

Plastic Response to Impact of Beams with Cracks: Simple Models for Cracked Piping

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Abstract

Simple applied mechanics models of beams with cracks can be very useful in exploring the effects of various parameters on the structural dynamics of cracked piping. The size and location of cracks, the ratio of impacting mass to pipe mass, and the loading and support conditions are some of the important variables that can be treated analytically in order to identify worst case conditions for more sophisticated analysis via large computer models. The preliminary use of simple analytical models is discussed, and an example is given whereby the worst case location of a crack in a cantilever beam subject to impact is calculated.

1. Introduction

Cracks and other geometric discontinuities can have dramatic effects on the response of beams and piping loops to impact loading. Catastrophic fracture can result in brittle materials while gross ligament yielding can occur in ductile materials, and at load levels considerably below those that could be safely sustained by an uncracked beam or pipe. While specifying a tough material obviates the fast fracture mode and makes a structural member highly tolerant of cracks and flaws, increased ductility also means that stable cracks can provide favored sites for the development of plastic hinges that can result in extreme deflection responses. These in turn can threaten the stability of the crack.

In order to develop elementary models for and an understanding of the dynamic behavior of cracked piping loops, the effect of a stable crack on the large permanent deformation of beams has been studied by means of a simple rigid-perfectly plastic model (e.g. [1], [2]). The uncracked ligament is assumed to yield at a bending moment that is a function of crack size. Thus, the crack, even though not at the section of maximum bending moment, can be the preferred site of yielding in an otherwise uniform beam, and the subsequent plastic response can be considerably affected. Not only can the deformed shape of the beam be significantly altered, but also the magnitudes of maximum transverse deflection and plastic hinge rotation can be greatly increased. Even though they do not propagate, when cracks are sufficiently large, the deformations they cause can be intolerable. When the crack-opening angle, which is directly related to the plastic hinge rotation, is excessive, crack growth can accompany the gross plastic beam response.

In such cases a simple model coupling the beam deformation and stable crack growth can be applied [3].

Free-free, simply-supported and cantilever beams with cracks have been studied under a variety of loading conditions. The moving plastic hinges, which give plasticly-deformed beams their characteristic curved shapes, have been found to have their range of travel altered by the presence of cracks [1]. In the familiar problem of a cantilever beam loaded transversely at its tip, for example, the presence of a crack can entirely suppress the development of a moving plastic hinge. The number of plastic hinges has also been found to depend upon the size and location of cracks [2].

By employing the simple models developed for the beam with a stable crack it is possible to solve a variety of problems to a considerable extent in closed form. This enables one to understand the mechanical phenomena involved in the problem, to ascertain the relative importance of various parameters (crack size, crack location, ratio of impact mass to beam mass, ratio of impact energy to beam strength, etc.) and to identify worst case configurations. Once this has been done for a class of problems, a more sophisticated computer model can be employed if it is desired to obtain more precise quantitative results for special cases. The simple model results also provide solutions to benchmark problems against which more complex and less intuitive computer models may be compared.

The analytical predictions of the simple model have been confirmed experimentally for the case of the cracked cantilever beam [4]. Different numbers of plastic hinges have been obtained by introducing appropriate cracks, and the familiar pattern of an uncracked cantilever has been found to be significantly altered for certain combinations of crack size and location.

2. Worst-Case Identification

The efficacy of the simple-model approach may be demonstrated by considering the problem of identifying the worst-case crack in a ductile cantilever struck transversely at the tip. This problem may be taken as the idealization of a segment of a piping loop attached to (cantilevered out from) a pressure vessel nozzle. By allowing the attached tip mass to be a parameter in the simple model, the extremes of a pipe alone and a pipe section with a heavy pump attached may be considered at one and the same time. Furthermore the model may easily be adapted to the problem of pipe whip. An illustration of the use of the simple model to identify worst-case problems recently appeared among reports of current research in the International Journal of Fracture [5], as follows.

Given a ductile cantilever beam subject to dynamic forces acting transversely at its tip, what crack location and orientation will result in the greatest likelihood of instability? While it may be clear that the worst-case crack orientation will be one for which the crack front experiences the greatest tensile stress, where along the beam such a crack will undergo the greatest crack-opening displacement may be less trivial a question.

The cantilever beam with attached tip mass is a familiar configuration in applied mechanics, and its analysis provides fundamental insight into such important practical

problems as damage due to impact and pipe whip. When the beam's material is modeled as rigid-perfectly plastic and the structure is assumed uniform and flawless, such treatments as those of Parkes [6], Bodner and Symonds [7], and Ting [8] give results which capture nicely the phenomena observed in experiments reported in conjunction with their own [6,7] and subsequent work [9,10]. The presence of a stable crack at any position along the cantilever beam, as shown in Fig. 1, will cause a stationary plastic hinge to develop at the cracked section, with the size and location of the crack determining whether subsequent hinges develop.

