A Uniaxial Cyclic Elastoplastic Constitutive Law with a Discrete Memory Variable

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1. INTRODUCTION.

At the present time the study of the cyclic elastoplastic constitutive laws is focused on the non proportional loading, but for uniaxial loading some problems are still with us[1]. For example the possibility for a law to describe simultanously the ratcheting in non symmetrical load-controlled test, elastic and plastic shakedown in symmetrical and non symmetrical one. We present here a law which in addition to previous phenomena describes the cyclic hardening in a pushpull test, the cyclic softening after overloading and also the dependence of cyclic-strain-stress-curve(c-s-s-c) on the history of loading. These are the usual properties of a 316 stainless steel at the room temperature. This law uses an internal discrete memory variable:the plastic strain at the last unloading, represented by $\varepsilon^p{}_n$. In the other hand the choice of all macroscopic variables is justified by a microscopic analysis. A first version of this law is proposed in[2], but there was some problem with the representation of c-s-s-c[3], so a modified one is proposed here. This law has been also extended to the threedimensional case[4].

2. THE MICROSTRUCTURE UNDER CYCLIC LOADING.

2.1 Plastic shakedown.

When the cross sliping of dislocations is possible for a metal (easy cross slip for pure Al and pure Cu, difficult cross slip for a 316 stainless steel) the microscopic structure is characterised at law cyclic amplitude by permanent slip bounds and at higher amplitude by cell structure, whose mean size decreases with an increasing amplitude of loading. But when the amplitude of loading decreases the cell structure is stable (at room temperature). The cell structure seems also to be detected for a monotonic loading[3]. As before the cell size decreases when the maximal stress or strain increases. We suppose that the mean cell size is defined by the maximal stress supported by the material in its history[3]. The asymptotic form of the curves showing mean cell size in fonction of the amplitude loading suggests us to make the hypothesis that there exists a minimal cell size which depends only on the material and not on the loading[3]. During cycling, dislocations pile up on the obstacles(walls), stabilisation is obtained when the numbers of dislocations created and anihilated are equal. At stabilised state the plastic deformation is created by dilocations which sweep away the cell volume and then are anihilated by dislocations of opposite sign. Once a stabilised state is obtained if we increase the maximal stress or the amplitude of loading, smaller cells will be obtained. That means new obstacles are created on which dislocations have to be pile up again to obtain a new stablised state. That means usually more than one cycle is needed to get a new stabilised state. This is macroscopically illustrated by a pushpull test. We call this: interaction between dislocation density and cell size.

2.2 Ratchetting.

Different experiments show that at room temperature ratcheting is obtained at a nearly fixed maximal stress, independently of the amplitude of loading[5],[3]. Fig. 1 shows an experimental result about ratcheting at room temperature for a 316. After some cycles, there is a constant gap between each traction or compression curve. If we increase of a small quantity, the maximal sress we get practically a plateau. The metal becomes practically perfectly plastic .This suggests us the use of a cyclic ultimate stress S_r in relation with minimal cell size [3]

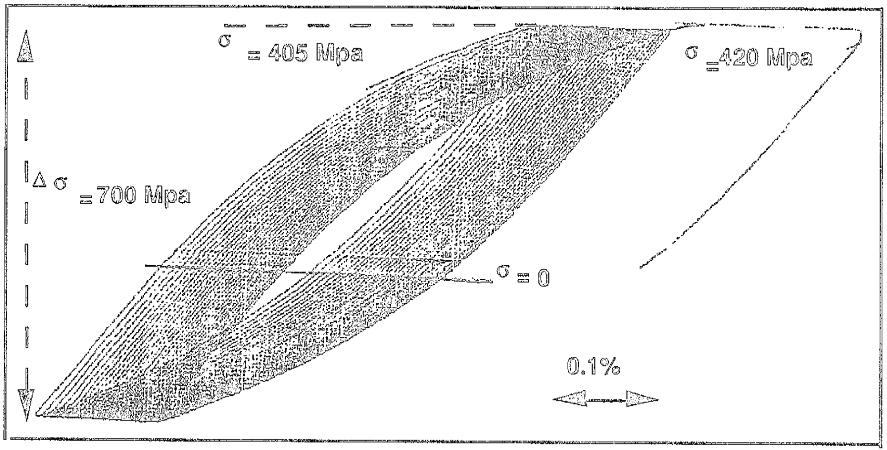


Fig . 1 Rachetting and overload effect on rachetting for a 316.

