THE ANALYSIS OF PIPING SYSTEMS FOR CREEP

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SUMMARY

Until recently piping system design was justified in terms of purely elastic analysis principles. In evaluating the safety and reliability of high temperature systems, in particular those associated with LMFBR development, inelastic behaviour must be admitted and the possibility of the accumulation of strain in the highly stressed locations accounted for.

The purpose of this paper is to provide a survey of the current situation and to attempt to explain and clarify a number of misconceptions which have arisen in connection with approximative techniques. Recently there have become available several Finite Element computer codes for the analysis of vessel type structures which can deal with any geometry, loading condition and material behaviour. However, a finite element analysis of a large diameter complex piping system for high temperature reactor service is simply not feasible economically to the detail and accuracy of a pressure vessel analysis; in any case, the aim is to produce a flexible system. Even those simplified finite element methods which have been designed to cope principally with piping are not practical for the initial stages of design for elevated temperature.

Ideally, a simple flexibility beam approach, similar to those classically used in elastic analysis, would be desirous. Such an approach has so far only been achieved using simplifications to the creep response. The two simplified methods which are primarily used are known generally as Kachanov's Method and the Method of Superposition of States. In this paper the reliability of these approximations is examined for simple piping components under combined loading; these also serve to clarify a number of misconceptions which have arisen over the methods. It is then discussed not only how these simplified methods can be adapted to the analysis of piping systems but also how a more general beam type approach could be constructed, and what sort of information would be necessary for their successful implementation.

In summary, this paper analyses the motivation for a simplified approach to the initial stages of elevated temperature piping design, examines the implications for such an approach and provides a possible solution to the problems which arise.

1. Introduction

Increased operating temperatures in nuclear reactors have demanded that creep effects be incorporated into the design and verification of their basic structural components. Perhaps the most formidable problem which needs to be resolved in connection with this is the stress analysis of the principal piping loops. The purpose of this paper is to provide a personal survey of the current situation with particular emphasis on simplified methods of analysis. It is an extension of some previous reviews [1, 2, 3] and is an attempt to bring these publications up to date.

If elastic behaviour alone is admitted the stress analysis of a complex piping circuit can be systematised and automated through the use of matrix methods [4] or finite element techniques [5]; indeed there are many computerised methods available which adequately perform such an analysis. In classical situations various flexible elements are employed in the design in order to cope with expected mechanical and thermal expansions in the system. The most important of these components is the curved pipe, or piping elbow - it is well known that it may deform considerably more than an equivalent length of straight pipe. A measure of the resiliance of the curved pipe is usually given by the "flexibility factor", being the ratio of the rotation of its ends under a given bending moment to the end rotation of a similar straight pipe under the same moment. Both the matrix and some simplified finite element methods include the curved pipe by equating its energy to a scalar multiple of that of the equivalent straight pipe - this scalar multiple is identical to the flexibility factor.

It was natural when it was realised that elevated temperature creep had to be accounted for to attempt to extend the classical methods to include the inelastic behaviour. Initial steps were taken by E L Robinson [6] and W Gorzynski [7] for simple loops neglecting bends. This work was later amplified to include the effect of curved pipes by J Spence [8], using creep flexibility factors which were derived for in-plane bending. These analyses, of necessity, were based on a simple approximation to the creep response and no attempt was made to generalise the analyses for complex systems. A step in this general direction was taken by E C Rodabaugh and G H Workman [9] using a flexibility type matrix method. However in order to make the analysis tractable, a number of simplifying assumptions were adopted which remain to be completely verified - in particular the uncoupling of the effects of the combined loading [10]. A better attempt, but one which avoided the combined loading problem, was suggested by T L Johnstone [11, 12] in his analyses of a typical LMFBR pipe run, again basing the analysis on the flexibility matrix method in a manner similar to [8]. Each of the above mentioned publications relied on an approximation to the creep response known as "superposition of states" which shall be described later. An alternative simplification, based on a method suggested by L M Kachanov, was used by J Valenta et al [13, 14] in a study of the rupture characteristics of a pipe run. Both of these approximations, superposition of states and Kachanov's method, were made in order to make the analysis possible by a simple flexibility matrix method using beam theory. However the main obstacle to practical success is the paucity of information on the combined loading problem. There has been some amount of activity, particularly in the USA, in trying to overcome this difficulty by constructing a simplified finite element method of analysis of a general complex piping circuit to include inelastic, and in particular creep, effects.

