COMPUTATION OF DEFORMATIONS AND STRESSES IN GRAPHITE BLOCKS FOR HTR CORE SURVEY PURPOSES

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SUMMARY

Stresses and deformations in graphite fuel elements for HTRs are caused by the temperature distribution and by irradiation under influence of creep, shrinking, thermal strains, and elastic deformations. The material behaviour can be treated as linear in the following sense: If the irradiation is given, i.e. the shrinking deformations are known, solutions which consider stresses and deformations resulting from thermal effects only, can be superimposed to any complete solution. Hence, it is allowed to distinguish between the stress field induced by the global distribution of a mean temperature and by irradiation, which can be considered as distributed smoothly, on the one hand, and the stresses caused by local variations of the temperature on the other hand. Here, only the first effect will be considered.

In this case the global deformations and the stress distribution in a prismatic fuel-element containing regularly distributed axial holes for the coolant flow and the fuel sticks, can be computed in the following manner: The block with its holes is treated as an "effective" homogeneous continuum with an equivalent global behaviour. Assuming that the fourth-order-tensor of the elastic constants is proportional to the corresponding tensor in the constitutive equations for creep, only the effective strains are of interest. The values of temperature and dose may be given in n points of the block at certain points of time. Then, the inelastic nonthermal strains are integrated by a Runge-Kutta-procedure in the n points. When interpolated and combined with thermal strains, they are incompatible. Hence, they produce elastic deformations which cause creep and can be computed by use of a Ritz-polynomial-series by help of a specific principle of the minimum of potential energy.

Excessive computing time can be avoided easily since the influence of the local variation of the elastic constants within the block is almost neglibible and, therefore, of practically no importance for the determination of the elastic strains. By this reason some matrices can be calculated a priori, and the elastic deformations are obtained by multiplications of these matrices rather than inversions. Therefore, this method is particularly suited for the computation of deformations and stresses for reactor core survey purposes where a large number (up to 7000 blocks) have to be treated.

1. Introduction

If deformations and stresses in graphite fuel- or reflector elements of HTR cores have to be calculated, shrinking caused by irradiation as well as thermal strains, elastic deformations, and creep has to be considered. The deformations are sufficiently small to allow the application of a geometrically linear theory. The strain tensor $\underline{\underline{\varepsilon}}$ (number of bars under a symbol = order of the tensor) is composed of four parts:

$$\underline{\underline{\varepsilon}} = \overset{s}{\underline{\underline{\varepsilon}}} + \overset{t}{\underline{\underline{\varepsilon}}} + \overset{e}{\underline{\underline{\varepsilon}}} + \overset{c}{\underline{\underline{\varepsilon}}} \tag{1}$$

(s: shrinking, t: thermal, e: elastic, c: creep). The field of the inelastic strains

$$\hat{\underline{\underline{\varepsilon}}} = \overset{s}{\underline{\underline{\varepsilon}}} + \overset{t}{\underline{\underline{\varepsilon}}} + \overset{c}{\underline{\underline{\varepsilon}}} \tag{2}$$

is generally not compatible. Therefore, they cause elastic strains. While the inelastic strains or their rates with respect to the neutron dose Φ can be found as functions of local quantities, the computation of the elastic strains is much more time-consuming. For this reason, it is of particular interest to simplify especially this part of the computational procedure. In this paper, a procedure is proposed, which is suitable for core survey purposes.

2. Constitutive equations

The four effects which have to be considered are assumed to be represented by the following constitutive equations:

a) <u>shrinking:</u>

$$\partial \overset{s}{\underline{\underline{\varepsilon}}} / \partial \Phi = \underline{\underline{f}} \; (\Phi, T) \tag{3}$$

(T: temperature),

b) <u>thermal strains:</u>

$$\overset{t}{\underline{\underline{\varepsilon}}} = \underline{\underline{g}} \; (\Phi, T) \, , \tag{4}$$

c) <u>elastic deformations and stresses:</u>

$$\underline{\underline{\sigma}} = \underline{\underline{\underline{\underline{C}}}} \; (\Phi, T) \cdot\cdot \; \overset{e}{\underline{\underline{\varepsilon}}} \, , \tag{5}$$

d) <u>creep:</u>

$$\partial \overset{c}{\underline{\underline{\varepsilon}}} / \partial \Phi = c \; (\Phi, T) \; \overset{e}{\underline{\underline{\varepsilon}}} \, . \tag{6}$$

The quantity $\overset{t}{\underline{\varepsilon}}$, is a function of Φ and T, and can change instantaneously with T, whereas in the case of $\overset{s}{\underline{\varepsilon}}$ and $\overset{c}{\underline{\varepsilon}}$ only the rates are functions of Φ, T, and $\overset{e}{\underline{\varepsilon}}$, resp. Their sum will be denoted by

$$\overset{o}{\underline{\varepsilon}} = \overset{s}{\underline{\varepsilon}} + \overset{c}{\underline{\varepsilon}} . \qquad (7)$$