3 DEFINITION OF MACROSCOPIC VARIABLES TAKING ACCOUNT OF MICROSTRUCTURE.

Here we define the macroscopic variables in relation with microscopic ones.

a- the density of dislocations: as it is supposed usually, it is related to cumulated plastic strain λ.
b- the minimal mean cell size is related to ratcheting stress Sr which is considered as a constant of material in room temperature.
c- the mean cell size is related to maximal stress(absolute value) σpic supported in the load history. It is used under $S_r-\sigma_{pic}$ forme.
d- the interaction between the mean cell size and the density of dislocations (a new repartition of dislocations for each mean cell size): we use a combination of λ and $S_r-\sigma_{pic}$. A particulary simple form is $\lambda(1-\sigma_{pic}/S_r)$.
e- cells sweeped by active dislocations: gliding of dislocations produces the plastic deformation. At stabilised state (cell mean size and dislocation density fixed) the amplitude of plastic deformation (in a strain controlled test) is proportional to the cell surface (or volume) sweeped by the active dislocations. It seems so suitable to take as variable the amplitude of plastic deformation. But to have a variable which makes sense also in a stress controlled test, we take $\varepsilon^p-\varepsilon^p_n$ as variable. This gives the amplitude of strain in a strain controlled test.

4 UNIAXIAL CONSTITUTIVE LAW.

4.1 Natural introduction εp_n

In the case of ratchetting, after some deformation the traction and compression curves are translated at each cycle of a constant quantity. So we may write that:

$\sigma_t=f(\varepsilon p-\varepsilon p_{2n})$ and $\sigma_c=g(\varepsilon p-\varepsilon p_{2n+1})$ with $(f\neq g)$, and $f', f'', g'>0$ and $g''<0$

where εp_{2n} (resp εp_{2n+1}) is the plastic strain at the last unloading on the compression (resp. traction) curve. We know that there is no ratcheting phenomena at a symmetric loading (for $\sigma_{min}=-\sigma_{max}$ we obtain shakedown). With this, it is possible to show [3] that the general form of traction and compression curves in the ratchetting state are:

$$\sigma_t=(\varepsilon p-\varepsilon p_{2n})*F(|\varepsilon p-\varepsilon p_{2n}|) + R(|\varepsilon p-\varepsilon p_{2n}|)$$
$$\sigma_c=(\varepsilon p-\varepsilon p_{2n+1})*F(|\varepsilon p-\varepsilon p_{2n+1}|) - R(|\varepsilon p-\varepsilon p_{2n+1}|)$$

We will study here the simplest cases F=0 and F=K where K is a non zero constant. In the threedimensional case a combined Mises criterum is given in the deviatoric space by $\sqrt{1.5(s_{ij}-\alpha_{ij})(s_{ij}-\alpha_{ij})}-R=0$. This gives in the uniaxial case $\sigma_t=\alpha+R$ in traction and $\sigma_c=\alpha-R$ in compression, where $\alpha=15\alpha_{11}$. It is now clear that in the case of ratchetting, for F=0 we have $\alpha=0$. So the traction and compression curves in the ratchetting state are:

$$\sigma_t= R(|\varepsilon p-\varepsilon p_{2n}|) \qquad \sigma_c= - R(|\varepsilon p-\varepsilon p_{2n+1}|)$$

For the second case (F=K=constant) we have:

$$\sigma_t=K(\varepsilon p-\varepsilon p_{2n}) + R(|\varepsilon p-\varepsilon p_{2n}|) \qquad \sigma_c=K(\varepsilon p-\varepsilon p_{2n+1}) - R(|\varepsilon p-\varepsilon p_{2n+1}|)$$

4.2 Case K=O

We studied this case in [2]. The difficulty in this case comes from the bad representation of the cyclic strain stress curve after prehardening[3]. But for this case it is easier to show the connection between different experimental phenomena and different variables used. So we explain it again briefly here. We are going to show how the progressive introduction of different variables permits us to get different experimental phenomena.

