Transactions of the 15th International Conference on
Structural Mechanics in Reactor Technology (SMiRT-15)
Seoul, Korea, August 15-20, 1999

A Study on the Axial-Flow-Induced Vibration of Fuel Rod

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Abstracts; An analytical model has been proposed to predict the axial-flow-induced vibration of a PWR fuel rod, which is modeled as a simply-supported and a single-spaned Euler-Bernoulli beam subjected to axial force. Comparisons between the proposed model and those of the previous researchers such as Chen and Kanazawa have been made. It was discovered that the natural frequencies of the fuel rod model are varied greatly as a function of its burnup, and the vibration contributions of modes higher than the fundamental one are insignificant. The calculated rms amplitude of the fuel rod model increases about three times as the flow velocity increases from 5 to 8 m/s. The maximum displacement is anticipated to be within the first cycle when the maximum compression force occurs due to the difference between the minimum internal rod and the maximum system pressure.

1. INTRODUCTION

PWR fuel rods are exposed to reactor coolant at a flow velocity up to 8 m/s. The coolant flow produces energy to induce the vibration of the fuel rods, which may result in structural damage like fretting wear. Since the coolant flows normally parallel to the rods, the vibration is called as axial-flow-induced vibration. It is widely accepted that the primary excitation mechanism is the randomly fluctuating pressure acting on the surface of the rods, and the vibration amplitudes y_{rms}/D, typically 10^{-2}, rarely exceed 10^{-1} [1].

The inside of the fuel rod is pressurized by helium gas up to 26 bars to prevent it from being crushed by a high coolant pressure of 150 bars in the reactor. The internal pressure of the fuel rod increases due to fission gases released from UO_2 pellets as burnup increases. Therefore, the fuel rods are subjected to tension in air and to compression after being loaded into the reactor, which gradually decreases as burnup increases.

The purpose of this paper is to propose a flow-induced vibration(FIV) model of a simply-supported cylinder subjected to axial force in incompressible flow parallel to it, and to estimate the axial force effect or the vibration of a fuel rod. For the cylinder model, the equation of motion derived by Paidoussis and simplified by Kanazawa[3] is used, and the cross-spectral density function on turbulent wall pressure reported by Chen[2] is utilized. In addition, the normal mode method is employed to solve the random vibration problem represented by the simplified equation.

2. MATHMETICAL MODEL AND ANALYSIS

2.1 Equation of Motion and Natural Frequency

Consider a simply-supported rod subjected to random fluctuation pressure p(x,θ, t) induced by incompressible fluid parallel to the x-axis in Fig. 1.

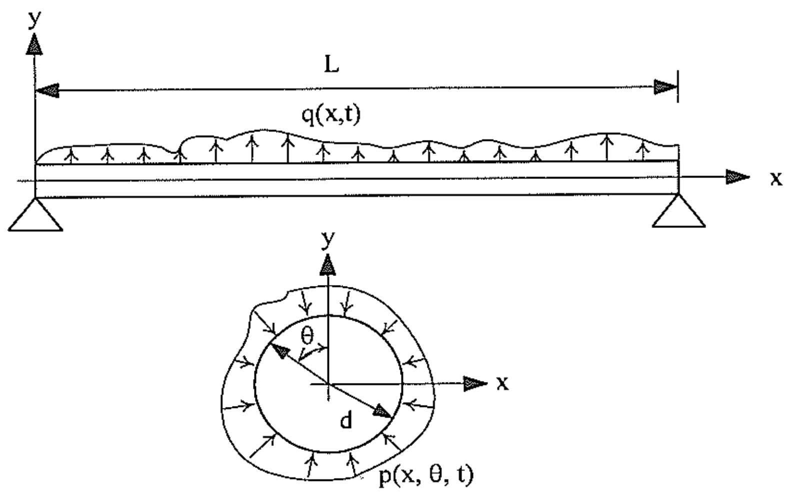

Figure 1 Simply Supported Rod Subjected to Random Fluctuation Force and Pressure Distribution on the Surface of Rod

The rod has flexural rigidity EI and mass per unit length m. The vibration is induced to the rod by the random pressure fluctuation in the turbulent flow. The random fluctuation pressure p(x,θ,t) can be converted to the line force q(x,t).

