Application of a New Thermoelastic Thick Shell Theory to Solar Boiler Tubes

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SUMMARY

The classical thin shell theory relies on Kirchhoff's hypotheses which notably lead one to neglect normal stresses in the "thickness direction", shearing stresses on surfaces parallel to the mean surface, warping and extension of fibers normal to the same surface.

As is well known, the thermoelastic constitutive equations contradict these assumptions, which are no longer acceptable when the shell is not "very thin" and when the thermal gradient through the thickness becomes noticeable.

The thick shell theory proposed here was initially meant to describe the behavior of solar boiler tubes, but it is also applicable to piping elements encountered in most energy conversion systems, such as fossil-fired power plants, nuclear reactors, etc.

For the sake of simplicity, the presentation is restricted to the study of a long tube subjected to an intense heat flux on one side; this tube carries a working fluid and the temperature distribution in the metal is not axisymmetric.

By generalizing the asymptotic expansion theory of Reiss to the thermoelastic case, it is shown that the higher order theory immediately following the classical thin shell approach should be based upon a displacement field whose radial component is a second order polynomial in the thickness coordinate z, whereas the tangential component is a third order polynomial in the same coordinate; the coefficients of these two polynomials are the generalized displacements of the theory.

Then a "generalized" Kirchhoff hypothesis is used, which consists in requiring the shearing stresses or deformations to vanish on the inner and outer tube surfaces only. This allows one to express two of the generalized displacements in terms of the others.

The so-constructed displacement distribution is sufficiently general to account for warping and extension of fibers normal to the mean surface as well as shearing on surfaces z = constant; these effects are usually neglected.

1. Introduction

Stress analysis of moderately thick piping structures subjected to noticeable temperature changes reveals the inadequacy of thin shell theory, notably due to the fact that expansion of fibers normal to the mean surface is not allowed.

For isothermal problems, several theories have been proposed. Let us mention, for instance, theories in which the displacement is assumed to be a quadratic function of the "thickness variable" z [1,2], or theories in which the "tangential" displacement is cubic in z, whereas the "normal" displacement is independent of z, [3] for shells, [4] for plates.

None of these assumptions can lead to a consistent theory when thermoelastic behavior is accounted for.

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For the sake of simplicity our investigation is restricted to a long cylindrical tube with annular cross section, prevented from expanding axially and bending. The tube state will be assumed to be a plane strain one, in which stresses and deformations do not vary along the axis. In this special case, comparison with results deduced from the three-dimensional thermoelastic theory will be easier, but transposition of the proposed theory to more general situations entails no difficulty.

In the tube with mean radius R and thickness h , a temperature distribution symmetrical about the $\theta = 0$ (π) line is assumed to exist, where θ denotes the circumferential angular coordinate. In order to fix ideas, this distribution $T(\theta, z)$ is supposed to be linear with respect to the radial or "thickness" coordinate z such that $z = 0$ on the tube mean surface

$$T(\theta, z) = \tau_0(\theta) + z\,\tau_1(\theta) \qquad\qquad -h/2 \leq z \leq +h/2$$

where τ_c and τ_1 are the mean temperature and through-the-wall gradient respectively. Although this assumption is not essential, its validity for a thick tube $(h/R \simeq 0.40)$ subjected to a stationary temperature distribution has been assessed in [5].

Before giving the kinematical assumptions underlying our theory, we will briefly present some useful results obtained through an asymptotic expansion procedure. The role of asymptotic expansions in shell theories has long been recognized, but few practical consequences have resulted, as far as thick thermoelastic shells are concerned.

2. Asymptotic expansion

As is done by Reiss [6], let us introduce the small parameter

$$\varepsilon = (h/2R)^{1/2}$$ and the dimensionless variables

$$\zeta = \frac{z}{R\varepsilon^2} \; , \; \varphi = \theta/\varepsilon$$ pertaining to the cylindrical coordinate system (r, θ, x) with $r = R + z$; x would be the variable in the axial direction.

The inner and outer tube surfaces are then described by $\zeta = \pm 1$

Let u_z, u_θ denote the physical displacements in radial and circumferential directions respectively, $\bar{w} = u_z/R$ and $\bar{v} = u_\theta/R$ are the corresponding dimensionless displacements.

If $\sigma_\theta, \sigma_z, \sigma_{z\theta}$ are the physical stress components related to the cylindrical coordinate system, then $\bar{\sigma}_i = \frac{\sigma_i}{E}$ (with $i = \theta, z, z\theta$) will denote their dimensionless counterparts.

The dimensionless temperature distribution is in the form

$$\alpha T(\zeta, \varphi) = \alpha T_0(\varphi) + \zeta \alpha T_1(\varphi)$$

where α is the thermal expansion coefficient.

