ON SEISMICALLY INDUCED VIBRATIONS OF PRESSURE VESSELS WITH CUTOUTS AND CRACKS

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SUMMARY

It is known that exceedingly conservative design practices were used in nuclear power plants where detailed knowledge of the system behavior was not available for some dynamic loading and hypothetical accident cases. Since such practices are costly, there is a great incentive for gaining additional knowledge about the effects of dynamically and in particular seismically-incuded vibrations on nuclear systems. Earthquake motion is one example of a possibly dangerous loading condition, since it is known that the frequencies of such motions are usually in the region of the natural frequencies of reactor vessels.

In this paper the dynamic response of finite circular cylindrical elastic shell representing a pressure vessel is investigated due to periodic and seismic vibration of the form $q_z = q(x,\phi) \cos (\omega t+\psi)$. where $q(x,\phi)$ is the amplitude of a seismically induced load, ω is the forcing frequency and ψ is the phase constant. The dynamical form of more accurate Morley's equations are developed for a circular cylindrical shell. Then, these governing equations are utilized to investigate the elastic behavior of a circular cylindrical vessel with an elliptical cutout or an axial crack under the seismically induced load. The distribution of force and moment stress resultants and corresponding stress concentration factors and the stress intensity factors around the crack tip were examined.

I. Introduction

Since their inception, nuclear power plants have been designed and constructed with public safety as a paramount concern. Excessively conservative design practices were used in many areas where detailed knowledge of the system behavior was not available. Since excessively conservative design practices are costly, there is great incentive for gaining additional knowledge about the effects of dynamically and in particular seismically induced vibrations on pressure vessels in nuclear reactors. It is known that the natural frequencies of reactor vessels are usually in the region of that of earthquake motion [1].

Moreover, the existence of cutouts and cracks may change considerably free and forced vibration characteristics of the vessel. For any given material under a specified stress field, there is a crack length of a certain initial value for which the crack will become self propagating. If this length is ever reached, either by penetration or by the growth of a small crack, complete loss of the vessel may occur.

In the existing studies on the dynamic response of cracked spherical [2] and cylindrical shells [3], simpler but less accurate Donnell's shallow shell equations were utilized. However, this situation puts restrictions on the range of shell curvatures encountered in the most practical applications. The set of equations introduced first in static form by Morley [4] is proven to yield more accurate results through the wide range of shell curvatures where as Donnell's equations are not adequate to describe the shell behavior. However, due to the relative complexity of the static form of Morley's equations in comparison to Donnell's formulation, very few authors have attempted to utilize Morley's equations exclusively to static problems.

In this paper the dynamical form of more accurate Morley's equations are developed. These equations are utilized for the first time, according to the best of the authors' knowledge, to investigate the behavior of a finite elastic circular cylindrical shell with cutouts and cracks under the seismically induced dynamic load. Analytical results are incorporated into a computer program to determine the distribution of force and moment stress resultants and corresponding stress concentration factors in the inner, middle and outer surfaces of the shell. The distribution patterns around and away from an elliptical cutout are illustrated in graphs as a function of shell geometric, material and loading parameters such as forcing frequency ω, shell curvature parameter βa, Poisson's ratio ν, and minor to major axis ratio $d=b/a$ of the elliptical cutout. The important case of an axial crack as the limiting form of an elliptical cutout is also investigated and stress intensity factors around the crack tip as well as their attenuation away from the crack zone are examined and illustrated. The results in the special case of a circular hole and static internal pressure are compared with the relevant work available in the literature.

2. Governing Equations

The axial, circumferential and radial shell coordinates $\overline{X}$, $\overline{Y}$, $\overline{Z}$, associated displacements $\overline{U}$, $\overline{V}$ and $\overline{W}$ and corresponding non-dimensional values are shown in Figure 1 for the circular cylindrical shell with an elliptical cutout

$$x = \frac{\overline{X}}{R}, \quad \phi = \frac{\overline{Y}}{R}, \quad z = \frac{\overline{Z}}{R}$$

$$u = \frac{\overline{U}}{R}, \quad v = \frac{\overline{V}}{R}, \quad w = \frac{\overline{W}}{R} \tag{1}$$