The following Equation Of Motion(EOM) is formulated by additionally considering axial force based on the EOM simplified by Kanazawa[3].

$$EI\frac{\partial^4 y}{\partial x^4}+(m_f V^2 - T_o)\frac{\partial^2 y}{\partial x^2}+C\frac{\partial y}{\partial t}+M\frac{\partial^2 y}{\partial t^2}=q(x,t) \tag{1}$$

Where, m_f is the added mass of fluid,

M is the total mass(sum of the m_f and rod mass)

T_0 is the axial force (tension),

$C\frac{\partial y}{\partial t}$ is the viscous damping force which was equated to the sum of the forces F_s, defined below[3].

$$F_s = -\frac{1}{2}\rho D V^2 C_f \frac{\partial^2 y}{\partial x^2}(\frac{L}{2} - x) + 2m_f V\frac{\partial^2 y}{\partial x \partial t} + \rho D V^2 C_f \frac{\partial y}{\partial x} + \frac{1}{2}\rho D V C_f \frac{\partial y}{\partial t} + \frac{1}{2}\rho D C_D \left|\frac{\partial y}{\partial t}\right| \cdot \frac{\partial y}{\partial t} \tag{2}$$

The C_f and C_D in Eq. (2) are the profile and skin drag, respectively, for a rod in cross flow. Since it is well known that the rod in axial flow is weak damping, the damping term in Eq. (1) can be neglected for simplicity in obtaining natural frequencies and mode shapes. Eq. (3) is obtained from Eq. (1) by the separation of the spatial variable.

$$EI\frac{d^4\phi}{dx^4} + (m_f V^2 - T_o)\frac{\partial^2\phi}{\partial x^2} - M\omega^2\phi = 0 \tag{3}$$

By using the boundary conditions of a simply-supported beam, the following natural frequency equation is obtained .

$$\omega_n^{\ 2} = \left(\frac{n\pi}{L}\right)^2 \frac{T_0 - m_f V^2}{M} + \left(\frac{n\pi}{L}\right)^4 \frac{EI}{M} \qquad n = 1,\ 2,\ 3,\ .... \tag{4}$$

The following Frequency Response Function (FRF) $H_n(\omega)$ can be found by considering a rod response and the forcing function as a harmonic one.

$$H_n(\omega) = \frac{1}{M\omega_n^{\ 2}\left(1 + i2\zeta r - r^2\right)} \tag{5}$$

Where, $r = \omega/\omega_n$ and $\zeta = C/C_c = C/(2M\omega_n)$

2.2 Solution of the Equation of Motion[3, 4]

The frequency response function $H_n(\omega)$ for the rod can be obtained by using the following Fourier transform of the impulse response function $h_n(t)$.

$$H_n(\omega) = \int_{-\infty}^{+\infty} h_n(t) e^{i\omega t} dt \qquad n = 1,\ 2,\ 3,\ ... \tag{6}$$

The forcing function q(x, t) is represented by an infinite sum of the normal modes.

$$q(x,t) = \sum_{n=1}^{\infty} F_n(t) \sin\frac{n\pi x}{L} \tag{7}$$

The response of the rod to a random forcing function q(x, t) can be denoted as following statistical average E[ ] by modifying the Duhamel integral with respect to the deterministic loading.

$$E\left[y_1(x,t)y_2(x',t')\right]=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{L}\sin\frac{n\pi x}{L}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}h_m(t-\tau)h_n(t'-\tau')\frac{4}{L^2}$$
$$\int_0^L\int_0^L E\left[q_1(x,\tau)q_2(x',\tau')\right]\sin\frac{m\pi x}{L}\sin\frac{n\pi x'}{L}dx\,dx'\,d\tau\,d\tau' \quad (8)$$

Once the excitation process is considered to be weakly stationary and weakly homogeneous under the assumption that the forces acting on the rod are due to fully-developed turbulence, the relationship that the cross-correlation function of excitation R_{qq} is equal to the inverse Fourier transform of the cross-spectral density function of excitation Φ_{qq} can be used. Thus, the rod response in the form of Eq. (8) is expressed as follows:

$$R_{yy}(x,x',\tau)=\frac{4}{L^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{L}\sin\frac{n\pi x'}{L}\int_{-\infty}^{+\infty}H_m(\omega)H_n^*(\omega)$$
$$\int_0^L\int_0^L\Phi_{qq}(x-x',\omega)\sin\frac{m\pi x}{L}\sin\frac{n\pi x'}{L}e^{i\omega\tau}dx\,dx'\,d\omega \quad (9)$$

