Proposal for substituting the factor K_e of the ASME Code for structures under thermal loads

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ABSTRACT

The plastic strain concentration factor K_e of the ASME Code is usually overly restrictive for fatigue problems caused by cyclic thermal loads. Alternative factors are derived on the basis of a cantilever beam under displacement-controlled loading. It is shown that they provide upper bounds for many practical design situations, and thus are proposed for substituting the ASME approach.

1 INTRODUCTION

Fatigue analyses are usually performed on the basis of a purely elastic material model, even if plastic material behaviour is involved. In order to preclude a possible underestimation of the fatigue damage, the fictitiously elastically calculated strain range is multiplied by a plastic strain concentration factor before entering the fatigue curve. The ASME B&PV Code (1986), just as other design codes, provides such factors, called K_e. However, it is well known that the approach of the ASME Code has some shortcomings in so far as it is unsafe in certain cases, like moderately stressed notches, and overly conservative in others, especially if the loading is thermally induced.

2 DIFFERENT TYPES OF THERMAL LOADS

Three different types of thermal loading, frequently encountered in designing nuclear power plants, may be distinguished: a temperature gradient through the wall, a temperature gradient along the length, and constrained mean thermal expansion of a structure. Structures subjected to temperature gradients can typically be idealized as plates or cylindrical shells of any geometric relations and boundary conditions. The plastic behaviour of circular cylindrical shells loaded by any temperature gradient will be traced in the following.

3 CYLINDRICAL SHELL UNDER RADIAL TEMPERATURE GRADIENT

A temperature gradient, applied axisymmetrically through the wall of a cylindrical shell, represents an equibiaxial strain-controlled loading

remote from the boundaries. Plastic strain concentration is originated only by the different Poisson's ratios due to elasto-plastic as compared to purely elastic behaviour. Hübel (1987) has derived plastic strain concentration factors in closed form, depending on stress level and hardening parameters, and has demonstrated that they never exceed 1.40 (or 1.61, depending on the definition of effective strain) for materials with elastic Poisson's ratios of $\nu = 0.3$.

4 CYLINDRICAL SHELL UNDER AXIAL TEMPERATURE GRADIENT

For an axisymmetric axial temperature gradient, however, besides stress level and material properties, the plastic strain concentration factor depends on many different parameters like the shape of the gradient, the ratio of wall thickness to mean radius, the length and the boundary conditions of the shell, and is therefore much more complicated to derive.

The ratio of wall thickness to mean radius does not become an effective parameter if the cylindrical shell is thin-walled. But even for increasing wall thickness, where shear deformation is no longer negligible, this parameter may conservatively be disregarded, because shear tends to relieve the plastic strain concentration at the surface of the wall.

With respect to the shape of the gradient, a maximum of stresses is certainly caused by a steplike change of temperature.

4.1 Long cylindrical shell

If the cylinder is sufficiently long to allow us to disregard boundary conditions, the maximum stresses and plastic strains are found exactly at the section of the temperature step, where axial bending is prevented. If mean thermal expansion is admitted freely in axial direction, the stress state is uniaxial in that section. Moreover, due to the antisymmetrically distributed loading, the hoop membrane strain is known, independent of the material model (if material properties are temperature-independent), and thus, the loading is uniaxial strain-controlled.

The plastic strain concentration factor therefore equals unity at the highest stressed location of a long cylindrical shell subjected to an axial steplike change of temperature. However, the plastic strain concentration factor may exceed unity in sections adjacent to this one, although the plastic strains themselves will never exceed those at the temperature step.

So far we have disregarded boundary conditions. But severe axial temperature gradients are often associated with stiffness differences, an extreme example of which is a cylinder, which is flanged rigidly, thus preventing axisymmetric rotation in case of a temperature difference between flange and cylinder. Hübel (1987) has derived plastic strain concentration factors by Finite Element analyses which reflect this boundary condition by applying a radial displacement at the clamped end of a long cylindrical shell.

4.2 Short cylindrical shell

Reducing the length of the cylinder now, the boundary conditions at the other end become increasingly important.

