## ABSTRACT

KAY, WARREN DAVID. Using Bin Packing Optimization to Investigate the Transportation Challenges Impeding School Start Time Changes in Wake County Public Schools. (Under the direction of Brandon M. McConnell and Kristin A. Thoney-Barletta).

In 2014 the American Academy of Pediatrics (AAP) recommended later school start times (after 8:30 AM) for adolescents to promote their health and academic success. Wake County Public School System (WCPSS) is the largest school district in North Carolina, with nearly 200 schools and over 160,000 students, and all its middle school and high school students start school at 8:30 AM or earlier. In the 2023–2024 school year, a national bus driver shortage exacerbated daily student transportation challenges and delayed the school board's ability to prioritize school start time changes. This study uses a mathematical optimization model following a bin packing approach to minimize the number of buses required to complete all the routes in the nine sub-districts within Wake County. Scenarios that changed the school start times to abide by AAP recommendations were generated and tested using bus route information from the 2023–2024 school year. Preliminary results show that areas with a higher concentration of magnet and special schools require a greater increase in the number of buses needed to swap elementary and high school start times. The results also indicate significant interactions between the sub-districts, which challenge WCPSS's ability to logistically isolate a sub-district to use as a case study for start time changes. This research investigates the current relationship between transportation requirements and school start times. It provides analysis that could inform decision-making and planning as WCPSS hires more bus drivers and pursues the best outcomes for its students.

Using Bin Packing Optimization to Investigate the Transportation Challenges Impeding School Start Time Changes in Wake County Public Schools

by
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# DEDICATION

To the countless math teachers in my life who have intentionally poured into their students' lives under the guise of the humble title of "Mathematician."

# BIOGRAPHY

Captain Warren Kay grew up in Goshen, Indiana, before attending the United States Military Academy. He graduated in 2016 and was commissioned as a second lieutenant in the Engineer Regiment of the United States Army. He has served in platoon and company-level positions within a Stryker Brigade Combat Team at Joint Base Lewis-McChord, Washington, and as a company commander for a Clearance Company at Fort Johnson, Louisiana. Upon completing his academic requirements at North Carolina State University, he will join the Department of Mathematical Sciences faculty at the United States Military Academy at West Point.

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To Kevin Hart — thank you for taking the time to explain and demonstrate TIMS. Your work continues to improve transportation efficiency across North Carolina.

To my wife and children — you have endured difficult seasons in order to be able to enjoy the past two years together. Thank you for letting me be a student again while also always being ready to make the most of our opportunities here.

**Disclosures:** The views expressed in this paper are those of the author and do not reflect the official policy or position of the United States Army, the Department of Defense, or the United States Government. The author used Grammarly for spelling, phrasing, and grammar assistance. The author used ChatGPT and Microsoft CoPilot for assistance in scraping the publicly available bus route data for Wake County Public Schools, designing and debugging functions, fixing LaTeX code, and making LaTeX tables. The author validated all outputs to ensure correctness.

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CHAPTER

# 1

# INTRODUCTION

Wake County Public School System (WCPSS) provides public education for over 160,000 students in the area surrounding Raleigh, North Carolina, making it the largest school district in the state of North Carolina (Wake County Public School System 2024e; NC Department of Public Instruction 2024). The rapid student population growth in parts of Wake County caused 25 of the district's 199 schools to reach their capacity during the 2023–2024 school year (Wake County Public School System 2024e). Although WCPSS is a single Local Education Agency (LEA), it is further divided into nine districts as depicted in Figure 1.1, each of which elects a single school board member to serve on the WCPSS school board (Wake County Public School System 2024b). Funding and resources, namely school buses, are shared across the entire county and are allocated to meet the collective needs of the unique districts.

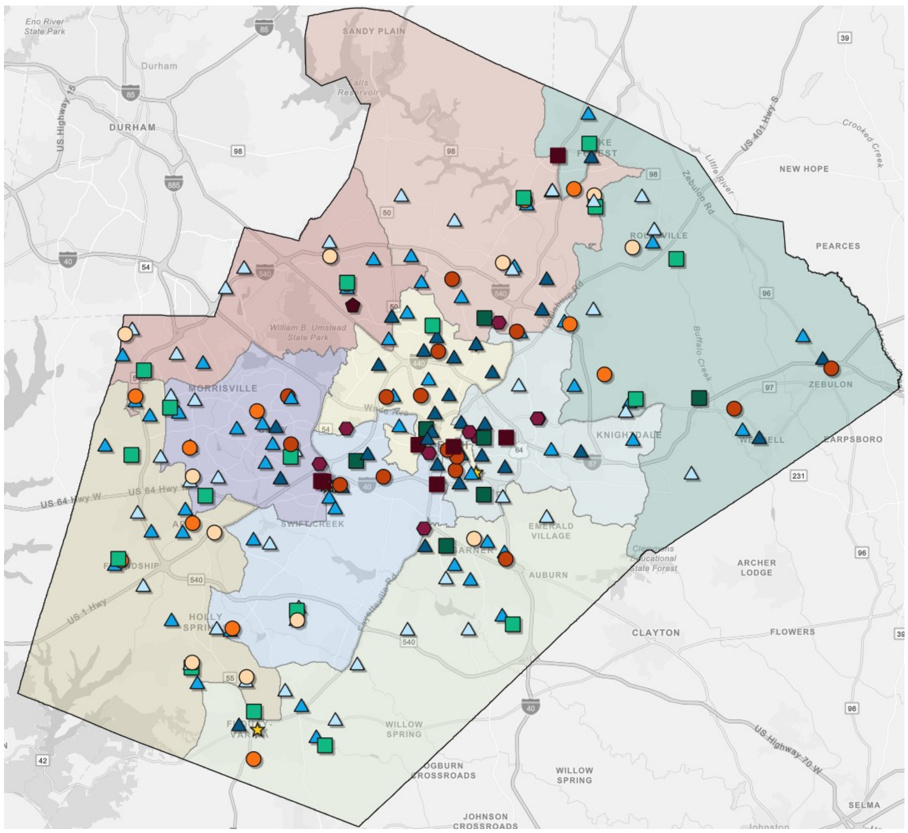


Figure 1.1: Map of WCPSS with sub-district borders and facility locations (Wake County Public School System 2024a).

## 1.1 Background

### Transportation Information Management System

Since 1992, as required by North Carolina law, all school districts in North Carolina have been using the Transportation Information Management System (TIMS) to improve the efficiency of their school bus transportation (NC Department of Transportation 2024a). While the exact method and algorithms used by Education Logistics, or Edulog (the software vendor) are private, the system's ability to improve school bus routing and scheduling is evident in the numerous case studies available on the TIMS website (NC Department of Transportation 2024b). When considering the size and the complexity of WCPSS, it is clear that TIMS offers advantages to district transportation planners who previously had to rely heavily on experiential knowledge and manual scheduling.

### Historical Relationship Between Transportation and School Start Times in WCPSS

A common practice that allows school districts to maximize the number of routes that a single bus can complete is the use of a tiered busing system. In an efficient three-tier system, a bus could theoretically complete at least one elementary school, one middle school, and one high school route in a single morning. Owens et al. (2014) note that every district that changed school start times adopted a tiered busing system as part of their implementation strategy. Wake County Public School System has utilized a three-tiered system since 1995, nearly two decades before much of the research advocating for later high school start times was published (Chou and Thomas 2017). Additionally, the transportation department in WCPSS has been responsible for proposing school start times for school board approval for at least a decade before the school system considered significant start time changes in 2022–2023 (Chou and Thomas 2017). Such transportation operating procedures have allowed WCPSS to manage a large increase in student population over the last 30 years. As a result of the procedures adopted by Wake County, the savings discovered by other school districts that changed their start times may not be realized by Wake County if it changes its start times since the savings achieved typically occurred in districts that implemented a tiered busing system *after* changing school start times.

## School Bus Driver Shortage

Since 2010, there has been a general decline in the total number (public sector and private sector) of school bus drivers nationwide (Economic Policy Institute 2023). A 2021 survey by the National Association for Pupil Transportation, the National Association of State Directors of Pupil Transportation, and the National School Transportation Association confirmed that the significant loss of school bus drivers following the COVID pandemic was felt across the country; approximately two-thirds of the districts that responded identified the bus driver shortage as their number one concern (National Association of State Directors of Pupil Transportation Services 2021). Near the start of the 2023–2024 school year, WCPSS stated that this nationwide shortage translated to a local shortage of 17 bus drivers, which would affect 2,000 students in its district (ABC11 News 2023). In a 2018–2019 report, Wake County was already the highest-ranked LEA in North Carolina for the average number of routes that its bus drivers completed daily (NC Department of Public Instruction 2019). The solution to the bus driver shortage in WCPSS involved adding *more* routes per bus driver and scheduling some routes to deliver students after their school's start time (ABC11 News 2023; WRAL News 2024).

## WCPSS Start Time Proposals

As the increase in student population in Wake County has created the need for more schools in WCPSS, the school board has had opportunities to refine its processes for establishing bell schedules or school start times. In April 2016, the WCPSS school board established start times for five new schools and approved changes to the bell schedules of six existing schools after hosting an online discussion forum and a series of meetings (Spectrum News Staff 2016). Although comments from parents offered inconclusive feedback, school board members emphasized the tiered busing schedule and the need to be as efficient as possible with funding as decisive factors for the changes (Spectrum News Staff 2016).

At a Student Achievement Committee Meeting in September 2022, WCPSS school board members reviewed a WCPSS-internal 2021 literature review focused on early school start times and a 2020 journal article by Lenard et al. (2020) to stimulate the consideration of a new bell schedule for the entire district. The meeting reviewed three proposed bell schedules: (1) shift all three tiers to a later start time for all levels, (2) swap elementary and high school tier assignments, and (3) swap elementary and high school tier assignments and consider additional time shifts for all three tiers (Wake County Board of Education

2024a). Additionally, a timeline was presented, which involved collecting stakeholder input through surveys, focus groups, and town halls from September 2022 through March 2023, culminating with the goal of adopting a well-researched and efficacious 2024–2025 WCPSS bell schedule in April 2023 (Wake County Board of Education 2024a). Prior to the April 2023 decision, the WCPSS school board reviewed the survey results of 12,951 parents, 3,423 school employees, and 1,200 students and found that the feedback was once again inconclusive (The News & Observer 2023). The consensus from the survey was that respondents connected to high school students were supportive of moving to a later start time, respondents connected to elementary school students did not want to move to an earlier start time, and WCPSS did not have enough bus drivers to condense the district's tiered schedule to satisfy the stakeholders in both groups (WRAL News Staff 2023b). Thus, WCPSS completed the 2023–2024 school year and started the 2024–2025 school year without the school board making any changes to the bell schedule, citing the importance of the transportation situation by saying that the "transportation piece is so critical that we have to get everything in line first" (WRAL News Staff 2023a).

## 1.2 Recent Efforts in Relevant Districts

While there is an ongoing effort to collect data from school districts across the country that have changed their school start times, Owens et al. (2014) present a study of 34 districts that changed their high school start times before 2015. From considering the ten key messages offered in Owens et al. (2014)'s blueprint for successful school start time changes, we find Boston Public Schools (BPS) and Durham Public Schools (DPS) present recent and relevant efforts for shaping our investigation of WCPSS.

### 1.2.1 Boston Public Schools

To appreciate the challenge that WCPSS faces, one must account for the size and complexity of the school district. Of the 14 school districts in the United States that are larger than WCPSS, six are in states where laws mandate high school start times no earlier than 8:30 AM (California and Florida) (National Center for Education Statistics 2019). This makes WCPSS one of the country's largest school districts that would attempt to change its high school start times without a state mandate forcing the change. Additionally, WCPSS offers a host of magnet and special schools that add complexity when determining the transportation dependencies across school levels and sub-district boundaries. In 2016, a group of researchers from the Massachusetts Institute of Technology used a mathematical model to address goals for BPS that were nearly identical to the ones the WCPSS school board holds for its district. For the Boston district of 54,000 students and 125 schools, the researchers developed a model that could construct bus routes to improve efficiency and modify school start times to align with the research recommending later start times for high school students (Bertsimas et al. 2020). The results demonstrated the potential for substantial savings ($12 million) in annual transportation costs while also shifting almost all (94%) of its high school students to the desired starting time (Bertsimas et al. 2020). Considering that the transportation situation in WCPSS appears to be one of the deciding factors for school start time changes, one might think that a solution that can satisfy these competing goals, if available, would be quickly approved in a school district. However, the researchers noted that their solution would have changed 85% of the schools' start times with a median change of one hour, which generated enough resistance among parents that Boston Public Schools chose not to implement the proposed solution (Bertsimas et al. 2020). To account for the outcome in BPS, our investigation of WCPSS considers scenarios that change the minimum number of school start times. These scenarios may be more appealing for parents and students across the district, although they do not minimize the potential transportation costs.

