Transient Analysis in Piping Network Including Fluid-Structure Interaction and Cavitation Effects

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ABSTRACT
The report refers on the first analyses of the dynamic response of a piping network, taking into account fluid-structure interaction, by means of a new computer code. It is shown that interaction effects between pipe and internal fluid can be very important in the case of internal pressure variation due to a rapid pump trip off as well as in the case of seismic excitation. Moreover, cavitation effects that can lead to high pressure impulses, are taken into account.

1. INTRODUCTION

Transient phenomena in piping networks can be caused by two different excitations: the "internal" one, due for example to pump trip off, and the "external" one, due to seismic ground motion. These phenomena can be analysed with different degrees of accuracy: for example neglecting interaction between fluid and pipe and cavitation in the fluid. Often these hypotheses lead to inaccurate response evaluation. Fluid-structure interaction is mainly due to the sharp variation of the fluid momentum in corrispondence of elbows, tees and closed ends; moreover the interaction is due to the so called "breathing effect" (a pipe section enlargment caused by an internal pressure variation) and "friction effect" between pipe wall and fluid.

The influence of these aspects on the response of a very simple system was analysed. The preliminary study showed a strong influence on the system response of the breathing effect and of the axial component of the seismic excitation; consequently the development of the computer code FESTIN was started.

2. RESPONSE OF A SIMPLE NETWORK TO SEISMIC EXCITATION

The response of the system in Fig.1 to a recorded ground motion, Fig.2, acting along the axial direction was computed by two different codes: the first one, a 1-D hydraulic transient code

(Capozza,1986), is based on the Method of Characteristics (MOC) and the second one, a 2-D axisymmetric finite element code (CISE, 1986) linear in structural and fluid behaviour, accounts for fluid-structure interaction. The two computed pressure histories at the dead end are reported in Fig.3. Response differences may be qualitatively explained by the fact that fluid-structure interaction generates new frequencies inside the seismic spectrum range(see Fig.2b).

Further analyses have been carried out with the 1-D hydraulic code to evaluate cavitation effects on pressure transient, Fig.4. Since the interaction effects are extremely important a new computational method was developed.

3. "FESTIN": A COMPUTER CODE FOR DYNAMIC ANALYSIS OF PIPING NETWORKS RESPONSE

The structure of the code consists of:

- A linear structural solver based on the finite element method: this module required the formulation of a special pipe finite element accounting for axi-symmetric section straining and Poisson coupling.
- A fluid transient solver based on non-linear water-hammer 1-D equations and solved by the MOC.
- A "coupling" algorithm between structure and fluid.

These three modules are briefly described in the following paragraphs.

3.1 A new pipe finite element accounting for breathing and Poisson effects

In order to take into account the internal pressure effects on the pipe response, a new finite element has been formulated: besides the usual twelve beam degrees of freedom (six displacement components and six rotations) we added four degrees of freedom (two radial displacements and rotations) for a total of sixteen. The displacement field $\mathbb{U}(s,\Theta,t)$ can be expressed in term of the unknown nodal displacements $\mathbb{d}$:

$$\mathbb{U}(s,\Theta,t) = \mathbb{N}(s,\Theta)\ \mathbb{d}(t) \qquad (1)$$

with s,Θ the axial and circumferential coordinate respectively, while t is time.

The stress-strain constitutive law (assumed linear) is expressed in terms of generalised variables σ and ϵ as:

$$\sigma = \mathbb{D}\ \epsilon \qquad (2)$$

where the matrix $\mathbb{D}$ is defined according to the Kirchoff-Love theory neglecting the shear strain on the pipe wall.

Applying the virtual work method we have:

$$\mathbb{M}\mathbb{d} + \mathbb{K}\mathbb{d} = \mathbb{f}_S + \mathbb{f}_V + \mathbb{f}_C \qquad (3)$$

with $\mathbb{M}$,$\mathbb{K}$ mass and stiffness matrices and f_s, f_v, f_c the vectors of generalised components of volume, surface (including fluid pressure) and external concentrated loads respectively.
The damping matrix $\mathbb{C}$ is assumed of the Raylegh type:

$$\mathbb{C} = \alpha\ \mathbb{K} + \beta\ \mathbb{M} \tag{4}$$

The solution of equation (3) is performed with a direct integration scheme making use of Hilbert, Hughes, Taylor algorithm (1977).

