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An enriched beam finite element for accurate piping analysis

Millard A.
CEA, France

SUMMARY

Standard structural piping analysis is performed by means of a beam approach, using classical finite elements with modified inertia properties. These modifications are necessary to account for the increased flexibility under bending moments, which is observed in pipe elbows. They are based on the classical Von Karman's theory, which describes the ovalization of the pipe cross-sections by treating the bends as shells, and condenses the local displacements in terms of the usual beam global unknowns (mean fiber displacements and rotations). As a consequence, this method in unable to account for the propagation of the ovalization along the pipe line, and leads to an over-estimation of the stress intensification factors in the pipe bends.

The element which is proposed in this paper can be seen as a compromise, since it is used as a beam element but it can supply detailed information as in a shell approach. The basic idea is to enrich the classical beam theory by means of displacements describing the cross-section deformations. Such elements have been already proposed in the litterature, and at the Smirt conferences in particular. The originality of the element proposed here comes from the following features :

- the discretized displacements fields are the local shell midsurface displacements
- for this purpose, Fourier series expansions are used
- the unknowns are the shell displacements at some pre-determined points around the cross-section
- beam displacements are recovered through a penalty approach
- special attention has been brought to the junction between adjascent non coplanar elements

1. INTRODUCTION

Pipings are one of the most used structural components in a nuclear power plant. According to the various circuits, they can be subjected to various types of loadings, as dead weight, internal pressure and temperature elevation in case of normal operating conditions, or seismic accelerations, following jet forces in case of faulty conditions. The usual design of piping lines is based on a beam type approach [1], in which the basic kinematic hypothesis considers that any cross-section behaves as a rigid brody. This leads to an over-stiff response of the piping as has been shown by the pionneering work of Von Karman [2]. Indeed, in the pipe bends, the cross-sections tend to ovalize and give way to an additional flexibility which is well known to pipings designers who take advantage of it to compensate thermal expansion effects in long straight parts by providing loops in the circuits. Unfortunately, the deformation of the cross-section is accompanied by a stress concentration which can be important for pipe bends with a small thickness. From a computational point of view, the beam approach can still be applied with modified bending inertia for the pipe bends :

$$I' = \frac{I}{k} \quad \text{with} \quad k \geq 1 \tag{1}$$

where I stands for the bending inertia of the pipe cross-section and k is a global flexibility factor for the pipe bend. A usually adopted expression for k in the litterature [1] is :

$$k = \frac{1.65}{\lambda} \tag{2}$$

where λ is a dimension less coefficient, characteristic of the pipe elbow, defined as :

$$\lambda = \frac{e.R}{r^2} \tag{3}$$

R, r and e respectively stand for the bend radius, pipe radius and pipe thickness as shown on figure 1.

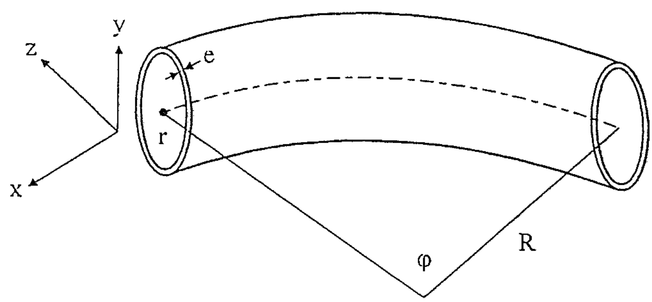


Figure 1
Geometric parameters of a pipe elbow

Local stresses are calculated a posteriori from the stress resultants over the cross-section (normal force, bending moments, etc.) by means of stress intensification factors. This approach has been extended in the past by Roche and colleagues [3], [4] for the treatment of inelastic material behaviours such as plasticity or creep. Their idea was to formulate the material behaviour law directly in term of stress resultants and internal pressure. For example, the yield stress was calculated as the following quadratic sum :

$$\sigma^* = \sqrt{\alpha_n^2 \sigma_n^2 + \alpha_b^2 \sigma_b^2 + \alpha_t^2 \sigma_t^2 + \alpha_p^2 \sigma_p^2} \tag{4}$$

where the σ_i represent the contributions of the stress resultants N, M_b, M_t and pressure p :

$$\sigma_n = \frac{N}{2\pi r e} \qquad \sigma_b = \frac{M_b}{\pi r^2 e} \qquad \sigma_t = \frac{M_t}{\pi r^2 e} \tag{5}$$

$$\sigma_p = \frac{p_r}{e} \tag{6}$$

and α_i are some coefficients derived from limit analysis considerations.