3. Integration

The integration of the differential equations (3) and (6) in connection with eqs.(4) and (5) and together with the usual equations of equilibrium, can be performed numerically (e.g. by a Runge-Kutta-procedure) with $\overset{o}{\underline{\varepsilon}}$ (eq. (7)) as variable if T and Φ are given:

If $\overset{o}{\underline{\varepsilon}}$ is known for a certain instant, $\hat{\underline{\varepsilon}}$ is determined, too, by eq. (4). The elastic strains $\overset{e}{\underline{\varepsilon}}$ in eq. (6) can be calculated from $\overset{e}{\underline{\varepsilon}} = \underline{\varepsilon} - \hat{\underline{\varepsilon}}$ and by use of a modified principle of minimal potential energy, which, if the volume forces are neglected, can be written as

$$\int_I \{\frac{1}{2}\, \underline{\varepsilon} \cdot\cdot \underline{\underline{C}} \cdot\cdot \underline{\varepsilon} - \underline{\varepsilon} \cdot\cdot \underline{\underline{C}} \cdot\cdot \hat{\underline{\varepsilon}}\}\, dV - \oint_S \underline{s}\ \underline{u}\ dA \Longrightarrow \min \qquad (8)$$

(I: Interiour of the body, V: Volume, S: surface of the body, $\underline{s}$: stress vector at the surface, $\underline{u}$: displacement vector, A: area). The strains $\underline{\varepsilon}$ have to satisfy the equation

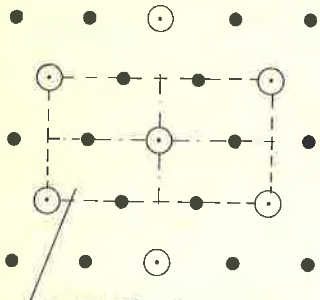
$$\varepsilon_{ij} = \frac{1}{2}\,(u_{i/j} + u_{j/i}) \qquad (9)$$

($_/$ denotes covariant derivatives).
The principle (8) may be applied in connection with power-series or with a finite element-method. If the field $\overset{e}{\underline{\varepsilon}}$ (x_i) is known, rates of $\overset{o}{\underline{\varepsilon}}$ with respect to Φ are determined by eqs. (3) and (6). Hence, the use of the Runge-Kutta-procedure is possible.

Due to the fact, that the consideration of the detailed dose- and temperature distribution would result in excessive computing time, the following simplifications are introduced.

4. Separation of local and global parts of the strains

The fields of each type of strain, as well as the temperature field, contain local and global parts because of the large number of channels for coolant flow and fuel, which



rectangular "cell" of the hole-pattern.

Fig. 1

are regularly distributed according to Fig. 1.

The field of Φ and the global parts of $\overset{g}{\underline{\underline{\varepsilon}}}$ and $\overset{t}{\underline{\underline{\varepsilon}}}$ show only small variations between two holes, whereas the induced elastic strains $\overset{e}{\underline{\underline{\varepsilon}}}$ and creep strains $\overset{c}{\underline{\underline{\varepsilon}}}$ show, due to the boundary conditions at the channel surfaces for the corresponding stresses, considerable variations within one rectangular "cell".

The additional assumption, that the variation of c (Φ, T) in eq. (6) has only small influence on the global part of $\overset{c}{\underline{\underline{\varepsilon}}}$ makes the field of $\partial\overset{c}{\underline{\underline{\varepsilon}}}/\partial\Phi$ proportional to $\overset{e}{\underline{\underline{\varepsilon}}}$ within one cell. As a consequence, the local constitutive equation (6) can also be directly applied to the global parts of $\overset{c}{\underline{\underline{\varepsilon}}}$ and $\overset{e}{\underline{\underline{\varepsilon}}}$.

The global part of $\overset{e}{\underline{\underline{\varepsilon}}}$ is identical with the "effective" strain of a homogenous continuum containing the corners of the cells as a grid. Also effective stresses are introduced as mean values of the stresses in the surfaces of the "cells". In this way, the original body with its holes is replaced by an "effective" continuum. This reduces the number of parameters in a power-series- or finite element-application of principle (8) considerably. But only the global part of the stresses is calculated. A second application of principle (8) within one cell allows the computation of local values. This paper is restricted to the global part only.

5. Constitutive equations of the "effective" continuum

In establishing the constitutive equations of the linear elastic "effective" continuum, numerous symmetry conditions have to be considered. It can be shown, that

1. the effective continuum is "classical", i.e. the tensors of strains and stresses are symmetric, couple-stresses do not appear,

2. the body has the symmetries of an hexagonal crystal, hence it is transversally anisotropic.

