# STRAIN AND ULTIMATE STRENGTH OF CONCRETE UNDER TRIAXIAL STRESS

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Three years of experimentation including the construction time for the test machine have been needed to measure:

- the deformation of concrete cubes (14 x 14 x 14 cm) under combined compression stresses up to 500 bars at temperatures of 20°C, 40°C and 60°C;
- the ultimate strength of concrete cubes (7 x 7 x 7 cm) under combined compression stresses up to 2,000 bars and under two compression stresses and one tension stress.

One type of concrete was investigated which had been previously thoroughly studied under uniaxial stress; creep and shrinkage properties had been more particularly investigated before it was selected to be the concrete used in the construction of the prestressed concrete pressure vessel of nuclear reactor BUGEY I. Therefore instrumentation in situ should support the research made on specimens.

Three different limits are emphasized as a prominent result of the investigation under triaxial stresses:

- the "quasi-elastic" limit under which the concrete behaves as a purely elastic material,
- the crack initiation limit under which the concrete shows a much larger bearing capacity than when submitted to an uniaxial stress. This limit can be detected by sonic tests on the experimented specimen,
- the ultimate strength limit which correspondens to the flow of material, deformation extending without increase of stresses imposed.

Corresponding curves are traced and an analytical approach is proposed.

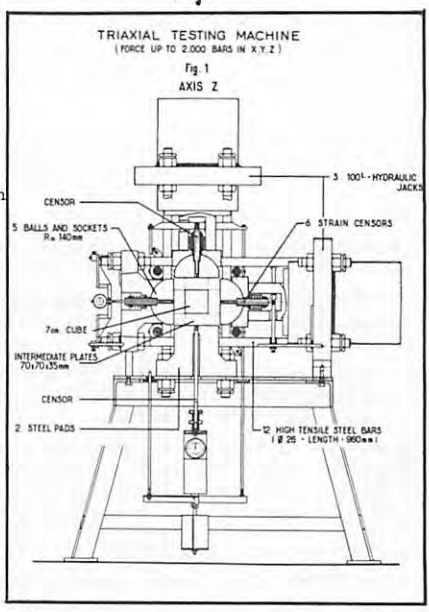
As a first result of these experiments, tentative criteria and increased permissible stresses might be ruled for concrete submitted to multiaxial compressive stresses. However, extreme care should be taken in this way, when tension is imposed in one direction while the two others are under increasing compression.

. Ultimate state or yield point.

The testing program has been carried out as a part of the Research and Development program of "Compagnie Industrielle de Travaux", (CITRA) from June 1967 until June 1969. It went on until December 1970 as a part of R & D Program of SECN (Société d'Etudes des Caissons Nucléaires) supported by EDF (Electricité de France) and CEA (Commissariat à l'Energie Atomique). It has been performed at the Laboratory of "Centre d'Etudes Scientifiques et Techniques" (CEST) in Grenoble.

2 - DESCRIPTION OF THE TESTING MACHINE
(see Fig. 1 and 2)

Fig: 1




- In its initial setting, the machine is capable of applying a 100 metric tons compression in each of the 3 orthogonal directions.

- Maximum size permissible for cubic specimens is 14 x 14 x 14 cm. The maximal average stress supported by each side of the cubic specimen is then 500 bars.

- Deformations of the cube are measured by means of displacement gauges according to the 3 main directions. These gauges measure the relative displacement of the two opposite sides of the specimen The fingers of gauges are in close contact with the specimen surface.

- The stresses are measured either by manometers connected to the jacks, or by extensometers set on the carrying columns which then work like dynamometers.

- The loading-plates rest on ball and socket joints. The loading system moves freely in ground plan so that its setting versus the vertical loading axis is easy.

Fig: 2

- Pad is inserted between the plates and the specimen in order to avoid any friction effect. The pad consists of 4 aluminium sheets thickness 4/10, each face being lubricated with talc.

- Two kinds of specimen have been used :

. 14 x 14 x 14 cm cubic specimen (from 0 to 500 bars)

. 7x7x7 cm cubic specimen (from 0 to 2 000 bars ; mainly used for the ultimate tests.)

- A subsidiary setting allows to produce along one of the principal direction of the testing machine, a 4 T tension and a 12 T compression, i.e. either a maximal tensile or compressive stress of 80 bars, 240 bars respectively, on a 7x7x7 cm cubic specimen.
- The tension is applied on the 7x7x7 cm specimen by mean of two metallic plates which are stuck to the specimen by a thermoplastic resin on one side, and bounded to the loading machine plates on the other side.
- The compressions produced by the two other 100 metric tons jacks (i.e. 2000 bars on a 7x7x7 cm cubic specimen) are applied on the specimen through a support system performing a sliding plan of the cube along the tension direction.
- The supporting system is made of a bearing-cage set on the plate from one hand, and supported by a rectified high-grade steel plate in direct contact with the concrete specimen from the other hand.
- The hydraulic system is supplied by a motor-pump group flowing out 0,12 ℓ/mn. The control of outflows in each one of the three circuits is hand-ajusted. It is possible to connect 2 or 3 circuits. The loading progression is performed at a speed of about 5 bars/sec.