Perhaps the most successful of these is the MARC system [15]. In this straight pipes can be treated as standard beam elements, or cylindrical shell of revolution elements, with the curved pipe modelled by what has been called the "von Karman pipe bend element" [16]. No approximation to the overall creep response is made. Nevertheless this simplified finite element method suffers from the serious disadvantage of being excessively expensive, e.g. twenty three hours of computer time reported in [17]. Also it has been stated that the von Karman pipe bend element cannot deal with torque loadings, or loads varying along the axis [16]. Nevertheless there has been considerable enthusiasm for the element [18, 19]. A more simple finite element approach has been presented by G H Workman and E C Rodabaugh [20]. Unfortunately like their earlier paper [9], this also contained some curious simplifications [21].

It is apparent that while the finite element method may provide the most accurate model of the deformations, it is too costly at present to be efficiently used in the initial stages of design. For this reason the need for a simplified method of analysis should be obvious; since some success has been gained with a beam type approach, it would seem advisable to pursue this further. The overall manner in which such a flexibility method should be approached, with a general development of the analysis and an explanation of the simplified techniques, shall be presented in the following. Afterwards some evidence for these approximations shall be discussed. Indeed the analysis section may be missed initially.

2. Analysis

2.1 Statement of the problem

In order to identify a starting point for our discussions, let us examine the problem as it arises in elasticity. Consider a piping system anchored at one end 0 (Fig 1) and free at the other end A ; a force $\underline{F}_A$ is applied to the free end A and the system responds elastically with a displacement $\underline{\delta}_A$. The problem is to determine $\underline{\delta}_A$ given $\underline{F}_A$ or vice versa. The linearity of the static response allows these to be related by the matrix equation

$$\underline{\delta}_A = [A]\underline{F}_A \tag{1}$$

where [A] is called the flexibility matrix; this is usually readily derived from a consideration of Castigliano's Theorem where, if U is the strain energy of the system

$$\underline{\delta}_A = \frac{DU}{D\underline{F}_A} = \frac{D}{D\underline{F}_A}\int_0^A u\, ds$$

The basis of the simple beam approach is to express the energy density u at a point S in the system as a function of the loads acting at that point. Since these loads may be related statically to $\underline{F}_A$, u is a function of $\underline{F}_A$ and the matrix [A] can be evaluated. If we consider the rotated loading system $\underline{F}^R = (F_T\ F_N\ F_B\ \ M_T\ M_N\ M_B)$ at the point S in the directions $\underline{T}$, $\underline{N}$ and $\underline{B}$ (Fig 1) then for the straight pipe simply

$$u = \frac{1}{2EI}(M_B^2 + M_N^2 + (1+\nu)M_T^2)$$

ignoring the effect of the direct forces F_T F_N and F_B compared to the energy in bending. For a curved pipe the energy density takes the form

$$u = \frac{1}{2EI}(K_B M_B^2 + K_N M_N^2 + K_T M_T^2)$$

where $K_B = K_N$ are the flexibility factors, and $K_T = 1 + \nu$ assuming the curved pipe responds like a straight pipe to torque. There are approximations here of course, for example the effect of flanges and connections are roughly included by suitable redefinition of the flexibilities.

Let us now consider the creep problem: the free end A is fixed after a displacement of $\underline{\delta}_A$ through application of a force $\underline{F}_{A0}$. Due to environmental conditions the material creeps resulting in a relaxation of the forces $\underline{F}_A(t)$ in time at A. The problem is to determine $\underline{F}_A(t)$ so that expressions for the accumulation of strain - the most important consideration - may be derived. If we try and apply the same procedure as we did in elasticity, problems soon arise. Since relaxation is being considered elastic effects continue at all times to be significant; thus the relationship between the fixed displacement $\underline{\delta}_A$ and the relaxed force system $\underline{F}_A(t)$, if any, will include the time derivatives of these quantities. Furthermore the nonlinearity of the creep response to applied stress makes such a relationship nonlinear as well. However it is not at all clear how such a relationship may be derived. Consider a straight pipe under bending; whereas in elasticity there is a simple relation between curvature and moment, no such (closed form) expression exists for creep with elastic effects included. So if the relationship cannot be discovered for the simplest case there can be little hope for more complex situations. The reason for this difficulty in creep is the presence of stress redistribution. Any relation between displacement and force must also implicitly contain the stress.

