# Effects of Ambient Humidity on Moisture Transfer in Concrete Slabs

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Summary

A diffusion theory with a linear or a nonlinear coefficient of diffusivity is insufficient for the characterization of the drying behavior of light-weight concrete slabs. A new mechanistic mathematical model, based on irreversible flow of heat and mass, is applied to study the effects of ambient humidity on moisture movement in concrete slabs. Implicit finite difference calculations for a light-weight concrete slab yield the global average drying rates under various values of ambient humidity, and yield the histories of the distributions of moisture in the slab.

## I. Introduction

Because concretes have excellent thermal and radiative resistance, they are frequently used as components of reactors. Water content in porous concrete structures increases thermal-conductivity and attenuates the neutron radiation significantly. Thus, it is essential to study the distributions of moisture in concrete structures. A diffusion theory with a linear or nonlinear coefficient of diffusivity as used in the past is inadequate for the description of mass and heat transfer in a porous concrete slab [1]. Transfer of mass of all phases and the transfer of heat must be considered simultaneously. Particularly in the pendular stage, (low water saturation) experiments show that diffusion, capillary, and evaporation-condensation actions are the governing mechanisms for mass transfer. A comprehensive mathematical model describing the simultaneous multi-phase moisture and heat transfer in porous concretes has been constructed by using the principle of non-equilibrium irreversible flows of heat and mass [2]. In this paper, the theory developed is applied to study the effects of ambient humidity on the drying of a light-weight concrete slab. Various environmental relative humidities (10%, 30%, and 50%) are imposed as ambient conditions on a light-weight concrete slab of 2.54 cm (1 inch) thickness.

The set of governing differential equations for heat and mass transport in concrete comprises three nonlinear partial differential equations. An implicit finite difference scheme is employed for obtaining numerical results. The numerical solution of the simultaneous algebraic equations thus obtained yields values of moisture, temperature, and pressure distributions at the pre-assigned grid points throughout the domain investigated.

The results show differences during constant rate, funicular, and pendular states. Different shapes of moisture distribution occur in each period. The time required to dry the slab for a specified moisture content at the middle plane of the slab is calculated for various ambient cases. The average moisture content stored in the slab is calculated as a function of time. As expected, the migration of the moisture content in the slab approaches a quasi-equilibrium state more quickly for the slab subjected to a higher relative ambient humidity than one subjected to a lower ambient humidity. The distribution curves of moisture content for lower ambient relative humidity are not always smooth convex curves such as those obtained from a diffusion theory. This phenomenon results from the existence of the capillary and evaporation-condensation mechanisms of the mass transfer, which depend on the relevant material characteristics (the topology of solid matrix, interface phenomena among solid, liquid, gaseous vapor and air, and liquid-vapor equilibria). The average drying rates for various ambient humidities are also calculated. A decrease of ambient relative humidity effectively accelerates the rate of drying of a concrete slab.

## II. Equilibrium Sorption in Concrete

A wet porous system is a multi-phase medium. In general it contains all three phases. The liquid and gaseous phases are contained in the porous system with the surface of the solid matrix as their boundaries. In the present paper, the solid matrix is considered, in a macroscopic sense, as a rigid body with homogeneous and isotropic pore structures. The properties of the liquid water phase in the porous system are different, to some extent, from those of the bulk free water. Layers of water molecules next to the surface of the solid matrix are tightly held by van der Waals' attraction. The adsorbed water film has a limited mobility and can move by a process of surface diffusion. In the narrow capillary pores, the

liquid water beyond the adsorbed liquid film is subject to capillary force and has a higher degree of mobility than the adsorbed liquid film. Convection is the primary transport mechanism for this kind of liquid water. In the gaseous phase, the air and vapor mixture can be transported by both molecular diffusion and molar convection on a macroscopic scale. The water vapor concentration in the system can be determined from the equilibrium sorption relation which is obtained from experimental data for a light weight concrete as shown in Fig. [1]. The equlibrium sorption relation depends on the characteristics of the porous system, such as the porosity, the specific surface of the solid matrix, and the pore size distribution. Vassilou and White [3] have proposed that the equilibrium sorption relation could be expressed as a functional relation between the moisture content $\theta$ and the "equivalent pore diameter" r.

$$\theta = \theta(r) \quad \text{or} \quad r = r(\theta) \tag{1}$$

in which the moisture content can be approximated as

$$\theta \equiv \frac{m}{\varepsilon} \simeq 1 - \frac{\varepsilon_g}{\varepsilon} \tag{2}$$

because the water content in the gaseous phase is negligible in comparison to that in the liquid phase. The $\varepsilon$ denotes the porosity of the porous system, $\varepsilon_g$ the volume fraction of the gaseous phase, and m the volumetric moisture content.