## 3 - STRAIN OF CONCRETE UNDER TRIAXIAL LOADING

Loading is slowly and progressively applied.

- Strain measurement occurs right in the same time as loading application. Efforts applied according to the 3 main directions of the specimen are compressions. Loading program has been previously scheduled.

### 3.1. Concrete characteristics

| | |
|---|---|
| - Agregate ("granulats de Meximieux" (Ain)) | |
| . Sifted gravel 4/15 | 1156 kg |
| . Sifted sand 0/5 | 647 kg |
| - CPAC cement (Val d'Azergues) | 400 kg |
| - Water content | 150 kg |
| - Pozzolith n° 8 | 0,8 kg |

### 3.2. Specimen definition

The tested specimen is cubic (14x14x14 cm). It is casted in a high-grade steel mould the sides of which are tooled and rectified with a high order of accuracy. The only free side of the cube is rectified after unmolding.

At testing time the concrete is 28 or 90 days old.

Elaboration and preservation are performed as follows :

- After casting into the mould : the specimen stays 24 hours in the mould covered with a plastic sheet.
- After unmolding : the specimen is dipped 7 days under 20° C water. Then the side which did not contact the mould walls is rectified.
- After tooling and until testing time : the specimen is preserved in a polyethylenic envelope

at 20° C.

3.3. Testing principle.

- The specimen is subject to a compression reaching up 500 bars, with reference to each its 3 orthogonal axis. A part from the heterogenety of concrete, we can assume that the distributor effect of the press-plates and of support system favours the development of a stress field nearly homogeneous at specimen scale.

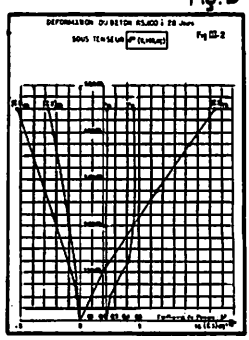
Measures have mainly concerned study of the increments of the 3 strain-components due respectively to the 3 assigned increments of stress-components. We measured :

$(\Delta\varepsilon_1)_{\Delta\bar{\sigma}_1}$ , $(\Delta\varepsilon_1)_{\Delta\bar{\sigma}_2}$ , $(\Delta\varepsilon_1)_{\Delta\bar{\sigma}_3}$

$(\Delta\varepsilon_2)_{\Delta\bar{\sigma}_1}$ , $(\Delta\varepsilon_2)_{\Delta\bar{\sigma}_2}$ , $(\Delta\varepsilon_2)_{\Delta\bar{\sigma}_3}$

$(\Delta\varepsilon_3)_{\Delta\bar{\sigma}_1}$ , $(\Delta\varepsilon_3)_{\Delta\bar{\sigma}_2}$ , $(\Delta\varepsilon_3)_{\Delta\bar{\sigma}_3}$

Fig: 3



The increments $\Delta\bar{\sigma}_i$ have got a value of 25 bars. Starting from a loading state characterized by the tensor $\bar{\bar{\sigma}}$ ( $\sigma_1$ , $\sigma_2$ , $\sigma_3$ ) the increments were scheduled in the following manner :

- First step : $\bar{\bar{\sigma}}$ ( $\sigma_1$ , $\sigma_2$ , $\sigma_3 + \Delta\sigma_3$ )
- Second step : $\bar{\bar{\sigma}}$ ( $\sigma_1$ , $\sigma_2 + \Delta\sigma_2$ , $\sigma_3 + \Delta\sigma_3$ )
- Third step : $\bar{\bar{\sigma}}$ ( $\sigma_1 + \Delta\sigma_1$ , $\sigma_2 + \Delta\sigma_2$ , $\sigma_3 + \Delta\sigma_3$ )

During each test, we assigned limit-values $\bar{\sigma}_1$ and $\bar{\sigma}_2$ to respectively $\sigma_1$ and $\sigma_2$ . These values were chosen successively 50 bars, 100, 150, 200, 250, 300, 350, 400 bars. The limit value for $\sigma_3$ is always 500 bars. We entered them : $\bar{\bar{\sigma}}$ ( $\bar{\sigma}_1$, $\bar{\sigma}_2$, $\sigma_3$ )

$\bar{\bar{\sigma}}$ (50, 150, $\sigma_3$ ) for instance.