### 1.2.2 Durham Public Schools

In addition to the logistical challenges that WCPSS faces, the results of the 2023 survey indicate that WCPSS faces a formidable challenge in uniting stakeholders tied to elementary school students with stakeholders tied to high school students. Aside from a general resistance to change, one explanation for the lack of support for changing school start times is WCPSS's bordering school district to the northwest, DPS. In 2016, Durham Public Schools moved all its high schools to 9:00 AM or 9:15 AM start times, citing the same reason for change as WCPSS identified in its Student Achievement Committee Meeting in September 2022 (Durham Public Schools 2024). The following year, a survey conducted by the Education Policy Initiative at Carolina found that 60 percent of elementary and high school parents believed the change had a negative impact on children and families (Herald-Sun 2017). Five years later, the DPS Board of Education published a bell schedule that preserved the later start times for high schools and solidified the three-tier system (Indy Week 2024). By formally establishing elementary schools as the earliest tier, elementary students were set to end school at 2:15 PM, and parents of more than 700 students in Durham County were left without reliable after-school childcare (Indy Week 2024). Although WCPSS and DPS require district-specific strategies for implementing school start time changes, such vocal and proximal discontent likely hinders WCPSS's attempts to build a consensus for change. To account for the influence of DPS, our investigation of WCPSS considers the number of bus drivers needed as a proxy for the known costs of hiring and training bus drivers over the less-defined requirements of generating after-school childcare options.

## 1.3 Research Objective and Structure of Paper

The complexity involved in efficiently planning the daily student transportation in WCPSS and its critical role in making meaningful changes to bell schedules indicated the need for a thorough investigation of the bus scheduling procedures in WCPSS. While TIMS appears to function well as a tool for assigning students to stops, generating efficient bus routes, and providing bus drivers with their daily schedules, WCPSS has not used it to offer the public a clear picture of the costs associated with school bell time adjustments. Since TIMS is only available for use by school transportation planners in Wake County, developing a mathematical model offers researchers and stakeholders a means to efficiently experiment and discover characteristics of the situation in WCPSS that may be critical for prudent decision-making. A year after the WCPSS school board missed its April 2023 goal of publishing an updated bell schedule, "administrators and school board members said the ongoing shortage of bus drivers and parental ambivalence about schedule changes means they can't significantly alter high school times" (The News & Observer Staff 2023). If WCPSS can provide transportation for its students, how significant is the bus driver shortage when changing high school start times? How many additional bus drivers are required to significantly alter high school times? How could new bus drivers be assigned to implement changes to high school times gradually? What changes can be made to high school times that do not require additional bus drivers? An accurate model of the school bus schedule in WCPSS is not intended to replace TIMS; our model uses route information generated by TIMS as input. However, investigation using the mathematical model presented in this research helps answer the questions that have impeded WCPSS's ability to update its bell schedules to best support its students' success.

This research aims to investigate the current allocation and scheduling of school buses in WCPSS to illuminate the relationship between student transportation requirements and school start times. Starting with an accurate model of WCPSS transportation under standard conditions, we set our objective to minimize the total number of buses required and observe the changes resulting from different bell schedules. Both the model of the current transportation operations and scenario testing are used to offer insights to further the consideration of school start time changes in WCPSS. The remainder of the paper includes a review of relevant literature, an explanation of the data and methodology used for modeling, a comparison of the model to 2023–2024 data for validation, and the results and conclusions from our work.

CHAPTER

# 2 LITERATURE REVIEW

The relevant literature for this paper includes a summary of the research relating school start times to student performance—reasons *why* WCPSS may want to adjust its current school start times—and a more thorough review of the methods and findings related to the School Bus Routing Problem (SBRP)—*how* WCPSS may find a suitable solution.

## 2.1 Student Performance in Relation to School Start Time

Since the AAP published its recommendation in 2014, numerous studies have been published that support the conclusion that adolescents benefit from later school start times (generally 8:30 AM or later) (Adolescent Sleep Working Group et al. 2014). A selection of these studies is shown in Figure 2.1. While some studies show improvements in math and reading test scores (Edwards 2012) and course grades (Fuller and Bastian 2024), others suggest that the benefits of later start times are more evident by improved attendance, improved enrollment rates, less sleeping in class, less student-reported depression, and reduced vehicular accident rates (Wahlstrom 2002; Lenard et al. 2020; Morgenthaler et al. 2016). Four studies that support the AAP recommendation are particularly relevant because they were conducted using data from North Carolina schools, which include WCPSS (Bastian and Fuller 2023; Fuller and Bastian 2024; Edwards 2012; Lenard et al. 2020). Lastly, when considering school start time scenarios for WCPSS, it is worth noting that Bastian and Fuller (2023) recommend that if a school level needs to start earlier to accommodate other school levels, then elementary schools are the advisable choice.

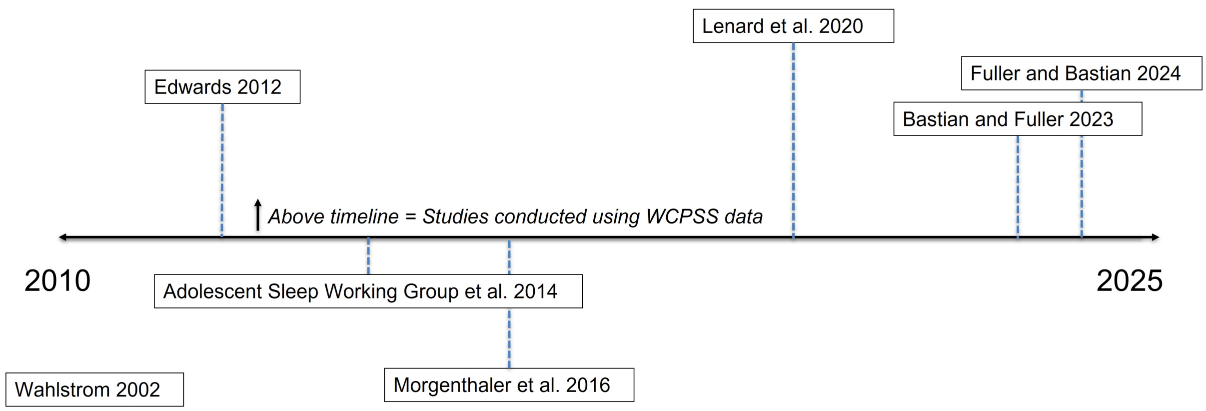


Figure 2.1: A timeline of the literature presented on student performance in relation to school start time. Papers above the timeline are studies that include WCPSS data.

## 2.2 School Bus Routing Problem (SBRP)

The SBRP has its roots in the Vehicle Routing Problem (VRP), but an increase in interest and publications within the last two decades has given this problem its distinction in the literature. The SBRP's connections to the Pickup and Delivery Problem with Time Windows (PDPTW) and the Dial a Ride Problem (DARP) are evident in papers such as Dumas et al. (1991) and Psaraftis (1980). Additionally, Zeng et al. (2022) use the combinatorial nature of the SBRP to highlight the similarities to the Machine Scheduling Problem and the Bin Packing Problem. Our review of the SBRP will begin with the general problem formulation and then rely heavily on the review papers of Park and Kim (2010) and Ellegood et al. (2020) to outline the body of work that has been published on this problem.

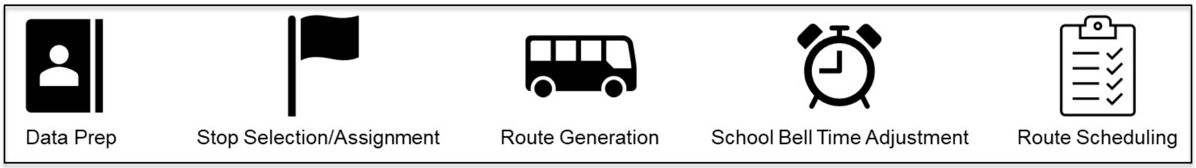


Figure 2.2: The five sub-problems of the SBRP established by Desrosiers et al. (1981).

### 2.2.1 SBRP Formulation

The widely accepted sub-problems within the SBRP—depicted by Figure 2.2—have been defined by Desrosiers et al. (1981) as data preparation, bus stop selection or student assignment, bus route generation, school bell time adjustment, and route scheduling. Data preparation involves structuring all the information pertaining to students, schools, and road networks that describe the scope of the problem and its constraints. Bus stop selection or student assignment is the sub-problem that seeks to optimize the location of the bus stops according to safety and walking distance considerations. The bus route generation sub-problem attempts to find the optimal route(s) to connect the bus stops into a single trip. As Park and Kim (2010) and Ellegood et al. (2020) point out, the school bell time adjustment and route scheduling sub-problems are only considered in a multi-school scenario. In the multi-school scenario, the optimal solution may be affected by different bell schedules and student destinations (schools). The SBRP is typically formulated as a combinatorial problem requiring heuristics or algorithms specific to the studied school district.

Of the 40 publications that consider a multi-school SBRP, only 32 use real-world data, and only 20 of those publications focus on an area within the United States (Ellegood et al. 2020; Park and Kim 2010). The problem size for the 40 multi-school SBRP studies ranges from two schools and 140 students to 200 schools and 20,000 students (Ellegood et al. 2020; Park and Kim 2010). This paper considers WCPSS, which has 199 schools, approximately 90,000 student bus rider requests, and 1,745 bus routes (WRAL News 2023; Wake County Public School System 2024e).

Each of the sub-problems within the SBRP offers a research area that is ripe for optimization, but the predominant focus has been on the bus route generation and the route scheduling sub-problems (Park and Kim 2010). The most common objective for studies of these two sub-problems is some form of cost minimization. This can be in the form of fixed costs (number of buses), variable costs (distance/time traveled), or a multi-objective approach that includes both objectives (Ellegood et al. 2020). Other objectives are more likely to be considered in studies that use real-world data, as they typically have concrete bus transportation policy decisions to consider. Examples of these objectives are total student walking distance, shared bus stops, safety factors, and the number of bus transfers (Ellegood et al. 2020). Though this paper uses real-world data, the bus driver shortage served as a primary motivation for this study. Therefore, our model seeks to minimize the number of buses (bus drivers) as its sole objective.

Excellent tables are available in Park and Kim (2010) and Ellegood et al. (2020), which categorize 93 SBRP-related publications since 1969. Several of the SBRP characteristics provided in the literature (homogeneous/heterogeneous bus fleet, mixed student load policies, maximum walk or ride time policies, earliest pickup time, school drop-off time-window policy, etc.) would be relevant for a study of the TIMS outputs used in WCPSS, but they are all assumed to be fixed and accounted for in the input data for this research. Because the primary focus of this paper is an investigation of WCPSS school start times and bus driver requirements, we will further narrow our literature review to the research with contributions to the school bell time adjustment and the route scheduling sub-problems.

### 2.2.2 SBRP Literature with Bell Times and Route Scheduling Focus

Only three SBRP papers in the last two decades have addressed the school bell time adjustment sub-problem (Bertsimas et al. 2019; Mandujano et al. 2012; Fügenschuh 2009). Both Mandujano et al. (2012) and Fügenschuh (2009) focus on solving the SBRP in rural areas where, according to Bodin and Berman (1979), the bus route generation problem becomes more important than the route scheduling problem as a bus in a rural area tends to reach a travel time constraint before its capacity constraint. Ellegood et al. (2020) distinguish this characteristic of the SBRP as the service environment of the problem (urban, rural, or both). Thus, regarding the service environment, our research primarily supplements the Bertsimas et al. (2019) study of the urban BPS district. Bertsimas et al. (2019) show how tiered start times like those in WCPSS and those suggested in Owens et al. (2014) can actually create sub-optimal bus requirements if the tier system does not account for geography and route linkage/compatibility. Bertsimas et al. (2019) develop an algorithm called BiRD (biobjective routing decomposition), which decomposes the overall SBRP into the single-school and multi-school sub-problems and selects optimal routing scenarios from a group of generated routing scenarios that are placed on a Pareto frontier for their two objectives: number of buses and average riding time. The success of their algorithm in reducing transportation costs was briefly described in Section 1.2.1. As their study is the most prominent example of the successful application of an SBRP algorithm to large-scale, real-world data, it is worth restating one of their conclusions: “Intuitively, what is optimal for one school may not be optimal for the entire system, motivating the biobjective decomposition approach" (Bertsimas et al. 2019).