Introduction of εp_n: with this variable in a load controlled test, we can modelize the plastic shakedown, the elastic shakedown (in one cycle), the ratchetting (in one cycle). We have many choices for α and R. One of the simplest one is [2]:$\alpha=C_\infty\, \varepsilon p$ and $R=R(|\varepsilon p-\varepsilon p_n|)$ with $C_\infty \geq 0$ and $R'>0$, $R''<0$

For a simulation between σ_{max} and σ_{min} we get:

$$\sigma_{max}= C_\infty\, \varepsilon p_{2n+1} + R(|\varepsilon p_{2n+1} -\varepsilon p_{2n}|) \quad \text{in traction}$$
$$\sigma_{min}= C_\infty\, \varepsilon p_{2n+2} - R(|\varepsilon p_{2n+2} -\varepsilon p_{2n+1}|) \quad \text{in compression}$$

This constitute a recurent relation. For $X=\varepsilon p_{2n}$, $Y=\varepsilon p_{2n+1}$, we get two curves:

$C1 \Leftrightarrow 1.5.C_\infty.Y + R(|Y-X|)-\sigma_{max}=0$ and $C2 \Leftrightarrow 1.5.C_\infty.X-R(|Y-X|)-\sigma_{min}=0$

With an initial state of deformation εp_0 (fig. 2) the <u>plastic shakdown</u> is obtained if $C_\infty\neq 0$ and $\sigma_a=(\sigma_{max}-\sigma_{min})/2 > R(0)$. The <u>elastic shakedown</u> is obtained if $C_\infty\neq 0$ and $\sigma_a \leq R(0)$, because for $\sigma_a=R(0)$ the intersection of the two curves is on the first bisector and so $\varepsilon^p_{max}=\varepsilon^p_{min}$. For $C_\infty=0$, C1 and C2 are two parallel straight lines, ratchetting is obtained (fig. 3) except for $\sigma_{min}=-\sigma_{max}$ where the two lines are superposed and the shakedown is obtained.

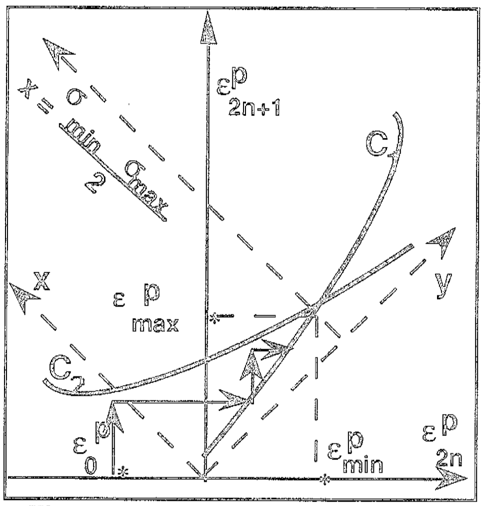


Fig. 2 Plastic shakedown

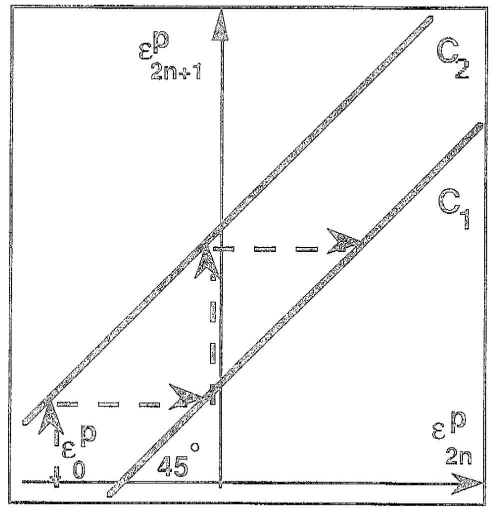


Fig. 3 Ratchetting

Introduction of λ. The introduction of this variable permits to get <u>the hardening and the softening</u> for strain controlled test, and the ratchetting and the elastic shakedown in more than one cycle. As a simple choice[2] we may take: $\alpha=C(\lambda).\varepsilon^p$ and $R=f(\lambda)g(|\varepsilon^p-\varepsilon^p_n|)$ with $C(\infty)=C_\infty>0$ and $f>0$ and $f(\infty)=1$.

In a strain controlled test for each cycle we may write:
$\sigma^n_{max} = C(\lambda)\ \varepsilon^p_{max} + f(\lambda_n)\ g(\Delta\varepsilon^p)$ (σ^n_{max} is the stress associated with ε^p_{max}). The difference for two cycles n and n+1 gives us:

$$\Delta\sigma_{max} = [C(\lambda_{n+1})-C(\lambda_n)]\varepsilon^p_{max} + [f(\lambda_{n+1}) - f(\lambda_n)]\ g(\Delta\varepsilon^p)$$

We take (at least for great λ) $C'<0$ and $f'>0$. For $\Delta\varepsilon^p_{max} << \Delta\varepsilon^p$ we will have $\Delta\sigma_{max}>0$ and so we get the hardening. For $\Delta\varepsilon^p_{max} >> \Delta\varepsilon^p$ we have $\Delta\sigma_{max}<0$ and so we get the softening. For a load controlled test when $\lambda\to\infty$ C1 and C2 approche their limits asymptotically and different load controlled phenomena are obtained in more than one cycle.