In the case at hand, equilibrium equations can be written as

$$\bar{\sigma}_{z,\zeta} + \varepsilon \bar{\sigma}_{z\theta,\varphi} + \varepsilon^2 [\bar{\sigma}_z - \bar{\sigma}_\theta + \zeta \bar{\sigma}_{z,\zeta}] = 0 \qquad (1)$$

$$\bar{\sigma}_{z\theta,\zeta} + \varepsilon \bar{\sigma}_{\theta,\varphi} + \varepsilon^2 [2 \bar{\sigma}_{z\theta} + \zeta \bar{\sigma}_{z\theta,\zeta}] = 0 \qquad (2)$$

When inner fluid pressure is negligible, boundary conditions on inner and outer surfaces read as :

$$\bar{\sigma}_z(\varphi, \pm 1) = 0 \qquad \sigma_{z\theta}(\varphi, \pm 1) = 0 \qquad (3,4)$$

Plane strain $(\varepsilon_x = 0)$ thermoelastic constitutive equations are :

$$\bar{v}_{,\zeta} + \varepsilon \bar{w}_{,\varphi} + \varepsilon^2 \{\zeta \bar{v}_{,\zeta} - \bar{v} - 2(1+\nu)\bar{\sigma}_{z\theta}\} - \varepsilon^4 \{2\zeta(1+\nu)\bar{\sigma}_{z\theta}\} = 0 \qquad (5)$$

$$\bar{v}_{,\varphi} + \varepsilon \{\bar{w} - (1+\nu)\alpha T - (1-\nu^2)\bar{\sigma}_\theta + \nu(1+\nu)\bar{\sigma}_z\}$$
$$- \varepsilon^3 (1+\nu)\zeta\{\alpha T + (1-\nu)\bar{\sigma}_z - \nu \bar{\sigma}_\theta\} = 0 \qquad (6)$$

$$\bar{w}_{,\zeta} - \varepsilon^2 (1+\nu)\{\alpha T + (1-\nu)\bar{\sigma}_z - \nu \bar{\sigma}_\theta\} = 0 \qquad (7)$$

As is usual in the bending theory of shells, we look for asymptotic expansions of the form :

$$\bar{w} = \sum_{n=0}^{\infty} \varepsilon^n w^n \qquad \bar{v} = \varepsilon \sum_{n=0}^{\infty} \varepsilon^n v^n \qquad (8)$$

$$\bar{\sigma}_\theta = \sum_{n=0}^{\infty} \varepsilon^n \sigma_\theta^n, \quad \bar{\sigma}_{z\theta} = \varepsilon \sum_{n=0}^{\infty} \varepsilon^n \sigma_{z\theta}^n, \quad \bar{\sigma}_z = \varepsilon^2 \sum_{n=0}^{\infty} \varepsilon^n \sigma_z^n \qquad (9)$$

The above expressions are substituted into the equilibrium equations eq. (1,2), constitutive equations eq. (5,6,7), and boundary conditions eq. (3,4). Coefficients of the same power of ε are equated, yielding systems of equations satisfied by the stress and displacement coefficients v^n, w^n, σ_i^n of all orders n. Thanks to the boundary conditions, these systems can be solved "with respect to ζ", i.e. their solutions are obtained in terms of ζ and unknown functions of φ, by using a method whose principle is outlined in [6].

The N^{th} order theory is defined as the approximate solution obtained by retaining only the $N+1$ first terms in each of the above asymptotic expansions. Such a "solution" is a set of functions whose dependence with respect to ζ has a known form.

The zeroth and first order solutions are both characterized by $\bar{w}$ independent of ζ and $\bar{v}$ a linear function of ζ ; this displacement field thus has the same form as that of the classical thin shell theory.

The second order solution leads to a second order polynomial in ζ for $\bar{w}$, and a third order polynomial for $\bar{v}$; moreover, if thermal effects are disregarded, the coefficient of ζ^2 in $\bar{v}$ is zero.

These preliminary results will now serve as a guide to construct a consistent thick shell theory.

3. Kinematical assumptions

As the first order solution corresponds to the thin shell theory, it is natural to try a displacement field similar to that of the second order solution, i.e. to use as a starting point a circumferential displacement u_θ cubic in z and a radial displacement quadratic in z :

$$u_z(z,\theta) = w(\theta) + z\,\psi_z(\theta) + z^2\chi_z(\theta) \qquad (10)$$

$$u_\theta(z,\theta) = v(\theta) + z\,\psi_\theta(\theta) + z^2\chi_\theta(\theta) + z^3\rho(\theta) \qquad (11)$$

The seven functions $w, \psi_z, \chi_z, v, \psi_\theta, \chi_\theta$ and ρ are the generalized displacements of the theory, v and w being the circumferential and radial displacements of a point on the mean surface respectively. Let us point out that in an isothermal theory the z^2 term in u_θ would be absent, a result consistent with the assumptions of Levinson [4], which are in other respects different from ours.