If we allow the shell to deform freely at this end, the hoop membrane

strains will dominate more and more over the axial bending strains, resulting in lower plastic strain concentration factors than derived for the long cylinder, and asymptotically reaching the ring solution. A radially expanded ring is an uniaxially stressed, kinematically determinate structure, the plastic strain concentration factor of which is unity.

On the other hand, if we prevent radial deformation and axisymmetric rotation at this end, the loading is increasingly carried by axial bending while shortening the cylinder, causing higher plastic strain concentration factors than a long shell does, tending towards the behaviour of a cantilever beam with solid rectangular cross-section under displacement-controlled loading.

This is illustrated in Fig. 1, where the effects of the length of the cylinder and of different boundary conditions are shown for some examples (assuming elastic-perfectly plastic material) by plotting the plastic strain concentration factor K_ε versus the fictitiously elastically calculated effective stress $\sigma^{f.el}$, normalized by the yield stress σ_y.

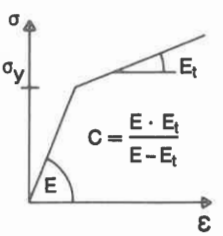
5 CANTILEVER BEAM UNDER DISPLACEMENT-CONTROLLED LOADING

The highest stressed location of a cantilever beam with solid rectangular cross-section, subjected to a displacement-controlled single load, provides an upper bound for the plastic strain concentration factor of the highest stressed location of a circular cylindrical shell of arbitrary geometric relations and boundary conditions, subjected to an axisymmetric axial temperature gradient of any shape. (This is not necessarily true for lower stressed locations, where the plastic strain concentration factors may be even higher, although the plastic strains themselves remain smaller than at the location of maximum stress).

The plastic strain concentration factor for this cantilever beam, K_ε, can be derived by purely analytical means and may be expressed in parameter form (parameter λ) in terms of material data and of the normalized fictitiously elastically calculated stress level, $\sigma^{f.el}/\sigma_y$.

In case of a bilinear stress-strain relation (hardening parameter C/E) we get

$$(1)\quad \begin{cases} K_\varepsilon = \lambda \dfrac{\sigma_y}{\sigma^{f.el}} \\[2ex] \dfrac{\sigma^{f.el}}{\sigma_y} = \dfrac{\frac{20}{9} - \frac{2}{\lambda} + \frac{2}{9\lambda^3} - \frac{1}{9}\frac{C}{E} + \frac{C}{E}\lambda^2 + \frac{4}{9}\left(\frac{C}{E}\right)^2\lambda^3 + \frac{2}{3}\frac{C}{E}\ln\lambda}{\left[1 - \frac{1}{3\lambda^2} + \frac{2}{3}\frac{C}{E}\lambda\right]^2} \end{cases}$$



matching the results obtained by Radomski and White (1968) in case of elastic-perfectly plastic material (C/E = 0).

6 PROPOSAL FOR PRACTICALLY APPLICABLE FACTORS K_ε

6.1 Global structural behaviour

In a concrete design situation, specific plastic strain concentration factors for the particular structure and kind of loading at hand (such as those briefly discussed in chapters 3 and 4) are usually not available to the designer. In this case, the upper bound solution provided by

the cantilever beam, eq. (1), is to be employed. The advantage of the bilinear stress-strain model used there, is that it can easily be adapted in some way to the real cyclic behaviour of any material, relating a specific yield stress range $2\sigma_y$ and hardening parameter E_t (or C) to each value of the elasto-plastic strain range, which in turn can be expressed as a product of the fictitiously elastically calculated peak strain range S_p/E and the plastic strain concentration factor K_ε. By virtue of eq. (1) we can then plot K_ε for the axial temperature gradient as a (usually temperature-dependent) function of S_p.

In order to cover the radial temperature gradient according to chapter 3 too, the plastic strain concentration factors given by Hübel (1987) are to be evaluated in the same way. If they exceed those derived by eq. (1), which is likely for moderate stress levels, only the higher of the two shall be considered further.

6.2 Notch effects

So far, we have only considered the global behaviour of structures like cylindrical shells and cantilever beams, and have disregarded local plastic behaviour arising from notch effects, which may exist at the intersection of a cylindrical shell and a flange, for example.