Introduction of σ_{pic}. In the case of ratchetting we have $C_\infty=0$. This has to be obtained for $\sigma_{pic}=S_r$ and $\lambda=\infty$. So we may introduce $S_r-\sigma_{pic}$ in the expression of $C(\lambda)$. In the other hand in a strain controlled test we want to describe the pushpull test. For these two reasons, λ is replaced by $\tilde{\lambda}=(1-\sigma_{pic}/S_r)\lambda$. The final expressions are given in[2]

4.2 CASE $K\neq0$

As before the ratchetting is a limit case of the plastic shakedown in a load controlled test when the maximal stress reaches the value S_r. As a simple choice we may take: $\alpha = C_\infty\ (S_r\varepsilon^p-\sigma_{pic}\varepsilon^p_n)$ and $R=R(|\varepsilon^p-\varepsilon^p_n|)$,where C_∞ is a constant. The curves analogous to C1 and C2 are:

$$C1 \Leftrightarrow C_\infty |S_r Y - \sigma_{pic} X| + R(|Y-X|) - \sigma_{max} = 0$$
$$C2 \Leftrightarrow C_\infty |S_r X - \sigma_{pic} Y| - R(|Y-X|) - \sigma_{min} = 0$$

The same analysis as before shows that[3]: for $S_r \neq \sigma_{pic}$ we have plastic shakedown when $\sigma_a > R(0)$ and elastic shakedown when $\sigma_a \leq R(0)$, while for $S_r = \sigma_{pic}$ and $\sigma_m \neq 0$ we have ratchetting.

Cyclic stress strain curve. For a stabilized limit state obtained in a load or strain controlled test the traction and compression curves are:

$$\sigma_t = C_\infty |\sigma \varepsilon^p - \sigma_{pic}\, \varepsilon^p{}_{min}| + R\, |\varepsilon^p - \varepsilon^p{}_{max}|$$
$$\sigma_c = C_\infty |\sigma \varepsilon^p - \sigma_{pic}\, \varepsilon^p{}_{max}| + R\, |\varepsilon^p - \varepsilon^p{}_{max}|$$

This gives us

$$\sigma_{max} = C_\infty | S_r \varepsilon^p{}_{max} - \sigma_{pic}\, \varepsilon^p{}_{min}| + R\, |\varepsilon^p{}_{max} - \varepsilon^p{}_{min}|$$
$$\sigma_{min} = C_\infty | S_r \varepsilon^p{}_{min} - \sigma_{pic}\, \varepsilon^p{}_{max}| - R\, |\varepsilon^p min - \varepsilon^p max|$$

After important prehardening σ_{pic} is different from σ_{max} or σ_{min}, so we get:

$$y = C_\infty\ |S_r + \sigma_{pic}| x + R(2x) \qquad \text{where } y = \Delta\sigma/2 \qquad \text{and } x = \Delta\varepsilon^p/2$$

With this relation the cyclic strain stress curve after prehardening is upper than the cyclic curve without prehardening [3], which is given by:

$$y = C_\infty . S_r\ x + R(2x)/(1 - C_\infty . x) \qquad \text{with} \quad x = (\Delta\varepsilon^p/2) < (1/C_\infty x)$$

Introduction of λ.. A similar analysis as in 4.2 may be given[3]. Some results seem fit better the experiments. For example in the first model(K=0), in a strain controlled test the relaxation was independent of the prehardening :$\sigma_m = C_\infty \varepsilon^p$, while here it depends on the prehardening : $\sigma_m = C_\infty (1 - \sigma_{pic}/S_r) \varepsilon^p{}_m$. The Fig. 4,5,6,7,8 show simulation of different experimental phenomena. The expression used are:

$$\alpha = g(\tilde{\lambda})\ (S_r \varepsilon^p - \sigma_{pic} \varepsilon^p{}_n) \quad , \quad R = f(\tilde{\lambda})\ R(|\varepsilon^p - \varepsilon^p{}_n|) \quad \text{where } \tilde{\lambda} = \lambda(1 - \sigma_{pic}/S_r)$$