Suppose that the material creep is governed by a simple nonlinear Maxwell relation such that if the stress field $\underset{\sim}{\sigma} = (\sigma_s \ \sigma_\theta \ \sigma_{s\theta})$ is known at some instant the corresponding rate of increase of creep strain $\dot{\underset{\sim}{\varepsilon}}_c = (\dot{\varepsilon}_s \ \dot{\varepsilon}_\theta \ \dot{\varepsilon}_{s\theta})$ can be evaluated according to

$$\dot{\underset{\sim}{\varepsilon}}_c = \underset{\sim}{f}(t, \underset{\sim}{\sigma}) \tag{2}$$

where $\underset{\sim}{f} = (f_s \ f_\theta \ f_{s\theta})$ for notational simplicity. Such a simple constitutive model has been chosen here solely for clarity. The rate of change of creep strain can be treated as an initial strain in the system and it remains to evaluate the rate of change of resultant forces at A. From simple elastic considerations the rate of change of deformation at A is given by

$$\dot{\underset{\sim}{\delta}}_A = [A]\dot{\underline{F}}_A + \dot{\underline{\delta}}_A^R(\dot{\underset{\sim}{\varepsilon}}_c) \tag{3}$$

similarly to (1) where $\dot{\underline{\delta}}_A^R(\underset{\sim}{\varepsilon}_c)$ is the displacement at A resulting from the creep strain in isolation. However since relaxation is being considered $\dot{\underline{\delta}}_A = \underline{0}$ remains fixed so that

$$\frac{d}{dt}\underline{F}_A = -[A]^{-1}\dot{\underline{\delta}}_A^R(\underset{\sim}{f}(t, \underset{\sim}{\sigma})) \tag{4}$$

It is easily seen that the presence of the redistributing stress is verified. In fact it can be shown [22] that the redistribution of stress is governed by an equation of the form

$$\frac{d}{dt}\underset{\sim}{\sigma} = \underline{R}(\underline{f}(t, \underset{\sim}{\sigma})) \tag{5}$$

where $\underset{\sim}{R}$ is an 'operator' defined by what has been called the 'residual elastic problem'. Let $\underset{\sim}{\varepsilon}^*$ be a strain field, then $\underset{\sim}{R}(\underset{\sim}{\varepsilon}^*)$ is the stress field obtained by solution of an equivalent elastic problem on the piping system with no displacements specified at A but with $\underset{\sim}{\varepsilon}^*$ as an initial strain field.

As it is seen that (4) and (5) constitute a coupled system of first order ordinary differential equations in time, which can be formally resolved given the initial conditions

$$\underset{\sim}{\sigma}(0) = \underset{\sim}{\sigma}_o \qquad \underline{F}(0) = \underline{F}_{A0} \tag{6}$$

These must be solved in order to trace the relaxation of forces. If beam theory is to be used then the equivalent deflections $\underline{\delta}_A^R$ can be expressed as system integrals of the creep strains with modified flexibility factors in the case of curved sections; similarly the operator R can be developed using modified stress concentration factors. It is important to realise that such considerations only involve linear elastic studies. The system (4)and (5) must be then reduced to a finite system of ordinary differential equations for computer implementation; this can be achieved by replacing the necessary integrals by finite sums. However the equations (4) and (5) with (6) are quite general and can be used formally for any discrete or direct model of the piping system. For example, we may use finite elements or simplified finite elements, so that equations for the stresses at the nodal points, then the operator R and the vector function $\underline{\delta}_A^R$ assume simple matrix forms obtained by linear elastic analysis.

Although a beam theory approach is entirely possible on the lines described above, it would be desirous in the initial design process to make a further simplification to the creep response aimed at eliminating the presence of the stress in (4). In the following sections we shall discuss two such approximations - "Kachanov's Method" and the "Method of Superposition of States".