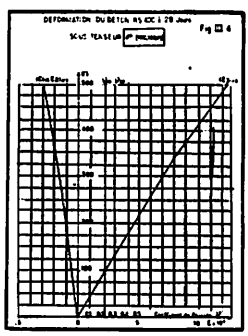
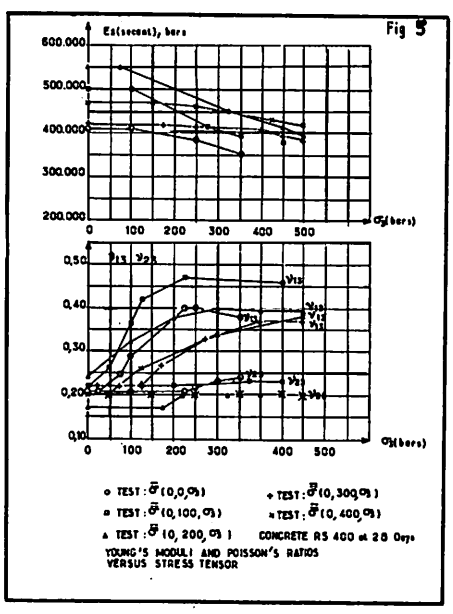


Fig: 4




3.4. Results

Some of these results are printed out in the annexed diagrams. (Fig. 3 to Fig. 7). The stress-strain curves :

$\left[\sigma_3 - (\varepsilon_3)\sigma_3 \, , \, \sigma_3 - (\varepsilon_2)\sigma_3 \, , \, \sigma_3 - (\varepsilon_1)\sigma_3\right]$

are drawn for several states of stress tensor.

Diagrams showing the development of strain secant modulus $E_3 = \sigma_3 / (\varepsilon_3)\sigma_3$ and of Poisson's ratios $\nu_{13} = -\dfrac{(\varepsilon_1)\sigma_3}{(\varepsilon_3)\sigma_3}$

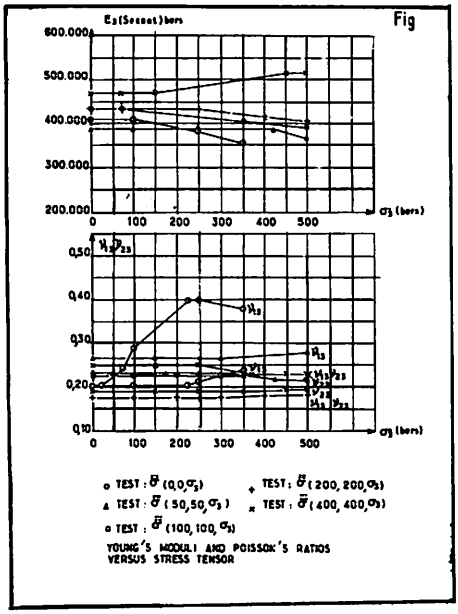
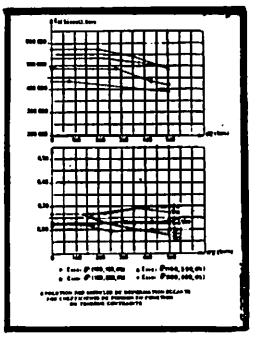



Fig:6

Fig:7



and $\nu_{23} = -\frac{(\varepsilon_2)\sigma_3}{(\varepsilon_3)\sigma_3}$ have been made out from these curves and classified versus the stress-state of the specimen.

- Biaxial stress : $\bar{\bar{\sigma}}$ ( $0, \sigma_2, \sigma_3$ )
- Cylindrical tensor stress:

$\bar{\bar{\sigma}}$ ( $\sigma_1 = \sigma_2$ , $\sigma_3$ )

- Any tensor stress : $\bar{\bar{\sigma}}$ ( $\sigma_1, \sigma_2, \sigma_3$)

Let us note :

3.4.1. The fairly good linearity of the $\left[\sigma_3 - (\varepsilon_3)\sigma_3\right]$ curves shows that, for an assigned state of loads, ( $E_3$ ) secant modulus of deformation is scarcely varying versus $\sigma_3$ . (limits of $\sigma_3$ up to 350-500 bars).

3.4.2. In cylindrical tensor, the slight incidence of lateral biaxial compression on the value and evolution of secant modulus $E_3$ (limits of $\sigma_1 = \sigma_2$ from 0 up to 400 bars).

It is rather difficult to give evidence of a hardening ratio of the secant modulus $E_3$ due to the lateral compression, at least for slight values of $\sigma_3$ .

3.4.3. Poisson's ratio $\nu_{23}$ remains unchanged ( $\nu_{23}^{average} = 0,20$) whatever $\sigma_1 \leq \sigma_2 \leq$ 400 bars may be and for any value of $\sigma_3 \leq$ 500 bars.

3.4.4. Poisson's ratio $\nu_{13}$ varies erratically :

- As $\sigma_1 = \sigma_2$ , $\nu_{13} = \nu_{23} = 0.20$ (average) for any value of $\sigma_3$
- As $\sigma_1 < \sigma_2$ , $\nu_{13} > \nu_{23}$ :

. As $\bar{\sigma}_1 = 0$ , $\nu_{13}$ progressively increases (from 0,20 to 0,40) with $\sigma_3$ and it depends of $\sigma_2$ . Then it stays unchanged (= 0,40) for $\sigma_3$ = up to 500 bars.