In addition, by using the fact that the cross-correlation function of the rod response R_{yy} is equal to the inverse Fourier transform of a cross-spectral density function of the rod response Φ_{yy} as the same case of forcing function, and it has both longitudinal and lateral component (Figure 1), the rod response to a random pressure can be expressed as follows:

$$\Phi_{yy}(x,x',\omega)=\frac{4}{L^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{L}\sin\frac{n\pi x'}{L}H_m(\omega)H_n^*(\omega)$$
$$\int_0^d\int_0^d\int_0^L\int_0^L\Phi_{pp}(x-x',z-z,\omega)\sin\frac{m\pi x}{L}\sin\frac{n\pi x'}{L}dx\,dx'\,dz\,dz' \quad (10)$$

Finally, the mean square value of the vibration amplitude of rod is obtained by integrating Φ_{yy} over all frequencies. Thus

$$\overline{y^2}(x,x')=\int_0^{+\infty}\Phi_{yy}(x,x',\omega)d\omega \quad (11)$$

2.3 Turbulent-wall-pressure Cross-spectral Density Function

Corcos[5] proposed the following expression for the cross-spectral density function of the fluctuating wall pressure.

$$\Phi_{pp}(\bar{x},\bar{z},\omega)=\phi(\omega)\,A\left(\frac{\omega\bar{x}}{U}\right)B\left(\frac{\omega\bar{z}}{U}\right)\cos\left(\frac{\omega\bar{x}}{U}\right) \quad (12)$$

Where, U is convection velocity: $\frac{U}{V}=0.9-0.06\frac{\omega d_h}{2\pi V}$ [3]

Introducing Chen's expression for the A and B [2] in Eq. (12), and substituting Eq. (12) into Eq. (10), one can obtain the following Eq. (13).

$$\Phi_{yy}(x,x',\omega) = \frac{4}{L^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin\frac{m\pi x}{L}\sin\frac{n\pi x'}{L}H_m(\omega)H_n^*(\omega)\int_0^d\int_0^d Exp\left[-0.55\left|\frac{\omega \bar{z}}{U}\right|\right]dz\,dz'$$
$$\int_0^L\int_0^L \phi(\omega)\,Exp\left[-0.1\left|\frac{\omega \bar{x}}{U}\right|\right]\cos\left(\frac{\omega \bar{x}}{U}\right)\sin\frac{m\pi x}{L}\sin\frac{n\pi x'}{L}dx\,dx' \tag{13}$$

The lateral component in Eq. (13) can be simplified by using the expression $z = d\cdot\theta/2$.

$$\int_0^d\int_0^d Exp\left[-0.55\left|\frac{\omega \bar{z}}{U}\right|\right]dz\,dz' = \frac{d^2}{4}\int_0^{2\pi}\int_0^{2\pi} Exp\left[-0.275\frac{\omega d}{U}\left|\theta-\theta'\right|\right]\cos\theta'\cos\theta\,d\theta' d\theta$$
$$= d^2\chi^2 \tag{14}$$

Introducing non-dimensional parameters ξ and ξ' by dividing x and x' respectively by the rod length L, Eq. (14) can be expressed as following Eq. (15).

$$\Phi_{yy}(\xi,\xi',\omega) = 4d^2\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\sin m\pi\xi\sin n\pi\xi'\,H_m(\omega)H_n^*(\omega)\chi^2\,\phi(\omega)\,Jn^2 \tag{15}$$

$$\text{Where,}\quad Jn^2 = \int_0^1\int_0^1 Exp\left[-0.1\frac{\omega L}{U}\left|\xi-\xi'\right|\right]\cos\left(\frac{\omega L}{U}\left|\xi-\xi'\right|\right)\sin(m\pi\xi)\sin(n\pi\xi')d\xi d\xi' \tag{16}$$

For the power spectral density $\phi(\omega)$ in Eq. (15), the following fitted curve[2] is used here.

$$\phi(\omega)/\rho^2V^3d_h = 5.86\times10^{-6} + 3.61\times e^{(-S/1.87)} + 1.81\times e^{(-S/0.133)} \tag{17}$$

Where, S means Strouhal number ($S = \omega d_h/2\pi V$).

