FAST — A SHELL CODE USING ASYMPTOTIC RESULTS

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Abstract

Closed form asymptotic results for the solutions of the equations for thin shells are incorporated into the computer code FAST (Functional Algorithm for Shell Technology) which is under development. The stresses and deflections from the linear theory for an axisymmetric shell structure with axisymmetric loads are calculaced in FAST-1. Several examples are discussed, which have been run on FAST-1 and on STAGS, a commercially available finite difference code. The advantage of FAST is clearly demonstrated. The computational cost is reduced by an order of magnitude. Of greater importance is the reduction in input preparation time - hours reduced to minutes, and the elimination of dependence on user judgment. This is because the user prescribes only the endpoints of a shell segment and not any interior mesh spacing, as must be done for all finite difference and finite element codes. Therefore, FAST should be useful in several areas for which all other codes are too unwieldy and expensive, particularly in preliminary design and safety evaluation studies.

1.. Introduction

Asymptotic, i.e. perturbation, analysis providesa powerful tool for the calculation of thin shell structures. It seems, however, that the information available from such analysis at a low cost in effort has not been utilized in any of the many finite difference and finite element shell codes which have been developed. Our main tenet is that the straightforward application of finite difference or finite element methods to thin shells is inherently inefficient. To demonstrate this, the computer code FAST-1 has been developed in which full utilization of asymptotic methods is made. The theory and some examples are given in Steele, et al. [1]. In the present work the concentration is on further example problems.

Flaherty and O'Malley [2] have shown that asymptotic analysis can be used advantageously for the numerical calculation of boundary value problems for stiff systems of nonlinear equations. Our objective is to demonstrate the advantage for shell analysis provided by FAST. For comparison we chose the finite difference code STAGS, which was developed by the Lockheed Palo Alto Research Laboratory, and which is available with several computer service companies. This code enjoys a good reputation for reliability and ease of use, and, according to employees at three computer service companies, has a higher usage than other available shell codes.

2. Present Capability

Since the development of FAST is recent, the range of problems which can be treated is limited in comparison with STAGS and many other codes. FAST-1 calculates the results from linear theory for the shell of revolution of linear elastic material with axisymmetric loading. The geometry and loading can be fairly general, so that a broad class of practical problems of pressure vessels, piping, and domes is encompassed.

The meridian curve can consist of one or more "paths" which can be multiply connected. Each path is a simply connected curve which can consist of a sequence of the segments shown in Table I. The examples which are discussed in this paper are all "one path" problems. For FAST the endpoints of the segments are the only stations prescribed by the user. For the vessel shown in Fig. 1, which consists of a cylinder with an elliptical and conical head, the entire geometrical input requires the four lines shown in Table II. The three segment shells shown in Figs. 2 and 3 require similar information, as shown in Tables III and IV.

Material properties (E, NU) and thickness (TH) can be prescribed for each segment, but for the present examples are taken as constant for the entire shell, as shown in Tables II-VI.

The following loads can be prescribed:

DLR,DLZ,DLN,DLT	distributed loads in radial, axial, normal, and meridional directions;
LLR,LLZ,LLN,LLT	line loads
LM	line moment
DLS	snow load, for which DLR = 0 and DLZ = (DLS) $\cos\phi$

SG	specific gravity of shell wall
NG	number of "g's" axial acceleration, for which DLR = 0 and DLZ = SG × NG × TH × 9.8 × 10^3 (Nm^{-3})
RPM	number of revolutions per minute of spinning about axis, for which DLZ = 0 and DLR = R × SG × TH × $(RPM)^2 \times (\pi/3)^2 \times 10$ (Nm^{-4})
SGF	specific gravity of contained fluid
ZF	axial coordinate of free surface of fluid, for which DLT = 0 and DLN = 0 for $Z < ZF$ and DLN = SGF × NG × (Z - ZF) × 9.8 × 10^3 (Nm^{-3}) for $Z > ZF$

The loads can be prescribed segment by segment or can be prescribed for the entire shell with one statement by the user. This simplifies the input for the frequently occurring cases of "snow", fluid, gravity, and pressure loading. In addition, loading due to spinning of the entire structure is obtained by prescribing only "RPM".

The input information is reduced to the minimum by the elimination of any prescription by the user of mesh point locations interior to a given shell segment. A mesh is used in FAST primarily for numerical integration over the segment, with the appropriate spacing chosen from the knowledge of the rate of decay of solutions, i.e. boundary layer widths, and the rate of geometric variation. The stress, strain, and displacements are calculated at each of these interior mesh points. The user has the following choices for output:

(1) Complete listing of stress, strain, displacement at all edge and interior points of all segments.