Then the dynamical form of Morley's equations were developed through the equations of motion, kinematics of deformation of a shell element and the stress resultant-displacement relations. Thus the stated equations for the unknown displacements u, v, and w are given in uncoupled form as follows [5]:

$$\nabla^4 u = \frac{\partial^3 w}{\partial x \partial \phi^2} - \nu \frac{\partial^3 w}{\partial x^3} - \frac{1}{\nu_m} \frac{\partial^2 q_x^*}{\partial \phi^2} - \frac{\partial^2 q_x^*}{\partial x^2} + \frac{\nu_p}{\nu_m} \frac{\partial^2 q_\phi^*}{\partial x \partial \phi} + G \frac{3-\nu}{2\nu_m} \frac{\partial^2}{\partial t^2} (\nabla^2 u)$$
$$+ \frac{G}{\nu m} \left(\nu \frac{\partial^3 w}{\partial x \partial t^2} + \frac{\partial^2 q_x^*}{\partial t^2} - G \frac{\partial^4 u}{\partial t^4}\right) \tag{2}$$

$$\nabla^4 v = -(2+\nu) \frac{\partial^3 w}{\partial x^2 \partial \phi} - \frac{\partial^2 q_\phi^*}{\partial \phi^2} - \frac{1}{\nu_m} \frac{\partial^2 q_\phi^*}{\partial x^2} + \frac{\nu_p}{\nu_m} \frac{\partial^2 q_\phi^*}{\partial x^2} +$$
$$G \frac{3-\nu}{2\nu_m} \frac{\partial^2}{\partial t^2} (\nabla^2 v) - \frac{\partial^3 w}{\partial \phi^3} + \frac{G}{\nu_m} \left(\frac{\partial^3 w}{\partial \phi \partial t^2} - G \frac{\partial^4 v}{\partial t^4} + \frac{\partial^2 q_\phi^*}{\partial t^2}\right) \tag{3}$$

$$\nabla^4 (\nabla^2+1)^2 w + 12(1-\nu^2) \left(\frac{R}{h}\right)^2 \frac{\partial^4 w}{\partial x^4} = \frac{G}{\nu_m} (\nabla^2+1)^2 \frac{\partial^2}{\partial t^2} \left(\frac{3-\nu}{2} \nabla^2 w - G \frac{\partial^2 w}{\partial t^2} \right)$$
$$+ \frac{G}{k\nu_m} \frac{\partial^2}{\partial t^2} \left(\frac{3-\nu}{2} \nabla^2 w - \frac{\partial^2 w}{\partial \phi^2} - \nu^2 \frac{\partial^2 w}{\partial x^2} - \nu_m \nabla^4 w\right) - \frac{G^2}{k\nu_m} \frac{\partial^4}{\partial t^4} \left(w + G \frac{\partial^2 w}{\partial t^2} - \frac{3-\nu}{2} \nabla^2 w\right)$$
$$+ \frac{1}{k} \left[\nabla^4 q_z^* + \nu \frac{\partial^3 q_x^*}{\partial x^3} + (2+\nu) \frac{\partial^3 q_\phi^*}{\partial x^2 \partial \phi} + \frac{\partial^3 q_\phi^*}{\partial \phi^3} - \frac{\partial^3 q_x^*}{\partial x \partial \phi^2}\right] - \frac{G}{k\nu_m} \frac{\partial^2}{\partial t^2} \left(\frac{3-\nu}{2} \nabla^2 q_z^* \right.$$
$$\left. + \nu \frac{\partial q_x^*}{\partial x} + \frac{\partial q_\phi^*}{\partial \phi} - G \frac{\partial^2 q_z^*}{\partial t^2} \right) \tag{4}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial \phi^2}, \quad G = \frac{R^2 \rho h}{D}, \quad D = \frac{Eh}{1-\nu^2}$$
$$k = \frac{h^2}{12R^2}, \quad (q_x^*, q_\phi^*, q_z^*) = \frac{R}{D} (q_x, q_\phi, q_z) \tag{5}$$

Here E is the Young's modulus ν Poisson's ratio and ρ is the density of the material of the shell. R and h represent the radius and thickness of the cylindrical shell respectively. q_x^*, q_ϕ^*, q_z^* are the components of the mechanical loading in the x, ϕ, and z directions.