This approach has proved to be efficient and is still used for design calculations. However, it sometimes leads to over conservative estimations of the stresses, and it is not sufficient in cases where an accurate knowledge of stresses and strains is required, as for example in the evaluation of safety margins.

One of the main reasons comes from the fact that the ovalization is not uniform along a pipe elbow, because of the restraining effect of the adjascent components which can be straight parts or rigid flanges, among others. Moreover, the curvature discontinuity between a pipe bend and a straight part generates some distorsion of the cross-section. Some elementary solutions have been proposed in [5] which show for example the reduction of a bend flexibility induced by staight parts or rigid flanges, for various bend angles (figure 2).

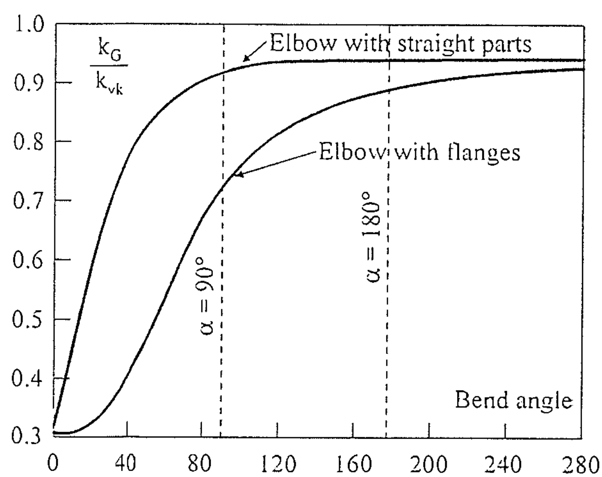


Figure 2
Variation of the bend global flexibility, with bend angle

The non uniform behaviour along the bend can be clearly seen either on figure 3 which shows the variation of a local flexibility factor $k_l(x)$ (calculated for an elementary bend portion) along a flanged bend, for various bend angles, or on figure 4 which depicts the evolution of cross-section rotation ψ and ovalization amplitude a along a bend welded to two straight parts, compared to the solutions where the propagation of ovalization is neglected.

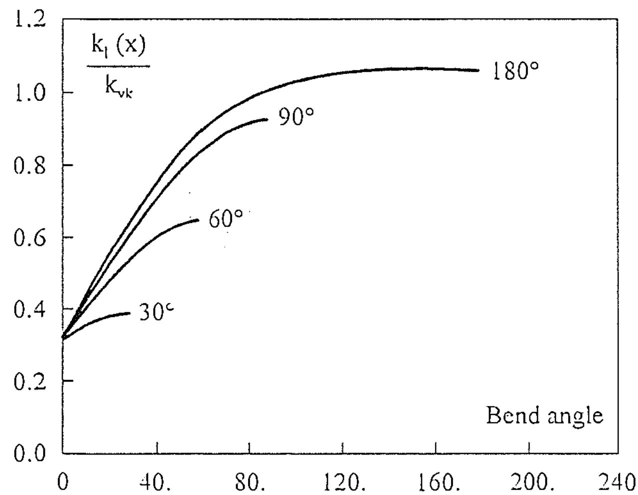
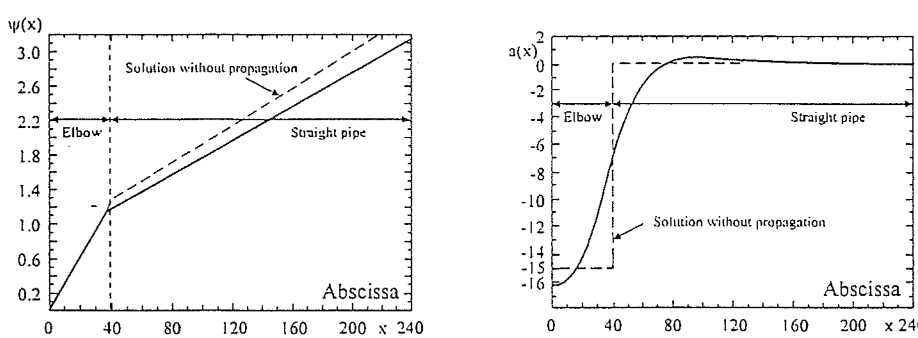