The constitutive equations in an Euclidean coordinate system x,y,z with z parallel to the holes read (bar denotes "effective"):

$$\left.\begin{aligned}
\bar{\sigma}_{xx} &= a\,\bar{\varepsilon}_{xx} + b\,\bar{\varepsilon}_{yy} + c\,\bar{\varepsilon}_{zz}\\
\bar{\sigma}_{yy} &= b\,\bar{\varepsilon}_{xx} + a\,\bar{\varepsilon}_{yy} + c\,\bar{\varepsilon}_{zz}\\
\bar{\sigma}_{zz} &= c\,\bar{\varepsilon}_{xx} + c\,\bar{\varepsilon}_{yy} + d\,\bar{\varepsilon}_{zz}\\
\bar{\sigma}_{yz} &= e\,\bar{\varepsilon}_{yz}\\
\bar{\sigma}_{zx} &= e\,\bar{\varepsilon}_{zx}\\
\bar{\sigma}_{xy} &= (a-b)\;\bar{\varepsilon}_{xy}
\end{aligned}\right\} \qquad (10)$$

The same type of constitutive equations is (due to the anisotropy of the material) valid in the original graphite, too. If, with changing dose and temperature, the proportionalities a:b:c:d:e remain constant and if the variance of the absolute values of the moduli is of small interest, it is possible to calculate the effective moduli a priori. This can be done by appliying finite-element-methods to one cell under special loads.

6. Influence of the absolute values of the elastic moduli on the elastic strains

When calculating the global parts of stresses in any specific point of the block, the local values of Φ and T have a great influence. But when the principle (8) is applied, the result will be the same if the moduli and their proportionalities a:b:c:d:e remain spatially constant but may vary in time, because the non-vanishing terms of principle (8) are then multiplied by constant factors and the surface integral vanishes, since $\underline{s} \equiv \underline{0}$ is valid.

With respect to St.-Venant's principle, it can be assumed additionally that also a local variance of these absolute values will not influence the result too much. Especially when developing a survey-code, it is allowed to replace $\overset{e}{\underline{\varepsilon}}$ by those values, which result from computations with $\underline{\underline{C}}$ = const. Then, the Green's function $\underline{\underline{A}}$ ($\underline{r}$, $\underline{x}$) in the linear expression

$$\overset{e}{\underline{\underline{\varepsilon}}}\ (\underline{r}) = \int^{I} \underline{\underline{A}}\ (\underline{r}, \underline{x}) \cdot\cdot\ \underline{\varepsilon}\ (\underline{x})\ dV\ (\underline{x}) \qquad (11)$$

can be computed a priori. In technical applications, $\overset{e}{\underline{\varepsilon}}$ and $\hat{\underline{\varepsilon}}$ are given or of interest, resp., at n special points of the block only. In this case, eq. (11) reduces to

$$\overset{e}{\underline{\underline{\varepsilon}}}_{\alpha} = \sum_{\beta=1}^{n} \underline{\underline{A}}_{\alpha\beta} \cdot\cdot \hat{\underline{\underline{\varepsilon}}}_{\beta} \qquad (12)$$

where α, β denote the point numbers.

Numerically, the 6n coordinates of the n tensors $\overset{e}{\underline{\underline{\varepsilon}}}_{\alpha}$ and of $\hat{\underline{\underline{\varepsilon}}}_{\beta}$ form vectors with 6n coordinates, while $\underline{\underline{A}}_{\alpha\beta}$ leads to a 6n x 6n - matrix $\mathfrak{A}$ which can be calculated a priori. This is the most effective idea in order to reduce the computing time.

7. Short description of the programs

The computational work consists of three steps. A first program calculates the effective moduli, a second one determines the 6n x 6n - matrices with the coordinates of $\underline{\underline{A}}_{\alpha\beta}$ (cf. eq. (12)), and the third step is the Runge-Kutta-procedure for the tensors $\overset{o}{\underline{\underline{\varepsilon}}}$ in the n points of the block.

The effective moduli (for $T = \Phi = 0$) are computed by means of a finite-element-method. Triangular elements in the x,y-plane with quadratic displacement fields $\underline{u}$ (x,y) are used in the cases of two load systems. Then, the five moduli a to e are determined, since the Youngs modulus in z-direction can be given directly.

In the application of the theory developed above, the input-data are Φ and T (mean values) in 35 regions of the block.

It is assumed, that they represent neutron-dose and global part of the temperature-distribution in the centres of these domains. The strains $\hat{\underline{\underline{\varepsilon}}}$ calculated in these points, are interpolated by power-series. Principle (8) was applied by means of power-series, but finite-element-methods are also possible. In order to save computing time, the number of parameters, in the series was chosen so, that the matrix $\mathfrak{A}$ (6n x 6n) is reduced to two smaller matrices. This diminishes the number of multiplications within one Runge-Kutta-step.

The 210 x 210 matrix or the two smaller ones are prepared in the second program.

The final program works in the way which was described in sect. 3. If stresses are to be calculated they are determined by $\overset{e}{\underline{\underline{\varepsilon}}}$ and the local values of the effective moduli. The results are effective stresses which can be transformed into local stresses by applying concentration factors, which are obtained from the first program.