The dynamic response of a finite circular cylindrical shell representing a pressure vessel is investigated elastically due to seismic vibrations of the form $q_z = q(x,\phi)\cos(\omega t+\psi)$. Here $q(x,\phi)$ is the amplitude of a seismically induced load, ω is the forcing frequency in rad/sec and ψ is the phase constant. The method of solution is based on superposition of the following two solutions:

a. The Nominal Solution

The stress field caused by prescribed dynamic surface loads in a shell of finite length with no cutout.

b. The Residual Solution

The stresses in the shell caused by applied edge loads around the cutout otherwise free of an other applied loads. These loads are equal in magnitude but opposite in sign to those present in the shell without cutout at the cutout location.

II. Solution of the Problem

In accordance with the expression of q_z, the unknown displacements u,v, and w are assumed in the following form:

$$\begin{aligned} u &= U\,(x,\phi)\,\cos\,(wt+\psi) \\ v &= V\,(x,\phi)\,\cos\,(wt+\psi) \\ w &= W\,(x,\phi)\,\cos\,(wt+\psi) \end{aligned} \qquad (6)$$

Then q_z, u, v, and w expressions are substituted in the uncoupled set of partial differential equations, (2-4). Thus the nominal solution is obtained to describe the behavior of the finite shell with specified boundary conditions at both ends of the cylindrical shell. Then the residual solution to the homogeneous governing differential equations is obtained in such a way that when it is combined with the corresponding nominal solution the boundary conditions around the elliptical cutout are satisfied. A boundary point matching technique in the least squares sense is utilized in satisfying of the boundary conditions along the cutout.

III. Numerical Results

Due to the limited space for the article, only a few results are presented here.

Figure 2 shows the attenuation of the non-dimensional normal in-place membrane stress resultant $\overline{N}_\xi$ away from the cutout boundary for ω =300Hz and various minor to major axis ratios d=b/a=.8, .6, .4, .2, .08 of the elliptical cutout. There is an overall initial decrease in the stress level for the smaller values of d down to .4. For the values of d=.2, .1, and possible minimum value .08 there is a rather steep increase in the magnitudes of stresses in the zone ξ/ξ_o = 0 to 7. Observing this trend, one expects much higher stress magnitudes when d approaches zero value to represent an axial crack. It is also interesting to note that maximums of the curves shift towards the immediate vicinity of the top of the elliptical cutout for small d values.

In Figure 3, the non-dimensional bending stress $\overline{M}_\xi$ for ω=300 Hz is zero at the cutout boundary in accordance with the free boundary conditions. The stresses show an overall decreasing trend for the values of d=1 down to d=.4. For d=.2, the stresses show a rapid increase in magnitude and maximums of the curves shift towards the top of the elliptical cutout indicating a stress singularity for smaller d values.

Figure 4 shows the distribution of membrane stress concentration and stress intensity factor S_c around one quadrant of the elliptical cutout. The cutout location is indicated by the polar coordinate φ for a circular cutout and elliptical coordinate η is employed in the case of the elliptical cutout. For the forcing frequency of 100 Hz, various minor to major axis ratios d=b/a=1., .8, .6, .2, .08 are illustrated.

First, there is a decrease in the stress concentration level when the ratio d attains smaller values from 1. down to .8. There is a notable shift of the curve for .8 relative to the curve for d=1. towards the location $\varphi=\eta=0°$, i.e. to the tip of the elliptical cutout. On the other hand, a sharp increase in the magnitudes are followed by a more significant shift towards the tip of the elliptical cutout is observed for the ratics d=.6, .2 and d=.08 (Penny-shaped) crack.

This is accompanied by an increased values of the curvatures at the maxima of the curves. Observing this trend of increasing stress values towards the crack tip $\varphi=\eta=0°$, we may extrapolate an approaching stress singularity for the limiting zero value of the ratio d, as dictated by the elastic solutions. There is also an overall stress relief observed in the region $\varphi = \eta = 60°$ to 90° when the ratio d decreases. This may be explained physically, since the curvatures of the cutout curve in this zone becomes larger as the ratio d decreases approaching a line crack along the x-axis.