Figure 3
Local flexibility factor for a flanged elbow



Bend angle $2\alpha = 90°$ $r = 10$ $R = 50$
Straight part length $l = 20\ r$ $e = 1$ $\nu = 0.3$

Propagation of ovalization along an elbow with straight parts
Bend angle $2\alpha = 90°$ $r = 10$ $R = 50$
Straight part length $l = 20\ r$ $e = 1$ $\nu = 0.3$

Figure 4
Compared evolutions of rotation and amplitude of ovalization along an elbow + straight parts

Another reason which justifies the development of refined analysis, is the difficulty to formulate a global inelastic material model (at the scale of the cross-section) which reproduces the hardening properties as well as the various couplings between different stress resultants effects.

Finally, the last reasons come from the necessity to account for local phenomena like thermal gradients across the pipe thickness, non uniform circular distribution of temperature in case of fluid stratification (as it may occur in sodium fast breeder reactors), non linear modifications of the cross-section shape leading to buckling (Brazier's effect), etc.

In view of the nowdays currently avalaible computer ressources, one might be tempted to switch from a beam analysis to a shell analysis or even a tridimensionnal continuum analysis of a full piping line. This would lead doubtless to detailed and accurate results. Nevertheless, piping engineers are still favouring beam type input descriptions and output results which are more meaningful to them. This does explain the various proposals for enriched beam finite elements which can be found in the litterature, as well as the present one.

2. REVIEW OF SOME PREVIOUS ENRICHED BEAM FORMULATIONS

In this paragraph, some solutions presented in the past are reviewed and discussed.

2.1. Enriched multifiber approach

This approach, which has been developped some years ago [6], is devoted to an accurate description of the development of inelastic zones in a given cross-section. For this purpose, a special pipe bend element, based on a shell approach, has been developped. The shell mean surface is described by two parameters : the angle θ describing the circumference, and the curvilinear abscissa s along the bend mean line. A third parameter, z, varies across the shell thickness.

Following Von Karman [2], some simplifying hypothesis are assumed :

- plane cross-section remain plane
- inextensibility : $\varepsilon_{\theta\theta}$ $(z = 0) = 0$
- ε_{ss} is constant across the thickness
- r is small in front of R

These hypothesis are valid for many pipe bend geometries and loadings but may be too restrictive in some particular cases (for example in case of internal pressure).

The local displacements u, v, w (see figure 5) of the shell mean surface are then expanded in Fourier series. According to the first hypothesis, the normal displacement u can be expressed in terms of the global beam rotations ω_i and ω_o (i stands for in plane and o for out of plane) :

$$u(\theta, s) = r\cos\theta.\omega_i(s) - r\sin\theta.\omega_o(s) \qquad (7)$$

The additional unknowns which represent the amplitudes of the ovalization modes are condensed at the element level in terms of the beam degrees of freedom.

The method has been extended to tackle with non linear material behaviour. For this purpose, the cross is discretized into a certain number of integration points regularly distributed across the thickness and around the circumference (figure 6).