3.2 Fluid transient analysis

The matematical model of the fluid is defined under the following assumptions:
- 1-D flow dynamics;
- single-phase flow (liquid);
- constant values of density and friction factor;
- no heat transfer between the liquid and pipe walls.

Then the flow dynamics conservation equations of mass and momentum can be written in the form (Streeter and Wiley 1985) :

$$\text{(mass)} \qquad \frac{\partial H}{\partial t} + \frac{Q}{A}\frac{\partial H}{\partial x} + \frac{c^2}{gA}\frac{\partial Q}{\partial x} = \frac{Q}{A}\frac{dz}{dx} \tag{5}$$

$$\text{(momentum)} \qquad \frac{\partial H}{\partial x} + \frac{1}{gA}\frac{\partial Q}{\partial t} + \frac{Q}{gA^2}\frac{\partial Q}{\partial x} = -F + \frac{a}{g} \tag{6}$$

where:
H = piezometric head
Q = flow rate
A = pipe cross section area
c = wavespeed
a = axial seismic acceleration
g = gravity acceleration
x = spatial abscissa
z = elevation above sea level
$F = f\,Q|Q|/gA$ is the drag term due to friction
f = pipe friction factor
D = pipe diameter

When a vapour cavity forms inside the pipe, say at point P, this is treated as an intermediate new boundary with the relevant boundary condition given by:

$$H_p = p_{sat} + z_p \tag{7}$$

The evolution of the cavity volume W around point P is ruled by the unbalance of the liquid flowrate upstream and downstream the point P (Q_p^- and Q_p^+ respectively):

$$dW/dt = Q_p^+ - Q_p^- \tag{8}$$

When the cavity collapses an overpressure dp arises:

$$dp = c\ (Q_p^- - Q_p^+)/\ 2gA \tag{9}$$

The flowrate at point P immediatly after the cavity collapse is:

$$Q_p = (Q_p^- + Q_p^+)/2 \tag{10}$$

The boundary conditions are supplied by the mathematical models of the nodal elements (connections with fixed resistance, reservoirs with variable level, dead ends) which constitute the phisical boundary of the pipe.

3.3 Coupling algorithm

Fluid-structure interaction is considered through the breathing and the friction effects. The steps of the coupling iterative algorithm can be briefly described as follows:

(a) perform a fluid dynamic solution of the network (MOC) based on known variables at time t_n to predict fluid pressure and velocity at time t_{n+1};

(b) compute structural displacements of the network at time t_{n+1} due to the friction, pressure and external forces;

(c) check for convergence of structural displacements and forces of step (b), if convergence is satisfied update all the variables to the new values at time t_{n+1} and go to (a) for a new time step;

(d) else update wavespeed nodal values according to the formula:

$$c_i^{(n+1)} = \frac{c_\infty}{\sqrt{1 + \dfrac{\epsilon}{A}\dfrac{dA}{dp}}} \approx \frac{c_\infty}{\sqrt{1 + \dfrac{2\epsilon}{D/2+\delta_i^{(n)}}\ \dfrac{\delta_i^{(n+1)} - \delta_i^{(n)}}{p_i^{(n+1)} - p_i^{(n)}}}} \tag{11}$$

where c_∞ is the wavespeed in the infinite media, ϵ is the fluid bulk modulus, δ_i is the radial pipe displacement and i is the nodal index; then goto (a) for a new fluid computation at time t_n.

4. EXAMPLE

The system of Fig.1 subjected to the seismic excitation of Fig.2 has been analysed with the FESTIN code accounting for cavitation. The dead end pressure time history, reported in Fig.5, clearly show the smoothing effect on the system response if fluid-structure interaction is included in comparison with the results of the hydraulic code, Fig.4.

5. CONCLUSIONS

FESTIN is a computer code, under development, for transient analysis of piping network able to take into account fluid-structure interaction as well as cavitation.

Preliminary results obtained in the analysis of a very simple system are presented.

Developments of the FESTIN code will concern with the formulation of an elbow finite element allowing for a more realistic fluid-structure interaction computation. Further improvements will deal with fluid-structure coupling algorithm formulation.