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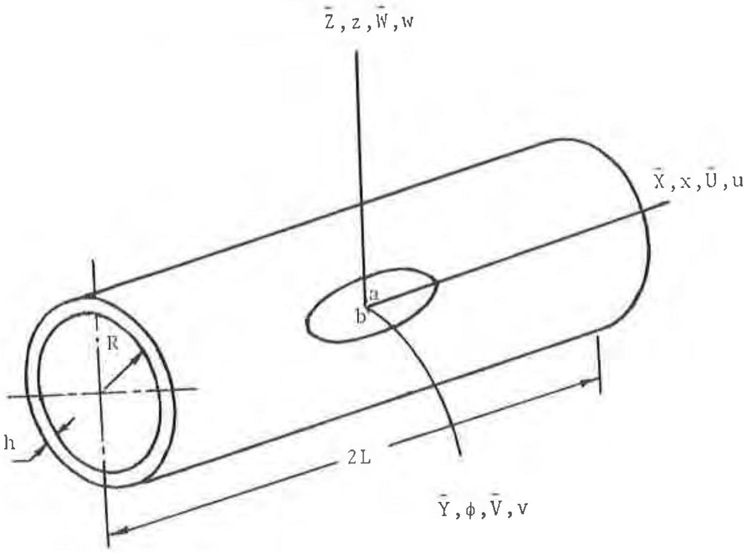

Figure 1. Geometry of the Cylindrical Shell.

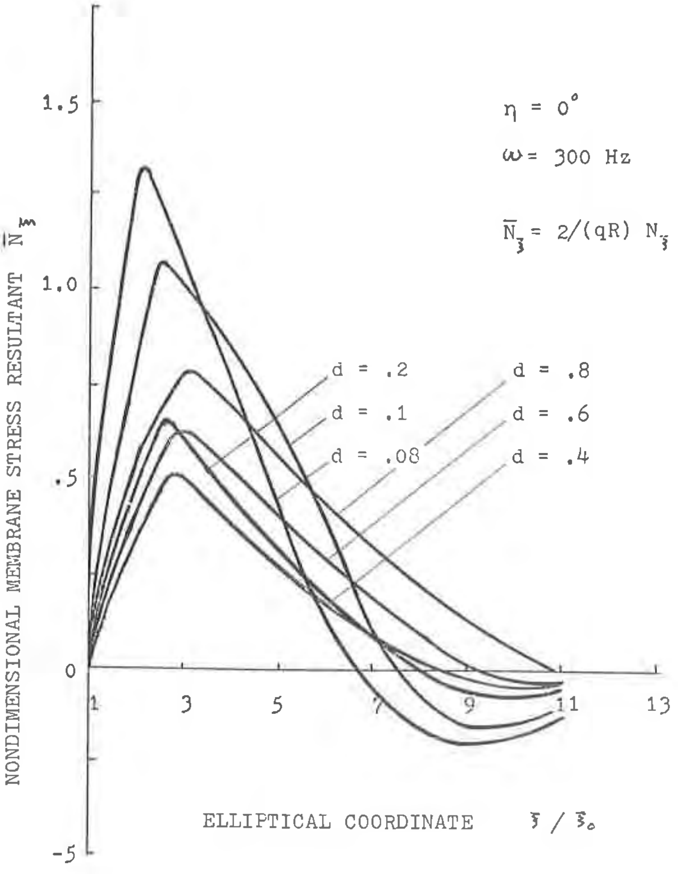

Figure 2 . Attenuation of nondimensional inplane Membrane Stress Resultant $\overline{N}_\xi$ away from the Elliptical Cutout.

