CYCLIC CHARACTERISTICS OF EARTHQUAKE TIME HISTORIES

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SUMMARY

From an engineering standpoint, an earthquake record may be characterized by a number of parameters, one of which is its "cyclic characteristics". The cyclic characteristics are most significant in fatigue analysis of structures and liquefaction analysis of soils where, in addition to the peak motion, cyclic buildup is significant. Whereas duration, peak amplitude and response spectra for earthquakes have been studied extensively, the cyclic characteristics of earthquake records have not received an equivalent attention. Present procedures to define the cyclic characteristics are generally based upon counting the number of peaks at various amplitude ranges on a record.

This paper presents a computer approach which describes a time history by an amplitude envelope and a phase curve. Using Fast Fourier Transform Techniques, an earthquake time history is represented as a projection along the x-axis of a rotating vector—the length the vector is given by the amplitude spectra—and the angle between the vector and x-axis is given by the phase curve. Thus one cycle is completed when the vector makes a full rotation. Based upon Miner's cumulative damage concept, the computer code automatically combines the cycles of various amplitudes to obtain the equivalent number of cycles of a given amplitude. To illustrate the overall results, the cyclic characteristics of several real and synthetic earthquake time histories have been studied and are presented in the paper, with the conclusion that this procedure provides a physical interpretation of the cyclic characteristics of earthquakes.

1. Introduction

"Cyclic characteristics" of an earthquake time history are defined by the total number of load cycles it contains, the distribution of cycles at various amplitude levels, the cumulative distribution of cycles with time, and the number of equivalent sinusoidal cycles with a reference amplitude such that the fatigue or liquefaction damage by the equivalent cycles is the same as that by the entire time history. These characteristics are of significance for phenomena such as soil liquefaction and structural fatigue, which are governed by the cyclic buildup of the entire time history rather than the single occurrence of a critical or failure load.

Cyclic characteristics of earthquake time histories have been studied by Seed et al.,(1) Annaki and Lee,(2) and Lee and Chan.(3) The above-referenced papers define occurrence of one-half of a cycle every time a peak is observed, while this paper represents an earthquake time history by the projection of a rotating vector of varying amplitude and phase. When this vector completes one rotation, one cycle is said to be transcribed. This concept of rotating vector has been studied by Farnbach,(4) as discussed in Section 2 below. The authors have developed a computer code which studies cyclic characteristics of a digitized earthquake time history, defines significant duration of the earthquake related to damage potential, and provides a histogram of cycle amplitudes.

2. Complex Envelope or Rotating Vector Representation

A time history can be described by the real part of its complex envelope. The concept of representing a time history in this manner has been used extensively in radar technology and mechanical vibrations, while Farnbach(4) has illustrated its application to seismic signal analysis. The following is a brief discussion of this concept.

For a time history a(t) defined at the discrete points $t = 0, 1\Delta t, 2\Delta t, \ldots, (N-1)\Delta t$, its complex envelope $a^*(t)$ is such that:

$$a(t) = R_e\ a^*(t) = \bar{a}(t)\ .\ \mathrm{Cos}\ \Phi(t) \qquad (1)$$

where the symbol R_e implies the real part, and the terms $\bar{a}(t)$ and $\Phi(t)$ are termed as the amplitude envelope and the phase envelope, respectively, of the time history a(t), and are equal to the magnitude and phase of the complex envelope $a^*(t)$. As illustrated in Figure 1, the time history can be physically viewed as the projection of a rotating vector with varying length $\bar{a}(t)$ and phase $\Phi(t)$. The $\bar{a}(t)$ and $d\Phi/dt$ are the instantaneous amplitude and the instantaneous frequency, respectively.

Equation (1) is insufficient to define the complex envelope since its imaginary part is undefined. Hence, an appropriate transform must be chosen to obtain $a^*(t)$ such that the resulting $\bar{a}(t)$ and $\Phi(t)$ are physically meaningful, i.e. the instantaneous amplitude $\bar{a}(t)$ and the instantaneous frequency $d\Phi/dt$ should agree reasonably with the amplitudes measured at time history peaks and the frequency estimated from the number of zero amplitude crossings. For instance, for a sinusoidal signal a_o Sin ωt, the magnitude of the amplitude envelope should be constant at all times and equal to a_o, and the phase $\Phi(t)$ should increase at a constant rate of ω.