Ellegood et al. (2020) define four mixed-load (students destined for different schools on the same bus simultaneously) transportation policies. As previously stated, our research does not evaluate the mixed-load policies in WCPSS, but the details of the bus routes used in our study suggest that three of the four mixed-load transportation policies are used across the nine sub-districts in WCPSS. An examination of the traditional (Ellegood et al. 2015), interscholastic (Yao et al. 2016), and transshipment (Tang and Yan 2010) mixed load policies in WCPSS would be necessary to optimize the entire district but would require more detailed student rider information. The pickup/delivery mixed loading policy does not appear to be used in WCPSS.

As Shafahi et al. (2017) demonstrate, methods for solving the route scheduling sub-problem are often directly influenced by the optimality of the bus route generation sub-problem. Due to computational complexity, it is natural to examine these two SBRP sub-problems sequentially. However, Shafahi et al. (2017) show that the optimal solution to a bus route generation problem that seeks to minimize the number of buses used can lead to routes that do not link efficiently and thus generate sub-optimal solutions to the route scheduling sub-problem. Considering the importance of examining these two sub-problems together, two trends arise: (1) studies using exact solution methods (Kim and Park 2013; Schittekat et al. 2013; Faraj et al. 2014; Chen et al. 2015; de Pádua Agripa Sales et al. 2018; Mokhtari and Ghezavati 2018; Shafahi et al. 2017) are typically limited to smaller problem sizes or synthetic data and (2) solution methods rely heavily on heuristics (construction or improvement) and metaheuristics (Ellegood et al. 2020). The list of metaheuristics applied to the SBRP has grown significantly in the last decade to include now Ant Colony Optimization, Genetic Algorithm, Harmony Search, Estimation of Distribution Algorithm, Particle Swarm, Scatter Search, Greedy Random Adaptive Search Procedure (GRASP), Variable Neighborhood Search, Simulated Annealing, Tabu Search, and Iterated Local Search (Ellegood et al. 2020).

This paper addresses two gaps in the literature identified by Ellegood et al. (2020): (1) it uses real-world data in a school district that is larger than any of those in the current literature, and (2) it studies different sub-districts to highlight the need for characteristic-based solution approaches. This paper also adds to the limited number of studies of the school bell time adjustment sub-problem, which is likely to increase in importance as states adapt policies to change high school start times, as is the case in California (California State Legislature 2019).

CHAPTER

# 3

# DATA AND METHODOLOGY

## 3.1 Structure of Wake County Public School System Transportation Data

Wake County Public School System offers the complete data necessary for this model through its Transportation page on its website (Wake County Public School System 2024d). Bus routes are listed by tier (elementary, middle, or high school) and by school for all Wake County schools. An example of the data available for a single route can be found in Appendix C, and all of the route data used for this research can be found at Kay et al. (2025). Thus, an aggregate of the following information for each 2023–2024 route in Wake County was compiled: bus name (e.g., “Abbotts Creek ES 01 Red"), route start time, and route end time (the time listed for arrival at its designated school). The bus and route names are used interchangeably throughout this paper because one of the premises of the research and modeling was to preserve the routes that WCPSS had constructed using TIMS. Therefore, “Abbotts Creek ES 01 Red" represents both a physical bus identifiable by Abbotts Creek Elementary School students and a route formed by its prescribed stops. School district assignment information and school start times were collected from Wake County Public

School System (2024b) and Hui (2024). Two categories of outliers were found in the data: multiple routes listed by the same bus name and routes with incomplete start and end times. Routes with incomplete start and end times were excluded from the model. See Appendix C for further details regarding handling outliers and Section 3.4 for an explanation of combining routes based on bus names. In total, this research considered 1,745 routes across the nine districts within Wake County.

## 3.2 Assumptions

As previous work has demonstrated, numerous complexities can be explored to optimize school bus routing according to a particular set of objectives. However, one of the goals of this research was to simplify the dynamic and complex problem set facing decision-makers in WCPSS. As such, several assumptions were necessary for this research. They are categorized according to their relation to the typical sub-problems found in the school bus routing problem.

### 3.2.1 Stop Assignment

We assumed that each stop in the routes adheres to the applicable WCPSS School Board policies. Specific policies cover Student Transportation Services and align WCPSS transportation services with state and federal laws regarding the necessary characteristics of a bus stop and which students can be assigned to the authorized stop locations (Wake County Board of Education 2024b). To maintain the legality and feasibility of stop assignments in the model solutions, the model did not modify or add any stops or stop assignments.

### 3.2.2 Route Generation

We assumed that the routes generated using TIMS are feasible and optimized for minimum travel time. We also assumed that WCPSS reports start and end times for the routes that are accurate according to the routes that the buses will drive. We did not adjust the provided route travel times according to factors such as predicted traffic volume, maximum or average school bus driving speed, or the loading times at each stop. It is assumed that the impact of these factors was accounted for when WCPSS transportation coordinators generated the routes using TIMS. During model experimentation, when route start times were adjusted to reflect a later school start time, we assumed that any change in traffic patterns was not significant enough to change the optimal route or its travel time.

### 3.2.3 Route Scheduling

The most significant assumptions in this research were made within this sub-problem. Here, we began by analyzing a period of the school year with the greatest demand for bus drivers, such as the early fall (September), when most of the traditional calendar, modified calendar, and year-round calendar schools are in session. We assume that there would be a different optimal schedule — likely with fewer buses required — for the periods of the school year when the various year-round or modified calendar schools are not in session. Similar to much of the previous research on the school bus routing problem, this research only solves the morning optimization problem. It is assumed that the afternoon routing problem could be solved with an equal number of buses and bus drivers as the morning problem.

This research did not consider afternoon extra-curricular bus requirements or planning factors for the number of surplus buses and drivers required to compensate for bus maintenance or driver absences. We also assumed that each bus would be able to arrive at the prescribed time to the first stop of its first route, regardless of its afternoon routes. The model does not specify the depot locations necessary to implement a given solution. Additionally, we assumed that the current routes are already optimal according to bus capacity constraints. The model does not designate the bus size required for a given route or consider the current WCPSS bus fleet or bus driver qualifications. This means an individual familiar with the different sizes of buses in the WCPSS fleet would need to validate the model's proposed solution before implementation. Lastly, during model validation, inter-route travel times (the time required for a bus to drive from its last stop — a school—to the first stop of another route) were generated using Google Maps. We assumed that the generated inter-route travel times were approximately equal to the travel times for a route that a bus would take to approach its first stop from the correct direction of travel.

## 3.3 Model Formulation

### Indices & Sets

$r \in R$: Set of routes to be completed. These correspond to route names that are the same as the bus names given by WCPSS.

$b \in B$: Set of available buses. Bus indices are arbitrarily assigned to buses in the fleet of a given sub-district within WCPSS.

### Input Data

$\mathbf{T} = [t_{i,j}] \quad \text{where } \mathbf{T} \in \mathbb{R}^{|R|\times|R|}$

A matrix representing travel times (in minutes) from the endpoint of route $i$ (a school) to the first stop of route $j$. Initially, every element of $\mathbf{T}$ is set to 0, defining a zero-minute transition time model that establishes a lower bound. Alternative models were tested by setting all $t_{i,j}$ values to 6 minutes for a constant transition time assumption and by using Google Maps travel time estimates for an actual transition time model.

$\mathbf{C} = [c_{i,j}] \quad \text{where } \mathbf{C} \in \{0,1\}^{|R|\times|R|}$

A compatibility matrix is created that compares the end times ($e_i$, $e_j$) and travel times ($t_{i,j}$, $t_{j,i}$) to start times ($s_i$, $s_j$) of all routes in the model to determine if they overlap in time. Routes $i$ and $j$ are compatible if $s_j \geq e_i + t_{i,j}$ *or* $s_i \geq e_j + t_{j,i}$. If the routes $i$ and $j$ are compatible, $c_{i,j}$ is set equal to 1. Alternatively, if routes $i$ and $j$ are incompatible, $c_{i,j}$ is set equal to 0. In the zero-minute transition time model ($t_{i,j} = t_{j,i} = 0$), the conditions for compatible routes simplify to ($s_i < s_j$ *and* $s_j \geq e_i$) *or* ($s_j < s_i$ *and* $s_i \geq e_j$).

### Decision Variables

$x_b$: Binary variable indicating if bus $b$ is used.

$$x_b = \begin{cases} 1 & \text{if bus } b \text{ is used,} \\ 0 & \text{otherwise.} \end{cases}$$

$a_{r,b}$: Binary variable indicating if route $r$ is assigned to bus $b$.

$$a_{r,b} = \begin{cases} 1 & \text{if route } r \text{ is assigned to bus } b, \\ 0 & \text{otherwise.} \end{cases}$$

## Objective Function

$$\min \sum_{b \in B} x_b \tag{3.1}$$

## Constraints

$$\sum_{b \in B} a_{r,b} = 1, \quad \forall r \in R \tag{3.2}$$

$$a_{r,b} \leq x_b, \quad \forall r \in R,\ b \in B \tag{3.3}$$

$$a_{i,b} + a_{j,b} \leq 1, \quad \forall b \in B, \forall i, j \in R \text{ such that } c_{i,j} = 0 \tag{3.4}$$

The objective function (3.1) seeks to minimize the total number of buses a district utilizes. The first constraint (3.2) requires that every route in the district be covered, and the second constraint (3.3) links the binary decision variables to prevent a route from being assigned to a bus that is not being utilized. The final constraint (3.4) references the compatibility matrix, which is generated as input data for the model. In this matrix, all diagonal entries and entries for compatible routes are 1, while entries for overlapping routes are 0. Thus, the last constraint (3.4) ensures that routes that overlap in time are not assigned to the same bus. This model follows a standard formulation for a Bin Packing with Conflicts model, except there is no constraint for bin capacity. In this model, the bin capacity would be the number of routes a bus can complete in a single morning. However, the fixed input data (start and end times of routes), combined with the compatibility matrix, prevents the model from assigning an excessive number of routes to a bus. This eliminates the need for typical capacity constraints found in a Bin Packing model.

## 3.4 Scenarios Tested

This research tested several scenarios to offer a comprehensive list for WCPSS decision-makers to consider. Each scenario tested is listed in Table 3.1 was tested. Scenarios were constructed with the following considerations:

1. In accordance with the recommendations from Adolescent Sleep Working Group et al. (2014), high school students should start school no earlier than 8:30 AM. A scenario with high school start times before 8:30 AM or after 9:15 AM would be insufficient or unrealistic.

2. An effort should be made to minimize the total number of school start times that are changed. As Bertsimas et al. (2019) demonstrate, dramatic start time changes for the majority of the student population may require fewer buses, but it is challenging to garner the necessary support to implement such start time changes. When possible, adherence to tiered start times should be considered.

3. The experience in Durham Public Schools suggests that swapping elementary school and high school start times could easily create a crisis for parents seeking after-school care. However, Bastian and Fuller (2023) recommend that elementary schools start earliest in a three-tier system. Both scenarios — those that leave the elementary start times unchanged and those that move elementary start times to the first tier — should be considered.

Scenarios were tested by adjusting school start times for specific tiers of schools. To replicate the current staggered arrival times of buses, a change to a school start time corresponded to an equal change in the route start time for every route that serviced the affected school. For example, a high school route that delivered students to school at 7:15 AM with a school start time of 7:25 AM in the base scenario would deliver students to school at 9:05 AM for a 9:15 AM start time in the *Swap ES and HS* scenario.

Here, it is pertinent to discuss handling some route outliers previously identified in Section 3.1 as multiple routes listed by the same bus name. For multiple routes listed by the same bus name in the same sub-district, a single combined route was formed for use in the model. For multiple routes listed by the same bus name in *different* sub-districts, each sub-district maintained a listing of the route and bus name for sub-district experimentation. None of the combined routes were adjusted in scenario testing. For example, if *Middle School Route A* and *Elementary School Route B* were both listed by *Bus C*, then it was assumed that *Bus C* picked up a mixed load of students with inter-mixed stops for *Middle School A* and *Elementary School B* students. The final two stops for *Bus C* would be *Middle School A*, immediately followed by *Elementary School B*. We were unable to decompose such bus routes effectively, and thus, the school start times for *Middle School A* and *Elementary School B* remained fixed for all ten scenarios. This prevented the model from violating assumptions regarding bus capacity, route optimization, and feasibility.