. As $\bar{\sigma}_1$ = 100 bars, $\nu_{13} = \nu_{23}$ as long as stress tensor remains spheric. When $\sigma_2$ and $\sigma_3$ become greater than $\sigma_1$ , the $\nu_{13}$ and $\nu_{23}$ values diverge. ( $\nu_{13} > \nu_{23}$ ). We can point out : from one hand, a hardening of the material as concerns the principal stress $\sigma_2$ and a stabilizing of $\nu_{23}$ near 0,20 ; from the other hand, sinking of the material as concerns $\sigma_1$ (then, $\nu_{13}$ increases and stays near 0,30).

## 4 - ANALYSIS OF CONCRETE BEHAVIOUR UNDER TRIAXIAL LOADING

This analysis concerns a progressive and slow loading (5 bars/sec.) starting in the natural state of the specimen and completed until the dislocation or ultimate state of the material is achieved.

In order to reach this ultimate state of the specimen in all the loading cases we had scheduled, we had to reduce the ultimate strength of the specimen down to 380 bars in uniaxial compression at 28 days. This was obtained in reducing the cement ratio.

An extensive series of tests on this "R-300" concrete was then performed :

- For a triaxial compression state.
- For a loading state characterized by two compressive stresses and one tensile stress.

### 4.1. Concrete composition.

| | |
|---|---|
| - Gravel 4/15 Meximieux | 1 285 kg/m3 |
| - Sand 0/5 Meximieux | 718 kg/m3 |
| - CPAC cement (Val d'Azergues) | 300 kg/m3 |
| - Water | 158 kg/m3 |

### 4.2. Definition of the specimen

The specimen is a 7 x 7 x 7 cube. Concrete is cast into high-grade steel moulds just as for the 14 x 14 x 14 cm specimens.

After moulding, the specimen stays one day inside the mould. Then it is unmoulded and dipped under water during 7 days and finally conserved until the test day in a tight polyethylene sack. Concrete is tested at 28 days. At least, four specimens are picked up from the same pour. Four specimen is the minimal number necessary to define a point of the ultimate strength field of the material.

### 4.3. Loading principle

The stress state in a specimen will be noted :

$\bar{\bar{\sigma}} ( \sigma_1 , \sigma_2 , \sigma_3 )$ with $( \sigma_1 \leq \sigma_2 \leq \sigma_3 )$

or $\bar{\bar{\sigma}} ( \alpha_1 , \alpha_2 , \alpha_3 )$ with $( \alpha_i = \frac{\sigma_i}{\sigma^\circ} )$ and $\sigma^\circ$ = uniaxial compression strength.

We assume compression positive.

Suppose one of the components $\sigma_i$ (resp. $\alpha_i$) of the stress state is assigned. We write it : $\bar{\sigma}_i$ (resp. $\bar{\alpha}_i$).

The loading schedule planned for the tests on triaxial compression is split in 3 steps :

$\bar{\bar{\sigma}} ( \sigma_1 = \sigma_2 = \sigma_3 )$ $\quad \sigma_1 \leq \bar{\sigma}_1$

$\bar{\bar{\sigma}} ( \bar{\sigma}_1 , \sigma_2 = \sigma_3 )$ $\quad \sigma_2 \leq \bar{\sigma}_2$

$\bar{\bar{\sigma}} ( \bar{\sigma}_1 , \bar{\sigma}_2 , \sigma_3 )$ $\quad \sigma_3 \leq \bar{\sigma}_3 \leq \sigma_3^*$

$\sigma_3^*$ represents a certain limit in the concrete behaviour for assigned values of $\bar{\sigma}_1$ and $\bar{\sigma}_2$.

So $\sigma_3^u$ = ultimate value of the component $\sigma_3$ in the ultimate loading state (failure or dislocation):

$$\bar{\bar{\sigma}}^u ( \bar{\sigma}_1 , \bar{\sigma}_2 , \sigma_3^u )$$

With triaxial loading made of one tension and two compressions, the loading schedule differs considerably and is performed in two steps :

$\bar{\bar{\sigma}}$ ( 0 , $\sigma_2 = \beta \cdot \sigma_3$ , $\sigma_3$ ) $\quad \sigma_3 \leq \bar{\sigma}_3$