Farnbach(4) points out that the negative of the Discrete Hilbert Transform of a(t) would satisfy the above conditions as well as the linearity implied in Equation 1. Cizek(5) has provided the techniques for computing the Discrete Hilbert Transform from the Discrete Fourier

Transform (Bergland,(6) Singleton(7) and Cochran et al.(8)) and, accordingly, complex envelope computation procedure involves the following steps (Farnbach(4)):

(1) Calculation of the Discrete Fourier Transform (DFT) of a(t), as S*(k), $o \leq k < N$.

(2) Multiplication of this transform by two for indices $o < k < N/2$ and by zero for indices $N/2 < k < N$, leaving the transform unchanged for $k = 0$ and $k = N/2$.

(3) The inversion of the resulting DFT leads to a*(t).

3. Complex Envelope Concept to Define Cycles

The complex envelope allows a time history to be separated into its amplitude and phase elements. The authors have used this facility to define the occurrence of a cycle based on $\Phi(t)$. Whenever $\Phi(t)$ between any two time points differs by 2π, one complete cycle is said to have been transcribed between these two points. It follows that this definition allows the consideration of cases involving fractions of a cycle also, with corresponding increments in $\Phi(t)$ which are fractions of 2π.

However, existing definitions for the occurrence of a cycle have been based upon either (1) number of zero amplitude crossings, or (2) number of peaks observed.

The number of zero amplitude crossings concept is based upon counting the number of times a signal crosses the zero amplitude line, and the total cycles are one-half of this number. A disadvantage of this method is that the distribution of cycles with amplitude is not studied.

The number of observed peaks concept has been discussed by Seed et al.,(1) and consists of counting peaks within the absolute value of a signal, each peak counting as one-half a cycle. The amplitude of the half-cycle is that of the peak. The procedure gives rise to some problems, as illustrated in an example in Figure 1. The small peak in the middle of the time history would seem to have little physical significance, but based on the observed peaks concept, it would count as one-half a cycle. The difficulty arises from the fact that the observed peaks concept looks at the original time history, which combines and, hence, confuses the amplitude and phase information. For this reason, the definition proposed herein based on the phase-envelope cycle concept seems to be more fundamentally meaningful.

4. Computer Code to Evaluate Cyclic Characteristics

The authors have developed a computer code which evaluates the cyclic characteristics of a given digitized time history. Although fatigue and liquefaction failures are more directly related to a stress than to an acceleration time history, Annaki and Lee(2) point out that it may be more convenient to base the computation of equivalent number of cycles on acceleration rather than stress time histories. This approach is considered appropriate because of the direct proportionality between acceleration, force and stress. Thus, even though the computer code discussed below can analyze either the acceleration or stress time history, the numerical examples consider only acceleration time histories. The concept of a complex envelope and capabilities of the above computer code are illustrated by numerical examples presented in Figures 2 and 3, and in Tables 1, 2 and 3. The four examples of earthquake time histories considered in Tables 1, 2 and 3, are an artificial time history derived from the El Centro event (1940), the T-14 Ancona earthquake (June 1972), the Pasadena Record (Kern County earthquake, 1952) and the Cholame Shandon Record No. 8 Parkfield earthquake (1966). Table 1 presents the general characteristics of the time histories taken as examples; Table 2 presents their total and equivalent number of cycles; and Table 3 presents the total and significant earthquake durations.

4.1 Example of Complex Envelope Computation

Figure 2 presents an example of complex envelope computation. The example time history, plotted in Figure 2a, and listed in Table 1, was artificially generated and consists of 2,048 points at 0.01-second spacing, and an acceleration peak of 0.25 g. Figures 2b and 2c present the amplitude and phase envelopes of the time history, respectively. Similarly, complex envelopes of the other examples shown in Table 1 were computed, but have not been illustrated graphically. Referring to Figure 2, the phase envelope generally rises continuously with time and the amplitude envelope varies according to the amplitude of the time history. It can be noted, that the instantaneous peak amplitude of the time history is only a projection of the amplitude envelope, thus the peak of the amplitude envelope will always be equal to or greater than the instantaneous peak amplitude of the time history.

