Analysis of a PWR nuclear containment shell structure due to wind force

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1 INTRODUCTION

The containment is a reinforced concrete structure in the shape of a cylinder with a hemispherical dome and a flat foundation slab. The analysis of the containment structure as a whole is carried out as an axisymmetrical structure using the Kalnins' "Static Analysis of Thin Elastic Shells of Revolution", computer program (Kalnins 1968). The computer program is applicable to the analysis of thin, elastic shells which are symmetric about one straight axis. The requirement of symmetry includes all geometric as well as physical parameters of the shell. However, the properties of the shell can have arbitrary variations along the meridian of the shell. This program is concerned with the general problem of deformation of thin, elastic shells of revolution, symmetrically or non-symmetrically loaded, and with the utilization of multisegment, direct numerical integration technique due to Adams' method (Kalnins 1964 and Hildebrand 1956). The shell is divided along the meridian into parts which are to be selected in such a way that shell properties, shell type and loads are constant over each part. Each part is again to be subdivided into a specified number of equal segments to facilitate direct, numerical integration which requires sufficiently short meridional segments of the shell. Segmental length may not exceed $\sqrt{Rh}$; where R is the average minimum radius of curvature and h is the average wall thickness. The loads on the shells may be asymmetric, in which case they are represented by Fourier Series in the circumferential variable.

Multisegment, direct numerical integration technique of shell analysis, as mentioned above, is used in this paper to predict displacements and stresses in a PWR containment shell structure under asymmetric wind pressure. The shell material has been assumed isotropic and perfectly elastic and concrete cracking has not been considered in the analysis.

2 GEOMETRY AND BASIC EQUATIONS

Equilibrium equations have been derived following the classical shell theory presented in (Reissner 1941). The position of a point of a shell of revolution is given by the coordinates θ, ϕ, ζ

measured along the triplet of unit vectors $\vec{t}_\theta$, $\vec{t}_\phi$, $\vec{n}$, respectively, as shown in Figure 1. The shape of the shell is determined by specifying the two principal radii of curvature R_ϕ, R_θ of the middle surface as functions of ϕ. Instead of R_θ, it is convenient to use the distance r from a point on the middle surface to the z axis; from Figure 1 it follows that

$$(1) \qquad r = R_\theta \sin\phi \ ,$$

If the generating curve of the middle surface is given by r=r(z) then

$$(2a) \qquad R_\phi = -\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{3/2} \Big/ \frac{d^2r}{dz^2} \ ,$$

$$(2b) \qquad R_\theta = r\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} \ ,$$

When referring to arbitrary shells of revolution, the governing system of equations (Reissner 1941) can be written in the following form: Equations of equilibrium (Figures. 1 and 2):

$$(3a) \qquad N_{\theta,\theta} + \frac{r}{R_\phi} N_{\phi\theta,\phi} + 2\cos\phi\, N_{\theta\phi} + Q_\theta \sin\phi + rp_\theta = 0 \ ,$$

$$(3b) \qquad N_{\theta\phi,\theta} + \frac{r}{R_\phi} N_{\phi,\phi} + (N_\phi - N_\theta)\cos\phi + \frac{r}{R_\phi} Q_\phi + rp_\phi = 0 \ ,$$

$$(4) \qquad Q_{\theta,\theta} + \frac{r}{R_\phi} Q_{\phi,\phi} + Q_\phi \cos\phi - N_\theta \sin\phi - \frac{r}{R_\phi} N_\phi + rp = 0 \ ,$$

$$(5a) \qquad M_{\theta,\theta} + \frac{r}{R_\phi} M_{\phi\theta,\phi} + 2\cos\phi\, M_{\theta\phi} - rQ_\theta = 0 \ ,$$

$$(5b) \qquad M_{\theta\phi,\theta} + \frac{r}{R_\phi} M_{\phi,\phi} + (M_\phi - M_\theta)\cos\phi - rQ_\phi = 0 \ ,$$