From the Kelvin equation, r is expressible as a function of temperature and the relative vapor pressure as

$$r = -\frac{2\sigma M_w}{\rho_w R} \cdot \frac{1}{T\ell n(P_v/P_v^0)} \tag{3}$$

where $\sigma$ denotes the surface tension, $M_w$ the molecular weight of water, $\rho_w$ density of water, and R the gas constant.

Thus, a functional relation for moisture content $\theta$, which is a function of temperature T, the relative vapor pressure $(P_v/P_v^0)$, and the mole fraction of water vapor of gaseous components $\emptyset$, can be established; $\theta = \theta(\emptyset, P_v/P_v^0, T) = \theta(\emptyset, P, T)$. This analysis assumes that practically all portions of the pore space with pore sizes larger than that given by the Kelvin equation (3) is accessible to the gaseous phase. From equations (1), (2) and (3), a relation between the volume fraction of moisture in gaseous phase and the water content is finally established as

$$\varepsilon_g = \varepsilon\{1 - \theta[r(\emptyset, P, T)]\} , \tag{4}$$

which must be determined experimentally. Eq. (4) characterizes the effects of the particular porous system on the mass transport of the porous system.

## III. Basic Equations for Heat and Mass Transfer in Cement Paste

The basic equations for heat and mass transfer, assuming that local thermal and chemical equilibrium exists in the porous system and that the gaseous vapor phase is the major factor in mass transfer, derived from laws of conservation of mass, momentum, energy, and the kinetic theory of ideal gases, for one-dimensional mass and heat transfer in a slab of thickness L, are [2]:

$$A_i \frac{\partial \phi}{\partial t} + B_i \frac{\partial P}{\partial t} + C_i \frac{\partial T}{\partial t} = D_i \frac{\partial^2 \phi}{\partial x^2} + E_i \frac{\partial^2 P}{\partial x^2} + F_i \frac{\partial^2 T}{\partial x^2} + G_i \left(\frac{\partial \phi}{\partial x}\right)^2 + H_i \left(\frac{\partial P}{\partial x}\right)^2 + I_i \left(\frac{\partial T}{\partial x}\right)^2$$
$$+ J_i \left(\frac{\partial \phi}{\partial x} \cdot \frac{\partial P}{\partial x}\right) + K_i \left(\frac{\partial \phi}{\partial x} \cdot \frac{\partial T}{\partial x}\right) + L_i \left(\frac{\partial P}{\partial x} \cdot \frac{\partial T}{\partial x}\right) , \quad (i = 1, 2, 3) \tag{5}$$

where the coefficients $A_i$, ..., $L_i$ are functions of the dependent variables $\phi$, P, T, and $\varepsilon_g$ [4,5].

The three basic Eqs. (5) contain four variables $\phi(x,t)$, $P(x,t)$, $T(x,t)$, and $\varepsilon_g(x,t)$. Thus, they should be combined with Eq. (4) to form a complete set of governing differential equations.

The boundary conditions on the surfaces of the slab are (j=0 for x=0; j=1 for x=L)

$$\frac{\partial\phi}{\partial x} = (-1)^j \alpha\left(\frac{RT}{PD}\right)(\phi - \phi_{oj}) \ , \quad \frac{\partial T}{\partial x} = (-1)^j \left(\frac{h}{K}\right)(T - T_{oj}) \ , \text{ and } P = P_{oj} \qquad (6a) - (6c)$$

where $\alpha$ denotes the mass transfer coefficient, D the diffusion coefficient, h the heat transfer coefficient, and k thermal conductivity. $\phi_{oj}$, $T_{oj}$, and $P_{oj}$, respectively, are the mole fractions of water vapor, temperature, and pressure of the surroundings of the porous system. For a slab surrounded by air at atmospheric pressure and temperatures between 0°C and 60°C, the transfer coefficients may be expressed by

$$\alpha = \alpha_F + 1.63 \times 10^{-5}[(T_\infty - T)/B]^{1/4}, \quad (7) \qquad h = 2.75 \times 10^8 \alpha + \sigma_s e(T_{ej}^4 - T_i^4)/(T_{ej} - T), \quad (8)$$

and

$$k = [k_g^n \varepsilon_g + k_\ell^n \varepsilon_\ell + k_s^n (1-\varepsilon)]^{1/n}, \qquad (9)$$

where $\alpha_F$ denotes the mass transfer coefficient due to forced convection, B is a characteristic length of the slab, $\sigma_s$ the Stefan-Boltzman constant, $T_{ej}$ the temperature of the enclosure of the system, and n the topological constant of the porous system. $k_g$, $k_\ell$ and $k_s$ are the conductivities for gas, liquid, and solid matrix respectively; $\varepsilon$, $\varepsilon_g$ and $\varepsilon_\ell$ are the porosity of the porous system, the volume fraction for gas, and the volume fraction for liquid, respectively.