4.2 Total Number of Cycles

Considering the definition of a cycle proposed in Section 3, the total number of cycles, TN, is obtained from the maximum of the phase envelope by:

$$TN = \frac{\Phi(T)}{2\pi} \tag{2}$$

where T is the total duration. Table 2 lists the total number of cycles for all the example time histories, according both to the number of peaks concept as well as the cycle definition proposed in Section 3. For the examples considered in Table 2, the total number of cycles from number of peaks concepts is about one and one-half times more than that from the definition proposed in this paper.

4.3 Histogram of Amplitude Envelope of Cycles

Between any two subsequent time points, say $j\Delta t$ and $(j+1)\Delta t$, it can be said that $(\Phi[(j+1)\Delta t] - \Phi(j\Delta t))/2\pi$ cycles of an average amplitude envelope value of $(\bar{a}[(j+1)\Delta t] + \bar{a}(j\Delta t))/2$ are transcribed. Thus going through all time points, the histogram of number of cycles with various amplitude envelopes could be generated. Such a histogram for the time history shown in Figure 2a is illustrated in Figure 3. It is observed that the number of cycles decreases with the amplitude envelope in close accord with exponential law. Indeed, three of the four examples of earthquakes presented in Table 1 give high values for the highest level of significance when checked with the standard statistical Chi Squared goodness-of-fit test(9) for exponential distribution. The Ancona event gives a poor fit, but this earthquake had a short duration of strong motion coupled with a long period of weak motions.

4.4 Number of Equivalent Cycles

The number of equivalent cycles, N_{eq}, refers to that number of cycles having a certain reference uniform amplitude $\bar{a}_{ref}$, which when applied, has the same potential for fatigue or liquefaction damage, as that of the irregular train of cycles for the actual time history. This concept is significant, since laboratory tests can usually be performed only with a uniform sinusoidal loading and, in order to apply the laboratory results to realistic irregular earthquake-type loads, some relationship must be established between potential effects of the sinusoidal and the irregular loading histories. A detailed discussion of the background and the general computational procedure for the number of equivalent cycles can be found in Annaki and Lee.(2)

Palmgren(10) and Miner(11) proposed a concept of relative damage to combine the influence of load cycles with varying amplitudes. Referring to Figure 4, if n_i cycles of amplitude $\bar{a}_i$ have been observed, and the number of cycles with amplitude $\bar{a}_i$ required to cause failure is N_i, the relative damage d_i is obtained as:

$$d_i = \frac{n_i}{N_i} \tag{3}$$

The Palmgren-Miner hypothesis considers that n_i cycles of amplitude $\bar{a}_i$ will have the same potential for damage as n_j cycles of amplitude $\bar{a}_j$ if, and only if, the relative damages for cases i and j are the same, i.e. $(n_i/N_i) = (n_j/N_j)$. Based on this principle, the equivalent number of cycles, n_{eqi}, of the predetermined reference amplitude $\bar{a}_{ref}$ required to produce the same potential damage as n_i cycles of amplitude $\bar{a}_i$, is equal to:

$$n_{eqi} = d_i\, N_{ref} = \frac{n_i}{N_i} \cdot N_{ref} \tag{4}$$

If the entire time history consists of several sets of n_i, $\bar{a}_i$, such that i = 1, ..., m, the total number of equivalent cycles N_{eq} are obtained as:

$$N_{eq} = \sum_1^m n_i \cdot \frac{N_{ref}}{N_i} \tag{5}$$

If the total number of equivalent cycles, N_{eq}, is greater than the number of cycles to failure, N_{ref}, liquefaction or fatigue failure will occur. Obviously, the number of equivalent cycles, N_{eq}, depends upon the reference amplitude, $\bar{a}_{ref}$, and the curve of amplitude, $\bar{a}$, versus the number of cycles to failure, N, as is shown in Figure 4. The reference amplitude for liquefaction analyses has been traditionally(1) fixed at 65% of the peak amplitude of a(t).