(2) Complete listing for a particular shell segment.

(3) Stress, strain, and displacement at a particular point.

(4) Maximum compressive and tensile stresses and displacement components, magnitude and location, in each shell segment.

(5) Maximums for a particular shell segment.

Generally, option (4) will be the most useful since scanning of reams of paper output on the part of the user for the most critical conditions is eliminated. Option (5) is useful in the detailed design of a particular segment.

The input can consist of the nominal structure and loading with a sequence of modifications of loading or configuration, for instance a sequence of values for a knuckle radius. The output (5) then gives a compact tabular form showing the variation in stress with the change in configuration.

3. Comparison with STAGS

Tables II - VI show the comparison with STAGS for the shells in Figs. 1 - 3 and two cylinder problems. FAST is on the IBM 370/168 at the Stanford University Computing Center while the version of STAGS which we used is that available with Boeing Computer Services on the CDC - CYBER 175 (incorrectly given as the CDC 6600 in [1]). In the Tables the actual CPU time and cost are given. However, the CDC machine is considered to be faster by a factor of 1.75, which gives STAGS an advantage in comparison of run time. The different overhead rates probably give an advantage to FAST in the direct comparison. Nevertheless, it is clear that FAST offers a reduction by a substantial factor in computational cost. The lowest reduction is in the case of the short shell in Table V, for which the boundary layers interact, and the advantage of the asymptotic analysis is decreased. For very short shells, FAST will again be at an advantage since accurate asymptotic approximations are used.

The reduction in input preparation time offered by FAST is primarily because the user need prescribe only the segment endpoints and not the internal mesh. In addition, the proper conditions at the apex of dome or cone are automatically applied in FAST, while a small rigid plug must be added by the user for STAGS.

The mind-numbing chore of mesh generation for finite difference and finite element codes can be alleviated by preprocessors, but the dependence on user judgment is not. What is not included in the Tables is the time and expense of initial runs on STAGS which were substantially in error,in one case by a factor of 100, because of a mesh spacing which was too coarse in the boundary layer. In FAST the dependence on user judgment is eliminated. Results to a prescribed level of accuracy, from 1% to 15%, are delivered on the first run.

In summary, the results of the present examples, as well as others, show the advantages of FAST-1:

(1) Guaranteed accuracy independent of user judgment.
(2) Reduction in input preparation time by an order of magnitude.
(3) Reduction in computer cost by an order of magnitude.
(4) Simple parametric variation for weight and cost optimization as well as safety evaluation.
(5) Adaptability for use in preliminary design.

At the present time, we can see no particular disadvantage to the approach in FAST. Our expectation is that the relative advantage of FAST will increase as the complexity level of the problem increases.

References

[1] STEELE, C.R., RANJAN, G.V., GOTO, C., and PULLIAM, T.H., "Computer Analysis of Shells of Revolution Using Asymptotic Results," Proceedings, Structures, Structural Dynamics, and Materials Conference, St. Louis, U.S.A., 1979.

[2] FLAHERTY, J. E. and O'MALLEY, R.E., Jr., "The Numerical Solution of Boundary Problems for Stiff Differential Equations", Math of Computation, 31, (1977), 66-93.

Table I - Available Shell Segments

Surface	Meridian	Designation	Numerical Parameter
Cylinder Cone Flat Plate	Straight Line	C	
Sphere	Circle	S	
Toroidal Knuckle*	Circle	K	R_1 (Radius of curv.)
Paraboloid	Parabola	P	
Ellipsoid	Ellipse	E	R-axis/Z-axis (Ratio of principal axes)
Hyperboloid	Hyperbola	H	$\Delta R/\Delta Z$ (Slope of asymptote)
Toroid, Clockwise			
minimum length	Circle	T1	R_1
maximum length	"	T2	"
Counterclockwise			
maximum length	"	T3	"
minimum length	"	T4	"

*For knuckle, the end points are not prescribed by the user, but computed to give a continuous slope.

Table II - Input and Results for Shell of Fig. 1

Geometry					Properties and Load		Results		
Sta.	Coor. R	Coor. Z	Curve	Param.	Quantity	Magn.		FAST	STAGS
							No. Stations	4	213
1	0.0	-4.0	E	1.11	E	.22E12	Setup Time(Min)	5	150
2	10.0	5.0	C		NU	.3	Max. Stress	1.725E8	1.725E8
3	10.0	10.0	C		TH	.01	CPU Time (Sec)	0.76	5.24
4	0.0	20.0			DLN	.1E5	Cost ($)	0.79	13.20