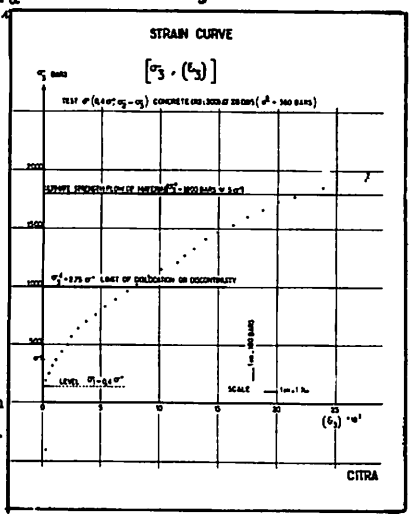
$\bar{\bar{\sigma}}$ ( $\sigma_1$ , $\bar{\sigma}_2 = \beta \cdot \bar{\sigma}_3$ , $\bar{\sigma}_3$ ) $\quad \sigma_1 \leq \sigma_1^u$

$\sigma_1^u$ is the ultimate value of the traction component $\sigma_1$ in the failure case of loading :

$\bar{\bar{\sigma}}^u$ ( $\sigma_1^u$ , $\bar{\sigma}_2 = \beta \cdot \bar{\sigma}_3$ , $\sigma_3$ )

These loading programs were chosen in order to delay at he most the failure instant in the concrete during a loading test.

Fig : 8




4.4. Mechanism of failure and dislocation of the material.

During several loadings in triaxial compression on several specimen we measured the strains $\varepsilon_1, \varepsilon_2, \varepsilon_3$ according to the 3 principal directions of the specimen and we recorded them on a diagram versus $\sigma_3$ (upper principal stress).

This diagram shows 3 distinct limits.

- A limit of reversibility : Starting from initial state (without any effort), we note a quasi linear answer of the material versus the loading until this first limit. This behaviour is characterized by "stiffness terms" (called also "accomodation" terms) which are studied in § 3.

- A limit of dislocation or discontinuity : As loading is going on, the stiffness of the material quickly decreases until it reaches a near constant but negligible value. This occurs when localized cracking starts into the specimen. Cracks are developping further on up to a second curvature can be noticed on the diagram.

- Loading going on, stiffness of the material seems to stay unchanged and then cracking probably extends everywhere in the specimen. This occurs until stiffness quickly decreases again. We can note either the sudden yield of the specimen or the "flow" of the material. Stiffness practically becomes zero. So we define the specimen yield point or ultimate strength.

These studies allow us to schedule our research program and to develop it, (for an assigned concrete), according to 3 directions.

- Definition of the quasi-elastic field bounded by the reversibility limit.
- Definition of the field bounded by the dislocation limit according to two different criteria.
- Definition of the ultimate strength field.

4.5. Limit of reversibility.

The limit of reversibility is determined as the limit beyond which the residual strain due to a progressively increasing loading state $\bar{\bar{\sigma}}$ ( $\lambda \cdot \bar{\sigma}_1, \lambda \cdot \bar{\sigma}_2$ , $\lambda \cdot \bar{\sigma}_3$ ), ( $\lambda \nearrow$ ) shows a sudden increase.

We have only studied this limit of reversibility under cylindrical tensor stresses such as $\sigma_1 \leq \sigma_2 = \sigma_3$

We have assigned some limit values for the first principal stress $\sigma_1$ :

i.e. $\bar{\sigma}_1 = K.\sigma^\circ$ with K = 0,0 - 0,2 - 0,4 - 0,6 - 0,8 - 1,0

Loading program run in two steps :

a - Firstly $\bar{\bar{\sigma}}$ ( $\sigma_1 = \sigma_2 = \sigma_3$ ) $\quad 0.1\,\sigma^\circ \leq \sigma_1 \leq K.\sigma^\circ$

b - Secondly $\bar{\bar{\sigma}}$ ( $\bar{\sigma}_1 = K.\sigma^\circ$ , $\sigma_2 = \sigma_3$ ) $\quad K.\sigma^\circ \leq \sigma_3 \leq \sigma_3^{rev.}$

Law of increment when loading (or decrement when unloading) as concerns stresses $\sigma_1$ (see. a.) and stresses $\sigma_2, \sigma_3$ (see. b.), is characterized by an increment of $0,1\,\sigma^\circ$

- Results have got a good order of accuracy and scarcely broke up. The values of $\sigma_2^{rev.}$ and $\sigma_3^{rev.}$ , which determine the limit of reversibility, stay unchanged and equal to $0,7\,\sigma^\circ$ as $\bar{\sigma}_1$ runs from $0,2\,\sigma^\circ$ to $\sigma^\circ$ .

The results partly determine the quasi elastic field curve for concrete with reference to the plan passing through $\sigma_3$-axis and the bisector of $\sigma_1$- and $\sigma_2$-axis, in stresses space.

