled response can be as low as 50% of the uncoupled response for mass ratio = 25% for 95.3% (Median+2β) Non-exceedance probability (NEP)

4. Risk consistent clipping of narrowband equipment response spectra

The estimation of the clipping factors based on the bandwidth and central frequency of the spectra correspond to harmonic type input motions which are extremely narrow banded in frequency content. In case of actual earthquake ground motions, there is considerable inter-mode interactions and the RMS severity ratios in such input motions are lower than sine beat type motions. This necessitates the development of a risk consistent framework for estimating the clipping factors.

Ground motions that are consistent with the site specific PSHA requirements are used to study the effect of broadband correction and modal interaction correction in case of floor spectra and incabinet spectra. Due to filtering of ground motion at different levels of the structure and equipment these spectra become more and more narrow banded in nature. The observations from this study are summarized as follows:

- Real earthquakes show random scattering of points unlike harmonic motions which exhibit a trend.
- Results from harmonic type motions are not representative of the behavior during actual earthquakes.
- Severity ratio for real earthquakes are over estimated if we use the EPRI [3] recommendations directly. Therefore, broadband correction factors are much lower as compared to harmonic type excitations.
- In case of real earthquakes, the equipment response includes interaction between different modes. Therefore, for all practical purposes, modal interaction correction may not be necessarily applicable to clipping of narrowband response spectra for seismic qualification of equipment.
- The combined effect of broadband and modal interaction correction for narrowband spectra gives lower overall value for the clipping factor in case of real earthquakes than the code recommended median values based on 1% NEP.
- The code recommends that the full coupling of C_B and C_{MI} gives an unconservative estimate of clipping factor. However, the present study reveals that in case of calculations based on actual earthquakes, the estimated clipping based on full coupling are lower than the median clipping factor values for a particular bandwidth to central frequency ratio.
- This necessitates the use of real earthquakes to develop a probabilistic framework for seismic qualification of equipment.

REFERENCES

[1] IEEE 344, 2004. “IEEE recommended practice for seismic qualification of class 1E equipment for nuclear power generating stations”.

[2] IEEE 693, 2005. “IEEE recommended practice for seismic design of substations”.

[3] EPR1, 1994. “Methodology for developing seismic fragilities.” TR-103959.

[4] EPRI, 2013. “Study of support amplification factors for various frequency and mass ratios of idealized equipment mounted on a support structure.” Palo Alto, CA, 3002000767.

[5] http://geohazards.usgs.gov/hazardtool/application.php