2.2 Kachanov's Method

Kachanov's method [23] is a special technique which requires only a knowledge of the elastic stressdistribution in a relaxing body. It assumes that the same proportional spatial distribution of stress will occur during relaxation; thus symbolically

$$\underset{\sim}{\sigma}(t) = \rho(t)\,\underset{\sim}{\sigma}_o$$

where ρ (t) is an unknown scaling factor to be determined. A suitable expression for may be established either by a Galerkin technique with substitution into (5) or perhaps more directly from a variational formulation [24]. The disadvantage with this simple approximation is that not all the stresses will relax in a similar manner - for example, one stress component may monotonically decrease while another may initially grow. A better approximation would be

$$\sigma_s(t) = \rho_s(t)\,\sigma_{so} \qquad \sigma_\theta(t) = \rho_\theta(t)\,\sigma_{\theta o} \qquad \sigma_{s\theta}(t) = \rho_{s\theta}(t)\,\sigma_{s\theta o}$$

and three equations for ρ_s , ρ_θ and $\rho_{s\theta}$ may be obtained again by a suitable direct method. It is expected that this approximation - which we should term a "Modified Kachanov Method" would be fairly easy to construct numerically. A still better approximation would be

$$\sigma_s^i(t) = \rho_s^i(t)\,\sigma_{so}^i \qquad \sigma_\theta^i(t) = \rho_\theta^i(t)\,\sigma_{\theta o}^i \qquad \sigma_{s\theta}^i(t) = \rho_{s\theta}^i(t)\,\sigma_{s\theta o}^i$$

where the superscript 'i' denotes the ith piping component.

2.3 The method of Superposition of States

The method of superposition of states has been proposed as an approximation for both forward creep and relaxation [24, 25]. Effectively the method assumes that the total deformation response of a structure to creep may be approximated by adding the pure elastic deformation state, found by ignoring creep, to the pure creep deformation state, found by ignoring elasticity.

This means that $\dot{\underline{\delta}}_A^R(\dot{\underset{\sim}{\varepsilon}}_c)$ the displacement at A corresponding to an imposed creep strain, can be approximated by the displacement rate found purely from creep calculations ignoring the continued presence of elasticity. Then a Castigliano type result can be construed and

$$\dot{\underline{\delta}}_A^R(\dot{\underset{\sim}{\varepsilon}}_c) \simeq \frac{D}{D\underline{F}_A} W(\underline{F}_A) = \dot{\underline{\delta}}_A^c(\underline{F}_A)$$

where W is the creep strain energy which should be related to the applied loading. Then on substitution into (4)

$$\frac{d}{dt}\underline{F}_A = -[A]^{-1}\underline{\dot{\delta}}^c_A(\underline{F}_A) \tag{7}$$

This equation is complete: use of the approximation of superposition of states requires no reference to the equation of stress redistribution. However it is important to realise that this is indeed an approximation. In order to evaluate $\underline{\dot{\delta}}^c_A$ using a pure creep calculation it must be assumed that $\underline{\dot{\delta}}^c_A$ is compatible with $\dot{\underset{\sim}{\varepsilon}}_c$; this is not generally true.

With this presentation of the analysis no examples or evidence have been given. Such considerations shall be discussed in the following section.

3 Discussion

In the construction of any simplified method of analysis some problems will disappear while new ones will be discovered. The simplifications to the creep response allow a time dependent creep analysis to be performed in a relatively simple manner; however their application to complex piping introduces new complications. Before these complications are discussed we shall present some evidence for the simplified methods.

3.1 Verification of the Simplified Methods for Relaxation

Let us consider the problem of the determination of the relaxation of bending moment M(t) on a curved pipe which is constrained to a given deformed curvature. The exact analysis of this problem, for a nonlinear Maxwell constitutive relationship, was given by the authors in Reference [26]; some results for a typical geometry are shown in Fig 2, along with all necessary information on the geometrical and material parameters. The relaxation of the bending moment can be estimated readily using Kachanov's method and superposition of states.