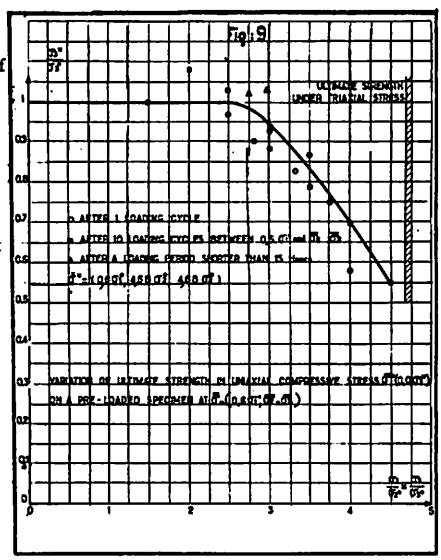
4.6. Limit of discontinuity or of dislocation

This limit has been placed on a level of internal cracking in the specimen causing a deterioration of some initial mechanical properties of the material. In order to choose the criteria of this limit, and after an assigned level of loading $\bar{\bar{\sigma}}$ ( $\bar{\sigma}_1$ , $\bar{\sigma}_2$ , $\bar{\sigma}_3$ and unloading, we have measured :

- The ultimate strength under uniaxial compressive stress along the direction of the upper principal stress $\sigma_3$ of the initial loading.
- The sonic propagation velocity through the specimen according to its 3-principal directions.

Thence, the limit of dislocation (or discontinuity) for an assigned scheduled loading is determined by the level of stress $\bar{\bar{\sigma}}^d$ , which causes, after unloading, a perceptible decrease of strengh under compressive stress in the specimen. (Fig. 9)

Fig. 9




We have systematically checked and found that the preliminary loading level, leading to decrease the sonic velocity through the specimen, corrected for a nulle loading state, used to take place just below the limit $\bar{\bar{\sigma}}^d$ defined above. Fig. 10.

We have developed the preliminary loading tests under triaxial compression. The loading schedule was the following :

$\bar{\bar{\sigma}}$ ( $\sigma_1 = \sigma_2 = \sigma_3$ ) $\quad 0 \leq \sigma_1 \leq k_1.\sigma^\circ$

$\bar{\bar{\sigma}}$ ( $k_1.\sigma^\circ$ , $\sigma_2 = \sigma_3$ ) $\quad k_1.\sigma^\circ \leq \sigma_2 \leq k_2.\sigma^\circ$

$\bar{\bar{\sigma}}$ ( $k_1.\sigma^\circ$ , $k_2.\sigma^\circ$ , $\sigma_3$ ) $\quad k_2.\sigma^\circ \leq \sigma_3 \leq \sigma_3^d$

Results $\sigma_3^d$ obtained with several states of stresses $\bar{\bar{\sigma}}$ ( $\sigma_1 = k_1.\sigma^\circ$ , $\sigma_2 = k_2.\sigma^\circ$ , $\sigma_3$ ) allowed to lay-out, in space of stresses ( $\frac{\sigma_1}{\sigma_0}$ , $\frac{\sigma_2}{\sigma^\circ}$ , $\frac{\sigma_3}{\sigma^\circ}$ ), the intersections of the surface

defining the limit of dislocation, with :

- Some planes such as :

$\sigma_3/\sigma^\circ = 0,0 - 0,1 - 0,2 - 0,6 - 0,8 - 1,0$. (Fig. 11)

- The plane passing through $\sigma_3$ -axis and the bisector of $\sigma_1 - \sigma_2$ -axis. (Fig. 12)

An important characteristic of this curve is that it is closed on itself. Its top seems to be on the equisector of the system ( $\sigma_1$ , $\sigma_2$ , $\sigma_3$ ) at the level this axis is cut by plane $(\sigma_1 + \sigma_2 + \sigma_3)^d \simeq 7.\sigma^\circ$

Results obtained give no sufficient evidences to completely determine this limit. Furthermore, some measurements should have to be checked or confirmed by other tests.

However, we may point out the fondamental concept of the limit of dislocation. It seems to start the beginning of a deep modification in the internal concrete structure which acts on its mechanical strength. Over this level, (actually distinct and deeply inside the ultimate strength field), the mechanical properties of the material go on decreasing until they reach a quasi-nul level (i.e. outflowing of the material.).

We therefore can say that the dislocation limit is characteristic of a limit state of reference for the field where concrete can be used as an elastic or quasi-elastic material.