From several fatigue and liquefaction tests one can conclude that log N is a linear function of log $\bar{a}$, as shown below:

$$\log N = \log \beta - \alpha \log \bar{a}$$

or

$$N = \beta(\bar{a})^{-\alpha} \tag{6}$$

Constants α and β are determined empirically from laboratory-defined failure curves. Constant β is merely a scaling factor whereas constant α determines the shape of the N versus $\bar{a}$ curve. Since N_{ref}/N_i and not N_i is significant in the computation of N_{eq}, the term β is of no significance in N_{eq} computations. From published test results,(1,2) and extensive in-house testing, it is concluded that the α value for liquefaction of most soils ranges from 2 to 6, with the average value around 3.5. For fatigue of steel structures, the typical α values may range from 5 to 30, depending upon the type of material, fabrication techniques, and the ratio of minimum to maximum stress.(12) Seed et al.,(1) have proposed the use of a standard N versus $\bar{a}$ liquefaction curve for all soils, for the purposes of computing N_{eq}. The authors experience tends to support this philosophy, except for unusual soils. Therefore, the standard N versus

$\bar{a}$ curve(1) is also used by the computer code discussed above. Table 2 presents the results using both the number of peaks concept and the phase envelope definition of cycles, and the difference between the two does not seem to be significant for the examples considered.

4.5 Cumulative Equivalent Cycles and Significant Duration

A typical earthquake record has an initial buildup period, a period of intense motion and, finally, a period of slowly dying out motion which may last several minutes. The variation of intensity of motion within the record is of considerable significance in determining that part of the motion that may impart the most damage. It is proposed that the number of equivalent cycles be considered as a measure of intensity of ground motion, since it is related directly to fatique or liquefaction damage potential. A curve, such as in Figure 2d, representing cumulative increase of equivalent number of cycles with time, can then be seen as representing cumulative fatigue or liquefaction damage. It can be seen from this figure that the significant duration of an earthquake can be defined as the period during which the cumulative number of equivalent cycles rises from 5% to 95% of the total number of equivalent cycles. This definition of significant duration is recommended by the authors.

Existing measures of ground motion intensity and significant earthquake duration are based upon one of the following two concepts:

(1) Duration of time history between two occurrences of a given critical amplitude.

(2) Duration based on cumulative energy of the integral $\int a^2 dt$.

The first concept has been discussed by several authors, e.g. Lee and Chan.(3) It postulates that a certain level of acceleration a(t), say either 0.05 g or some fraction of the peak, is considered critical, and the time period enclosed between the first and the last occurrence of this critical level is considered as the significant duration of the time history.

To impart a more scientific meaning to significant duration, Trifunac and Brady(13) proposed a definition based on energy. It can be shown(14) that $\int a^2 dt$ is related to the total energy imparted by a time history, on structures with uniformly distributed frequencies. Hence, the period over which $\int a^2 dt$ rises from 5% to 95% of its maximum value has been defined by Trifunac and Brady(13) as the significant duration of the time history.

Both the critical amplitude and the Trifunac and Brady(13) energy concepts for significant duration have the shortcoming that they are not directly related to the damage potential of a time history. The definition proposed herein is, however, linked more directly to the duration of the significant part of the damage potential and, hence, it is physically more meaningful. Table 3 shows that the proposed method gives a shorter period of significant duration than the Trifunac and Brady(13) method and, particularly, for records like Ancona and Cholame Shandon, which are characterized by a burst of a couple of cycles of strong motion, with the balance being of low level motion.

A natural extension to the above work, which is presently under development, is to include the influence of natural frequency of a structure in computing the significant duration and equivalent number of cycles for fatigue analysis. This can be accomplished by studying the response of a single-degree-of-freedom lumped parameter system with natural frequency equal to that of the structure. The significant duration and equivalent cycles concepts can then be applied to the acceleration response of the single-degree-of-freedom system. If the computation is repeated for several natural frequencies of the single-degree-of-freedom system, duration and equivalent cycle spectra similar to response spectrum can be developed.

5. Conclusions

Those characteristics of an earthquake time history which affect phenomena such as soil liquefaction and structural fatigue that are governed by the cyclic buildup of the entire time history rather than the single occurrence of a failure load, have been defined as the "cyclic characteristics" of the time history. The cyclic characteristics include the total number of cycles, the distribution of cycles at various amplitude levels, the number of equivalent cycles, cumulative distribution of the number of cycles and of the number of equivalent cycles, and significant duration of cyclic loading.

A method of studying cyclic characteristics has been presented where an earthquake record is represented as a projection of a rotating vector of length $\bar{a}(t)$ and phase $\Phi(t)$. One complete rotation of the vector represents one complete cycle within the time history. A computer code has been developed which allows the computation of the total number of cycles, the equivalent number of cycles, the histogram of numbers of cycles versus amplitude, and the significant duration of an earthquake. A definition of significant duration has been proposed which is related to damage potential. These concepts have been illustrated with examples of earthquake time histories.