A method, widely used to quantify plastic strain concentrations at notches, is Neuber's method, stating that the product of stress and strain is independent of the uniaxial stress-strain relation. Introducing the bilinear material model we get the plastic strain concentration factor

$$(2) \quad K_\varepsilon = -\frac{1}{2}\frac{E}{C}\frac{\sigma_y}{\sigma_{f.el}} + \sqrt{\frac{1}{4}\left(\frac{E}{C}\frac{\sigma_y}{\sigma_{f.el}}\right)^2 + 1 + \frac{E}{C}}$$

which may be higher or lower than K_ε for the cantilever beam, eq. (1). However, after adaptation to real material data, eqs. (1) and (2) often amount to similar values. If this can be validated for the particular material of interest, it appears sufficient from a practical point of view (although theoretically not justified), to only use either eq. (2) or the curve derived in chapter 6.1 for design purposes, whichever yields higher K_ε values. We then are relieved of discussing the complicated interaction of eqs. (1) and (2).

This procedure also avoids the potential unconservatism of the ASME approach, which is based on the secondary stress range S_n, neglecting plastic strain concentration due to peak stresses at notches.

7 EXAMPLE

The procedure outlined above is applied to a German austenitic stainless steel, similar to AISI 304 ss, the cyclic material data of which are bilinearized by the 10th cycle representation proposed by ORNL (1974).

The plastic strain concentration factors K_ε, derived according to the proposal presented in chapter 6, are usually some temperature-dependent function of S_p. In this particular example, however, K_ε can be represented very well by a single straight line in the entire temperature range between 450 °C and 650 °C and by another straight line for 20 °C, for strain ranges above 0.4 %. For lower strain ranges, cyclic material data are missing. But we know that K_ε starts to exceed unity as S_p exceeds twice the monotonic proportional limit. In order to fill the gap

in between, the straight lines are arbitrarily extrapolated down to four times the proportional limit. For values of S_p between two and four times this proportional limit, K_ε shall be gained by linear interpolation, which is felt to be a conservative assumption.

K_ε is plotted versus S_p in Fig. 2, along with the factor K_e of the ASME Code, for comparison.

The experimental results obtained by Radomski and White (1968) for the three-point-bending case of stainless steel roughly confirm the plastic strain concentration factor K_ε of Fig. 2.

8 CONCLUSION

The plastic strain concentration factors for the highest stressed location of a cantilever beam with solid rectangular cross-section have been shown to provide an upper bound for the highest stressed location of a circular cylindrical shell of any geometric relations, subjected to any axisymmetric axial temperature gradient.

Moreover, the specific behaviour of a cantilever beam indicates that these factors might be appropriate for any structure under any kind of displacement-controlled loading, if this load is mainly carried by bending of solid rectangular cross-sections. For example, piping systems loaded by anchor movements or mean thermal expansion, where the maximum stresses occur at the elbows in hoop direction, show this feature. In contrast to this, the factors derived herein may not be applied readily to other shapes of the cross-section, like an I-profile for example. Likewise, plastic redistributions of strains are different if the deflection-controlled loading is carried by membrane forces.

Nevertheless, in designing boilers and pressure vessels subjected to cyclic thermal loads of any kind, a considerable margin may be gained by substituting the K_e-approach of the ASME Code by the maximum of the plastic strain concentration factors for the cantilever beam, for the radial temperature gradient and for notches, adopting Neuber's method in order to supersede the potential unconservatism of the ASME Code with respect to notches.

However, these factors are not suitable for all practical design situations, especially not if substantial mechanical loads contribute to the stress range. Future establishment of a catalogue of similar factors for different kinds of structures and types of loading may help to provide plastic strain concentration factors for these design situations too.