If it is assumed that the stresses remain proportional to the elastic stresses then the relaxed bending moment is similarly proportional

$$M(t) = \rho(t)M(0) \tag{8}$$

It can be easily shown [24] that the scaling factor $\rho(t)$ is given by

$$\rho(t) = \left\{1 + (n-1)\frac{\overline{W}_e}{\overline{U}_e}\tau(t)\right\}^{-\frac{1}{n-1}} \tag{9}$$

with

$$W_e = \frac{B(t)}{n+1}\,\overline{W}_e \qquad U_e = \frac{1}{2E}\,\overline{U}_e \qquad \tau(t) = E\left(\frac{M(0)}{\pi b^2 h}\right)^{n-1}\int_0^t B(t)\,dt$$

where U_e is the elastic (complementary) energy of the curved pipe and W_e is the creep (complementary) energy evaluated using the elastic stress field. Comparison with the exact analysis is given in Fig 2. Clearly the Kachanov approximation predicts a faster relaxation of the bending moment in this example, but correspondingly overestimates the accumulated creep strain, which is attractive.

Of more immediate interest is the approximation obtained by superposition of states. Suppose that the curved pipe responds purely elastically to the moment M(t), then the observed end rotation would be

$$\gamma_e(t) = \frac{1}{E}\left(\frac{M(t)}{\pi b^2 h}\right) K_e \frac{\Theta}{b/a} \tag{10}$$

with reference to Fig 2, where K_e is the elastic flexibility factor. Similarly if it is supposed that the curved pipe exhibits only stationary (pure) creep then the rate of increase of end rotation would be

$$\dot{\gamma}_c(t) = B(t)\left(\frac{M(t)}{\pi h b^2}\right)^n K_c \left(\frac{\pi}{\int_0^{2\pi} \sin\phi^{1+1/n} d\phi}\right)^n \frac{\Theta}{h/a} \tag{11}$$

for a Norton power law, where K_c is a measure of the flexibility of the curved pipe compared to a similar straight pipe subject purely to stationary creep [27]. The approximation of superposition of states then assumes that the total end rotation γ_o may be obtained by simple addition of the pure elastic and creep parts, hence

$$\gamma_o = \gamma_e + \gamma_c$$

but since the overall end rotation is fixed $\dot{\gamma}_o = 0$ and

$$\dot{\gamma}_e + \dot{\gamma}_c = 0 \tag{12}$$

it follows on substitution of (10) and (11) into (12) and rearranging that

$$\frac{M(t)}{M(0)} = \left\{1 + (n-1)\frac{K_c}{K_e}\left(\frac{\pi}{\int_0^{2\pi} \sin\phi^{1+1/n} d\phi}\right)^n \tau(t)\right\}^{-\frac{1}{n-1}} \tag{13}$$

Again comparison with the exact analysis as demonstrated by Fig 2 is excellent.

Further analytical examples of the approximations can be discovered in the paper by Spence and Hult [24] for relaxation, and in Boyle and Spence [27] and Marriott and Leckie [25] for the forward creep situation. One criticism of the above comparisons is that the nonlinear Maxwell, or time hardening, creep constitutive model may be inadequate in some respects for the proper estimation of creep relaxation. However it is worth noting that comparisons of time hardening and strain hardening theories on a simple piping elbow by Mello, Griffin and Schelier [28] using the MARC finite element code were favourable for relaxation, Fig 3. An obvious improvement on these theories is to allow the creep strain rate at an instant to depend on the history of stress and stress rate. It is fairly easy to construct a suitable creep constitutive model which reduces to time hardening for constant stress; but the structural creep problems then reduce to functional differential equations. Some analytical comparisons on simple components are under examination by the authors. Nevertheless a more worthwhile study of the suitability of these time hardening models for relaxation would be experimental. Some helpful results have been obtained by Henderson at N.E.L. [29]. An interesting example is the comparison obtained for a circular bar subjected to pure torque by Henderson and Ferguson [30]. Spence and Hult [24] observed that load relaxation results obtained by superposition of states could be written in the form of a uniaxial relaxation test performed at some representative stress level. Henderson and Ferguson directly tested this for the thick tube comparing the result with the appropriate uniaxial relaxation test, Fig 4. The results are extremely favourable and contribute not only to the verification of a time hardening theory but also to superposition of states.