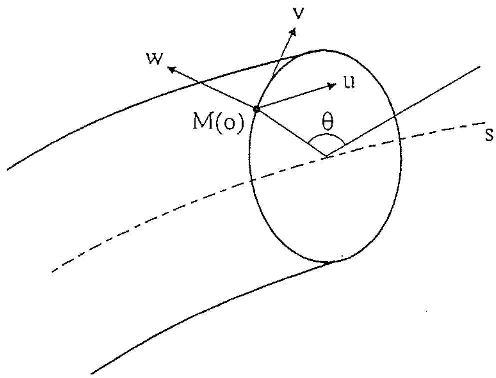


Figure 5
Parameters and local shell displacements of a pipe bend

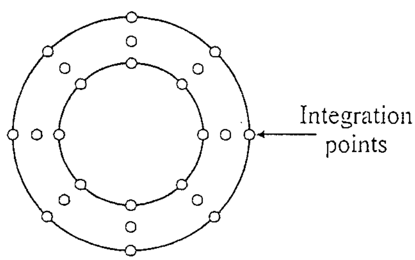


Figure 6
Integration points over the cross-section

At each of these points, any local material behaviour law may be used, as for example, some cyclic viscoplastic law to predict thermal ratchetting in LMFBR primary circuits.

The inelastic state of the cross-section is characterized by the knowledge of the stresses, strains and internal variables at all these points.

The implementation of such an approach within the frame of a beam type finite element program can be easily done when using the classical initial stress iterative method [8] : A first estimation of the ovalization unknown increments is obtained from the beam global kinematic. Then, local strains are obtained. The material behaviour laws are then used to produce local stresses. They are numerically integrated to evaluate residual forces at the shell and beam levels and a new iteration is done if convergence is not achieved.

The advantage of the method is double : First, the modification of the beam element flexibilities due to the development of non linearity in the cross-section, is automatically accounted for, and second, sophisticated material laws can be used for advanced analysis. It has been successfully used for LMFBR piping analysis, and favourably compared to other methods [7].

The major drawback is its inability to handle the propagation of ovalization and to represent cross-section distorsions.

2.2. "Victus" approach

The basic idea of this approach [9] is to consider the ovalization at a given point of a pipe, under a complex loading, as a superposition of elementary ovalizations caused by given localized point loads. The expressions for these influence functions are derived from the analytical works mentioned above [5] and may be written for example as follows :

$$\bar{a}(s, s_0) = A_1\, e^{-\mu|s-s_0|} \cos \mu\,|s - s_0| + A_2\, e^{-\mu|s-s_0|} \sin \mu|s - s_0| \tag{8}$$

where s_0 is the curvilinear abscissa where a bending moment $M(s_0)$ is applied. The two coefficients A_1 and A_2 are directly proportionnal to $M(s_0)$ when all non linearities are neglected :

$$\begin{aligned} A_1 &= a_1\, M(s_0) \\ A_2 &= a_2\, M(s_0) \end{aligned} \tag{9}$$

where a_1 and a_2 depend only on the geometry of the pipe (parameters e, r, R) and on Poisson's ratio ν.

It is therefore possible, by superposition, to determine the ovalization a(s) at any point, due to a bending moment which varies along the bend :

$$a(s) = \int_{S_1}^{S_2} \bar{a}(s, \tau)\, M(\tau)\, d\tau \tag{10}$$

The local flexibility factor is then given by :

$$k_l(s) = 1 + \frac{3\, a(s).EI}{4\, r\, R \quad M(s)} \tag{11}$$

and depends on the loading. Therefore, the calculation of the whole piping line by means of a beam type program becomes non linear. It can be easily solved by an iterative method, as Picard's method for instance. This method has proved to be very efficient for linear analysis, but is hardly extendible to non linear problems.

2.3. Other special pipe elbow elements

The idea of the enrichment of the beam displacements by shells displacements, capable of describing the cross-section deformation, is rather intuitive and has been adopted by other authors [10], [11], [12], within the frame of linear analysis.

The differences in the formulations come from the simplifying hypothesis, and from the shape functions adopted for the shell displacements.