9 REFERENCES

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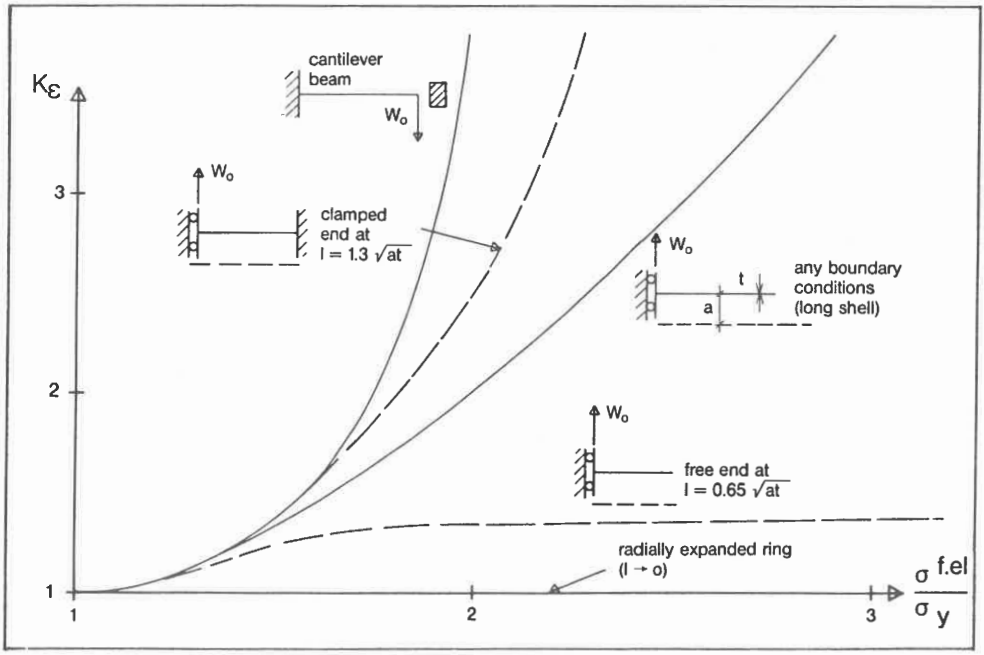

Figure 1. Plastic Strain Concentration Factors for Cylindrical Shells of Different Lengths l and Boundary Conditions; Perfectly Plastic Material Model, Poisson's Ratio $\nu = 0.5$.

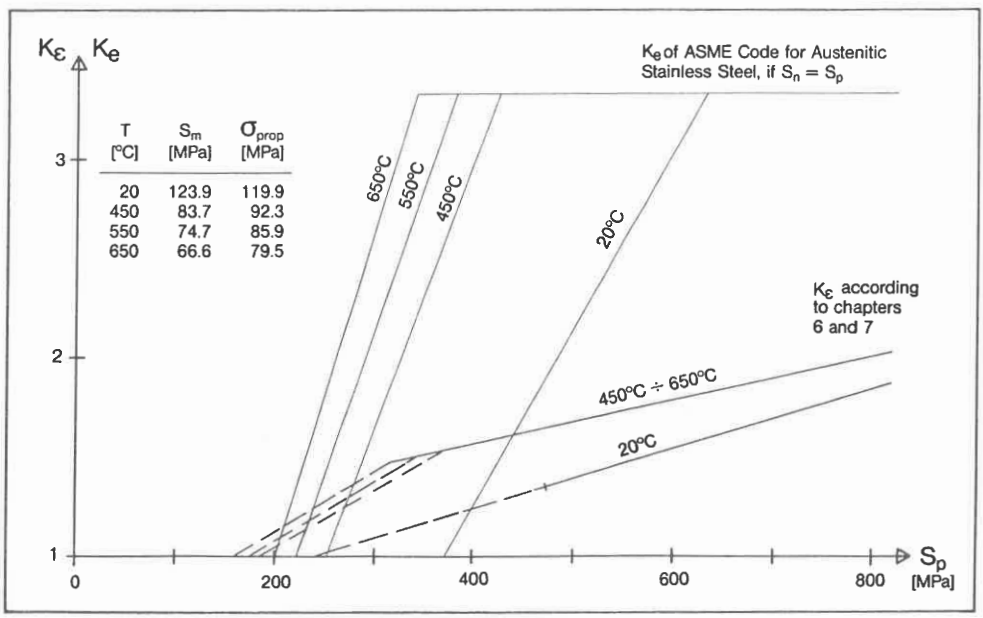

Figure 2. Plastic Strain Concentration Factors as Proposed in this Paper Compared to K_e of the ASME Code, German Austenitic Steel Similar to AISI 304 ss.