3. COMPARISON OF THE DEVELOPED MODEL WITH PRIVIOUS RESULTS

Comparison of the developed model with tests and analysis results of previous researchers such as Chen[2] and Kanazawa[3] is made to verify the model. The range of parameters used for their analyses and experiments are given in Table 1. Comparison of the developed model with Chen's one is shown in Figure 2, and with Kanazawa's test data in Figure 3 respectively. The displacement calculated by the developed model for Chen's case 1 is generally smaller than that by Chen's model with the exception in the range of low velocity as shown in Figure 2. Meanwhile, the displacement of the developed model for case 2 is obviously larger than that of Chen's model. The vibration amplitudes expected by the present model are larger than Kanazawa's test data. Discrepancy between the two is big in the range of low velocity as shown in Figure 3. However, it decreases with increase of flow velocity. The prediction of the proposed model is reasonable within flow velocity from 15 ft/sec to 30 ft/sec that is believed as the range of coolant velocity in PWR reactor.

Table 1 Parameters used by Chen and Kanazawa[2, 3, 6]

Cases	M_a (Kg/m)	d (mm)	L (m)	d_h(m)	EI (N/m^2)	V (m/s)
Chen						
Case 1	0.376	12.7	0.914	0.0381	55.06	0 ~ 20
Case 2	0.391	12.7	1.038	0.0381	111.5	0 ~ 20
Kanazawa						
Case 1	0.106	12.7	1.22	0.0381	39.64	0 ~ 20
Case 2	0.172	12.7	1.22	0.0381	150.73	0 ~ 20

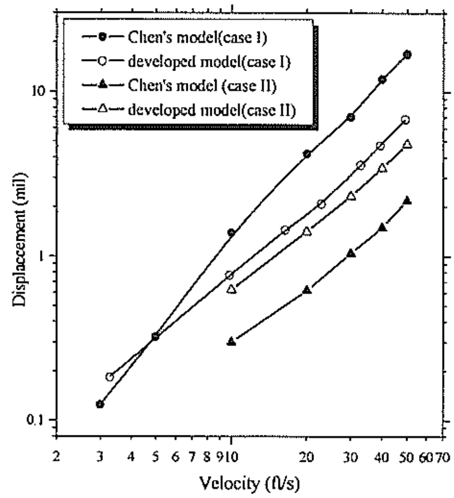

Figure 3 Comparison of the developed model with Chen's

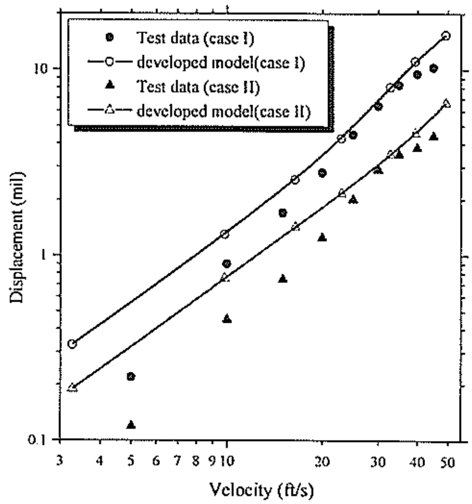

Figure 4 Comparison of the developed model with Kanazawa's data

4. VIBRATION ANALYSIS OF A FULE ROD MODEL

In present FIV analysis, fuel rod is modeled as a simply-supported and a single-spaned Euler-Bernoulli beam subjected to axial force. The ranges of the parameters given in Table 2 are believed as the environment in which the fuel rod typically operates. The internal pressure history of the fuel rod in FDR[7] is used for the calculation of axial force listed in Table 3. The natural frequencies obtained by Eq. (4) are given in the right side of Table 3.

The displacement PSD with increase of operation time at flow velocity 5 m/s is shown in Figure 5, and 8 m/s in Figure 6 respectively. The two picks in Figure5 and 6 occur due to the first and third natural frequencies. The root-mean-square (rms) displacements of the fuel rod according to operation day and flow velocity are obtained by integrating the displacement PSD from zero to 250 Hz where the third resonance pick is included. Since the first natural frequency is dominant, a one-mode(fundamental) approximation is sufficient to get an reasonable results from a practical engineering point of view.