Stress-strain relations:

$$(6a) \qquad N_\theta = K(\varepsilon_\theta + \nu\varepsilon_\phi) \ ,$$

$$(6b) \qquad N_\phi = K(\varepsilon_\phi + \nu\varepsilon_\theta) \ ,$$

$$(6c) \qquad N_{\theta\phi} = N_{\phi\theta} = (1 - \nu)K\varepsilon_{\theta\phi} \ ,$$

$$(7a) \qquad M_\theta = D(\kappa_\theta + \nu\kappa_\phi) \ ,$$

$$(7b) \qquad M_\phi = D(\kappa_\phi + \nu\kappa_\theta) \ ,$$

$$(7c) \qquad M_{\theta\phi} = M_{\phi\theta} = (1 - \nu)\, D\kappa_{\theta\phi} \ ,$$

Strain-displacement relations:

$$(8a) \qquad \varepsilon_\theta = \frac{1}{r}(u_{\theta,\theta} + u_\phi \cos\phi + w\sin\phi) \ ,$$

$$(8b) \qquad \varepsilon_\phi = \frac{1}{R_\phi}(u_{\phi,\phi} + w) \ ,$$

$$(8c) \qquad 2\varepsilon_{\theta\phi} = \frac{1}{r}(u_{\phi,\theta} - u_\theta \cos\phi) + \frac{1}{R_\phi} u_{\theta,\phi} \ ,$$

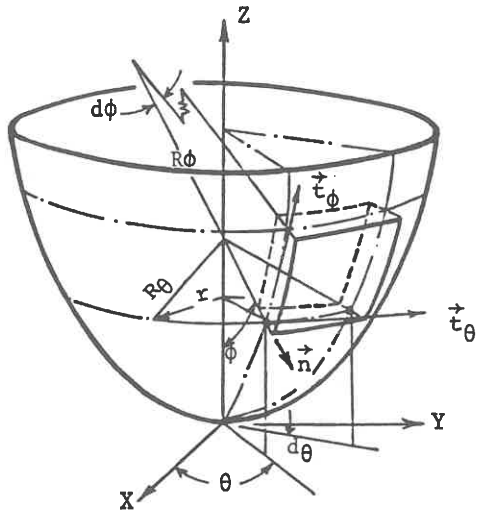


Figure 1. Element of a shell of revolution.

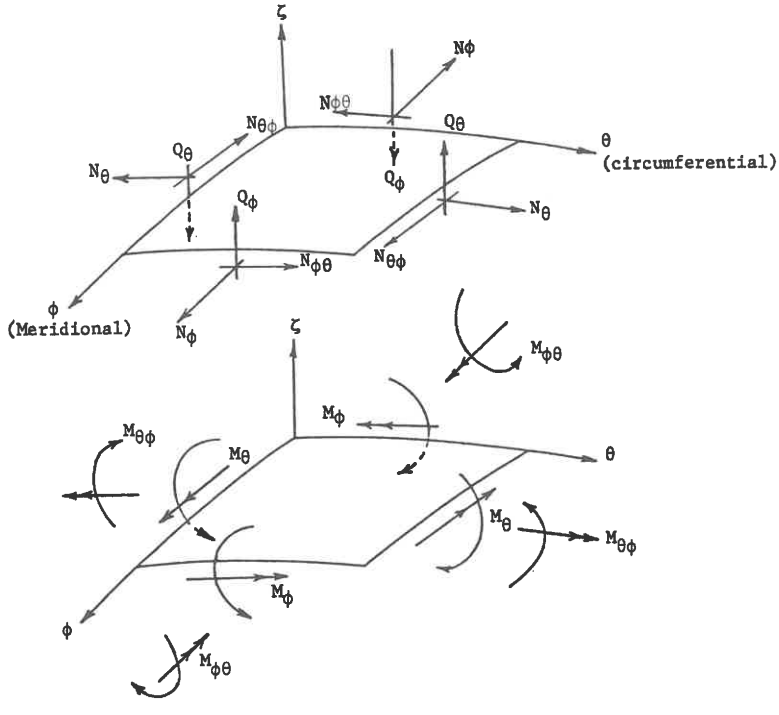


Figure 2. Internal forces on a shell element.