Table 3.1: School start times under different scheduling scenarios.

| **Scenario Name** | **School Start Time** | | |
|---|---|---|---|
| | **Elementary School** | **Middle School** | **High School** |
| Base | 8:20 - 9:20 AM | 7:30 - 8:30 AM | 7:20 - 7:25 AM |
| Swap ES and HS | 7:25 AM | 7:30 - 8:30 AM | 9:15 AM |
| All HS to 9:15 AM | 7:30 - 8:30 AM | 7:30 - 8:30 AM | 9:15 AM |
| All HS to 8:55 AM | 7:30 - 8:30 AM | 7:30 - 8:30 AM | 8:55 AM |
| HS to 9:15 AM, MS to 8:05 AM | 8:20 - 9:20 AM | 8:05 AM | 9:15 AM |
| HS to 9:15 AM, MS to 7:55 AM | 8:20 - 9:20 AM | 7:55 AM | 9:15 AM |
| HS to 8:55 AM, MS to 8:05 AM | 8:20 - 9:20 AM | 8:05 AM | 8:55 AM |
| HS to 8:55 AM, MS to 7:55 AM | 8:20 - 9:20 AM | 7:55 AM | 8:55 AM |
| Strict Tiers | 9:15 AM | 8:15 AM | 7:25 AM |
| Swap ES and HS with Strict Tiers | 7:25 AM | 8:15 AM | 9:15 AM |

CHAPTER

# 4 RESULTS

Preliminary analysis was conducted assuming there was no transition (travel) time from the end of one route to the beginning of the following route assigned to each bus. We attempted to solve the model formulated in Section 3.3 for all nine districts in WCPSS simultaneously. However, using Gurobi version 9.5.2 on a computer with 176 available threads and 1 TB of RAM, we received an error indicating our machine was out of memory. This is likely due to the memory consumption required to build and solve a model with a full-district-sized compatibility matrix (approximately 1,712 by 1,712). Additional experiments were conducted on single districts, and these smaller problems were found to be solvable, even on much smaller machines.

## 4.1 Zero-Minute Transition Time

As stated in Section 3.3, our model assumes no transition time by setting each $t_{i,j}$ equal to 0 and relying on equation (3.4) to ensure route compatibility. This model is referred to as the zero-minute transition time (ZTT) model, and the following sections describe the results from its use on progressively larger district combinations. Given the data that was publicly available, we were unable to deconstruct any of the bus routes. This significantly restricted the improvements we could make to the 2023–2024 bus requirements, as most of the methods for producing bus savings in the literature are improvement heuristics or algorithms applied to the bus route generation sub-problem. The results of the ZTT model computed for all nine districts in WCPSS simultaneously is a lower bound on the number of buses required to complete all of the published routes when the route start times are fixed. As this problem is not solvable, estimates of this lower bound will be computed (essentially, upper bounds on the lower bound).

### 4.1.1 Solving Districts Separately

The results in Figure 4.1 are from solving the ZTT model for each separate district for each scenario in Table 3.1. The crossing lines show that not every district has the same optimal school start time scenario. In fact, Districts 1, 6, and 7 show scenarios that require fewer buses than the base start times. It is important to understand that these ZTT model results do not represent guaranteed bus savings for the proposed bell time shifts. Instead, these results indicate a potential relationship between structural differences (bus route length, school types, school start times, etc.) and the current routes that produce a unique response to school start time changes. For example, Districts 4 and 6 have the highest concentrations of magnet schools, which requires buses in these areas to run longer routes to provide transportation for students who live in other districts. It is evident from the route information that WCPSS has already implemented some cost-saving methods (mixed loads and staggered start times) in these districts. Some structural differences in districts are provided in Appendix E.

Looking specifically at the comparison between the base scenario and the scenario that swaps elementary and high school start times, we can see the impact of the structural differences in the districts (see Table 4.1). Although responses to recent school start time adjustments in DPS indicate challenges exist when swapping elementary and high school start times (see Section 1.2.2), Bastian and Fuller (2023) recommend elementary schools for the earliest start time in a tiered system. Additionally, intuitively, one might think that swapping elementary and high school start times could be achieved without increasing bus requirements. However, Table 4.1 shows that swapping bell times requires more buses and significantly more buses in the districts with higher concentrations of magnet and special schools (Districts 4 and 6). Magnet and special schools require longer route times to bus students from peripheral districts into the middle of Wake County. With the 2023–2024

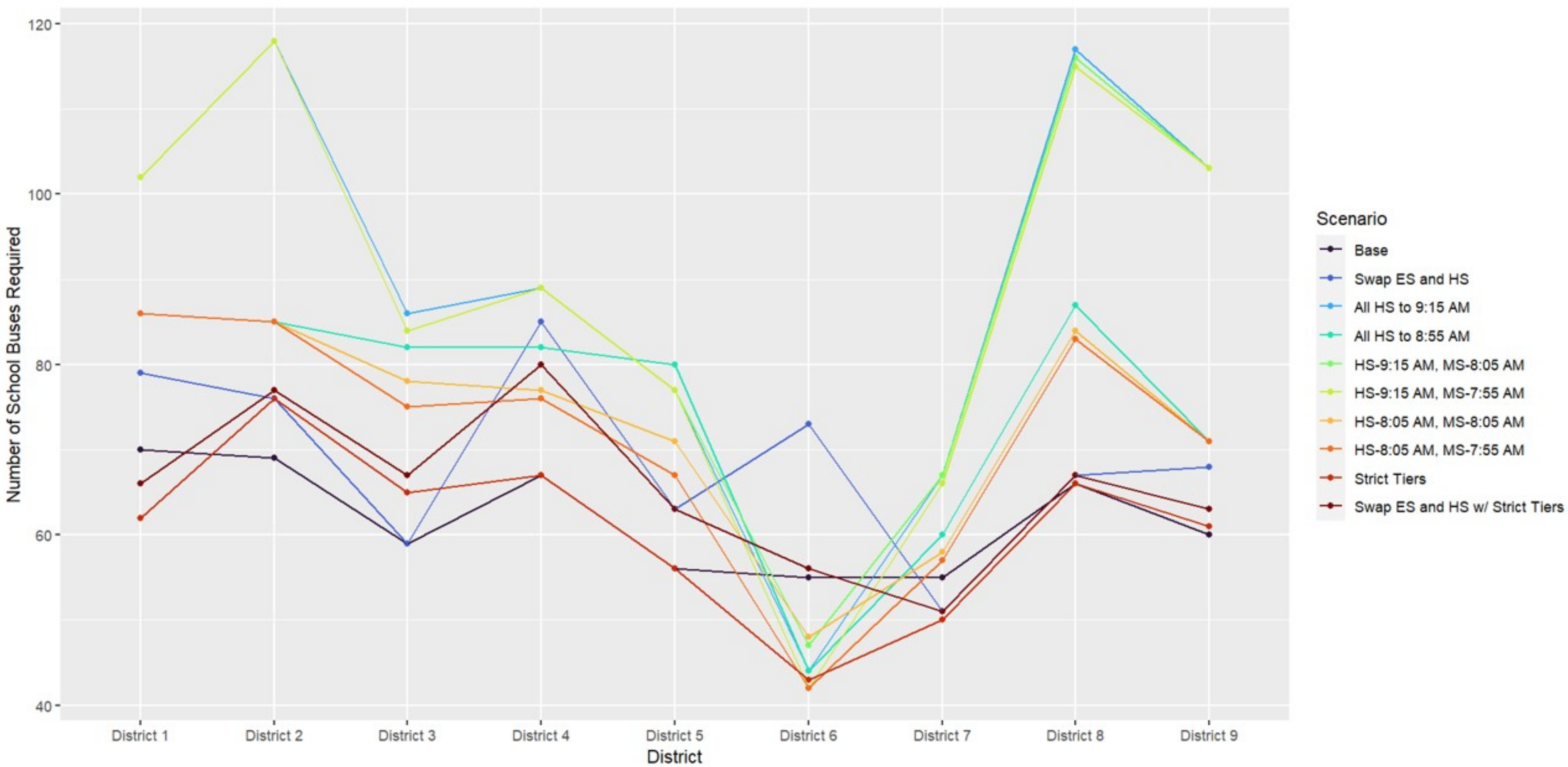


Figure 4.1: Number of buses required for each scenario using the ZTT model.

bus routes, these longer routes do not conflict with the routes from earlier tiers if they are scheduled for the first tier. In the first tier, the buses for magnet and special schools can run from about 5:45 AM to 7:25 AM instead of the route time window for the later tiers (7:25 AM to 8:15 AM or 8:15 AM to 9:15 AM).

Table 4.1: A comparison between the number of buses required in the Base scenario with the Swap ES and HS scenario using the ZTT model.

| **Scenario** | **Dist. 1** | **Dist. 2** | **Dist. 3** | **Dist. 4** | **Dist. 5** | **Dist. 6** | **Dist. 7** | **Dist. 8** | **Dist. 9** |
|---|---|---|---|---|---|---|---|---|---|
| Base | 70 | 69 | 59 | 67 | 56 | 55 | 55 | 66 | 60 |
| Swap ES and HS | 79 | 76 | 59 | 85 | 63 | 73 | 51 | 67 | 68 |
| *Increase* | *9* | *7* | *0* | *18* | *7* | *18* | *4* | *1* | *8* |

Here, it is worth noting that the total number of buses required in the Base case using the ZTT model to solve districts separately was 557. This is only three fewer buses than the number of buses WCPSS claimed to be using during the 2023–2024 school year (ABC11 News 2023). We expected that the lower bound from the ZTT model would be significantly lower than the actual number of buses in use since it is not feasible for a bus to transition from one route to the next with zero transition time. This led us to explore potential savings at the district boundaries, especially since WCPSS has not stated that buses are assigned exclusively to single districts.

### 4.1.2 Solving Two Districts Combined

Using the ZTT model, we examined three instances where schools appeared to be concentrated along district boundaries. Using just the base scenario, Table 4.2 shows the results of combining two adjacent districts for consideration as a single district. Even with the ZTT model, it was evident that savings are available — and likely occurring — by allowing buses to operate across district boundaries. As mentioned in Section 4.1.1, District 4 and District 6 have a unique relationship because buses are used to service magnet schools in both districts. The interaction between the two districts is evident by the magnitude of savings when solving the two districts as one combined district. In contrast, there is relatively limited savings in Districts 1/3 and Districts 8/9. Next, we examined the potential decrease in buses required by solving the ZTT model for two half-district combinations.

Table 4.2: Savings from three locations found by combining two adjacent districts in ZTT model.

| District | Buses Req. | District | Buses Req. | District | Buses Req. |
|---|---|---|---|---|---|
| District 1 | 70 | District 4 | 67 | District 8 | 66 |
| District 3 | 59 | District 6 | 55 | District 9 | 60 |
| Sum of separate districts | 129 | Sum of separate districts | 122 | Sum of separate districts | 126 |
| Combined in single district | 127 | Combined in single district | 111 | Combined in single district | 125 |
| *Savings* | *2* | *Savings* | *11* | *Savings* | *1* |

### 4.1.3 Solving Half-WCPSS Combinations

As expected, Table 4.3 shows increased savings when more districts are combined. The half-WCPSS combinations with the ZTT model result in 506 buses required for the entire WCPSS. As stated at the start of this chapter, we were unable to run the ZTT model for all nine districts simultaneously. However, based on the trend of the results in Sections 4.1.1–4.1.3, we expect the true lower bound to be less than 506 buses.

Table 4.3: Effect of larger district combinations on lower bound using ZTT model.

| District | Buses Req. | District | Buses Req. |
|---|---|---|---|
| District 1 | 70 | District 2 | 69 |
| District 3 | 59 | District 5 | 56 |
| District 4 | 67 | District 8 | 66 |
| District 6 | 55 | District 9 | 60 |
| District 7 | 55 | | |
| Sum of separate districts | 306 | Sum of separate districts | 251 |
| Combined (5 northern dist.) | 271 | Combined (4 southern dist.) | 235 |
| *Savings when combined* | *35* | *Savings when combined* | *16* |

## 4.2 Constant Transition Time

The ZTT model is helpful in observing the effect of combining districts on the minimum number of buses required, but its solutions are not feasible for bus schedules. To maintain simplicity in data collection and computation, we conducted further experimentation with a constant transition time (CTT) model. This formulation is supported by Zeng et al. (2022) for relatively compact school districts. We updated the transition time matrix **T** by setting $t_{i,j} = 6$ minutes, $\forall i, j,\ i \neq j$, to mirror an estimate of the average time to transition from one route to the next within a district. Table 4.4 mirrors the results displayed in Table 4.2 from combining two districts with the ZTT model but now using the CTT model. Similarly, Table 4.5 mirrors the half-WCPSS combinations of Table 4.3. These results show two trends: (1) the savings in the number of buses required in the CTT model also increase when solving for larger district combinations (like the ZTT model), and (2) at each level of district combination, the savings in the CTT model are greater than the savings in the ZTT model. This suggests that though we expect a model with actual transition times to require more buses per district solution, the savings available when solving district combinations may also be greater.