3.2 The Problem of Flexibility Factors

In the above whenever a curved pipe was mentioned the flexibility factor appeared as well. This is necessary in order to include the curved pipe in a beam formulation. Effectively the flexibility factor takes into account the added resilience of a curved pipe to bending obtained through cross sectional flattening. In linear elastic analysis the

the flexibility factors are well known and shall not be recounted here. However in creep analysis, when using the superposition of states approximation, flexibilities corresponding to the pure (stationary) creep deformations are required. If only in-plane bending is in evidence, with a uniformly applied moment, then suitable flexibility factors have been given by Spence, not only for the classical problem [31] but also for elliptical cross sections [32], short radius bends [33] and simply pressurised bends [34]. These factors were in the form of upper and lower bounds for a Norton power law of creep and were verified, using an alternative approach, in a later publication [27]. They have been employed in piping flexibility analysis by Spence [8], Johnstone [11, 12] and Rodabaugh and Workman [9, 20]. Unfortunately only a few experimental verifications of these factors are presently available. The tests performed by W I Griffith and E C Rodabaugh [35] are unhelpful since some rather ad hoc assumptions had to be made concerning the presence of plastic strains and the effect of tangent straight pipes. Nevertheless, once these empirical adjustments had been made the calculated deformations, derived from flexibility factors, were in reasonable agreement with the measured deformations. A better testing program was undertaken by A Imazu and his colleagues [36] at PNC in Japan. Measured values of the steady state creep deflection rate of the arm of a piping elbow are shown in Fig 5 with a comparison of a calculated deflection rate obtained using flexibility factors. The results obtained are pleasingly in agreement with each other.

The development of suitable flexibility factors in single load situations thus appears to have reached a fairly advanced stage. However it is in combined loading situations that the principal difficulties occur. Indeed this is true not only for the curved pipe but also for the straight. The problem confronting us is to establish a relationship between the pure creep displacement and the load. This can be deduced using an energy method identical to that used in elasticity, as mentioned previously. Then the problem is to find an expression for the creep complementary energy of the curved pipe under load. For in-plane bending alone the creep complementary energy per unit length of the pipe is given by

$$\frac{B(t)}{n+1} K_B \frac{M_B^{n+1}}{I_n}$$

where M_B = in-plane moment, I_n = (n + 1)th moment of area of cross section, $B(t)$, n = material creep parameters associated with a Norton n-power law of creep, and K_B is the flexibility factor for in-plane bending in creep derived by Spence [31].

For the problem of out-of-plane bending on one end of a curved pipe the creep complementary energy per unit length at angle θ is (Fig 6):

$$\frac{B(t)}{n+1} K_N(\theta) \frac{M_N^{n+1}}{I_n}$$

where M_N is the applied bending moment and $K_N(\theta)$ is an "energy factor" defined as the ratio of energy in a curved pipe to that of a similar straight pipe in creep [37]; when $\theta = 0$ $K_N = K_N(0)$ corresponds to the flexibility factor for a segment of a curved pipe under out of plane bending alone. There are two important points about this "energy factor" which should continually be borne in mind. In single load situations the energy factor is equivalent to the flexibility factor, but this is not the case for combined load situations as in the above where both out of plane bending and torque are present. Further, in linear elasticity the in plane and out of plane flexibility factors are identical; this again is not true for nonlinear creep. Thus this work on out of plane loading in creep pointed not to the development of flexibility factors but to that of energy factors. The problem was to determine a simple, useful expression for the complementary energy which requires the minimum number of parameters. Further these parameters should be readily calculable. Investigation into the general combined loading problem by the authors [38] showed that it was not easy to provide such a simple formulation for the energy. However they were useful in identifying simple approximations which could be employed in practice. The simplest such approximations is that due to Rodabaugh and Workman [9] being obtained on uncoupling the effects of the combined loads and adding their energies in isolation. This approximation will tend to underestimate the magnitude of the energy since it ignores the coupling effect. For the out of plane loading example mentioned above, this approximation is evaluated in Fig 6. Nevertheless such an approximation should not be used in isolation. In particular, if a lower bound is being used a compatible approximation which overestimates the magnitude of the coupling should be investigated as well for comparison. Such an overestimation, coupled with a number of other approximations including the lower bound, has been implemented in the creep piping program PACE developed by the authors [39] for the UKAEA. This approximation can be readily derived from observations on combined loading on a straight pipe [40]. This approximation is also shown in Fig 6.

