

THE EFFECT OF CREEP RATCHETTING ON THIN SHELLS

R. C. HIBBELER

Department of Civil Engineering, Union College, Schenectady, N.Y. 12308, U.S.A.

P. Y. WANG

*Reactor Analysis and Safety Division,
Argonne National Laboratory, Argonne, Illinois 60439, U.S.A.*

SUMMARY

The purpose of this paper is to study the behavior of thin shells, in particular, cylindrical and spherical shells, which are subjected to a long-time cyclic thermal gradient. Like many reactor components (shells) which are subjected to start-up and shut-down conditions, provided the temperature is high enough, the shell will exhibit a progressive growth with each cycle of temperature. This phenomena is often referred to as ratchetting and is caused by inelastic strains developed by creep. Although the thermal stress distribution is biaxial, it is possible to represent the material behavior using a simple uniaxial-stress model. Assuming thermal stress interaction occurs, the equations which determine the solution of the strain growth and stress per cycle are presented. The flexibility of the analysis provides a means for including the effects of an arbitrary temperature-cycle time and temperature dependence of material properties. A step temperature variation is considered.

For the range of stress and temperature, the creep rate is accurately approximated by the relation $\dot{\epsilon} = e^{-A/T} \sigma^m t^{-n}$, where T is the absolute temperature and A , B , m , and n are material constants. During each part of the temperature cycle it is necessary to satisfy the equilibrium and compatibility conditions for the model. At any instant, the total strain will depend upon elastic, thermal, and creep strain components in addition to prior inelastic creep strains accumulated during previous temperature cycles. Accounting for all these conditions, the relations describing the behavior of the material can be determined during an arbitrary j th cycle of temperature. In particular, the cases of material properties are considered which are used for reactor components. Where possible, a closed form solution is given for appropriate values of the creep law exponents n and m . For the general case, an algorithm for the computer-solution to the problem is given. Using the general solution, the analysis appears to offer a suitable compromise between accurate behavior description and analytical complexity.

Quite often reactor components in the form of thin cylindrical or spherical shells are subjected to cycles of temperature caused by "on-off" reactor conditions or power fluctuations. If the temperatures are high enough inelastic strains can be produced during each temperature cycle, and these, because of their irreversible nature, have a cumulative effect which causes progressive growth of the shell.

Several analytical models have been proposed to study the ratchetting behavior of materials [1,2,3]. For the study presented in this paper, a simple two-bar model is used [4,5]. The model essentially consists of two-bars A and B, having different material properties but equal cross-sectional areas. The bars are jointly attached to a fixed wall and weightless rigid bar guided by a roller mechanism. Such a model is useful for predicting the creep behavior of the thermal stress occurring across the shell thickness, since, due to symmetry, the thermal loading will cause uniform radial growth of the shell. Consequently, the model preserves this uniform strain behavior by utilizing the rigid bar attachment.

The shell material is assumed to follow a creep law having a form [6] of $\dot{\epsilon}_c = K\sigma^m t^{-n}$. At a given temperature, the thermal strain αT and creep constant K may vary throughout the shell thickness. It is assumed that each half-thickness of the shell has constant properties. Consequently, a correspondence of the material behavior is preserved in the model by requiring bars A and B to have the same material properties as a corresponding shell half-thickness. In the foregoing analysis, a uniform stepped-temperature cycle is considered so that recurrence relations can be established. (Only small alterations would be required for variable temperature cycles.)

The first part of any temperature cycle has a duration of t_1 . The material properties for both bars at this temperature consist of the modulus of elasticity E , and creep constants m and n . Each bar has a unique thermal strain $(\alpha T)_A$ and $(\alpha T)_B$, and creep constants K_A and K_B . During the second part of the cycle, t_2 , primed constants apply, i.e., E' , $(\alpha T)'_A$, $(\alpha T)'_B$, K'_A , and K'_B .

For the analysis, equilibrium requires that the stress in one bar be equal but opposite to the stress in the other, i.e., $\sigma_A = -\sigma_B$. Furthermore, the bar strains must be compatible, $\epsilon_A = \epsilon_B$. At a given instant the strain is defined as the sum of elastic strain, thermal strain, on-going creep strain, and cumulative creep strain, i.e., $\epsilon = \frac{\sigma}{E} + \alpha T + \epsilon_c + \Sigma \epsilon_c$, where again, $\dot{\epsilon}_c = K\sigma^m t^{-n}$. By successively applying these equations at each cycle, recurrence relations can be established to determine the stress and strain. The method of analysis is outlined in Ref. [4].