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TABLE 1

GENERAL CHARACTERISTICS OF THE EARTHQUAKE TIME HISTORIES CONSIDERED

TIME HISTORY IDENTIFICATION CHARACTERISTICS	ARTIFICIAL TIME HISTORY	ANCONA T-14 EAST JUNE, 1972	PASADENA RECORD KERN COUNTY EARTHQUAKE 1952	CHOLAME SHANDON NO. 8 (N50E) JUNE, 1966 PARKFIELD EARTHQUAKE
Peak Acceleration (g's), a_{max}	0.25	0.205	0.0572	0.237
Total Duration Considered (seconds)	20.47	14.38	15.98	23.98
χ^2 Test (Reference 9) - Highest Level of Significance for Exponential Fit to Cycle Amplitude Histogram	0.93	0.03	0.99	0.98

TABLE 2

TOTAL AND EQUIVALENT NUMBER OF CYCLES

TIME HISTORY IDENTIFICATION CHARACTERISTICS	ARTIFICIAL TIME HISTORY	ANCONA T-14 EAST JUNE, 1972	PASADENA RECORD KERN COUNTY EARTHQUAKE 1952	CHOLAME SHANDON NO. 8 (N50E) JUNE, 1966 PARKFIELD EARTHQUAKE
Total Number of Cycles based on Number of Peaks Concept (Seed et al. - Reference 1)	143	89	53	125
Total Number of Cycles (TN) based on Phase Envelope Concept proposed in this paper	83	71	26	81
Number of Equivalent Cycles based on Number of Peaks Concept (Seed et al. - Reference 1) with $\bar{a}_{ref} = 0.65\ a_{max}$ and				
Standard $\bar{a}$ versus N Curve (Seed et al. - Reference 1)	33	4.2	14	3.3
or $\alpha = 2.6$ (Equation 6)	43	5.1	16	6.2
or $\alpha = 4.0$ (Equation 6)	39	6.0	18	5.3
Number of Equivalent Cycles based on Number Phase Envelope Concept proposed in this paper with $\bar{a}_{ref} = 0.65\ a_{max}$				
and $\alpha = 2.6$ (Equation 6)	30	5.0	13	6.3
or $\alpha = 4.0$ (Equation 6)	36	5.9	19	6.0

TABLE 3

TOTAL AND SIGNIFICANT DURATIONS

TIME HISTORY IDENTIFICATION CHARACTERISTICS	ARTIFICIAL TIME HISTORY	ANCONA T-14 EAST JUNE, 1972	PASADENA RECORD KERN COUNTY EARTHQUAKE 1972	CHOLAME SHANDON NO. 8 (N50E) JUNE, 1966 PARKFIELD EARTHQUAKE
Total Duration Considered (seconds)	20.47	14.38	15.98	23.98
Trifunac and Brady (Reference 13) Significant Duration (seconds)	12.3	3.0	11.3	12.8
Significant Duration (seconds) proposed in this paper				
with $\alpha = 2.6$ (Equation 6)	12.0	1.8	9.2	6.0
with $\alpha = 4.0$ (Equation 6)	10.2	1.4	7.6	2.1