For the first one [10], [11], among the Von Karman's assumptions already presented in paragraph 2.1, only two are retained :
- plane cross-section remain plane
- inextensibility $\varepsilon_{\theta\theta}\ (z = 0) = 0$

The element proposed is a 4 node isoparametric element (figure 7).

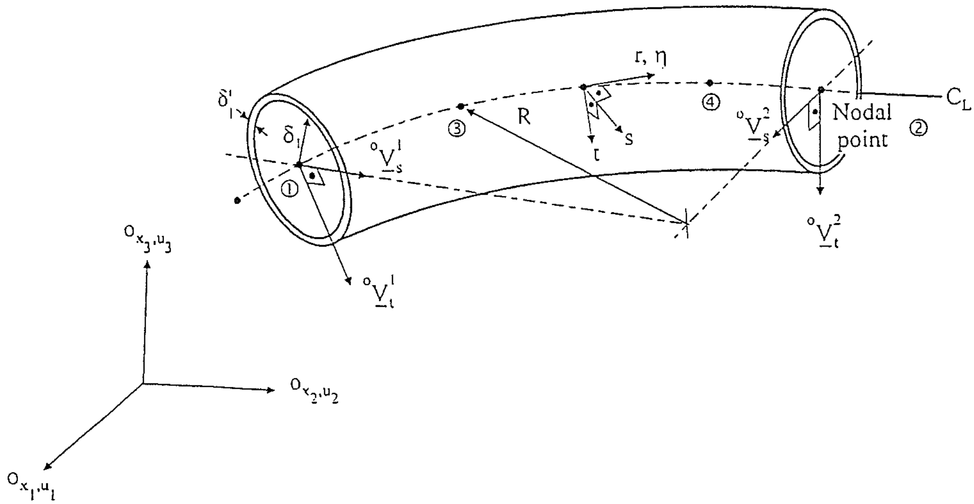


Figure 7
4 node isoparametric pipe bend element (from [11])

The global beam displacements are interpolated using cubic polynominals $H_i(s)$. The cross-section ovalization is described by the normal local shell displacement $w(\theta,s)$:

$$w(\theta,s)=\sum_{i=1}^{4}\left(\sum_{j=1}^{N_a}H_i(s)\,a_j^i\sin 2i\theta+\sum_{j=1}^{N_b}H_i(s).b_j^i\cos 2\,i\,\theta\right) \tag{12}$$

where i is the index associated with the beam nodes and j is the index associated with the number of terms retained in the Fourier development. a_j^i and b_j^i are the additional unknowns and represent the amplitudes of the in plane and out of plane ovalization modes. Notice that only even trigonometric terms have been kept in the Fourier developments.

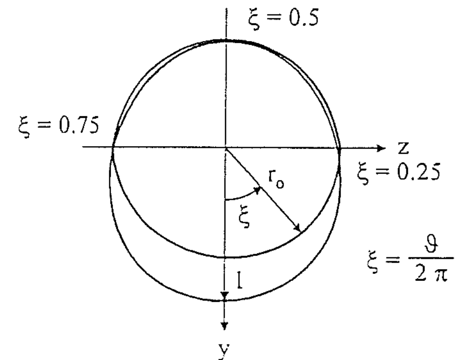
The interaction effects between two adjascent elements are treated differently for the ovalization and its derivative with respect to the s abscissa : The continuity of ovalization is achieved automatically through the continuity of the a_j^i and b_j^i coefficients. However, this is limited to some particular junctions like coplanar pipe bends for example.

The continuity of the derivative of the ovalization is ensured by a penality procedure, in order to avoid the introduction of additional degrees of freedom.