(9a) $$\kappa_\theta = \frac{1}{r}\left(\beta_{\theta,\theta} + \beta_\phi \cos\phi\right) ,$$

(9b) $$\kappa_\phi = \frac{1}{R_\phi}\beta_{\phi,\phi} ,$$

(10a) $$2\kappa_{\theta\phi} = \frac{1}{r}\left(\beta_{\phi,\theta} - \beta_\theta \cos\phi\right) + \frac{1}{R_\phi}\beta_{\theta,\phi} ,$$

(10b) $$\beta_\theta = -\frac{1}{r} w_{,\theta} + \frac{\sin\phi}{r} u_\theta ,$$

(10c) $$\beta_\phi = -\frac{1}{R_\phi} w_{,\phi} + \frac{1}{R_\phi} u_\phi ,$$

where,

ϕ, θ, ζ = coordinates of a point of shell

$\vec{t}_\phi, \vec{t}_\theta, \vec{n}$ = unit vectors tangent to coordinate curves (see Figure 1)

R_ϕ, R_θ = principal radii of curvature of middle surface

r = distance of a point on middle surface from axis of symmetry

E = Young's modulus

ν = Poisson's ratio

h = thickness of shell

$D = Eh^3/[12(1-\nu^2)]$

$K = Eh/(1-\nu^2)$

u_ϕ, u_θ, w = components of displacement of middle surface

β_ϕ, β_θ = angle of rotation of normal

p_ϕ, p_θ, p = components of surface loads

$N_\phi, N_\theta, N_{\theta\phi}$ = membrane stress resultants

$M_\phi, M_\theta, M_{\theta\phi}$ = moment resultants

Q_ϕ, Q_θ = transverse-shear resultants.

3 SURFACE LOADS

In order to reduce partial differential equations 3,4 and 5 into ordinary differential equations in ϕ by separation of variables, pressure components (p_ϕ, p_θ, p) are expressed by the following Fourier series in θ:

$$p_\phi = \sum_{n=0}^{\infty} \left(p_{\phi n} \cos n\theta + \bar{p}_{\phi n} \sin n\theta\right) ,$$

(11) $$p_\theta = \sum_{n=0}^{\infty} \left(p_{\theta n} \cos n\theta + \bar{p}_{\theta n} \sin n\theta\right) ,$$

$$p = \sum_{n=0v}^{\infty} \left(p_n \cos n\theta + \bar{p}_n \sin n\theta\right) ,$$

where $p_{\phi n}$, $\bar{p}_{\phi n}$, $p_{\theta n}$,..., are Fourier coefficients which are functions of ϕ only. For all the loadings considered in this paper, p_ϕ and p are symmetric about $\theta = 0$ plane, and p_θ is identically zero. This implies that

$$(12) \quad \bar{p}_{\phi n} = p_{\theta n} = \bar{p}_{\theta n} = \bar{p}_n = 0, \quad n=0, 1, 2, \ldots, \infty,$$

Furthermore, Fourier series (equation 11) are truncated after a finite number of terms (say N) for computational purposes. In view of these simplifications, pressure components are given by

$$p_\phi = \sum_{n=0}^{N} p_{\phi n} \cos n\theta,$$

$$(13) \quad p_\theta = 0,$$

$$p = \sum_{n=0}^{N} p_n \cos n\theta,$$

An inspection of equations 3,4,5 and 12 yields the following series expansions for the stress resultants:

$$(14) \quad \begin{bmatrix} N_\phi \\ N_\theta \\ N_{\phi\theta} \\ N_{\theta\phi} \\ M_\phi \\ M_\theta \\ M_{\phi\theta} \\ M_{\theta\phi} \end{bmatrix} = \sum_{n=0}^{N} \begin{bmatrix} N_{\phi n} \cos n\theta \\ N_{\theta n} \cos n\theta \\ N_{\phi\theta n} \sin n\theta \\ N_{\theta\phi n} \sin n\theta \\ M_{\phi n} \cos n\theta \\ M_{\theta n} \cos n\theta \\ M_{\phi\theta n} \sin n\theta \\ M_{\theta\phi n} \sin n\theta \end{bmatrix}$$

where all the Fourier coefficients, $N_{\phi n}$, $N_{\theta n}$, $N_{\theta\phi n}$, ...$M_{\theta\phi n}$, are functions of ϕ only.

It should be understood that Fourier coefficients of the loads produce the corresponding coefficients of the solution with the same circumferential wave number n_i. Above formulation of equations in sections 2 and 3 will form the foundation for the shell analysis described in this paper.

4 EXAMPLE PROBLEM

The cylindrical shell is 140 feet inside diameter, 4 feet thick and 156 feet high. The hemispherical dome is 70 feet inside radius and 3 feet thick shell. The foundation slab is 156 feet diameter and 14 feet thick (Figure 3). A basic wind velocity of 80 mph is used in the analysis. The variation of wind velocity with height and gust factors are established on the basis of the ASCE Task Committee Report (1961). Wind velocities selected for the various height zones above ground are:

0 - 50 FT.	80 mph
50 - 150 FT.	105 mph
150 - 400 FT.	135 mph

A gust factor of 1.10 is selected for the containment building. The wind pressure on the containment building is taken to be normal to the surface, and symmetrical about $\theta = 0$ plane. The normal wind pressure, p (lbs/ft^2), at any point on the surface is given by a product of a coefficient q representing vertical wind pressure profile at $\theta = 0$, and a coefficient C_{pe} representing circumferential wind pressure profile (Maher 1966).

$$p = qC_{pe} \quad (15)$$

where,

$$q\ (lbs/ft^2) = 0.002558\ (V \times \text{gust factor})^2 \quad (16)$$

wind velocity, V is expressed in miles per hour.

$$C_{pe} = \sum_{n=0}^{4} C_n \cos n\theta \quad (17)$$

where

$$C_n = \frac{1}{\pi} \int_0^{\pi} C_{pe}(\theta)\, d\theta \quad (18)$$

$$C_n = \frac{2}{\pi} \int_0^{\pi} C_{pe}(\theta) \cos n\theta\, d\theta \quad (18)$$

In the analytical model (Figure 4) for the containment structure, the cylindrical surface is divided into N=4 ring surfaces and the hemispherical surface is divided into N=14 ring surfaces. The region within each ring again is divided into M=9 equal regions for every $\pi/2$ sector. The load on each region is considered uniformly distributed. The Kalnins' shell program (Kalnins 1968) used in the containment analysis requires the distributed load be expressed in Fourier series in the circumferential direction (equations 15 thru 18). The coefficients C_{pe} for the various parts of the containment building (Figure 4) are plotted in Figure 5 and are given in equation forms below:

Parts 1 thru 4, $Cp_e = -0.475 + 0.3821 \cos\theta + 0.6326 \cos 2\theta + 0.3912 \cos 3\theta - 0.0324 \cos 4\theta$

Parts 5 and 6, $Cp_e = -0.4833 + 0.3835 \cos\theta + 0.6525 \cos 2\theta + 0.3956 \cos 3\theta - 0.0407 \cos 4\theta$

Parts 7 and 8, $Cp_e = -0.5147 + 0.3472 \cos\theta + 0.6481 \cos 2\theta + 0.3893 \cos 3\theta - 0.0471 \cos 4\theta$

Parts 9 and 10, $Cp_e = -0.5737 + 0.2715 \cos\theta + 0.6321 \cos 2\theta + 0.3611 \cos 3\theta - 0.0717 \cos 4\theta$