For the situation considered here, the relations for σ_B and ϵ_B during the j th cycle are given as follows:

First part of j th cycle

Stress

$$\sigma_B = \left[ct^{1-n} + \frac{1}{\sigma_{Bj}} (1-m) \right]^{\frac{1}{1-m}} \quad (1)$$

Strain

$$\epsilon_B = \frac{\sigma_B}{E} + (\alpha T)_B + K_B \left[\sum_{i=1}^{j-1} \Gamma_i (T_1, \sigma_{B_i}^1) \right] + K'_B \left[\sum_{i=1}^{j-1} \Gamma'_i (t_2, \sigma_{B_i}^2) \right]$$

Second part of *j*th cycle

Stress

$$\sigma_B^i = \left[c' t^{1-n} + \sigma_{B_j}^2 (1-m) \right] \frac{1}{1-m} \quad (3)$$

Strain

$$\epsilon_B^i = \frac{\sigma_B^i}{E'} + (\alpha T)_B^i + K_B^i \Gamma_j(t, \sigma_{B_j}^2) + K_B \left[\sum_{i=1}^j \Gamma_i(t_1, \sigma_{B_i}^1) \right] + K_B^i \left[\sum_{i=1}^{i-1} \Gamma_i(t_2, \sigma_{B_i}^2) \right] \quad (4)$$

where

$$C = E \left(\frac{\pm K_A - K_B}{2} \right) \left(\frac{1-m}{1-n} \right) \quad (5)$$

$$c' = E' \left(\frac{\pm K_A^i - K_B^i}{2} \right) \left(\frac{1-m}{1-n} \right) \quad (6)$$

$$\Gamma_k(t, \sigma_{B_k}^1) = \int_0^t \left[c t^{1-n} + \sigma_{B_k}^1 (1-m) \right] \frac{m}{1-m} t^{-n} dt \quad (7)$$

$$\Gamma_k(t, \sigma_{B_k}^2) = \int_0^t \left[c' t^{1-n} + \sigma_{B_k}^2 (1-m) \right] \frac{m}{1-m} t^{-n} dt \quad (8)$$

$$\begin{aligned} \sigma_{B_j}^1 &= \left[\frac{(\alpha T)_A - (\alpha T)_B}{2} \right] + \left(\frac{\pm K_A - K_B}{2} \right) \left[\sum_{i=1}^{i-1} \Gamma_i(t_1, \sigma_{B_i}^1) \right] + \\ &+ \left(\frac{\pm K_A^i - K_B^i}{2} \right) \sum_{i=1}^{i-1} \Gamma_i^i(T_2, \sigma_{B_i}^2) \end{aligned} \quad (9)$$

$$\begin{aligned} \sigma_{B_j}^2 &= \left[\frac{(\alpha T)_A^i - (\alpha T)_B^i}{2} \right] + \left(\frac{\pm K_A - K_B}{2} \right) \left[\sum_{i=1}^j \Gamma_i(t_1, \sigma_{B_i}^1) \right] \\ &+ \left(\frac{\pm K_A^i - K_B^i}{2} \right) \left[\sum_{i=1}^{i-1} \Gamma_i^i(T_2, \sigma_{B_i}^2) \right] \end{aligned} \quad (10)$$

For $\Gamma_k(t_1, \sigma_{B_k}^1)$, $\Gamma_k^i(t_2, \sigma_{B_k}^2)$ change the limit on the integrals defined in eqs. (7) and (8) to t_1 and t_2 respectively. When *m* is an even number choose the plus sign (+) for K_A and K_A^i , in eqs. (5), (6), (9), and (10). If *m* is odd, the negative sign (-) is used.

Closed form application of these equations can be obtained provided the integrals of eqs. (7) and (8) can be determined. In particular, for *m*=0, the creep law is of the form $\dot{\epsilon}_c = Kt^{-m}$, which is useful for describing transient creep. Here $\Gamma_k = t^{1-n}/(1-n)$. For *m*=1, logarithmic-exponential solutions arise and the analysis must be performed on this basis.