Table 4.4: Savings from three locations were found by combining two adjacent districts in the CTT model with a constant transition time of 6 minutes.

| **District** | **Buses Req.** | **District** | **Buses Req.** | **District** | **Buses Req.** |
|---|---|---|---|---|---|
| District 1 | 74 | District 4 | 67 | District 8 | 75 |
| District 3 | 69 | District 6 | 58 | District 9 | 63 |
| Sum of separate districts | 143 | Sum of separate districts | 125 | Sum of separate districts | 138 |
| Combined in single district | 132 | Combined in single district | 112 | Combined in single district | 135 |
| *Savings* | *11* | *Savings* | *13* | *Savings* | *3* |

Table 4.5: Effect of larger district combinations on lower bound using CTT model with a constant transition time of 6 minutes.

| District | Buses Req. | District | Buses Req. |
|---|---|---|---|
| District 1 | 74 | District 2 | 72 |
| District 3 | 69 | District 5 | 65 |
| District 4 | 67 | District 8 | 75 |
| District 6 | 58 | District 9 | 63 |
| District 7 | 62 | | |
| Sum of separate districts | 330 | Sum of separate districts | 295 |
| Combined (5 northern dist.) | 281 | Combined (4 southern dist.) | 249 |
| *Savings when combined* | *49* | *Savings when combined* | *46* |

## 4.3 Actual Transition Time

Due to the costs associated with converting the publicly available bus route data to coordinates that could be used in a model with actual transition times, we were selective in determining a district for scenario testing. As we saw in our ZTT model results, districts with higher concentrations of magnet schools and combined routes displayed anomalies during scenario testing. Due to the difficulty in decomposing combined routes into valid routes with accurate route times and updated school start times, we could not adjust the start times of schools with combined routes. This led us to select District 8 (no combined routes or deviations from tiered start times) as our district for actual transition time (ATT) model scenario testing.

In the ATT model, we updated the transition time matrix $\mathbf{T}$ according to its original definition in Section 3.3 — each $t_{i,j}$ value was set to the travel time (in minutes) from the end point of route $i$ to the start point of route $j$. Additionally, given the trends in bus savings that we established with the ZTT and CTT models, we returned to an examination of school start time scenarios for District 8.

The ATT model for District 8 shows an average increase of 26% for the number of buses required for each school start time scenario when compared to the ZTT model. A visual comparison of the ZTT and ATT models shows the results are consistent relative to each scenario (see Figure 4.2). The greatest differences between the ZTT and ATT model results were in the scenarios involving swapping elementary and high school start times. This is explained by the fact that the overlap period between the end of middle school routes and

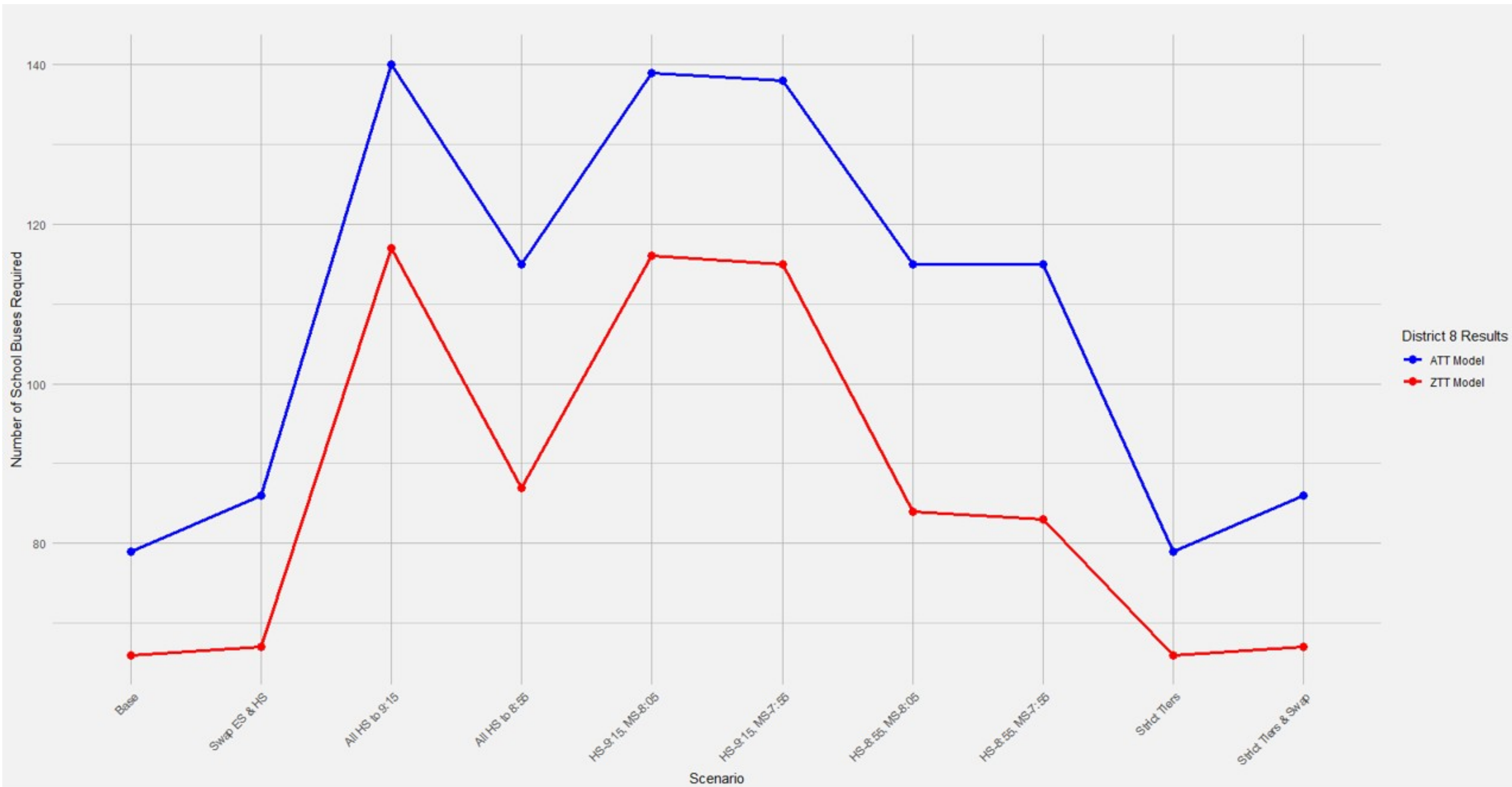


Figure 4.2: Comparison of the ZTT model results to ATT model results for District 8. Results show an average of 26% increase in the number of school buses required for each scenario.

the start of high school routes is exacerbated by the ATT model accounting for the actual travel times.

The solutions of the ATT model are easily displayed on a graph similar to a Gantt chart (see Figure 4.3), which shows 270 routes assigned to 79 required buses. The black vertical lines in Figure 4.3 denote the start times for the high (7:25 AM), middle (8:15 AM), and elementary (9:15 AM) schools, respectively. Because the ATT model accounts for inter-route transition time, these solutions, especially when displayed on the graph, are implementable as bus schedules in the prescribed district. Additionally, graphs of the ATT model solutions can be used by knowledgeable transportation specialists — whose expertise is required to implement any model solution — to identify opportunities to increase efficiency.

For example, a quick review of the District 8 base scenario solution revealed the opportunity to decrease the bus requirement by two by adjusting four route start times by no more than 10 minutes while still respecting the routing policies in place in WCPSS (seen in Figure 4.4). The improvements shown in Figure 4.4 were made by targeting buses that did not complete a route from each tier. In the top instance, one bus could only complete two high school routes and no middle school or elementary routes. By adjusting both high school routes 8 minutes earlier, the improved solution shows that a bus can complete the two adjusted routes *and* an additional high school, middle school, and elementary school route. In the middle instance, one bus could not complete a high school route. By adjusting the middle school route 10 minutes later, the improved solution shows that a bus can now complete a route from each tier. The bottom instance shows that a similar adjustment (6 minutes later) to a middle school route allows a bus in the improved solution to complete a route from each tier. Two total buses were saved in the improved solution by adjusting the four routes depicted in the top rows. Black vertical lines denote start times for the high (7:25 AM), middle (8:15 AM), and elementary (9:15 AM) schools, respectively. While somewhat small, this improvement illustrates several concepts related to the conclusions of this research.

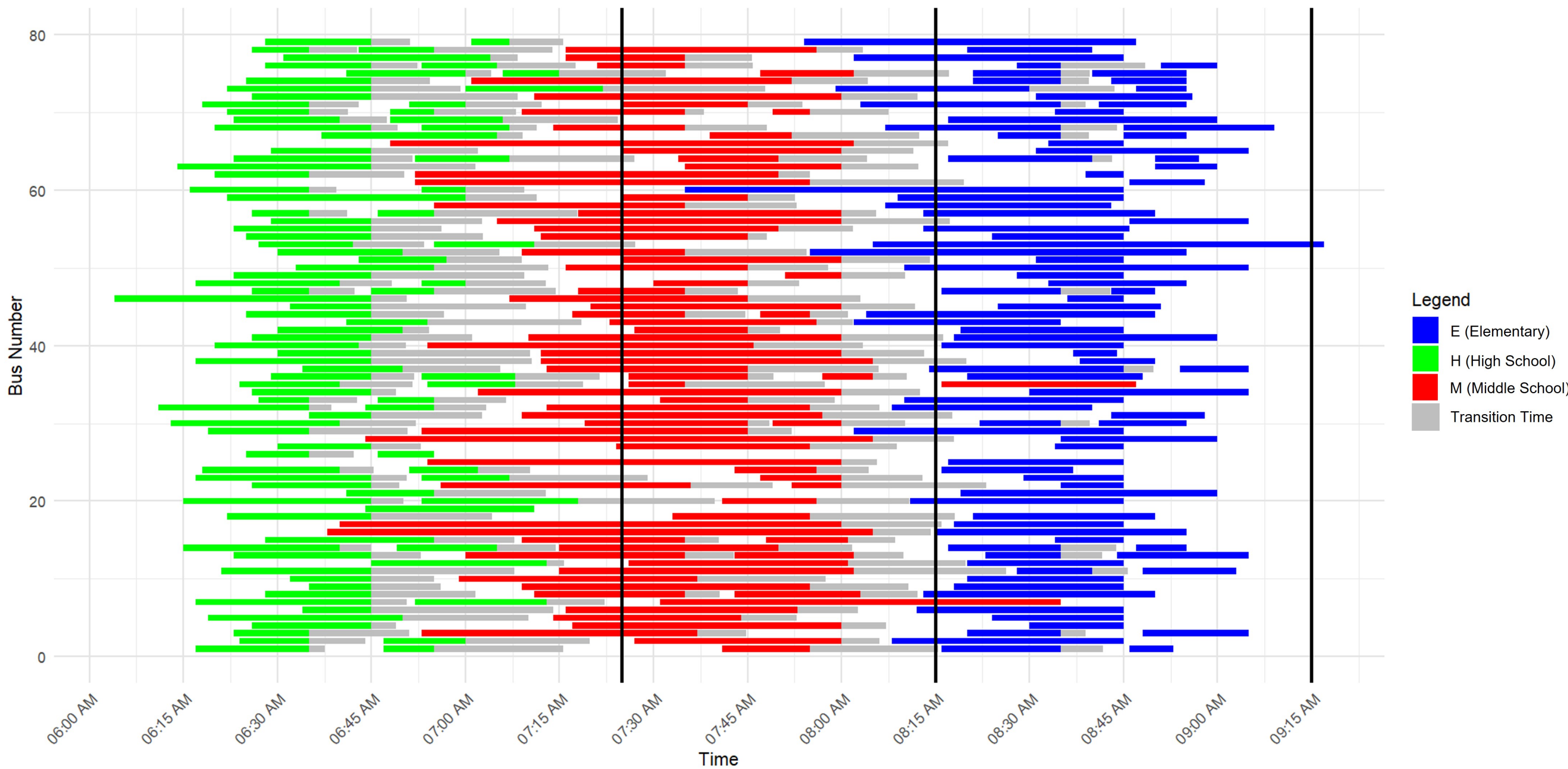


Figure 4.3: Bus schedule for District 8 base scenario using the ATT model.