It is also interesting to note that our assumptions are different from those of R. Kienzler, who uses a quadratic approximation in z , for elastic behavior [2].

It is also logical to replace the classical Kirchhoff hypothesis with a "general Kirchhoff hypothesis", amounting to impose the distorsion $\varepsilon_{\theta z}$ to be zero at $z = \pm h/2$, as is done by Levinson [4]; this allows to satisfy part of the stress boundary conditions on the inner and outer surfaces. The remaining boundary conditions bearing upon normal stress in the radial direction, $\bar{\sigma}_z = 0$ at $z \pm h/2$, are not used, as they would involve the temperature field and material properties; moreover they would not be appropriate for tubes subjected to noticeable internal or external pressure.

Then χ_θ and ψ_θ can be expressed in terms of the five other generalized displacements as :

$$\chi_\theta(\theta) = \frac{-1}{2R}\frac{d}{d\theta}\psi_z(\theta) - \frac{h^2}{4R}\rho(\theta) \qquad (12)$$

$$\psi_\theta(\theta) = \frac{v(\theta)}{R} + \frac{h^2}{8R^2}\frac{d}{d\theta}\psi_z(\theta) - \frac{1}{R}\frac{d}{d\theta}w(\theta) - \frac{h^2}{4R}\frac{d}{d\theta}\chi_z(\theta) + \left(\frac{h^4}{16R^2} - 3\frac{h^2}{4}\right)\rho(\theta) \qquad (13)$$

4. Equilibrium and constitutive equations

The density of virtual power of internal forces reduces to :

$$p = \bar{\sigma}_\theta\varepsilon^*_\theta + \bar{\sigma}_z\varepsilon^*_z + 2\,\bar{\sigma}_{z\theta}\,\varepsilon^*_{z\theta}$$

where the virtual deformation rates ε^*_θ, ε^*_z, $\varepsilon^*_{z\theta}$ are related to the five independent generalized virtual velocities through the usual strain-displacement relationships.

The principle of virtual power yields the macroscopic equilibrium equations [7] : integration through the tube thickness leads one to introduce generalized stresses, which are moments of order zero, one, two or three of stresses defined by :

$$\{N_\theta, M_\theta, V_\theta, R_\theta\} = \int_{-h/2}^{+h/2} \{1, z, z^2, z^3\} \bar{\sigma}_\theta \, dz$$

$$\{N_z, V_z\} = \int_{-h/2}^{+h/2} \{1, z\} \bar{\sigma}_z (1 + z/R) \, dz$$

$$\{N_{z\theta}, R_{z\theta}, K_{z\theta}, V_{z\theta}\} = \int_{-h/2}^{+h/2} \{1, z, z^2, z^3\} \bar{\sigma}_{z\theta} \, dz$$

The expressions of generalized stresses in terms of generalized displacements are obtained by integrating the plane strain thermoelastic constitutive equations over the tube thickness, using the strain-displacement relations and the kinematical assumptions eq. (10,11,12, 13) (see [8]).

5. Numerical results To assess the acurracy and the usefulness of the theory, two sorts of verifications were carried out :

The first test pertains to the THEM French solar boiler, whose working fluid is a mixture of molten salts. Its tubes are moderately thick $(h/R = 0.117)$; they are subjected to an asymmetrical solar flux around their circumference; this flux varies according to a cosine law along the sunside of the tube and the rear side is assumed to be insulated. The results provided by our theory were compared with those of the "exact" thermoelastic solution, as given in [9]; it is worthwile to note that our theory gives a sufficiently precise value of the normal stress resultant N_z in the radial direction, a quantity which is neglected in thin shell theories, together with an accurate value of the shearing stress resultant $N_{z\theta}$.

Moreover, in order to show that, even for this moderately thick tube, the classical thin shell theory already leads to a significant error, we calculated the values of the normal stress $\bar{\sigma}_x$ in the longitudinal direction as given by our theory and that of Chern and Pai [10]; they already differ by 12.5%.

The second test involves a steam solar boiler developed by the "Laboratoire d'Energétique Solaire" in Poitiers (France). This boiler uses a very thick tube $(h/R = 0.41)$. The value of the axial stress at the most heated point is obtained by the present theory within 2 percent of that given by the exact theory : even in this borderline case, the theory we propose is still applicable and renders it un-necessary to resort to the more complicated thermoelastic solution.

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