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Table 1. System frequencies [Hz]

Mode	2-D code	1-D code
1	2.136	2.340
2	6.906	7.011
3	11.200	11.674
4	13.010	16.345
5	15.136	21.008
6	18.710	25.680
7	21.980	30.343
8	25.047	35.014

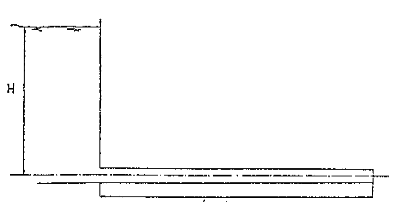

Fig.1 - Simplified network

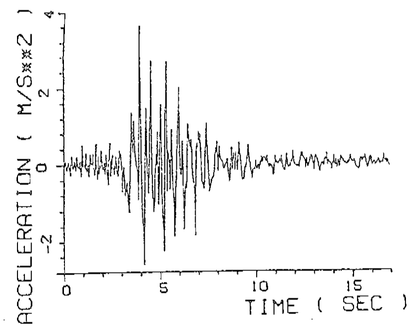

Fig.2a - Seismic acceleration time history

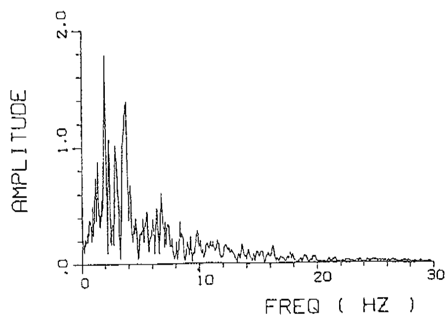

Fig.2b - Fourier spectrum

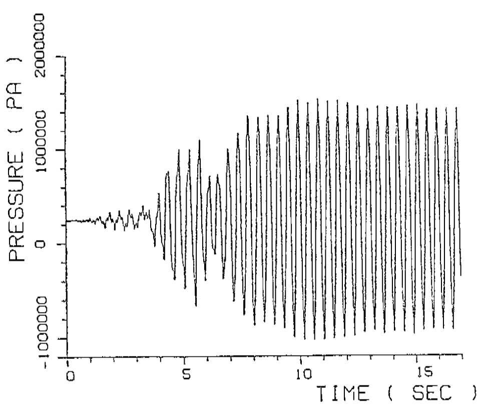

Fig.3a - Dead end pressure time history computed with 1-D hydraulic code switching off cavitation

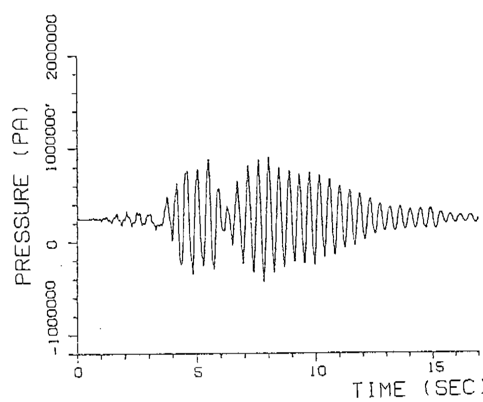

Fig.3b - Dead end pressure time history computed with 2-D fluid-structure code

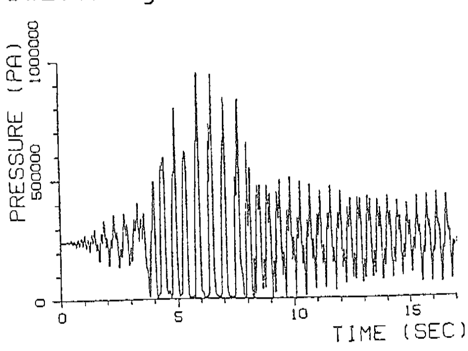

Fig.4 - Dead end pressure time history computed with 1-D hydraulic code including cavitation effects

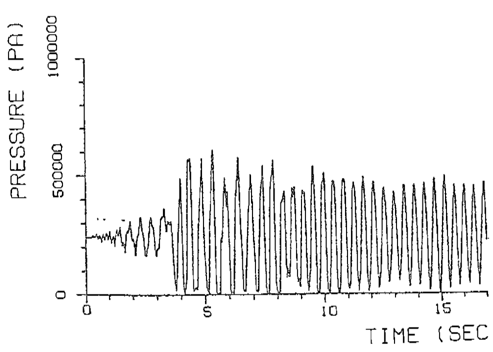

Fig.5 - Dead end pressure time history computed with code FESTIN including cavitation effects