The initial conditions used here are given by

$$\phi(x,0) = \phi_o \ , \quad P(x,0) = P_{atm} \ , \quad \text{and} \quad T(x,0) = T_o. \qquad (10)$$

Eqs. (4,5,...,10) form a nonlinear initial-boundary value problem for the simultaneous mass and heat transfer in a light-weight concrete slab.

## IV. Numerical Calculation and Conclusions

The given equations (4,5,...,10) are replaced by a set of algebraic, finite difference, backward-in-time equations relating $\phi$, P, and T to discrete points in the x,t space. For stable numerical solutions, the space and time differences must be chosen so that $\Delta t/(\Delta x)^2 \leq \frac{1}{2} C$, where C is a parameter determined by ratios of the coefficients $A_i,\ldots,K_i$. As C varies with time, the increment $\Delta t$ must be adjusted continually in order to provide both economy and stability of calculations.

As an example, mass and heat transfer were calculated for a light-weight concrete slab of thickness 2.54 cm (one inch). The system initially has a uniform temperature of 20°C, and a uniformly distributed moisture content of 0.096 $m^3/m^3$ (84 percent RH) throughout the slab, which is exposed to a constant ambient temperature of 23°C, but with different values of ambient relative humidity (RH) given as 10, 30, and 50 percent of RH. For comparison, three fractions of moisture content at the midplane of the slab, 99%, 88.5%, and 68.75% of the initial moisture content, are chosen in this study. The physical and geometrical constants, and boundary conditions for this problem are summarized in Table 1.

Figure 2 shows the average drying rates for the light-weight concrete slab with initally high moisture saturation, and subjected to natural drying under various values of ambient relative humidity (10, 30, and 50% of RH). The drying rate is high in the funicular satura-

tion state. The moisture movement at this state is relatively independent of the properties of the concrete, and strongly dependent on the ambient temperature and relative humidity of the surrounding air. As the pendular saturation state begins, the liquid threads in the

TABLE 1. VALUES OF THE CONSTANTS USED IN THE EQUATIONS (Light-Weight Concrete)

| | Type of Constant Symbol | Value | Unit | Symbol | Type of Constant Value | Unit |
|---|---|---|---|---|---|---|
| Phys. | R | $8.3149 \times 10^{-14}$ | Kg $m^2/s^2$ K mol | a | $1.209658 \times 10^{-11}$ | m $s^2$/Kg |
| Const. | $\sigma_s$ | $5.670 \times 10^{-8}$ | $Kg/s^3K4$ | b | 5080 | K |
| | | | | n | 0.25 | |
| | $(C_p)_a$ | $1.0063 \times 10^{3}$ | $m^2/s^2$ $K^2$ | $\beta$ | $0.167 \times 10^{-3}$ | Kg/$s^2$ K |
| | $(C_p)_s$ | $0.879 \times 10^{3}$ | $m^2/s^2$ $K^2$ | v | 1.0 | |
| | $(C_p)_v$ | $1.8646 \times 10^{3}$ | $m^2/s^2$ $K^2$ | $\sigma_o$ | $121.2 \times 10^{-3}$ | Kg/$s^2$ |
| | $(C_p)_w$ | $4.1793 \times 10^{3}$ | $m^2/s^2$ $K^2$ | $P_{atm}$ | $1.01325 \times 10^{5}$ | Kg/m $s^2$ |
| | D | $0.256 \times 10^{-4}$ | $m^2/s$ | $M_o$ | 0.096 (RH=84%) | $m^3/m^3$ |
| | e | 0.8 | | $T_o$ | 203.15 (20°C) | K |
| | $k_g$ | 0.02613 | Kg m/$s^2$ K | | case 1: $\phi_{00}$=0.013400 | mol/mol |
| | $k_s$ | 1.4422 | Kg n/$s^3$ K | | $\phi_{01}$=0.013400 | mol/mol |
| | $k_w$ | 0.616 | Kg m/$s^3$ K | | | |
| | $M_a$ | $28.962 \times 10^{-3}$ | Kg/mol | $\phi_{01}$ | case 2: $\phi_{00}$=0.008936 | mol/mol |
| | $M_w$ | $18.016 \times 10^{-3}$ | Kg/mol | | $\phi_{01}$=0.008936 | mol/mol |
| | $\varepsilon$ | 0.3 | $m^3/m^3$ | | | |
| | $\eta_g$ | $1.83 \times 10^{-5}$ | Kg/m s | | case 3: $\phi_{00}$=0.003548 | mol/mol |
| | $K_g^o$ | $2.50 \times 10^{-14}$ | $m^2$ | | $\phi_{01}$=0.003548 | mol/mol |
| | $\lambda$ | $2.4418 \times 10^{6}$ | $m^2/s^2$ | $T_{oj}$ | For all cases: $T_{00}$=296.15 (23°C) | K |
| | $\rho_s$ | $2.6 \times 10^{3}$ | $Kg/m^3$ | | $T_{01}$=296.15 (23°C) | K |
| | $\rho_w$ | $0.99707 \times 10^{3}$ | $Kg/m^3$ | $T_{ej}-T_{oj}$ | For all cases: $T_{eo}=T_{00}$ | K |
| Geom. Const. | L | 0.254 | m | | $T_{ei}=T_{01}$ | K |
| | B | 1.1 | m | $\alpha_f$ | 0.0 | mol/$m^2$ s |