Table 2 Parameters Used for The Fuel Rod Model

Flow Parameters	Value		Fuel rod Parameters	Value
Temperature (°C)	310		Density (kg/m^3)	6.6×10^3
Density (kg/m^3)	624		Inner diameter (mm)	8.22
Viscosity (kg/m-sec)	0.000075		Outer diameter (mm)	9.7
Flow velocity (m/s)	$5 \sim 8$		Span length (m)	0.62
Hydraulic dia.(mm)	11.6		Young's modulus (N/m^2)	7.98×10^{10}

Table 3 Operation Day vs. Axial Forces and Natural Frequencies

Operation Days	Axial force (N)	Natural Frequency (Hz)				
		1st	2nd	3rd	4th	5th
0	- 477	11.9	90.8	217.6	394.7	622.4
100	- 502	9.6	89.7	216.6	393.7	621.4
300	- 482	13.2	91.5	218.3	395.4	623.1
600	- 444	15.2	92.8	219.4	396.6	624.2
800	- 389	20.1	96.4	222.9	400.0	627.7
1000	- 305	22.3	98.4	224.8	401.9	629.6
1200	- 203	26.1	102.0	228.5	405.5	633.2

5. RESULTS AND DISCUSSION

An analytical model has been developed to predict the axial-flow-induced vibration of the PWR fuel rod, which is modeled as a simply-supported and a single-spaned Euler-Bernoulli beam subjected to axial force. The proposed method has used the EOM simplified by Kanazawa, utilized turbulent wall pressure PSD reported by Chen, and employed the random vibration theory. Comparisons between the proposed model and those of previous researchers have been made. The result shows that the proposed model reasonably predicts the vibration amplitude.

It is found that the natural frequencies of the fuel rod model are varied largely as a function of its burnup. Since the fundamental mode are dominant, it is sufficient to use only fundamental one in calculating the vibration amplitude of the rod model in view point of practical calculation. The calculated rms displacement increase three times with increase of flow velocity from 5 to 8 m/s. The maximum displacement of 0.12 mm for the rod model is anticipated to be around 100 days after reactor operation when the maximum compression occurs in the rod. It is well known that support springs of the rod are relaxed in proportion to its burnup, and that leads the support force to be weak. Therefore, it is said that the possibility of fretting wear damage on the fuel rod is stochastically high during the last cycle of it in terms of the spring relaxation while high during the first cycle in terms of rod vibration. It is judged that the burnup influence of the fuel rod should be closely scrutinized in studying on the fretting wear of the fuel rod as well as the flow-induced vibration of it.

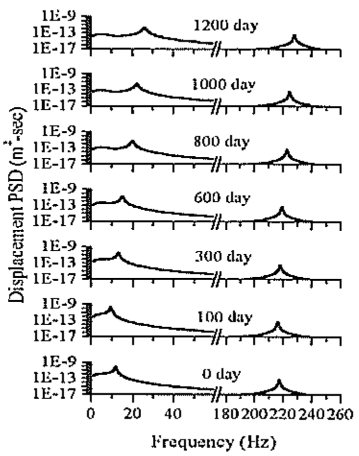

Figure 5 Displacement PSD at V=5

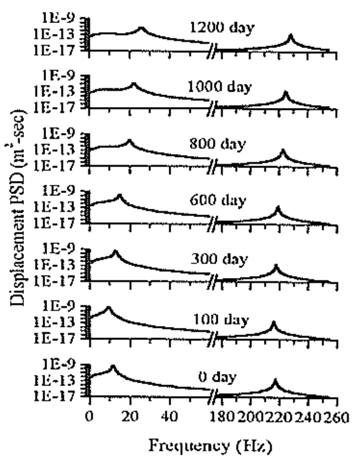

Figure 6 Displacement PSD at V=8

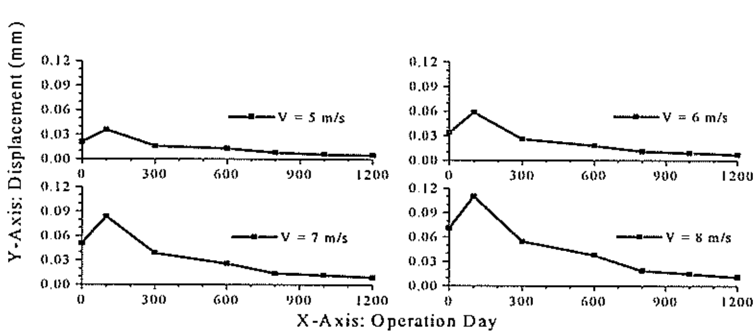

Figure 7 Vibration Displacement of Fuel Rod with Operation Time

Acknowledgment

This project has been carried out under the nuclear R&D program by MOST

6. REFERENCES

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