Table III - Input and Results for Shell of Fig. 2

Geometry					Properties and Load		Results		
Sta.	Coor. R	Coor. Z	Curve	Param.	Quantity	Magn.		FAST	STAGS
							No. Stations	3	138
1	0.0	-10.0	P		E	.22E12	Setup Time(Min)	4	120
2	10.0	-4.0	H	0.5	NU	.3	Max. Stress	1.652E8	1.587E8
3	10.0	4.0	P		TH	.01	CPU Time (Sec)	.61	3.75
4	0.0	10.0			DLN	.1E5	Cost ($)	.67	9.40

Table IV - Input and Results for Shell of Fig. 3

Geometry					Properties and Load		Results		
Sta	Coor.		Curve	Param.	Quantity	Magn.		FAST	STAGS
	R	Z					No. Stations	4	174
1	0.0	0.0	S		E	.22E12	Setup Time (Min)	5	150
2	10.0	2.6795	TI	4	NU	.3	Max. Stress	.201E9	.20E9
3	10.0	9.6077	S		TH	.01	CPU Time (Sec)	0.74	4.6
4	0.0	16.2872			DLN	.1E5	Cost ($)	0.69	14.40

Table V - Input and Results for short, thick cylinder, clamped at ends, with internal pressure.

Geometry					Properties and Load		Results		
Sta	Coor.		Curve	Constraint	Quantity	Magn.		FAST	STAGS
	R	Z					No. Stations	2	18
1	1.0	0.0	C	DR, RT	E	.22E12	Setup Time(Min)	3	40
2	1.0	0.7		DR, DZ, RT	NU	.3	Max. Stress	.17E6	.167E5
					TH	.1	CPU Time (Sec)	0.25	0.4
					DLN	.1E5	Cost ($)	0.28	1.32

Table VI - Input and Results for Cylinder under Gravity Loading.

Geometry				Properties and Load		Results		
Sta.	Coor.		Curve	Quantity	Magn.		FAST	STAGS
	R	Z				No. Stations	2	73
1	20.0	0.0	C	E	.22E12	Setup Time (Min)	3	60
2	20.0	8.0		NU	.3	Max. Axial Disp.	.1011E-4	.1096E-4
				TH	.1	Max. Comp. Stress	.908E6	.905E6
				SG	7.88	CPU Time (Sec)	0.27	1.38
				NG	1.0	Cost ($)	0.26	4.00

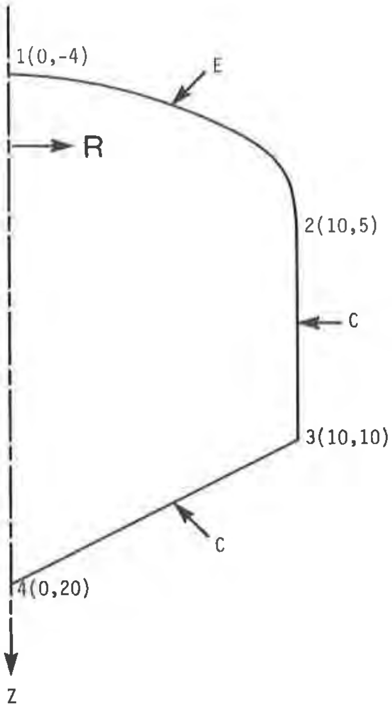

Figure 1.

Cylindrical Pressure Vessel with Ellipsoidal and Conical Heads.

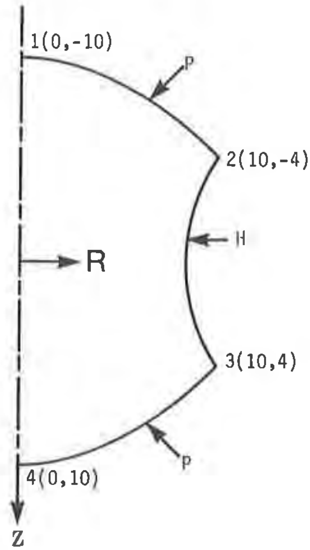

Figure 2.

Shell with Parabolic and Hyperbolic Meridional Segments.

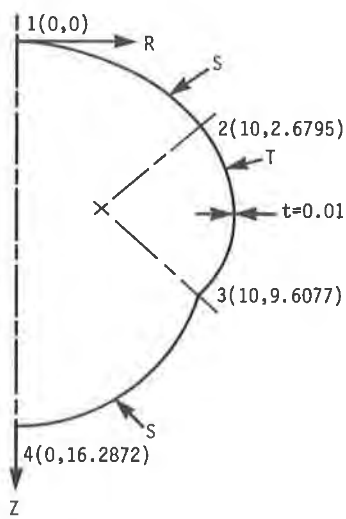

Figure 3.

Shell with Spherical and Toroidal Meridional Segments.