The implications of the beam's altered behavior for questions of crack stability is this: what may be considered worst-case locations for cracks according to a static or even a dynamic analysis of an uncracked beam may not in fact be the worst-case location under the dynamic loading of a cracked beam. Thus treatments such as that of Gomez [11], who varies the loading conditions but apparently not the nozzle crack location, or that of Sharma [12], who appears to consider cracks in nuclear piping only at a nozzle junction or an elbow, may not adequately answer the question of the stability of a crack at some other location. While these treatments may be definitive for nozzle and elbow cracks, under the loadings considered they may not assure the stability of a crack elsewhere in the piping loop, say at a sensitized weld.

To see that a worst-case crack, as defined by greatest crack-opening angle, can exist elsewhere than at the root of a cantilever beam, consider a beam of length l and of mass per unit length m with an attached heavy tip mass M , as shown in Fig. 1. A second plastic hinge will develop only if the uncracked ligament has a reserve resistance in bending that is greater than k/l times the uncracked beam's plastic limit moment M_p . If such a second hinge develops the beam will come to deform as shown in Fig. 1, where the crack-opening angle is $\theta = \phi$.

For a given amount of input energy, corresponding to a mass M impacting with velocity v , the two-hinge mode of deformation clearly results in a lesser crack-opening angle since the plastic hinge at the root absorbs energy at the rate $\dot{\phi} M_p$, where the superposed dot indicates the time derivative. Thus the single-hinge mode, as shown in Fig. 2, with $\phi \equiv 0$, must result in the ligament at the cracked section having to absorb the entire input kinetic energy $K = Mv^2/2$ and thus rotate the crack faces through the total angle θ_{max} . This angle may be determined from the equations governing the rigid-body rotation of the portion of the beam beyond the crack. These give for the angular acceleration

$$\ddot{\theta} = - \frac{3\gamma M_p}{(3 + \alpha)Mk^2} \quad (1)$$

with the initial conditions, at $t = 0$,

$$\dot{\theta} = \frac{3v}{(3 + \alpha)k}, \quad \theta = 0 \quad (2)$$

where $0 < \gamma \leq 1$ is the fraction of the uncracked beam's fully plastic moment that can be developed across the ligament at the cracked section and is thus a measure of crack size, and where $\alpha = mk/M$ is a measure of the ratio of the beam's rotating mass to the tip mass.

This may be integrated to give θ as a function of time, and the condition that θ is maximum where $\dot{\theta} = 0$ enables us to calculate the maximum angle θ , which is of course related to the maximum crack-opening angle, to be

$$\theta_{\max} = \frac{3K}{(3 + 2\alpha)\gamma M_p} \quad (3)$$

For a given material with fully plastic yield moment M_p , for a given crack size γ , and for a given input energy K , (3) clearly is maximum for minimum α . Thus the worst-case crack is the one that is closest to the tip and yet results in only a single plastic hinge at the crack location. This is a crack such that $k/l = \gamma$ [2].

For a pipe, a beam with a tubular cross section, with a crack penetrating the wall over the entire arc subtending an angle 2β ,

$$\gamma = \cos \frac{\beta}{2} - \frac{1}{2} \sin \beta \quad (4)$$

and Fig. 3 gives the worst-case locations for cracks of various sizes opened by a mass impacting the end of the beam. Conversely, given a critical crack-opening angle, the maximum size for a stable crack at a given location may be determined.

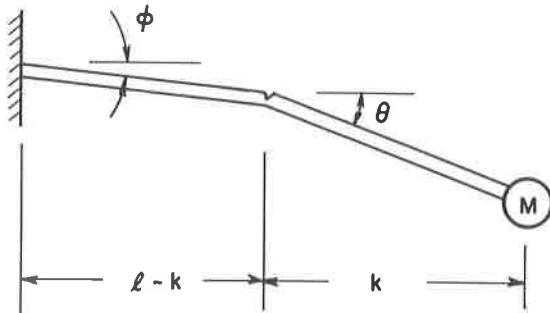
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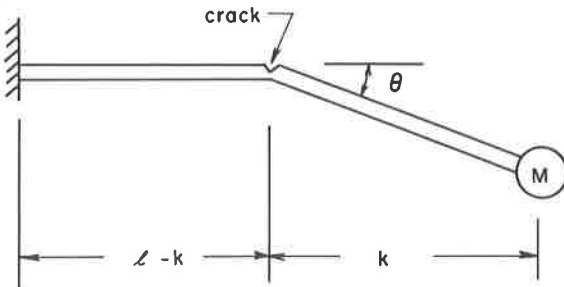
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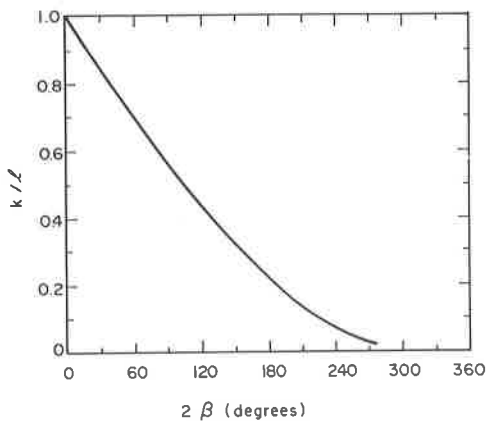
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1. Two-hinge mode of plastic response.



2. Worst-case, single-hinge response.



3. Worst-case locations for circumferential through-cracks in a cantilevered pipe.