A DAMAGE POSTULATE FOR NONPROPORTIONAL CYCLIC PLASTICITY

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This paper examines damage states and analysis for the multiaxial mechanical component of a general thermal-mechanical cyclic loading. Specifically, multiaxial histories developed under nonproportional cyclic plasticity synthesized through appropriate constitutive theories are used as a vehicle to discuss nonproportional damage processes and analysis. Nonproportional damage processes considered include both fixed and rotating principal planes. These are discussed in the context of an extension of a recent damage postulate for analysis of nonproportional action with fixed principal directions. It is shown that current data bases developed for use in damage analysis are, in general, inappropriate for applications to nonproportional cycling. The influence of transient inelastic response observed in studies of proportional cyclic plasticity on the damage state is also discussed.

1. INTRODUCTION

Reactor components are subjected to operational cycles which give rise to coupled thermal-mechanical loadings, often in the presence of an aggressive environment. The action of these loadings and the environment is manifest in various forms of damage to metallic elements, which ultimately reduces service capacity and jeopardizes safety. When the action of the environment is not a dominant factor, the damage process which leads to the formation of cracks is still very complex. As considered here, the damage process is a path dependent function of the loading history. Recent work [1] indicates that analysis of such complex damage processes is beyond current postulates which are shown to be inadequate because, being developed for proportional loadings, they fail to account for nonproportional action.

Examples of nonproportional action are illustrated in Figure 1. As portrayed there, nonproportional action is defined as any (cyclic) loading which gives rise to a nonradial path in deviatoric stress space. This interpretation differs somewhat from the classical definition [2] for reasons elaborated in [1,3]. Such nonproportional action sets up a path dependent deformation response history that must be traced throughout the history. Now both crack nucleation and growth damage in fatigue can be considered to be a consequence of slip [4-6]. It follows that, since stress and strain are measures of the propensity to slip, they will also serve as measures of the damage done as a result of such slip, as argued in [1,3,7]. It also follows that in general the damage process will be path dependent.

The purpose here is to examine complex damage states set up by nonproportional action. Specifically, only damage due to the multiaxial mechanical component of the more general thermal-mechanical cycle is examined in terms of a recently postulated framework which provides for path dependence [8]. That framework, advanced for analysis of problems with fixed principal directions, is extended to deal with rotating principal directions. Discussion of this damage postulate is illustrated in the context of nonproportional deformation response synthesized using an appropriate constitutive theory. Emphasis is placed on specific histories which serve to illustrate several key problems in damage analysis.

2. DAMAGE ANALYSIS

It is convenient to introduce the concept of a damage state which characterizes past history as a function of the mechanical condition of the material at a given instant. Consideration is restricted to isothermal rate independent processes in the portion of the damage process spent in crack nucleation. Changes in the damage state define the damage rate. It follows that the damage rate in a structure is a function of the change in the mechanical measures of damage over the path in some interval.

2.1 Damage Assessment

Damage assessment involves the comparison of the damage process in some component with that in some test specimen. Laboratory conditions define what is termed in reference damage process. Damage assessment of a component based on the reference damage data can be meaningful only if there is similitude between the damage rate processes. In this paper a damage parameter is postulated to establish the similitude between the reality of the structure and the laboratory idealization. By definition, equal values of the damage parameter mean equal damage rates for the damage processes being compared. The more simple the reference damage condition or the more complex the structural damage process, the more complex the damage parameter.

The reference damage process and damage parameter must be matched to resolve path dependent damage increments. Given a reference damage process for which a constant amplitude isothermal loading is repeated continuously until nucleation, it is assumed that each repetition of a given type of cycle is equally damaging. Cycles such as this are termed uniform cycles; all others are termed nonuniform. It remains to resolve the variable amplitude stress-strain history into segments which are identical to those of the uniform cycling reference data. As such, the history must be broken up into constant damage rate segments since the reference damage process develops such data. This parallels cycle counting in proportional plasticity damage analysis based on the concept of memory [1,3,9].

2.2 The Damage Postulate

Under the action of nonuniform cycling, the damage rate dD, with respect to the progression of the history, dt, incurred by some generic element of material is proportional to the corresponding increment in the octahedral shear and normal components of the energy, dU^c; taken with respect to the critical reference damage state.