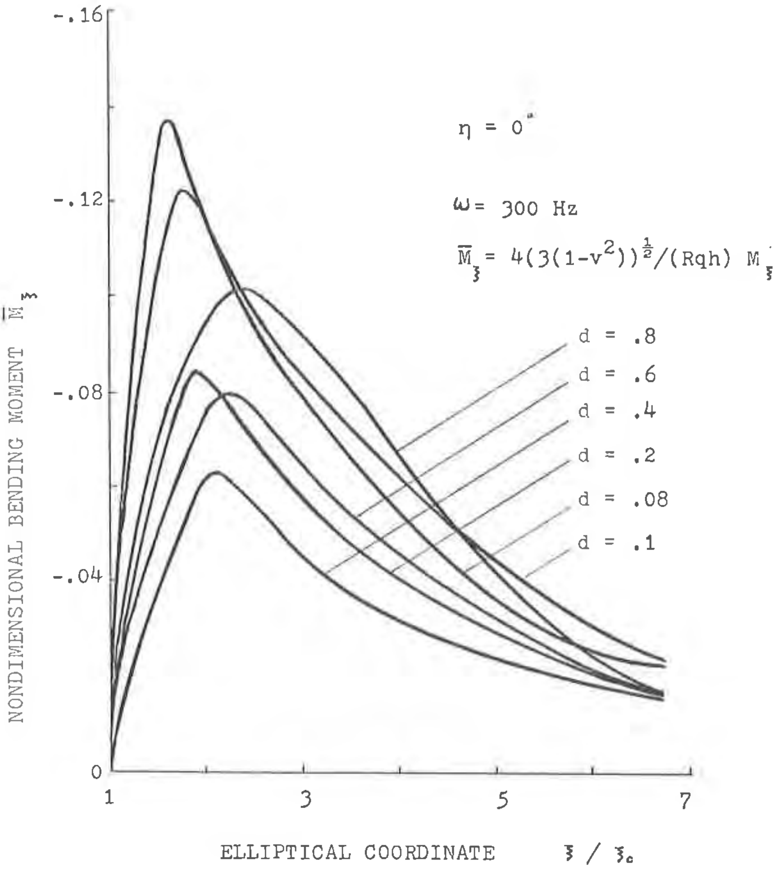

Figure 3 . Attenuation of nondimensional Bending Moment $\overline{M}_\xi$ away from the Elliptical Cutout.

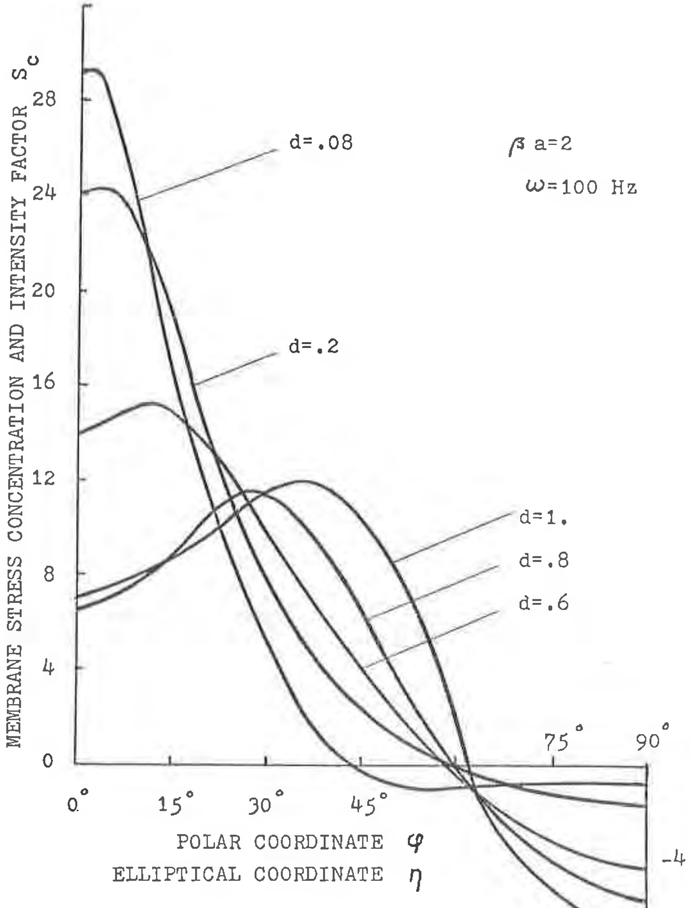

Figure 4 . Effect of the minor to major axis ratio d on the Distribution of Membrane Stress Concentration and Intensity Factor S_c along φ or η axis around one quadrant of the Elliptical Cutout.