pore space begin to break down, and capillary forces and even true adsorption set in. Then the internal characteristics of the porous concrete play the primary role in moisture movement. The onset of such a transience from the funicular state to the pendular state is determined by the ambient relative humidity. As is expected, the drying time required for removing a certain amount of moisture in a porous concrete can be shortened by lowering the ambient relative humidity. Besides, the amount of removable moisture depends strongly on the ambient humidity. Figure 3 shows the histories of moisture distribution of a light-weight concrete slab, dried naturally under various values of ambient humidity. The drying rates are higher, in general, in the surface regions during the funicular state. Lower ambient humidity increases the drying rate. As drying proceeds, the rate of drying in the interior region increases and surpasses the drying rate in the surface regions. To characterize the effects of ambient humidity in concrete slabs, three fixed relative moisture contents at the midplane of the slab (99, 88.5 and 68.75% of the initial moisture content) were

chosen as shown in Fig. 3. The drying times for attaining a specific moisture content at midplane of the slab are listed in Table 2. To attain a specific moisture content 68.75% of the initial moisture content of the slab, the required drying time is 200 hrs. for the case of an ambient humidity of 50% RH, and is only 69 hrs. for 30% RH. It is obvious that control of the ambient humidity is important for economy in the drying process.

TABLE 2.

Environmental RH, Drying time for attaining the specific moisture content at midplane of slab, hours (based on the initial moisture content)

| Ambient RH % | 99% | 88.5% | 68.75% |
|---|---|---|---|
| 10 | 4.26 | 19.57 | 41.95 |
| 30 | 4.32 | 26.98 | 69.28 |
| 50 | 4.99 | 48.61 | 199.84 |

V. References

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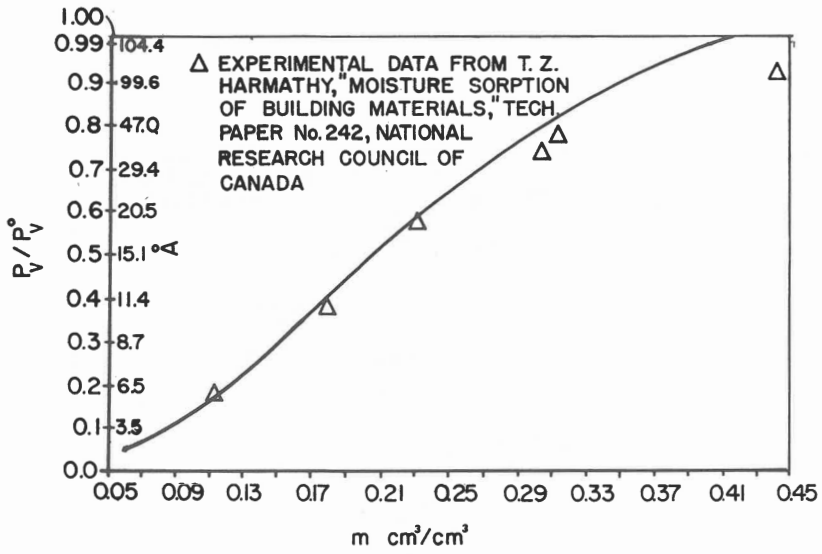