For values of *m*=2, *n*=1/2, then

$$\Gamma_k = 2\sigma_{B_k}^2 t^{1/2} / (c\sigma_{B_k}^1 t^{1/2} + 1)$$

If $m=2$, $n=1/3$,

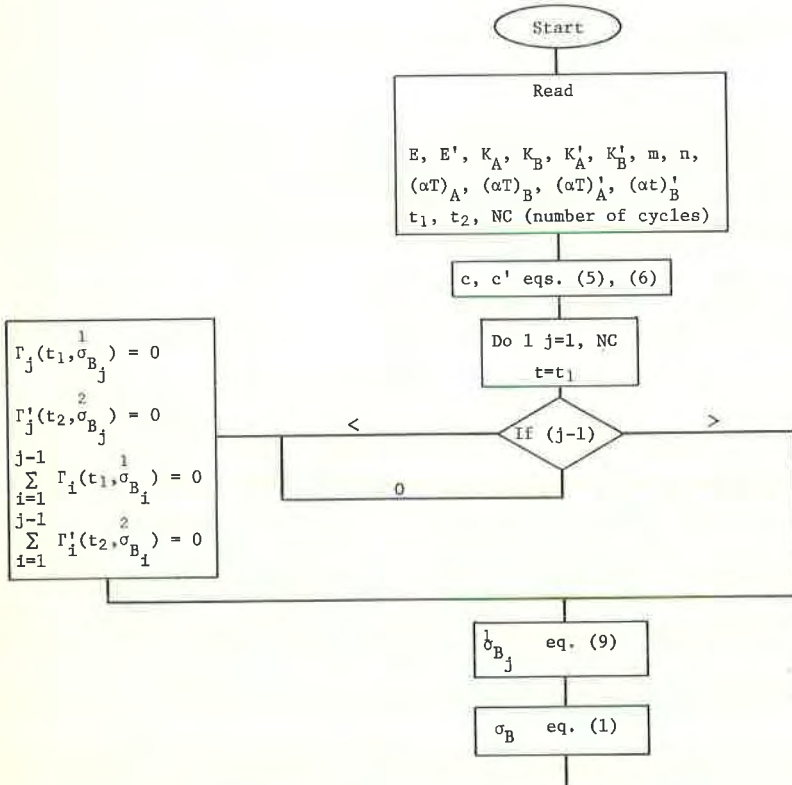
$$\Gamma_k = 3\sigma_{B_k}^2 t^{2/3} / \left[2(c\sigma_{B_k}^1 t^{2/3} + 1) \right]$$

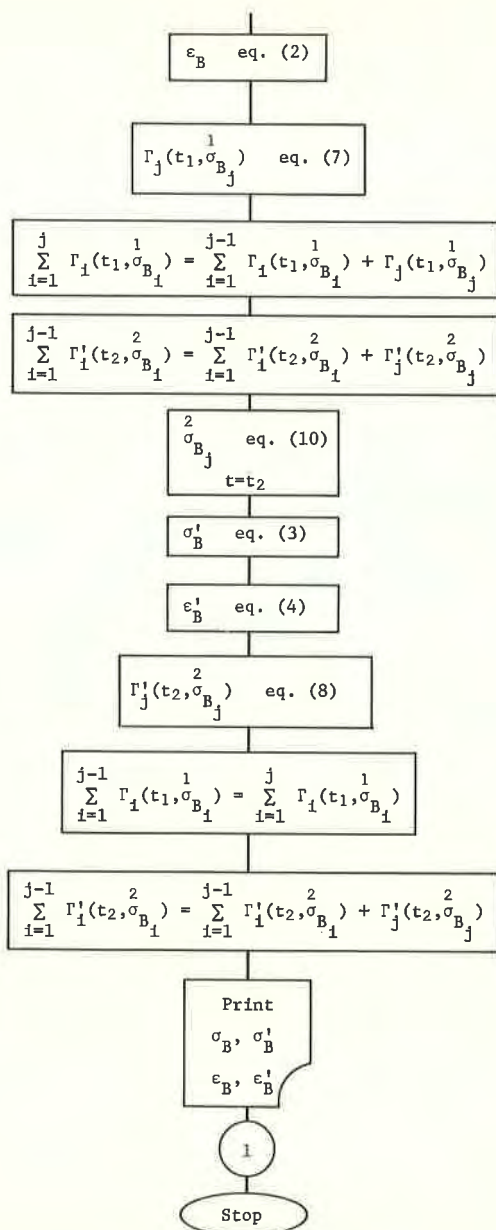
For most cases of metals, however, the value of m is very high. For example, for SAE 1035 steel the creep law is of the form [6]

$$\dot{\epsilon}_c = 6.03 \times 10^{-15} \exp(-4.768 \times 10^4/T) \sigma^{6.253} \epsilon^{-0.3329}$$

where T is the absolute temperature in degrees Rankine. This larger value of m (≈ 6) necessitates the use of numerical integration to obtain values of eqs. (7) and (8). For such cases, the solution can be computerized. A flow diagram outlining the procedure is given in fig. (1).

Depending upon the input data, computer output can describe the stress and strain in the shell at any given instant. Characteristic creep-step curves are formed which are similar in shape to those given in ref. [4]. For this study, however, a more general creep law has been utilized, and hence, the analysis provides a better description of material behavior.





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