The second enriched beam element, proposed in [12], is based on a combination of splines interplation in the circumferential direction, and strip theory interpolation in the longitudinal direction. Using the same notations as above, the hoop tangential displacement v is interpolated as :

$$v(\theta,s)=\sum_{i=1}^{4}N_i(\theta).\,V_i(s) \tag{13}$$

where N_i (θ) are splines functions defined at four predefined points on the cross-section. The expression of N_1 is given on figure 8.



$$N_1(\xi) = \begin{cases} -128\,\xi^4 + 144\,\xi^3 - 44\,\xi^2 + 1 & : \ 0 \le \xi \le 0.5 \\ -128(1-\xi)^4 + 144\,(1-\xi)^3 - 44\,(1-\xi)^2 + 1 & : \ 0.5 \le \xi \le 1 \end{cases}$$

Figure 8
Splines interpolation function (from [12])

The displacements $V_i(s)$ are obtained by means of cubic polynomials from the two node element nodal values $V_i(o)$, $V_i(l)$ and derivatives $\frac{dV_i}{ds}(o)$ and $\frac{dV_i}{ds}(l)$. The normal shell displacements w are obtained thanks to the mid surface circumferential inextensibility hypothesis.These shell displacements are added to the classical beam displacements, leading to a total of 14 degrees of freedom per node.

Unfortunately, nothing is said in [12] on the connexion between elements.
The main drawbacks of this approach are due to the limiting hypothesis (beam cross-sections remain plane, and circumferential inextensibility).

3. PROPOSAL OF A NEW ENRICHED BEAM ELEMENT

As a starting point, it is important to review the desired features of a new enriched beam element. For nuclear applications, the element should be applicable to very different geometries and loadings such as in PWR pipings (large thickness and high internal pressure) or in LMFBR pipings (small thickness, low internal pressure, high thermal gradients). In the latter case, the ovalization in pipe bends, and its propagation are more pronounced, leading to higher distorsions. As a consequence, circumferential strain must be considered (for internal pressure and thermal loadings), as well as warping of the cross-section, as shown in [5].

The element should be usable for straight parts as well as elbows, and for non coplanar configurations. Finally, the formulation must be easily extendable to material as well as geometrical non linearities.

The formulation proposed here can be seen as a compromise between the two previous ones, as well as a generalization of them. Following Von Karman [2], the first idea is to enrich the usual beam formulation by means of truncated Fourier series development of the three components u, v, w of the local mid surface shell displacements. The choice of trigonometric functions appear more straightforward than the splines functions, and the number of terms retained in the series may be easily adjusted. However, the connection between non coplanar elements, as illustrated on figure 9 may not be easily achieved. One possible solution is to enforce it by means of a penalty technique, as used in [11]. The solution preferred here is to introduce additional nodes distributed in the cross-section, as in [12], in order to ensure continuity automatically when assembling different elements. Of course, the number of additional nodes N is related to the number of Fourier modes.

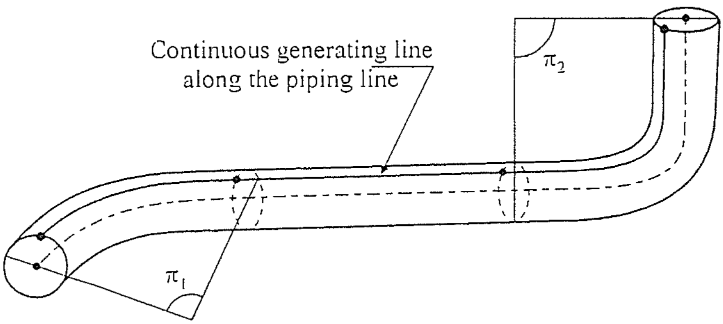
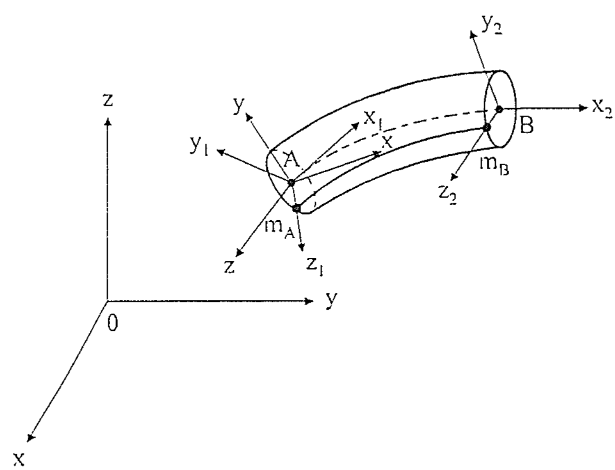