Fig. 1 Liquid-Vapor Equilibrium Curve for Concrete

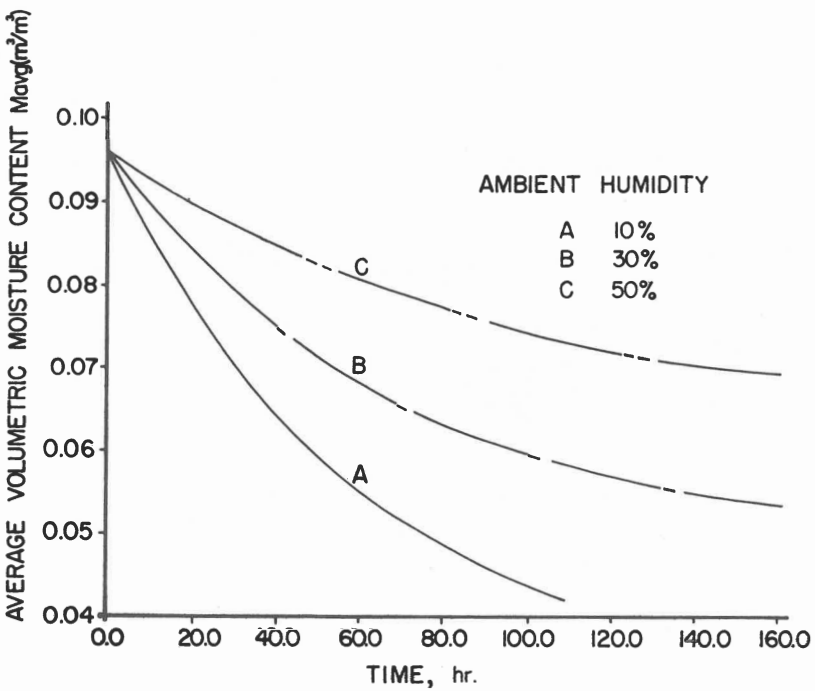


Fig. 2 History of the Average Moisture Content

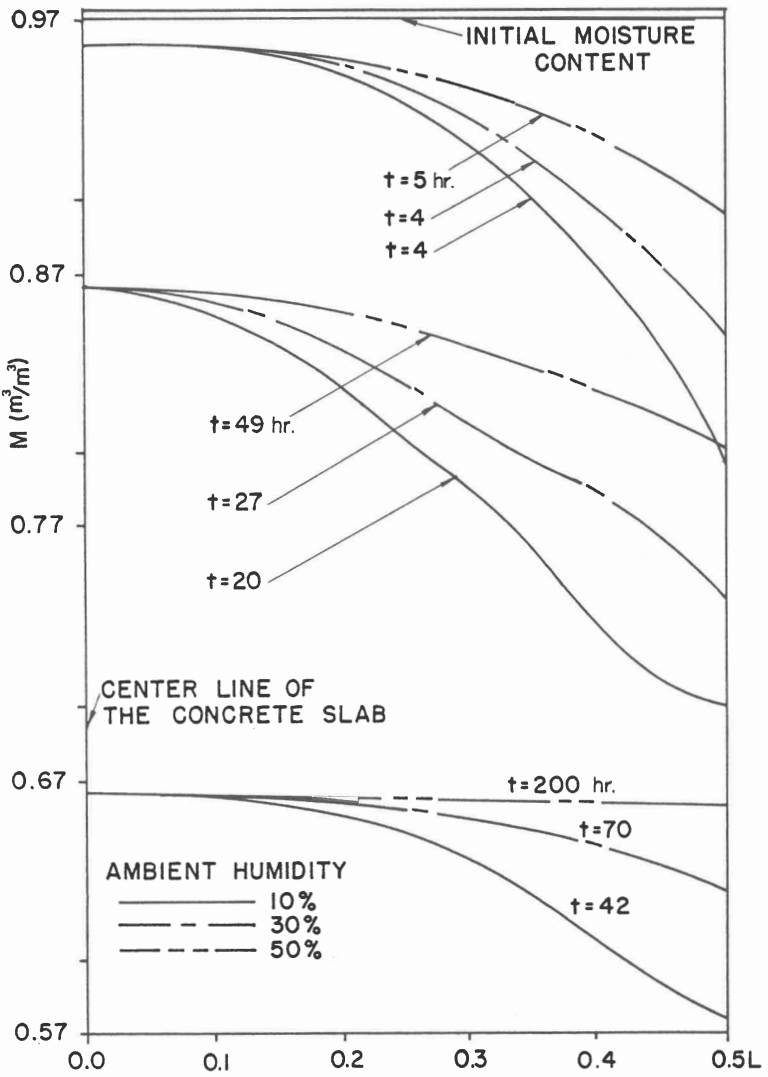


Fig. 3 Effects of Ambient Humidity on the Distribution of Moisture