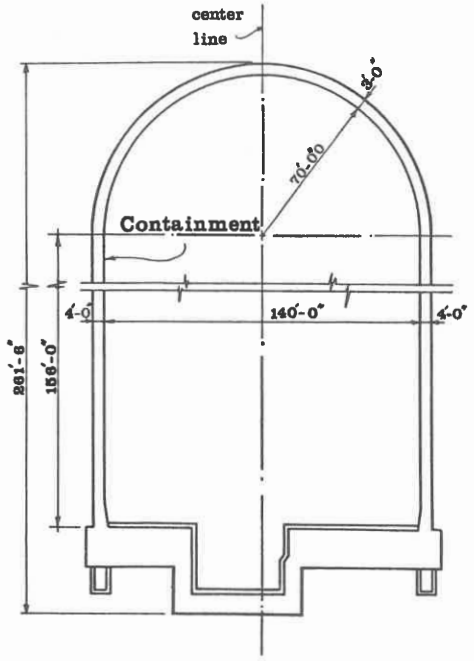


Figure 3. PWR containment building.

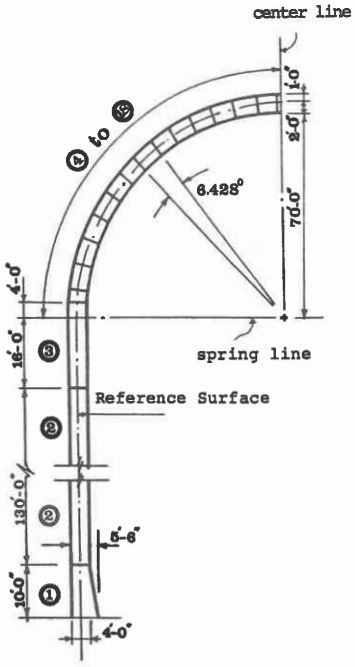
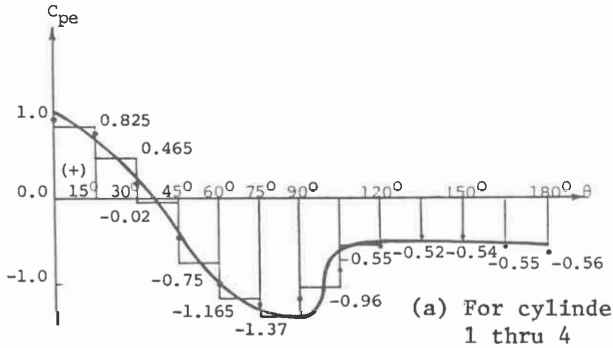
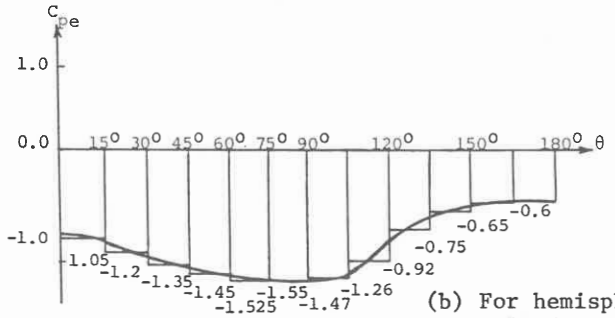


Figure 4. Analytical model.