3.3 Extensions to Plasticity

So far only elastic and creep deformations have been considered. A general analysis would not be complete unless plastic strains also are included. A plastic piping system flexibility analysis using a beam-type method has been suggested by R L Roche and his colleagues [41, 42, 43]. It is very similar to that described above in the sense that a simple approximation for the energy of the piping components under combined load is sought. Indeed for the in-plane bending situation Roche used effectively Findlay and Spence's [44] limit load calculations, obtaining good agreement with experiment. Moreover I W Goodall reports [45] that the creep energy factors for out of plane loading have proved effective also in the plastic case, by taking the exponent in the power law large and using the plastic analogue. If elastic-plastic strains are to be considered then the situation is more complex. An approximation which immediately presents itself is the possibility of using superposition of states, with suitable modifications to eliminate the time dependence. Thus the elastic-plastic behaviour of a piping system may be approximated by performing an elastic analysis and a separate rigid-plastic analysis. This approach is then ideally suited to a total deformation theory of plasticity of the Ramberg-Osgood type, with the added advantage that the flexibility and energy factors derived for pure creep with a power law can still be used. Indeed using this approach J Johnstone [46] reports excellent agreement with Mello and Griffin's [18] finite element solution for the plastic collapse of a piping elbow! Obviously this interesting aspect of the analysis warrants much further study.

3.4 The Problem of Material Data

The last, and arguably most ignored problem which arises is that of the availability of suitable material creep data. Allied to this is the observation that much of the available design information which has been developed for the creep of piping, e.g. energy factors, has been only for a simple power law of creep. The answers to these questions cannot be readily given here; however it can be said that they involve similar considerations. Recently there has been developed in the UK a technique known as the Reference Stress Method [47, 48, 49] which aims at relating structural behaviour to uniaxial test data; the application of the method to relaxation has been discussed by Spence and Hult [24]. This technique allows the adequacy of a set of creep data to be assessed [50] and also avoids problems associated with the choice of a single constitutive equation - in fact any such equation, in particular the power law, may be used initially. Literature on the Reference Stress Method is growing and further discussions can be found in the cited references.

4 Conclusions

No review can ever be complete in a contemporary field of research. Indeed further tests and verifications of the methods described in this paper are being performed at present in a number of countries. So the final role of these simplified techniques cannot be easily assessed. Certainly they are the only reasonable alternatives to the finite element techniques in the initial stages of design. If they are to be used with confidence further experimental studies must be undertaken, but there should also be a greater theoretical understanding of the approximations made to the creep response and their application to other forms of inelasticity. There are still numerous difficulties which must be overcome, not in the least of which is the generation of valid expressions for the creep energy of the piping components under the influence of combined loads. These difficulties, however, are not insurmountable.

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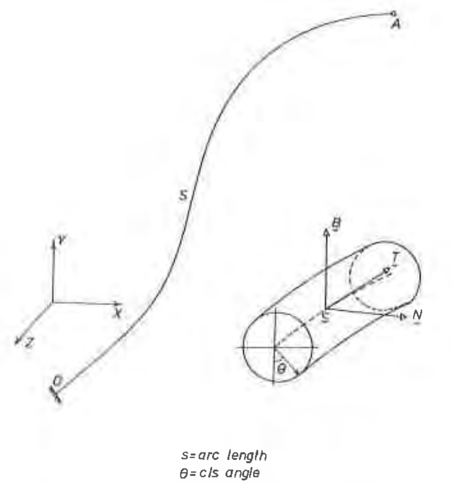

Fig 1 Two anchor piping system

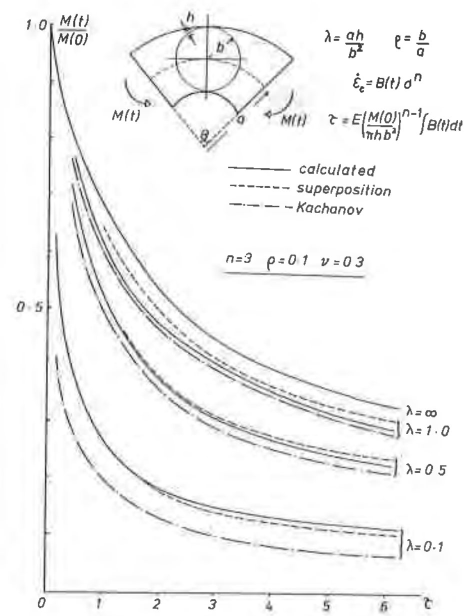

Fig 2 Comparison of estimations of moment relaxation in a curved pipe [26]