Fig: 10

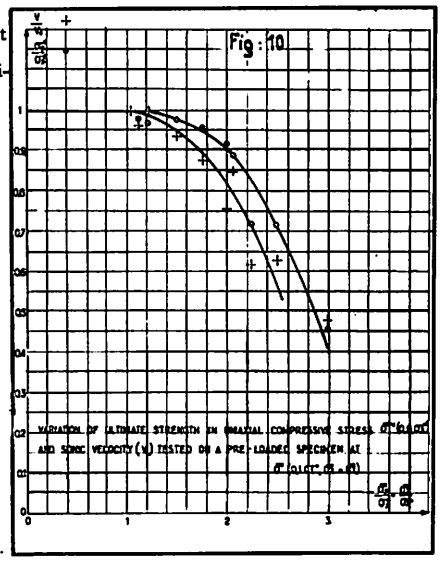
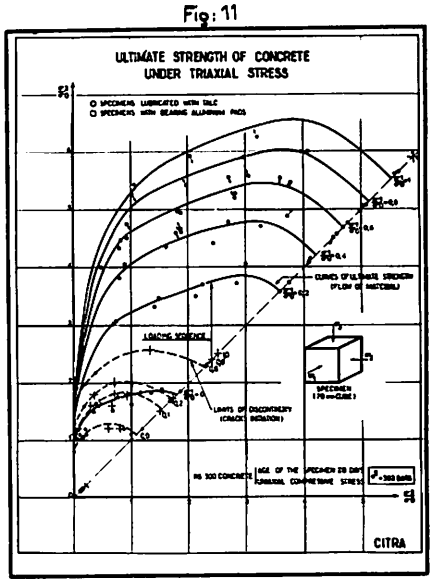
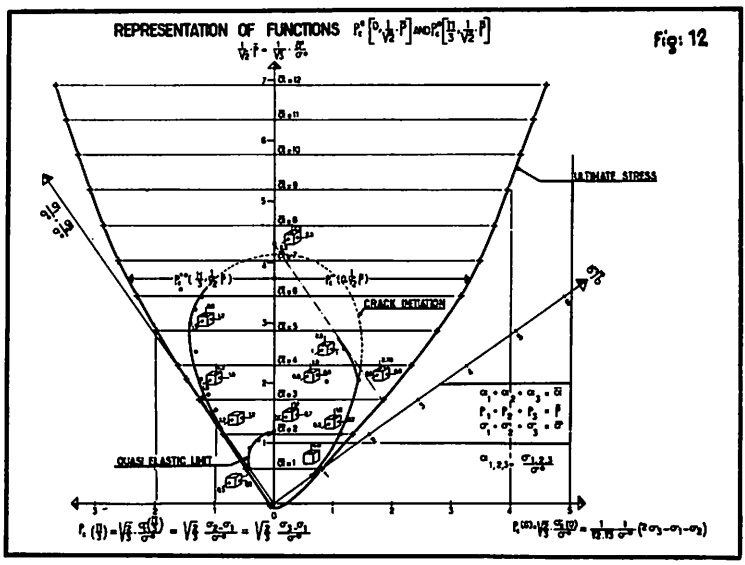


Fig: 11






4.7. Ultimate strength state under triaxial compression

Tests have been performed in the field limited by the following stress states :

$\overline{\overline{\sigma}}^{u}\ (\ 0,\ \overline{\sigma_2}\ ,\ \sigma_3^{u}\ )$ and $\overline{\overline{\sigma}}^{u}\ (\ \sigma^{\circ},\ \overline{\sigma_2}\ ,\ \sigma_3^{u}\ )$

with $0 < \overline{\sigma_1} < \sigma^{\circ}$

The values of $\sigma_1$ we have assigned were :

$\overline{\sigma_1}/\sigma_0$ = 0 - 0,2 - 0,4 - 0,6 - 0,8 - 1,0. and for each value of $\overline{\sigma_1}$

$\overline{\sigma_2}/\sigma_3^{u}$ = 0 - 0,2 - 0,4 - 0,6 - 0,8 - 1,0.

Where $\sigma_3^{u}$ is the ultimate strength in a cylindrical tensor stress state $\overline{\overline{\sigma}}^{u}\ (\overline{\sigma_1}, \sigma_2^{u} = \sigma_3^{u})$

Studying $\sigma_3^{u}$ has led us to define, in all cases, the ultimate strength state as :

$\overline{\overline{\sigma}}^{u}\ (\ \overline{\sigma_1}\ ,\ \overline{\sigma_2}\ ,\ \sigma_3^{u}\ )$

The loading program has been described in §. 4.3.

- Results obtained $\sigma^{u}\ (\ \overline{\sigma_1}, \overline{\sigma_2}, \sigma_3^{u}\ )$ permit to lay-out, in stress-space $(\ \sigma_1/\sigma^{\circ},\ \sigma_2/\sigma^{\circ}\ ,\ \sigma_3/\sigma^{\circ}\ )$, the intersections of the ultimate strength surface with :

- The different planes the levels of which are :

$\sigma_1/\sigma_0$ = 0,0 - 0,2 - 0,4 - 0,6 - 0,8 - 1,0. (Fig. 11)

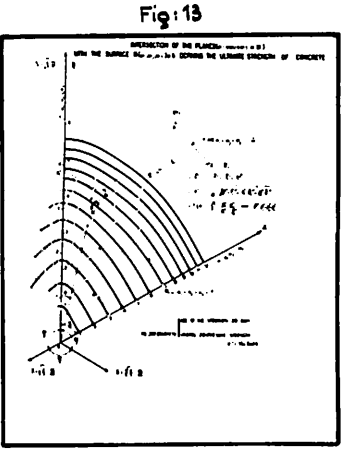
- The plane passing through $\sigma_3$-axis and the $\sigma_1$- $\sigma_2$-axis bisector (Fig. 12)

- The different planes perpendicular to the equisector of the system $(\ \sigma_1\ ,\ \sigma_2\ ,\ \sigma_3\ )$, the relation of which is given by :

$$\left(\frac{\sigma_1+\sigma_2+\sigma_3}{\sigma_0}\right)^{u} = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7,\ 8,\ 9,\ 10,\ 11,\ 12$$

(Fig. 13.)