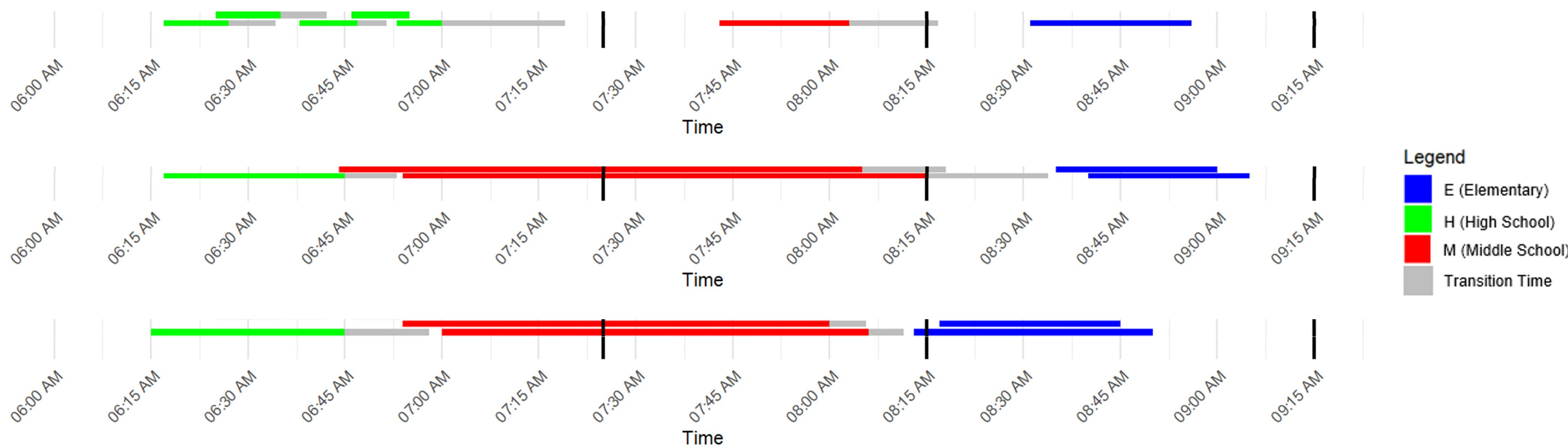


Figure 4.4: Improvements to the District 8 base scenario.

CHAPTER

# 5

# CONCLUSIONS

The school start times in WCPSS do not appear to be changing in the near future, despite evidence from numerous studies supporting Adolescent Sleep Working Group et al. (2014)'s recommendation for adolescents to start school no earlier than 8:30 AM. The bus driver shortage and school start times are easily tied to sub-problems of the SBRP (i.e., bus route generation/scheduling and school bell time adjustment). However, the barriers to change in WCPSS include some that can be addressed by SBRP solution methods, as well as many that are either fixed (e.g., school locations and capacities) or dominated by social (e.g., resistance to widespread or drastic bell time changes) or structural (e.g., magnet and special school transportation requirements) constraints. The conclusions in this chapter highlight observations from both the experimentation performed in this thesis and by comparing WCPSS's practices to recommendations from the SBRP literature.

1. None of the tested scenarios offer WCPSS the ability to start its high schools after 8:30 AM, using its 2023–2024 bus routes, without requiring additional buses. The intuition that WCPSS should be able to swap elementary and high school start times without requiring additional bus drivers was not supported by our ZTT model results, suggesting that ATT model results of all nine districts would require even more buses.

This may be due to the fixed start times of the routes. If the route start times had been allowed to vary within the constraints of WCPSS regulations, additional buses might not be needed to swap the elementary and high school start times. The result that additional buses would be required to swap elementary and high school start times could also be partly due to our data's structure; we could not decompose or restructure routes so that the start times of any school with a combined route were left unchanged. Another explanation for the results we observed is that the 2023–2024 bus routes had been optimized, at least partly for the current school start times.

2. The data from WCPSS shows that the districts in WCPSS clearly interact (share buses), and our results demonstrate that different districts achieve the minimum number of buses required in different school start time scenarios. While the size of WCPSS necessitated a decomposition approach in our research, optimizing each district in isolation performed poorly due to the shared buses.

3. WCPSS has already implemented several cost savings/bus minimization methods in the SBRP literature. This includes combining routes to generate mixed loads, establishing tiered start times, breaking the strict tiered start times for magnet schools with longer routes, and allowing buses to cross district boundaries. Unlike nearly every real school district studied in the SBRP literature, WCPSS already has a robust routing software system (TIMS) in use, and its policies allow for multiple types of mixed-load routing. This suggests that the improvement experienced by many districts in the literature might not be possible for WCPSS.

4. To minimize the number of buses required for any changes in school start times being considered, WCPSS should consider the following:

    Return to the bus route generation sub-problem within the SBRP. This would require using the TIMS routing software and the ability to generate multiple routing scenarios. (Our model results were limited by only being able to use the single routing instance available for the 2023–2024 school year.)

    Apply an appropriate improvement method to the TIMS routing procedures based on the district characteristics. The SBRP literature offers examples of bus route improvement methods for rural and urban scenarios, and WCPSS comprises both types of districts. Clearly, an improvement method applied to District 4 or 6, in the middle of Wake County, with dozens of magnet school routes, will produce different results when applied to District 8.

Establish school start time *windows*, i.e. not strict tiers. It is clear from the data that WCPSS already allows for a window of school bus arrival times. This helps reduce the number of buses that are currently required. However, the morning bus arrival window can only be extended so early before the gap between bus arrival and school start time is too large. Coupling the window of school start times with the bus arrival time windows is one way to expand the solution space and further minimize the number of required buses. As the minor adjustment to four bus routes in District 8 demonstrated, a route start time window (school bus arrival time window) can preserve the tiered structure while reducing the required buses.

Consider the combined effect of improvement methods across adjacent districts. Our ZTT model results on combined districts show that there is potential for greater savings when districts are combined. However, our scenario testing with the ZTT model and Bertsimas et al. (2019) show that the sum of the optimized decomposition of WCPSS must be compared to scenarios that consider the interactions across its elements (districts).

CHAPTER

# 6

# FUTURE WORK

The nature of the SBRP and its school-district-specific solutions create a discipline where further work will almost always be available. As student populations grow and school districts are asked to provide transportation with shrinking budgets, there is a need to equip school transportation coordinators with district-appropriate tools. In the case of WCPSS, we propose a few directions to further the research of this study.

1. *Student rider data:* Although this study contributed to the SBRP literature by modeling a large real-world school district, some bell time scenario testing was limited by limited student rider data. The home addresses and demographics of the students at each bus stop are understandably protected for their privacy. However, further research requires, at a minimum, the number of students at each bus stop and their destination school if one is going to generate different bus route scenarios. The SBRP literature recommends a joint study of the bus route generation and bus scheduling sub-problems; our findings support this recommendation. Modeling and manipulating both sub-problems is necessary to broaden the available solution space to the overall SBRP in WCPSS. Additionally, more detailed student rider data would allow researchers to create a more robust model for WCPSS.

2. *Planning versus operational use:* Several key factors impact the SBRP solution space for any particular day. In WCPSS, parents can notify the transportation office if their child is not riding the bus (Wake County Public School System 2024c). While this minor detail offers little cost savings in a compact region with multiple students at every stop, it could have meaningful impacts in rural areas with stops at single homes. Caceres et al. (2017) present relevant work concerning ridership uncertainty and assigning buses at or over their capacity. Further research would have to determine if the frequency of rider absence presents an opportunity for WCPSS to reduce bus requirements. Additionally, WCPSS has over 45 elementary and middle schools that operate on a year-round instructional calendar, which means students have approximately 15 days off after approximately 45 days of school. (Wake County Public School System 2024a). Future research could investigate the impact of the calendar cycles on bus driver requirements. Consistent bus routes lead to increased efficiency as bus drivers and coordinators grow accustomed to a set of optimal route assignments. Suppose there are significant changes to the set of optimal route assignments every 45 days of school. In that case, WCPSS may have difficulty completing bus routes, even if a model has determined them to be feasible. This concern warrants consideration for a study with a bi-objective approach: minimize the number of buses required while minimizing the changes to route assignments. As mentioned in Chapter 1, WCPSS also has many students attending overflow schools because their assigned school has reached its capacity ("capped"). As portable classrooms are moved throughout the district or spots become available, students can move from their overflow school to the capped school. Future research could include an analysis of the cost WCPSS incurs to bus students to overflow schools or extend the research to include a broader study of the impact of school zoning on the SBRP. Considering these factors can create a more robust model capable of planning and operational use.

3. *TIMS capabilities and algorithms:* The implementation and capabilities of TIMS have undoubtedly led to increased efficiency for transportation coordinators in North Carolina. Countless hours have been spent collecting, inputting, and editing data in TIMS to have accurate student rider and road network information. To attempt to tackle the SBRP in WCPSS without the data and structure provided by TIMS would be a significant undertaking. TIMS may be equipped to handle each SBRP sub-problem for a district the size of WCPSS. However, to decompose the problem, generate multiple scenarios, and experiment with various solution methods, researchers need to

understand the algorithms that TIMS uses. Ideally, future work would allow WCPSS to use as many of its current systems as possible by providing recommendations on the district-specific parameter tuning for TIMS' solution approach (Ellegood et al. 2020). Creating a model that can be used with TIMS would increase the likelihood of implementing it, distinguishing it from many of the academic endeavors in the SBRP literature.

4. *Windowed solution construction:* Due to its size, there is a high computational cost to solving the SBRP in WCPSS to optimality. However, we have demonstrated the ability to solve the route scheduling problem in districts to optimality using the 2023–2024 bus routes for fixed route start times. We have also shown that the sum of district optimal solutions will overestimate the total number of buses required, whether using the ZTT, CTT, or ATT model. Future work for the SBRP in WCPSS could use the ATT model and a windowed solution construction to solve a district to optimality, fix the solution, and then add a particular window size of bus routes to find an optimal solution to a slightly larger combination of districts. The exact number of routes to fix and the window size to add would need to be determined through experimentation to find the proper solution construction heuristic. Once the appropriate window size was established, multiple feasible solutions could be generated by varying the starting district. The windowed solution construction would generate a lower bound approximation — the minimum solution from the set of feasible solutions to the ZTT model – and a tight upper bound — the minimum solution from the set of feasible solutions to the ATT model — for the total number of buses required for all of WCPSS.

5. *Time-indexed formulation:* In addition to the windowed solution construction, another solution approach that would build on the concepts demonstrated in Figure 4.4 is a time-indexed formulation of the SBRP. Zeng et al. (2022) demonstrate this formulation by using binary variables to represent both school start times and route arrival times. We highlight the need to consider route start time windows in Chapter 5, and a time-indexed formulation can potentially optimize the school and route times in those established windows. While a time-indexed formulation is congruent with our recommendations regarding using time windows to establish better bounds, its main drawback is that it increases the size of the problem. In WCPSS, the problem size already limited our ability to solve a simple ZTT model for all nine districts simultaneously. A time-indexed formulation would have to be combined with an algorithm or heuristic — possibly the windowed solution construction described above — to suc-

cessfully apply it to WCPSS. See Appendix G for an example of a possible time-indexed formulation varying the route start times (route arrival times).

6. *Heuristic development for improving ATT model results:* In Figure 4.4, we demonstrated a way to improve the ATT model results (decrease the required number of buses) for the base scenario in District 8. This was accomplished by visual inspection of the Gantt chart of the model solution to identify buses that did not complete a route in each tier. The improvements shown in Figure 4.4 were made by adjusting route start (or end) times to reduce the total number of buses required. This is the same approach and objective as the time-indexed formulation described above. Future research could formulate a heuristic that could be applied to the results from our ATT model's current formulation or the windowed solution construction to achieve the same effect as the time-indexed formulation. An advantage of such an improvement heuristic method is that it would avoid the concerns of ballooning the problem size associated with the time-indexed formulation. A simple heuristic based on the visual inspection we completed for District 8 would:

   - Identify the subset of buses that do not complete at least one route in each tier (HS, MS, ES). Call this subset **B'** .
   - For each bus in **B'** , identify which tier is not represented in the routes assigned to the bus.
   - For buses not assigned HS routes, adjust the assigned MS route(s) to the latest feasible and acceptable school arrival time. For buses not assigned MS routes, adjust the assigned HS route(s) to the earliest acceptable route start time. For buses not assigned ES routes, adjust the assigned HS and MS route(s) to the earliest feasible and acceptable school arrival times.
   - Re-run the model with the updated route start/end times.