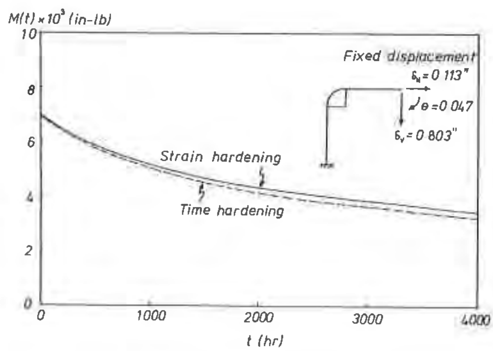

Fig 3 Comparison of time - and strain - hardening theories for moment relaxation in a curved pipe [28]

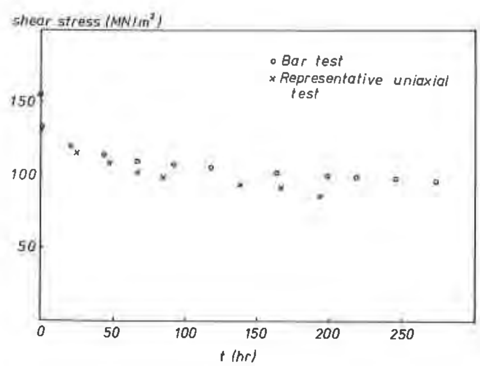

Fig 4 Comparison of superposition of states/representative stress prediction for torsional relaxation of 0.5% Cr 0.5% Mo 0.25% V steel at 575°C [30]

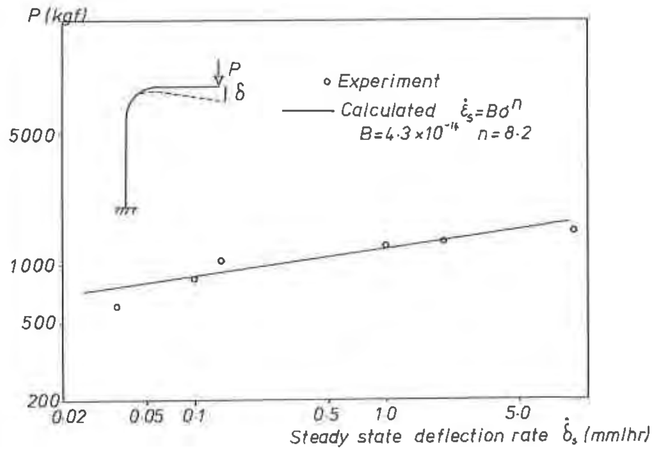

Fig 5 Experimental verification of steady state creep flexibility factor [36]

$$K_N(\theta) = \frac{\text{Energy per unit length of bend at } \theta}{\text{Energy per unit length of straight under } M_N}$$

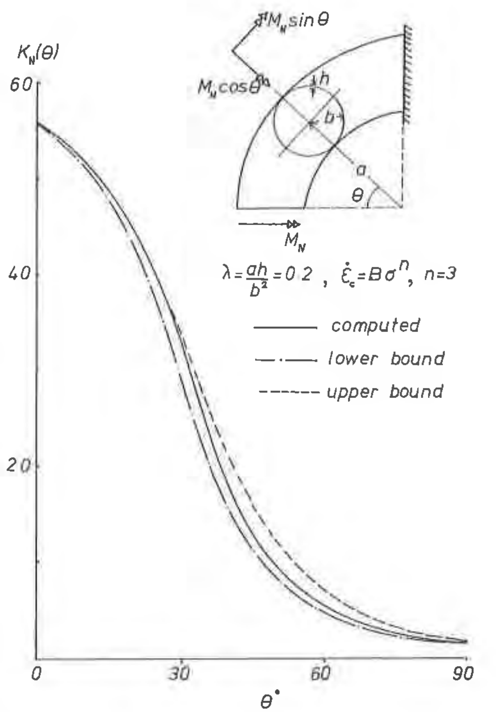

Fig 6 Energy factors in out of plane bending - comparison with simple approximations [38]