Let the triad $\hat{i},\hat{j},\hat{k}$ define a right-hand unit vector system that is coincident with the octahedral energy space, the directions corresponding, respectively, the U_1, U_2, and U_3. The increment in the energy resolved onto critical damage directions, $\underset{\sim}{dU}^c$ is given by [8]

$$\underset{\sim}{dU}^c = dU_1^c\,\hat{i} + dU_2^c\,\hat{j} + dU_3^c\,\hat{k} \;, \qquad (1)$$

so that the total energy and, therefore, the damage $\underset{\sim}{D}$ is given by

$$\underset{\sim}{D} = \underset{\sim}{U}^c = \oint_t^{t+\Delta t} dU^c \quad . \qquad (2)$$

The terms dU_1^c and dU_2^c denote the product formed between octahedral shear stresses and strains resolved onto the critical direction in the octahedral plane. The critical direction is defined with respect to Figure 2 by the direction related to the base vectors $\underset{\sim}{n}_1$ and $\underset{\sim}{n}_2$ associated with the stress state having the highest damage rate (lowest life) based on octahedral stress or strain. The term dU_3 denotes the product of the hydrostatic stress and strain. Further detail is provided in [8].

As defined by Equation (2), damage is proportional to the vector defined by the three scalar octahedral components of energy. While this provides for a geometric interpretation as a vector, the sense is arbitrary. This is because it is prescribed in terms of a reference stress state that defines the critical direction related in turn to arbitrary but orthogonal basic vectors on the octahedral plane (cf Figure 2). That is the vector denoted $\underset{\sim}{U}^c$, is a geometrical consequence; physically, energy remains a scalar.

The path traced out by the damage vector need never close in general. In such cases, the damage measure given by $\underset{\sim}{D}$ would be computed for segments of the history that compare "identically" to segments in the reference damage state developed under uniform cycling. These segments relate to circumstances under which the damage rate $d\underset{\sim}{D}/dt \propto d\underset{\sim}{U}^c/dt$ is essentially constant, such as in the reference uniform cycling case. This requirement, therefore, provides the rationale for breaking up the damage path into segments for which the damage rates are identical.

2.2.1 Fixed Principal Directions

The damage postulate represented by Equations (1) and (2) can be simply expressed for problems where the principal directions are fixed. It is prudent to first consider proportional cycling. Then, segments in the nonproportional history that can be identified as related to a proportional reference data base will be examined in relation to this analysis for proportional cycling.

For problems of proportional cycling the damage postulate leads to a parameter that is a function of distortion energy with provision to include the influence of a hydrostatic component if present [8]. It might be noted that such forms have been shown in recent independent reviews to provide the most consistent consolidation of a broad range of data [10,11]. There is, however, one major difference between the present postulate and the more usual distortion energy. In the present postulate, the octahedral energy computed for all stress states is multiplied by $\cos^2 \Theta_r$, where Θ_r is the angle between the direction for a given R ratio and the critical direction. Figure 3 maps several different R ratios as directions on the plane formed in terms of octahedral stress and the base vectors. These same directions exist in strain space. The direction in this plane associated with the stress ratio having the highest damage rate defines the critical direction. The value of Θ_r is thus the angle between this direction and the stress state of interest.

It follows that the damage parameter for proportional cycling is formed by the product of two scalars. One is the octahedral energy, U; the other is a function of the stress ratio, f(R). As such the parameter has the form

$$\frac{dD}{dN} = \frac{dU}{dN} \cdot f(R) \quad . \qquad (3)$$

Consider now nonproportional cycling; specifically the history synthesized using the Besseling constitutive model [12], presented in Figure 4. Details of this synthesis and the constitutive formulation may be found in [1,3]. The subscripts, ΘΘ and rr, in the figure denote tangential and radial directions, respectively. Note that the Path 0-1 represents an initial (proportional) loading; whereas the subsequent path, 1-2-3, represents a nonproportional loading cycle. The numbers at the tips of the hysteresis loops represent that point in the loading sequence.

The issue to be examined in view of the complex response of Figure 4 is how to resolve this hysteresis into segments which compare identically to that of the reference damage state. In this respect, consider the projection of the above stress history onto the damage plane, as shown in Figure 5. It is immediately apparent that the relatively simple representation of the proportional loading cases evident in Figure 3 as directions in the damage plane no longer exists. Note, however, that an apparent cycle is established for path 2-3-4-5. While the representation shown in Figure 5 is indeed more complex, the concept used to resolve damage in this case is still operative. That is, damage (energy) can still be resolved onto critical directions and summed throughout the history. Thus, the present formulation inherently defines segments of the history which compare identically to those of the reference damage state. While it defines these segments and provides in concept for their summation, there are no data available to assess the accuracy of this formulation in actual applications.

Once the hysteresis path is resolved into segments, the damage parameter expressed in Equation (3) can be applied. But one major problem remains: the formulation does not indicate how damage in one direction might interact with that in another. That is, damage may accrue in as many different directions as are activated by the nonproportional loading. Yet the usual reference state activates only one direction. Thus, mapping the damage under nonuniform cycling back onto the critical direction as previously postulated does not admit possible interaction.