   Future work on a more robust route start/end time adjustment heuristic could also consider buses with the greatest amount of bus-vacant travel and idle times, as well as the following proposition from SBRP literature: Given a set of routes with fixed operation times, the minimum number of buses to complete all the routes is equal to the maximum number of routes in operation during the same period (Desrosiers et al. 1986). Another benefit of this improvement heuristic is that it can be pursued and applied without requiring additional student rider data.

7. *Stakeholder input on bell times:* Bertsimas et al. (2019) demonstrate the ability to construct a graph depicting the trade-off between routing solutions (number of buses required) and the stakeholder preference scores for specific bell times. While this presentation of their model solutions ultimately did not lead to implementing a new bell schedule, it captured the same predicament that WCPSS faces. In the current discussion on school start time changes in Wake County, there is very little data tying stakeholder preference scores to specific bell schedules. This is partly because surveys in the community appear to offer only one alternative to the current start times (Hui 2024). Stakeholders have not been presented with specific bell schedules or their corresponding bus requirements. Assuming future research could determine bus requirements for an array of school start time scenarios, a new community survey showing the diverse set of scenarios may be able to generate the momentum necessary to bring change to school start times in WCPSS. Owens et al. (2014) offer findings that ought to be considered to maximize the utility of another round of stakeholder surveys.

8. *Equity concerns:* It is necessary to critically examine SBRP solutions to ensure that the disruptions caused by school bell time adjustments do not create disproportionate hardships for specific populations. Any model with a single objective to minimize the number of required buses has the potential to generate solutions that increase equity gaps while decreasing costs. This concern is particularly relevant in WCPSS, where busing students to specific schools has been part of the reassignment policies to address historic segregation in schools (Carter III et al. 2023). As we identified in Section 4.1.1, routes for magnet and special schools are typically longer and are likely to be placed in the earliest tier if start times are optimized. However, research suggests that adolescents would benefit from starting school in the latest tier. Therefore, by ignoring school demographics, a model solution could recommend moving several magnet and special schools (found in central Raleigh) to an arguably disadvantageous start time. Vendor transportation (non-yellow school bus) also plays a significant role in the special education transportation plan for WCPSS. The availability of vendor transportation throughout the typical morning commute period was not considered in this research. Any model solution that adjusts school start times to minimize the number of required buses but, as an unintended consequence, creates a hardship for special education students to receive adequate transportation should be reworked. As further research improves model robustness, it should account for the equity

concerns represented by these factors in WCPSS. For example, it may be necessary to include constraints that limit the magnitude of change in school start time that a specific school or type of school experiences. At the very least, we acknowledge that school bell time adjustments have the potential to impact families in WCPSS considerably, and future work should be aware of how the second and third-order effects are distributed.

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# APPENDICES

APPENDIX

# A

# ACRONYMS

A summary of all acronyms is documented in Table A.1.

Table A.1: A summary of acronyms used in alphabetical order.

| Acronym | Abbreviation |
|---|---|
| Actual Transition Time | ATT |
| American Academy of Pediatrics | AAP |
| Boston Public Schools | BPS |
| Constant Transition Time | CTT |
| Coronavirus Disease | COVID |
| Dial a Ride Problem | DARP |
| Durham Public Schools | DPS |
| Elementary School | ES |
| Greedy Random Adaptive Search Procedure | GRASP |
| High School | HS |
| Local Education Agency | LEA |
| Middle School | MS |
| Pickup and Delivery Problem with Time Windows | PDPTW |

| | |
|---|---|
| School Bus Routing Problem | SBRP |
| Transportation Information Management System | TIMS |
| Vehicle Routing Problem | VRP |
| Wake County Public School System | WCPSS |
| Zero-Minute Transition Time | ZTT |

APPENDIX

# B

# VARIABLES

A summary of all variables is documented in Table B.1.

Table B.1: A summary of all variables used in bin packing with conflict model

| Variable | Meaning |
|---|---|
| $a_{i,b}$, $a_{j,b}$ or $a_{r,b}$ | Binary variable indicating if route $i$, $j$, or $r$ is assigned to bus $b$ |
| $b$ | Arbitrary bus number within the set of buses available $B$ |
| $B$ | Set of available buses |
| $\mathbf{C}$ | Matrix composed of 0 (incompatible) or 1 (compatible) entries |
| $r$ | Arbitrary route number within the set of routes to be completed $R$ |
| $R$ | Set of routes to be completed |
| $x_b$ | Binary variable indicating if bus $b$ is used |

APPENDIX

# C

# BUS ROUTE DATA

Below is an example of a single morning route found on a WCPSS transportation web page (Figure C.1). The routes are updated throughout the school year, so the details of the 2023–2024 routes used for this study were captured and included in a data repository for future reference (Kay et al. 2025). The data included is sufficient to run the ZTT model. In order to run the ATT model, one would need the location of the first stop for each route (not included).

## C.1 Handling of Outliers

Although the WCPSS bus route web pages offered most of the data necessary for the model, there were a few outliers with incomplete data that were not included in the model. The routes and their reason for exclusion from the model are provided in Table C.1. Additionally, there were two distinguishable subsets of routes, which formed separate categories within a larger category considered combined routes.

One subset of the combined routes includes those that are listed by the same bus name for multiple schools in the same district. For example, the "Zebulon ES/MS 01 Red" bus

AM Routes

**ABBOTTS CREEK ES 01 RED**

*Route Start Time (1st Stop): 08:33 am*

ABBEY WOODS DR & SPRUCE TREE WAY

DUNN RD & ELEGANCE DR (DOORSIDE)

ALL POINTS VIEW WAY & PEPPERMILL DR

FALLS RIVER AVE & HOPETON AVE

FALLS RIVER AVE & HORNBECK CT (CLOSER TO BEDFORDTOWN DR)

1621 DUNN RD (DOORSIDE)

OCTOBER RD & WHITTINGTON DR

SUMMERTON DR & WOOD SPRING CT

HOLLOWGATE RD & WHITTINGTON DR

FALLS RIVER AVE & NELAND ST

COKESBURY LN & DUNN RD

HALLINGDALE CT & OAKBORO DR

SCHOOL: ABBOTTS CREEK ES 09:05 am

Figure C.1: An example bus route listed on WCPSS transportation web page.

is listed as a bus for Zebulon Elementary School and Zebulon Middle School, which are both District 1 schools. As WCPSS has designated a single bus to complete the list of stops for two schools, the author treated such instances as a single route. The other subset of combined routes includes those that are listed by the same bus name for two or more schools in different districts. This occurs on the borders of some districts. For example, there are several buses that pick up and drop off students from Fuller Elementary (District 4) and Washington Elementary (District 5). These instances were also treated as two separate routes (one in each district), even though it is evident from the names of the buses that WCPSS covers these routes with a single bus.

Table C.1: Routes excluded from the model.

| **School** | **District** | **Number of Routes** | **Reason Excluded from Model** |
|---|---|---|---|
| Knightdale High School | 1 | 26 | No route end times listed |
| River Oaks Middle School | 4 | Unknown | No routes listed |
| Wake Early College of Health and Sciences | 4 | 32 | No route end times listed |

APPENDIX

# D

# WAKE COUNTY SCHOOL START TIMES FOR 2023–2024

Below is a table showing the start times for each school in WCPSS for the 2023–2024 school year.

Table D.1: School start times in WCPSS for 2023–2024 school year.

| **School Name** | **Start Time** |
|---|---|
| Abbotts Creek Elementary | 09:15 AM |
| Adams Elementary | 09:15 AM |
| Alston Ridge Elementary | 09:15 AM |
| Alston Ridge Middle | 08:15 AM |
| Apex Elementary | 09:15 AM |
| Apex Friendship Elementary | 09:15 AM |
| Apex Friendship High School | 07:25 AM |
| Apex Friendship Middle | 08:15 AM |
| Apex High | 07:25 AM |

| | |
|---|---|
| Apex Middle | 08:15 AM |
| Athens Drive High | 07:25 AM |
| Aversboro Elementary | 09:15 AM |
| Baileywick Road Elementary | 09:15 AM |
| Ballentine Elementary School | 09:15 AM |
| Banks Road Elementary | 09:15 AM |
| Barton Pond Elementary | 09:15 AM |
| Barwell Road Elementary | 09:15 AM |
| Baucom Elementary | 09:15 AM |
| Beaverdam Elementary | 09:15 AM |
| Brassfield Elementary | 09:15 AM |
| Brentwood Elementary | 08:30 AM |
| Briarcliff Elementary | 09:15 AM |
| Bridges Elementary | 08:30 AM |
| Brier Creek Elementary | 09:15 AM |
| Brooks Elementary | 09:15 AM |
| Broughton High | 07:25 AM |
| Bryan Road Elementary | 09:15 AM |
| Buckhorn Creek Elementary | 09:15 AM |
| Bugg Elementary | 09:10 AM |
| Carnage Middle | 07:35 AM |
| Carpenter Elementary | 09:15 AM |
| Carroll Middle | 07:30 AM |
| Carver Elementary | 09:15 AM |
| Cary Elementary | 09:15 AM |
| Cary High | 07:25 AM |
| Cedar Fork Elementary | 09:15 AM |
| Centenial Middle | 07:30 AM |
| Combs Elementary | 08:30 AM |
| Conn Elementary | 09:15 AM |
| Creech Road Elementary | 09:15 AM |
| Davis Drive Elementary | 07:45 AM |
| Davis Drive Middle | 08:15 AM |

| | |
|---|---|
| Dillard Drive Elementary | 09:15 AM |
| Dillard Drive Middle | 08:15 AM |
| Douglas Elementary | 09:15 AM |
| Durant Road Elementary | 08:30 AM |
| Durant Road Middle | 08:15 AM |
| East Cary Middle | 08:15 AM |
| East Garner Elementary | 09:15 AM |
| East Garner Middle | 08:15 AM |
| East Millbrook Middle | 07:30 AM |
| East Wake High | 07:25 AM |
| Enloe High | 07:25 AM |
| Farmington Woods Elementary | 09:15 AM |
| Forest Pines Drive Elementary | 09:15 AM |
| Forestville Road Elementary | 09:15 AM |
| Fox Road Elementary | 09:15 AM |
| Fuller Elementary | 09:10 AM |
| Fuquay-Varina Elementary | 09:15 AM |
| Fuquay-Varina High | 07:25 AM |
| Fuquay-Varina Middle | 08:15 AM |
| Garner High | 07:25 AM |
| Green Elementary | 09:15 AM |
| Green Hope Elementary | 09:15 AM |
| Green Hope High | 07:25 AM |
| Green Level High | 07:25 AM |
| Harris Creek Elementary | 09:15 AM |
| Herbert Akins Road Elementary | 09:15 AM |
| Herbert Akins Road Middle | 08:15 AM |
| Heritage Elementary | 08:25 AM |
| Heritage High | 07:25 AM |
| Heritage Middle | 08:20 AM |
| Highcroft Drive Elementary | 09:15 AM |
| Hilburn Drive Academy | 08:15 AM |
| Hodge Road Elementary | 08:30 AM |

| | |
|---|---|
| Holly Grove Elementary | 09:15 AM |
| Holly Grove Middle | 08:15 AM |
| Holly Ridge Elementary | 09:15 AM |
| Holly Ridge Middle | 08:15 AM |
| Holly Springs Elementary | 09:20 AM |
| Holly Springs High | 07:25 AM |
| Hortons Creek Elementary | 09:15 AM |
| Hunter Elementary | 09:15 AM |
| Jeffreys Grove Elementary | 09:15 AM |
| Jones Dairy Elementary | 09:15 AM |
| Joyner Elementary | 08:30 AM |
| Kingswood Elementary | 09:05 AM |
| Knightdale Elementary | 09:15 AM |
| Knightdale High | 07:25 AM |
| Lacy Elementary | 08:30 AM |
| Lake Myra Elementary | 09:15 AM |
| Laurel Park Elementary | 09:15 AM |
| Lead Mine Elementary | 08:30 AM |
| Leadership (Men's) | 07:35 AM |
| Leadership (Women's) | 07:35 AM |
| Leesville Road Elementary | 09:15 AM |
| Leesville Road High | 07:25 AM |
| Leesville Road Middle | 08:15 AM |
| Ligon Middle | 07:35 AM |
| Lincoln Heights Elementary | 08:30 AM |
| Lockhart Elementary | 08:30 AM |
| Longview School | 07:25 AM |
| Lufkin Road Middle | 08:15 AM |
| Lynn Road Elementary | 08:30 AM |
| Martin Middle | 07:30 AM |
| Mary E. Phillips High School | 07:25 AM |
| Middle Creek Elementary | 08:30 AM |
| Middle Creek High | 07:25 AM |