$$g(\tilde{\lambda}) = g_\infty + g_1 \exp(-a\tilde{\lambda}) \quad , \quad f(\tilde{\lambda}) = 1 - m \exp(-a\tilde{\lambda})$$

with $R>0$, $R'>0.$, $R''<0$, $g_\infty > 0$, $g_1 > 0$, $0<m<1$, $a>0$

REFERENCES

[1] Chaboche,J.L. Time Independent Constitutive Theories For Cyclic Plasticity ,International journal of plasticity, Vol. 2, Num 2, pp.149-188.

[2].Taheri,S.,(1989). Une Loi de Comportement Uniaxial Cyclique Avec Variable à Memoire Discrète. 9° Congres Francais De Mécanique Metz France.

[3] Taheri,S.,(1990). Une Loi de Comportement Uniaxial En Elastoplasticité Pour le Chargement Cyclique, Note Interne E.D.F/D.E.R/HI-71/6812.

[4] Andrieux,S.,Taheri,S.,(1991) to be published.Plasticite 91 Grenoble France

[5] Pellissier-Tanon,A.,Bernard,J.L, Amzallag,C., Rabbe,P.(1980) Evaluation of the resistance of type 31$ stainless steel against progressive deformation. Int SYm. on low-cycle fatigue and life prediction France.

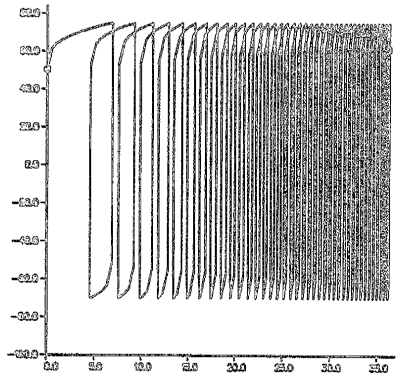


Fig. 4 Elastic shakedown

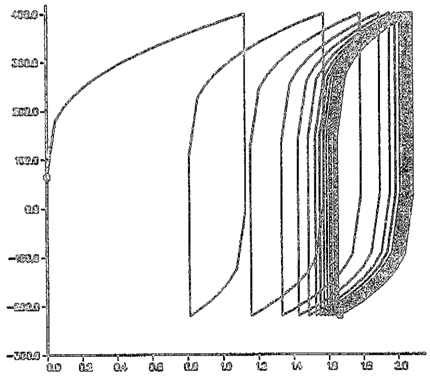
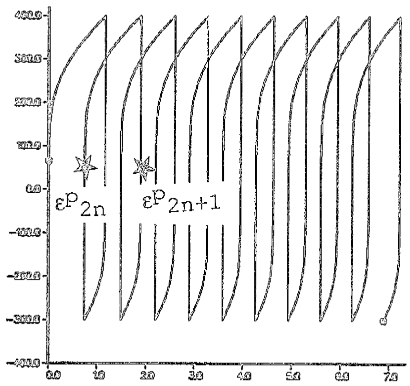


Fig.5 Plastic shakdown



Fif.6 Ratchetting

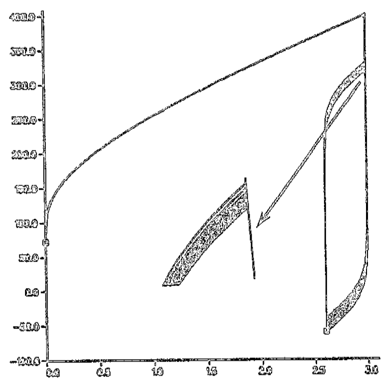


Fig. 7 Cyclic softening.

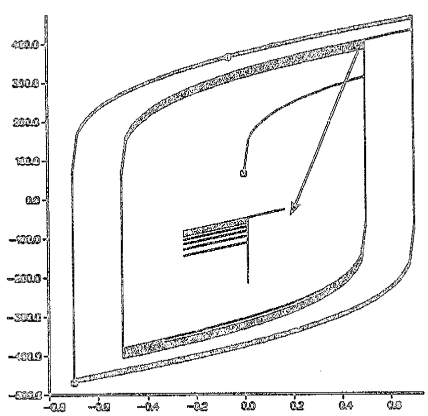


Fig. 8 Cyclic hardening.