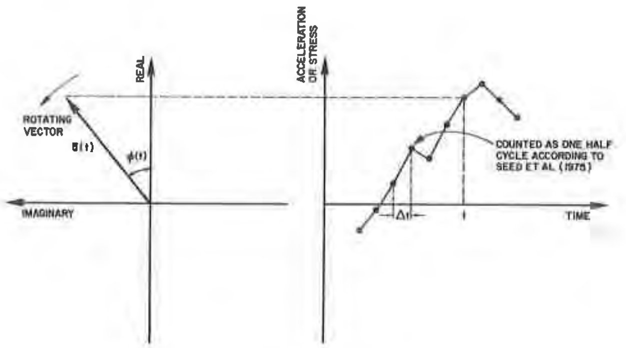

Figure 1 Definition of Complex Envelope

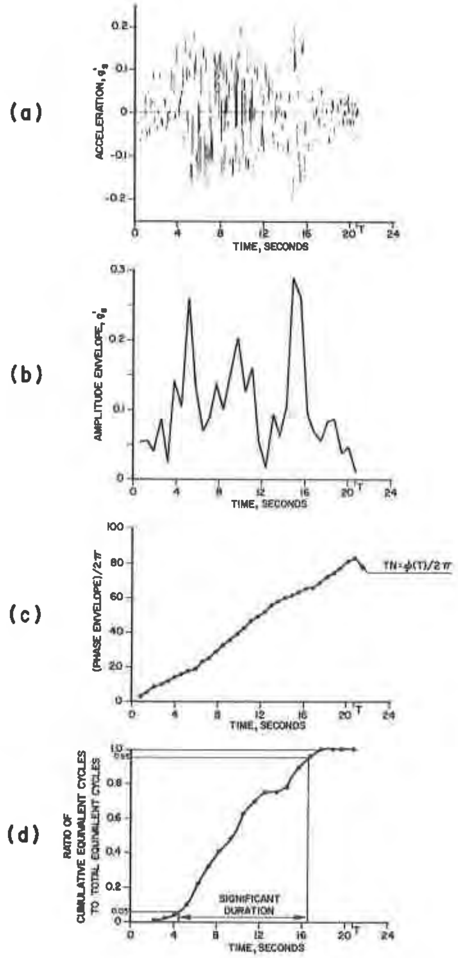

Figure 2 Amplitude and Phase Envelopes, and Cumulative Equivalent Cycles - for Artificial Time History shown in Table 1

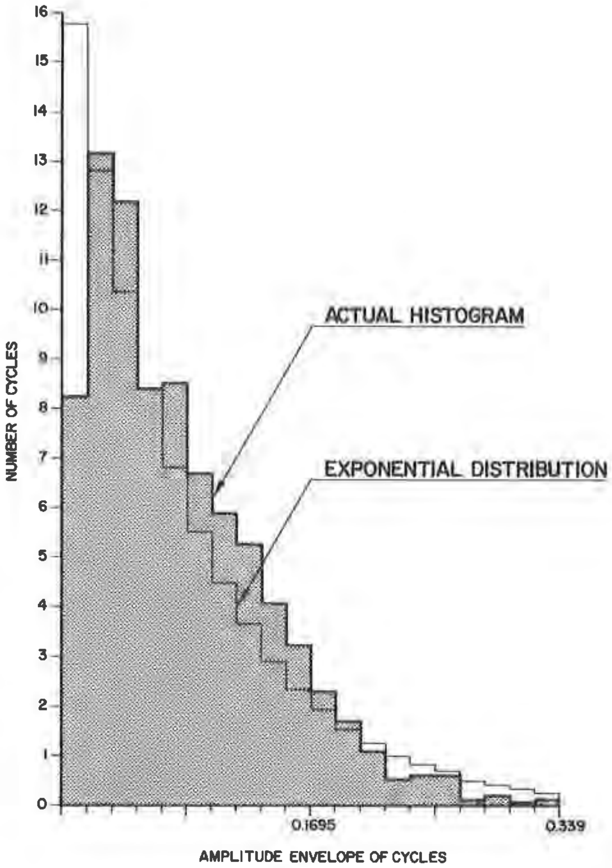

Figure 3 Actual and Exponentially Distributed Histograms of Number of Cycles versus Amplitude Envelope - for Artificial Time History shown in Table 1

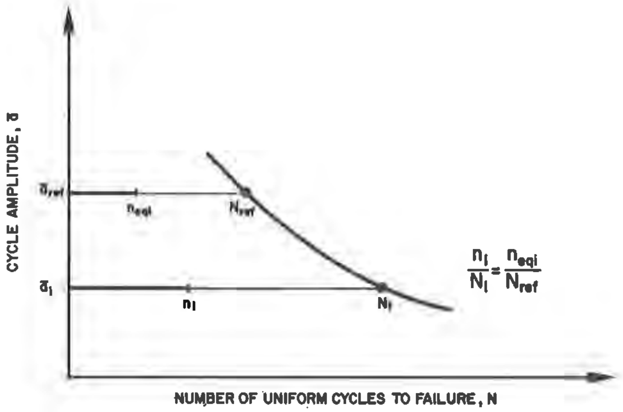

Figure 4 Typical Cycle Amplitude versus Number of Uniform Cycles to Failure Curve