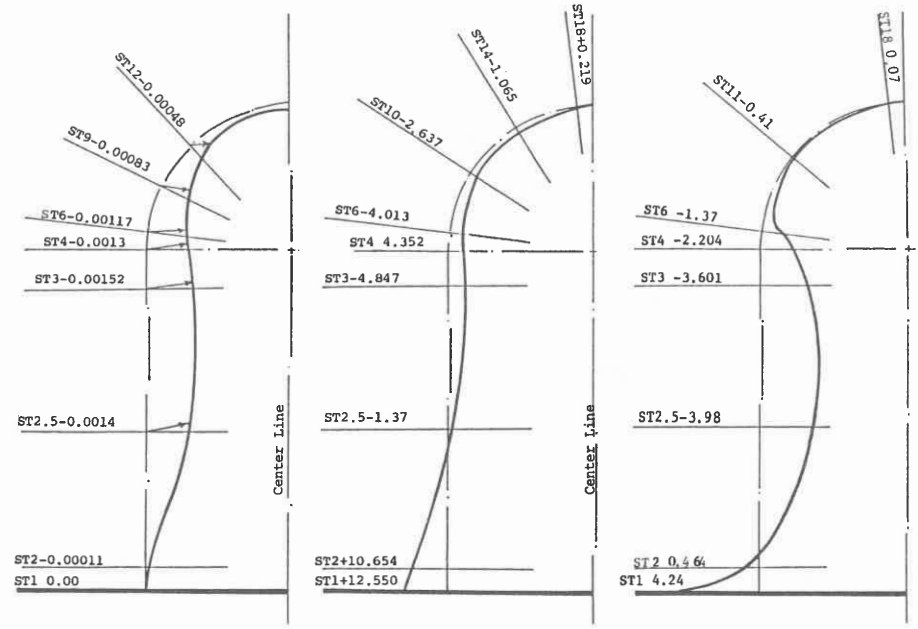


(a) For cylinder, parts 1 thru 4



(b) For hemisphere on cylinder, parts 15 & 16

Figure 5. Wind pressure coefficients.



(a) Deformation (ft) (b) N_ϕ (k/ft) (c) M_θ (ft.k/ft)

Figure 6. Results of the analysis at 0^o.

Parts 11 and 12, $Cp_e = -0.6746 + 0.1167 \cos\theta + 0.5568 \cos 2\theta + 0.2972 \cos 3\theta - 0.0937 \cos 4\theta$

Parts 13 and 14, $Cp_e = -0.8579 - 0.1403 \cos\theta + 0.4524 \cos 2\theta + 0.1856 \cos 3\theta - 0.08752 \cos 4\theta$

Parts 15 and 16, $Cp_e = -1.1479 - 0.3099 \cos\theta + 0.3385 \cos 2\theta + 0.0598 \cos 3\theta - 0.0276 \cos 4\theta$

Parts 17 and 18, $Cp_e = -1.50$ (uniform).

The surface forces are transformed to the Fourier series expression (equation 15) and are in the form applicable to the Kalnins' shell program (Kalnins 1968).

The containment structure analytical model used in the program is shown on Figure 4. The shell material has been assumed to be isotropic and perfectly elastic. Concrete properties used: f'_c = 5000 psi; $E_c = 4.0 \times 10^6$ psi; Poisson's ratio, ν=0.2. Properties of reinforcing steel used: f_y = 60,000 psi; $E_s = 29 \times 10^6$ psi. The results are presented in graphical form for $\theta=0^0$ in Figure 6.

SUMMARY AND CONCLUSIONS

The purpose of this paper is to summarize the analytical method, assumptions, procedures and the results of the analysis of a PWR containment shell structure due to the wind forces. The containment is a reinforced concrete structure in the shape of a cylinder with a hemispherical dome and flat foundation slab. A basic wind velocity of 80 mph is used in the analysis. The variation of wind velocity with height and gust factors are established. The wind pressure coefficients for the containment structure are obtained for all parts of the containment building.

This paper presents a method of solution for stresses and deformations at the various points of a thin axisymmetric concrete containment shell structure with variable thickness under asymmetric wind pressure loads. The analysis of the shell is performed using the Kalnins' "Static Analysis of Thin Elastic Shells of Revolution", computer program which utilizes the multisegment direct numerical integration technique due to Adams' method. The distributed surface load due to wind is expressed in Fourier series in azimuth angle. The results presented in this paper using multisegment direct numerical integration technique due to Adams' method of static shell analysis are computationally efficient and accurate provided that the length of each individual shell segment is less than the critical length of the original shell.

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