Figure 9
Configuration of two non coplanar elbows assembled to a straight part

For a given elbow element, it is necessary to consider four different sets of axis, as shown on figure 10. The global beam displacements are expressed in the global axis OXYZ, whereas the local nodal displacements are expressed in axis $A\,x_1\,y_1\,z_1$ and $B\,x_2\,y_2\,z_2$ attached to the elements nodes, with reference to a continuous generating line along the piping line as illustrated on figure 9.



OXYZ	global axis
Axyz	element local axis
$Ax_1y_1z_1$	node A local axis
$Bx_2y_2z_2$	node B local axis

Figure 10
Axis associated with the elbow element

Therefore, it is possible to write the local shell displacements interplations as :

$$u(\theta,s)=\frac{1}{N}\sum_{k=0}^{N-1}\left\{\sum_{n=0}^{N-1}\cos\left[n\left(\theta+\Omega-\frac{2k\pi}{N}\right)\right]\right\}.u_k(s)$$
$$v(\theta,s)=\frac{1}{N}\sum_{k=0}^{N-1}\left\{\sum_{n=0}^{N-1}\cos\left[n\left(\theta+\Omega-\frac{2k\pi}{N}\right)\right]\right\}.v_k(s) \qquad (14)$$
$$w(\theta,s)=\frac{1}{N}\sum_{k=0}^{N-1}\left\{\sum_{n=0}^{N-1}\cos\left[n\left(\theta+\Omega-\frac{2k\pi}{N}\right)\right]\right\}.w_k(s)$$

where Ω stands for angle (Az_1, Az). In the longitudinal direction, interpolation may be either quadratic, in which case a three node element will be used and shell continuity will be ensured only by means of translations, or cubic, in which case a two node element may be constructed, with nodal shell unknowns being displacements and their derivatives, to ensure the shell continuity on translations and rotations.

In the expression of the strain energy, all the terms are kept, as explained above :

$$W_{def}=\frac{1}{2}\int_0^l\int_0^{2\pi}\frac{Ee}{1-\nu^2}\left(e_{ss}^2+e_{\theta\theta}^2+2\nu e_{ss}.e_{\theta\theta}\right)+G.e.\gamma_{s\theta}^2 \qquad (15)$$
$$+\frac{Ee^3}{12\left(1-\nu^2\right)}\left(k_{ss}^2+k_{\theta\theta}^2+2\nu\, k_{ss}.k_{\theta\theta}\right)+\frac{Ge^3}{12}k_{s\theta}^2\, rd\theta ds$$

In view of non linear applications, the integration is performed numerically.

The element is being implemented in the general purpose finite element code Castem 2000 [13], [14]. For practical computations of piping lines, it will be necessary to prescribe boundary conditions on global quantities (beam displacements and stress resultants) and on local shell quantities (for example zero local displacements in case of a rigid flange).

4. CONCLUSION

The enriched beam element proposed in this paper may be seen together as a synthesis and a generalization of various previous works : It enables a more realistic description of ovalization and its propagation along general tridimensionnal lines made of elbows and straight parts. For this purpose, a special care has been taken in the choice of the additional unknowns and the suppression of usual limiting hypothesis. The resulting element possesses a large number of unknowns and may be used for linear as well as geometric and material non linear analysis to predict accurate states of stresses in the pipe. The main advantage over a full shell description of the pipe is the great simplification of input preparation and output post processing. However, it is limited to elbows and straight parts. In general piping lines, there are some other componants like tee junctions, bellows, etc. which cannot be modelled with this approach. One solution which can be proposed is to combine in the same analysis, a modelling of elbows and straight parts by means of enriched beam formulation, and a modelling of special componants, like tee junctions, by means of shell elements, the continuity between the two types of models being possible at the shell scale.

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