2.2.2 Rotating Principal Directions

The damage parameter given in Equation (3) could also be applied to rotating principal directions if segments of the history that relate uniquely to some reference data base could be identified. Unfortunately, such response maps onto a plane such as that shown in Figure 3 as continuously curvilinear paths. While it is possible to define piecewise linear segments to approximate this curvilinear response, the degree of approximation is uncertain and the logistics are complicated.

The general problem of out-of-phase cycling can be dealt with more simply in terms of Equation (3) if mean strains are zero [1,3]. For this case the damage postulate leads to a parameter that couples maximum and minimum values of the octahedral shear energy in terms of an amplitude with the corresponding hydrostatic values. Because maximum and minimum values occur at different times, they must in general be projected back onto the reference direction. The experimental results of [13] suggest, however, that this is unnecessary and that only the maximum value during the cycle need by considered. If such is the case, the form of Equation (3) could be employed in damage assessment for this class of problem.

Note that Equation (3) is not appropriate in the presence of mean strains. As shown in [1,3], mean strains create a more complex situation in which the occurance of the maximum values of the shear strain and the normal strain depend on the phase angle. As such, damage cannot be assessed from the simple consideration of a given maximum value in a given shear plane. In this case one must question whether proportional loading in which a single dominant damage plane is activated can be used to assess damage in terms of a two or three dimensional approach which does not uniquely match conditions on other apparently critical planes. The results presented in Figure 6 [1] can be used to illustrate this point. If the phase angle between shear and normal strains were unimportant, one might choose to assess damage in terms of the shear and the normal stain on the maximum shear plane even if as in this case they are out of phase. However, if the phase angle is important, the Z-Θ plane which couples the maximum normal strain with a large shear strain may be the most appropriate plane for damage assessment. Until experimental results are available, whether one particular plane or several planes must be used in damage assessment will remain an open question.

3. APPLICATION OF THE DAMAGE POSTULATE

Data suitable to illustrate the application of the damage postulate include proportional loading with fixed principal directions, and combined tension and torsion loading out of phase for rotating principal directions. Few data exist for nonproportional loading with fixed principal directions. Of the available results, very few represent comprehensive data sets. Others have shown the utility of the form of the present damage postulate for these cases [10,11,13]. Thus, only the data of Havard and Topper [14] will be examined here in detail. Consideration of these data is appropriate in that the postulate provides the rationale to break nonproportional histories into segments that compare to a proportional cycling data base. A first test of the postulate would therefore be the correlation of these proportional cycling data.

Havard, et al., considered thin tubular specimens tested under constant amplitude cycling of the magnitude of the principal stresses (uniform cycling) such that their ratio remained constant. The test material was normalized AISI 1018 steel. Figure 3 presents the projection of the stress states they examined mapped in terms of stress onto the reference damage plane. Of these directions, that associated with the stress ratio having the maximum damage rate defines both the reference damage state and the critical direction on this plane. The raw data suggests that the stress ratio $R = s_2/s_1 = 0.5$ defines the critical direction.

Applying the above damage parameter to the data of Havard, et al., for each combination of stress ratio and magnitude yields a high degree of data consolidation as is evident in Figure 7. The banding evident in a previous attempt to consolidate these data based on octahedral energy alone [15] is reduced. Nevertheless, some scatter remains; the scatter for the present consolidation achieved independent of empirical fitting parameters being about twice that achieved using an empirically calibrated form.

The results of Havard and Topper are also useful to illustrate the dependence of damage on transient material response. To this end the cyclically stable stress-strain response observed is compared to the response synthesized as a function of cummulative plastic strain using an iterative model based on the Prandtl-Reuss flow rule and von Mises criteria. Details of the formulation and results may be found in [16].

Results indicate that if stable material response is employed to calibrate the constitutive model, the predicted strains differ from that observed by as much as 90 percent. As reported in [3], errors are greatest at negative stress ratios for small strains; a probable consequence of the very extensive cycle softening at low inelastic strains and cyclic hardening at large inelastic strains exhibited by this steel. When this cyclic transient response is embedded in the constitutive equation, the errors in predicted response scatter with stress ratio at values less than 10 percent [16]. Damage assessed in terms of cummulative plastic strain show that computed damage based on each of the stable and transient models differs by more than 30 percent, there being no trend with stress ratio. At high strains, however, stable and transient models yield comparable damage estimates because of the very rapid cyclic hardening observed for uniaxial samples of this material. In view of these results it is clearly necessary to include transient response if accurate damage estimates are to be made for materials which significantly exhibit transient behavior.