| | |
|---|---|
| Millbrook Elementary | 07:45 AM |
| Millbrook High | 07:25 AM |
| Mills Park Elementary | 09:15 AM |
| Mills Park Middle | 08:15 AM |
| Moore Square Middle | 07:30 AM |
| Morrisville Elementary | 09:15 AM |
| Mt. Vernon | 08:30 AM |
| NC STEM | 07:25 AM |
| Neuse River Middle | 07:30 AM |
| North Forest Pines Elementary | 07:45 AM |
| North Garner Middle | 08:30 AM |
| North Ridge Elementary | 09:15 AM |
| North Wake College and Career Academy | 07:25 AM |
| Northwoods Elementary | 09:15 AM |
| Oak Grove Elementary | 09:15 AM |
| Oakview Elementary | 09:15 AM |
| Oberlin Middle | 08:15 AM |
| Olds Elementary | 08:30 AM |
| Olive Chapel Elementary | 09:15 AM |
| Panther Creek High | 07:25 AM |
| Parkside Elementary | 09:15 AM |
| Partnership Elementary | 08:30 AM |
| Penny Road Elementary | 09:15 AM |
| Pine Hollow Middle | 08:15 AM |
| Pleasant Grove Elementary | 09:15 AM |
| Pleasant Union Elementary | 09:15 AM |
| Poe Elementary | 09:05 AM |
| Powell Elementary | 09:15 AM |
| Rand Road Elementary | 09:15 AM |
| Reedy Creek Elementary | 09:15 AM |
| Reedy Creek Middle | 08:15 AM |
| Richland Creek Elementary | 09:15 AM |
| River Bend Elementary | 09:15 AM |

| | |
|---|---|
| River Bend Middle | 08:15 AM |
| River Oaks Middle | 07:30 AM |
| Rogers Lane Elementary | 09:15 AM |
| Rolesville Elementary | 09:15 AM |
| Rolesville High School | 07:25 AM |
| Rolesville Middle | 08:15 AM |
| Root Elementary | 09:15 AM |
| Salem Elementary | 09:15 AM |
| Salem Middle | 08:15 AM |
| Sanderson High | 07:25 AM |
| Sanford Creek Elementary | 09:15 AM |
| Scotts Ridge Elementary | 09:15 AM |
| Smith Elementary | 09:15 AM |
| South Garner High | 07:25 AM |
| South Lakes Elementary | 09:15 AM |
| Southeast Raleigh Elementary | 08:30 AM |
| Southeast Raleigh High | 07:25 AM |
| Stough Elementary | 09:15 AM |
| Swift Creek Elementary | 09:15 AM |
| Sycamore Creek Elementary | 08:30 AM |
| Timber Drive Elementary | 09:15 AM |
| Turner Creek Elementary | 09:15 AM |
| Underwood Elementary | 09:15 AM |
| Vance Elementary | 09:15 AM |
| Vandora Springs Elementary | 08:30 AM |
| Vernon Malone CCA | 07:25 AM |
| Wake Early College | 07:25 AM |
| Wake Early College of Information and Biotechnologies | 07:25 AM |
| Wake Forest Elementary | 09:15 AM |
| Wake Forest High | 07:20 AM |
| Wake Forest Middle | 07:30 AM |
| Wake Mens Academy | 07:35 AM |
| Wake Womens Academy | 07:35 AM |

| | |
|---|---|
| Wakefield Elementary | 09:15 AM |
| Wakefield High | 07:25 AM |
| Wakefield Middle | 08:30 AM |
| Wakelon Elementary | 09:15 AM |
| Walnut Creek Elementary | 09:15 AM |
| Washington Elementary | 09:20 AM |
| Weatherstone Elementary | 09:15 AM |
| Wendell Elementary | 08:30 AM |
| Wendell Middle | 08:15 AM |
| West Cary Middle | 07:30 AM |
| West Lake Elementary | 07:45 AM |
| West Lake Middle | 07:40 AM |
| West Millbrook Middle | 08:15 AM |
| White Oak Elementary | 09:15 AM |
| Wilburn Elementary | 09:15 AM |
| Wildwood Forest Elementary | 09:15 AM |
| Wiley Elementary | 09:15 AM |
| Willow Spring High | 07:25 AM |
| Willow Springs Elementary | 09:20 AM |
| Yates Mill Elementary | 09:20 AM |
| York Elementary | 09:15 AM |
| Zebulon Elementary | 08:30 AM |
| Zebulon Middle | 08:15 AM |

APPENDIX

# E

# STRUCTURAL DIFFERENCES OF SUB-DISTRICTS

The tables below show the structural differences in the nine sub-districts of WCPSS. Our study found that the sub-districts with more magnet/special schools typically had more combined routes and more schools breaking the strict tiered start times. Breaking a strict tiered start time occurs whenever there is a school that does not start at 7:25 AM (high school), 8:15 AM (middle school), or 9:15 (elementary school). This makes sense in WCPSS, as an effort has been made to provide transportation for students traveling into Raleigh to attend the magnet/special schools. Special schools are defined as those that are on WCPSS's list of early college and application schools. To provide bus transportation to the magnet/special schools in the Raleigh area, WCPSS must send buses further into adjacent sub-districts than it would for a sub-district without any magnet/special schools. To account for the longer routes, some of the start times in these sub-districts have already been adjusted. Note that District 8 does not have any magnet/special schools or combined routes. This led us to select District 8 for our ATT model study since its behavior was less likely to be affected by the bus-saving methods used to generate combined bus routes in the other sub-districts.

Table E.1: Relationship between magnet/special schools and combined routes.

| | # of Schools | # of Mag/ Spec (M/S) Schools | % M/S Schools | # of Routes | # of Combined Routes | # of Combined Routes for M/S Schools |
|---|---|---|---|---|---|---|
| **District 1** | 23 | 4 | 17.4% | 220 | 17 | 17 |
| **District 2** | 23 | 4 | 17.4% | 202 | 0 | 0 |
| **District 3** | 21 | 3 | 14.3% | 199 | 25 | 2 |
| **District 4** | 21 | 15 | 71.4% | 155 | 42 | 42 |
| **District 5** | 19 | 9 | 47.4% | 161 | 18 | 0 |
| **District 6** | 22 | 15 | 68.2% | 124 | 17 | 17 |
| **District 7** | 21 | 2 | 9.5% | 215 | 2 | 0 |
| **District 8** | 21 | 0 | 0.0% | 270 | 0 | 0 |
| **District 9** | 23 | 6 | 26.1% | 199 | 2 | 2 |

Table E.3: Relationship between magnet/special schools and breaking strict tiered start times.

| | # of Schools | # Mag/ Spec (M/S) Schools | # Schools Break Tier Times | % Break Tier Times | # M/S Schools Break Tier Times | % Break Tier Times that are M/S | % for Whole District |
|---|---|---|---|---|---|---|---|
| **District 1** | 23 | 4 | 5 | 21.7% | 2 | 40.0% | 8.7% |
| **District 2** | 23 | 4 | 2 | 8.7% | 1 | 50.0% | 4.3% |
| **District 3** | 21 | 3 | 4 | 19.0% | 1 | 25.0% | 4.8% |
| **District 4** | 21 | 15 | 7 | 33.3% | 6 | 85.7% | 28.6% |
| **District 5** | 19 | 9 | 5 | 26.3% | 2 | 40.0% | 10.5% |
| **District 6** | 22 | 15 | 9 | 40.9% | 6 | 66.7% | 27.3% |
| **District 7** | 21 | 2 | 3 | 14.3% | 1 | 33.3% | 4.8% |
| **District 8** | 21 | 0 | 0 | 0.0% | 0 | 0.0% | 0.0% |
| **District 9** | 23 | 6 | 3 | 13.0% | 1 | 33.3% | 4.3% |

APPENDIX

# F

# RUN TIME COMPARISONS OF ZTT AND ATT MODELS

All experiments listed in this Appendix were run on a 13th Gen Intel(R) Core(TM) i5-13400 system with 10 cores (6 performance and 4 efficiency), 16 threads, and 8.0 GB of RAM. The models used Gurobi 12.0.0 with a time limit of 10,800 seconds (three hours).

Table F.1: Run time comparisons of zero-minute transition time (ZTT) model and actual transition time (ATT) model using Gurobi solver.

| | **Gurobi Run Time (seconds)** | |
|---|---|---|
| **Scenario** | **ZTT Model** | **ATT Model** |
| Base | 11.817 | 551.805 |
| Swap ES and HS | 13.742 | 10803.077 |
| All HS to 9:15 AM | 7.376 | 57.889 |
| All HS to 8:55 AM | 24.346 | 1039.070 |
| HS to 9:15 AM, MS to 8:05 AM | 20.188 | 40.895 |
| HS to 9:15 AM, MS to 7:55 AM | 20.560 | 18.991 |
| HS to 8:55 AM, MS to 8:05 AM | 19.344 | 1029.165 |
| HS to 8:55 AM, MS to 7:55 AM | 17.016 | 817.450 |
| Strict Tiers | 10.943 | 556.549 |
| Swap ES and HS with Strict Tiers | 13.409 | 10802.724 |
| *Average* | *15.874* | *2571.762* |

APPENDIX

# G

# EXAMPLE OF TIME-INDEXED FORMULATION FOR SBRP

The example formulation below contains modifications to the model formulation presented in Section 3.3. Changes or additions to the original model formulation are indicated by *.

## G.1 Time-Indexed Model Formulation

### Indices & Sets

$r \in R$: Set of routes to be completed. These correspond to route names that are the same as the bus names given by WCPSS.

$b \in B$: Set of available buses. Bus indices are arbitrarily assigned to buses in the fleet of a given sub-district within WCPSS.

*$s \in S$: Set of possible route start times. Depending on the size of the time windows available and the complexity desired, this would be the time window divided by the

interval size. For example, Zeng et al. (2022) use 120 to represent a two-hour time horizon divided into one-minute intervals.

## Input Data

*$\mathbf{C} = [c_{i,j,k,l}] \quad \text{where } \mathbf{C} \in \{0,1\}^{|R\times S|\times|R\times S|}$

A compatibility matrix is created that compares the start and end times of all routes in the model to determine if they overlap in time. If the route $i$ at start time $k$ and route $j$ at start time $l$ overlap in time, then they are incompatible, and $c_{i,j,k,l}$ is set equal to 0. Alternatively, if route $i$ at start time $k$ and route $j$ at start time $l$ do not overlap in time, then they are compatible, and $c_{i,j,k,l}$ is set equal to 1.

## Decision Variables

$x_b$: Binary variable indicating if bus $b$ is used.

$$x_b = \begin{cases} 1 & \text{if bus } b \text{ is used,} \\ 0 & \text{otherwise.} \end{cases}$$

*$a_{r,b,s}$: Binary variable indicating if route $r$ is assigned to bus $b$ at start time $s$.

$$a_{r,b,s} = \begin{cases} 1 & \text{if route } r \text{ is assigned to bus } b \text{ at start time } s, \\ 0 & \text{otherwise.} \end{cases}$$

## Objective Function

$$\min \sum_{b\in B} x_b \tag{G.1}$$

## Constraints

$$* \sum_{b \in B} \sum_{s \in S} a_{r,b,s} = 1, \quad \forall r \in R \tag{G.2}$$

$$* \sum_{s \in S} a_{r,b,s} \leq x_b, \quad \forall r \in R,\ b \in B \tag{G.3}$$

$$* a_{i,b,k} + a_{j,b,l} \leq 1, \quad \forall b \in B, \forall i, j \in R, \forall k, l \in S \text{ such that } c_{i,j,k,l} = 0 \tag{G.4}$$

The objective function (G.1) still seeks to minimize the total number of buses utilized by a district. The first constraint (G.2) is updated slightly to include the new index of start times, but it still requires that every route in the district be covered. The second constraint (G.3) links the binary decision variables to prevent a route from being assigned to a bus that is not being utilized. The final constraint (G.4) references the compatibility matrix, which could require significant computation time and memory if the time window is large and the interval size is small. This matrix is still generated as input data for the model. In this matrix, all diagonal entries and entries for compatible routes are 1, while entries for overlapping routes are 0. Thus, (G.4) ensures that routes that overlap in time are not assigned to the same bus. Note that the size of the compatibility matrix **C** increases *quadratically* in $S$. This is the problem size concern mentioned in Chapter 6.