4. SUMMARY AND CONCLUSIONS

A damage postulate that provides a framework for the assessment of damage under cyclic nonproportional loading has been reviewed and discussed. A damage parameter developed from this postulate has been introduced and examined for applications to nonproportional cycling for each of fixed and rotating principal directions. Application of the postulate showed it suitable in the context of an extensive data set for proportional cycling. For this data set results reviewed indicated that care must be exercised to model material transient response to accurately assess fatigue damage.

The following conclusions can be drawn from the analysis and results reviewed in this paper. Problems may be encountered in applying damage parameters developed for proportional cycling. Data developed under proportional cycling may by inappropriate in damage analysis for nonproportional cases. In materials which exhibit significant cyclic transient material response, it must be accounted for if an accurate damage assessment is to be made.

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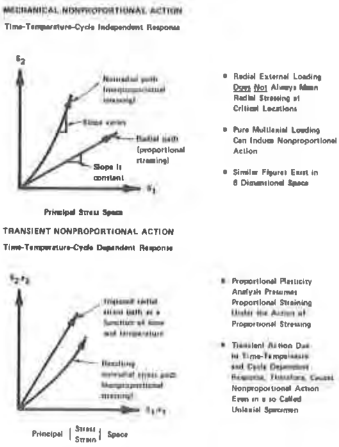
6. FIGURES



- Radial External Loading Does Not Always Mean Radial Stressing at Critical Locations
- Pure Multiaxial Loading Can Induce Nonproportional Action
- Similar Figures Exist in 6 Dimensional Space
- Proportional Plasticity Analysis Presumes Proportional Straining Under the Action of Proportional Stressing
- Transient Action Due to Time-Temperature and Cycle Dependent Response, Therefore, Causes Nonproportional Action Even in a so Called Uniaxial Specimen

Fig. 1. Schematics of Nonproportional Action

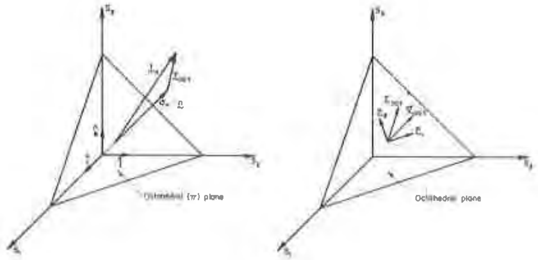


Fig. 2. The Octahedral Shear (Stress) Plane

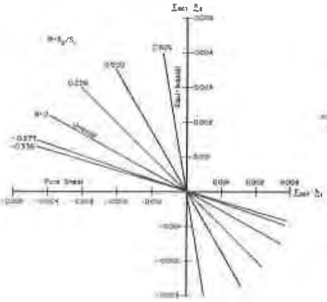


Fig. 3. Directions in Damage Plane

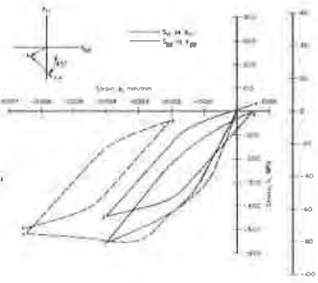


Fig. 4. Nonproportional Response

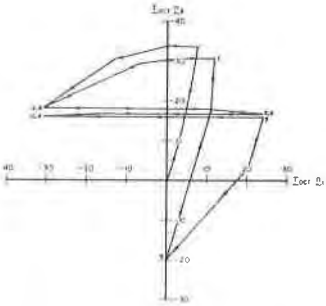


Fig. 5. Response of Fig. 4 Mapped onto Damage Plane

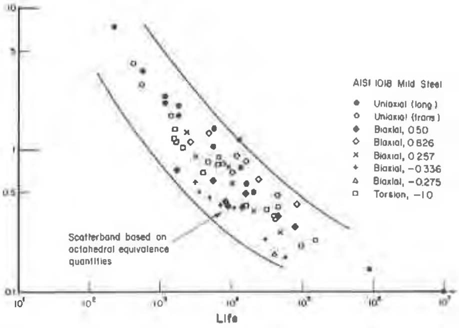


Fig. 6. Application of the Damage Postulate

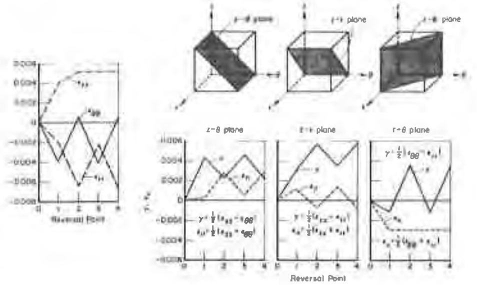


Fig. 7. Shear and Normal Strains on Various Shear Planes