Fig: 13



It is interesting to remind that this ultimate state here defined is according to :

- Either a sudden and straigt yield.
- Or an outflowing of the material. Strains become very large, the volume variations are nearly zero (it happens sometimes they are positive).

For certain cases, compression stress tests have been performed until yielding occurs, with specimen previously led at ultimate limit.

Results show that, after such a loading, this strength under uniaxial compression stress, becomes nearly zero.

4.8. Ultimate strength under triaxial stresses (2 compressions + 1 tension)

Let us note : $\sigma_1$ = tensile stress.

$\sigma_2, \sigma_3$ = compressive stress

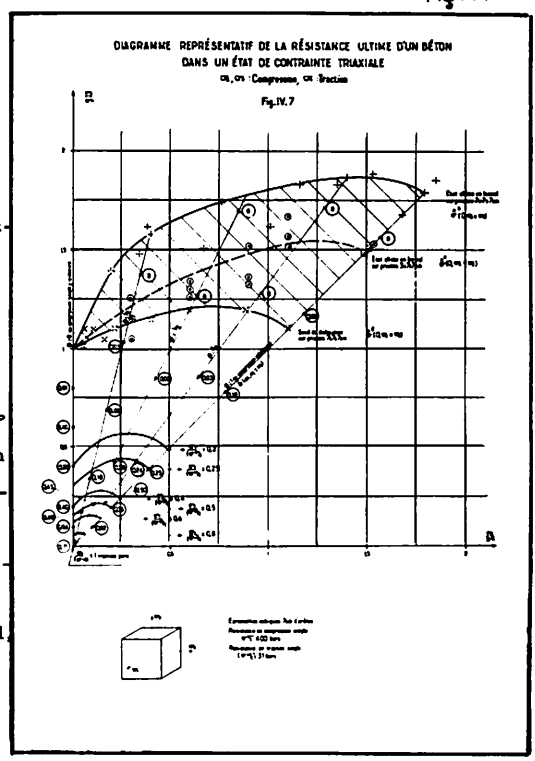
Loading schedule is the following :

- First step $\bar{\bar{\sigma}}\ (\ 0\ ,\ \sigma_2 = \beta.\sigma_3\ ,\ \sigma_3\ )$ $\qquad \sigma_3 \leq \bar{\sigma}_3$
- Second step $\bar{\bar{\sigma}}\ (\ \sigma_1\ ,\ \bar{\sigma}_2 = \beta.\bar{\sigma}_3\ ,\ \bar{\sigma}_3\ )$ $\qquad \sigma_1 \leq \sigma_1^u$

$\sigma_1^u$ is the ultimate value of $\sigma_1$, leading to the yield of the material under the stresses $\bar{\bar{\sigma}}^u(\ \sigma_1^u\ ,\ \bar{\sigma}_2 = \beta.\bar{\sigma}_3\ ,\ \bar{\sigma}_3\ )$

We assigned $\beta$ = 0, 1/4, 1/2, 3/4, 1. and we have looked for the ultimate biaxial states $\bar{\bar{\sigma}}^u(\ 0\ ,\ \sigma_2 = \beta.\sigma_3\ ,\ \sigma_3\ )$ under the ultimate state $\sigma_3 = (\sigma_3^u)_\beta$.

Fig : 14




The $\bar{\sigma}_3$ values we have assigned are given by :

$$\bar{\sigma}_3 = \alpha.\left(\sigma_3^u\right)_\beta$$

with $\alpha$ = 0, 0,01, 0,05 , 0,1 , 0,2 , 0,4 , 0,6 , 0,8 , 1,0.

Each of the ultimate state point of measurement is then determined by the stress state :

$$\bar{\bar{\sigma}}^u\left(\sigma_1^u\ ,\ \bar{\sigma}_2 = \alpha.\beta.\left(\sigma_3^u\right)_\beta\ ,\ \bar{\sigma}_3 = \alpha.\left(\sigma_3^u\right)_\beta\right)$$

The results obtained have allowed to plot in the stress-space $(\sigma_1/(\sigma^*)_c\ ,\ \sigma_2/\sigma^*\ ,\ \sigma_3/\sigma^*$ where $(\sigma^*)_c$ means the concrete strength under uniaxial compression, the intersections of the surface determining the ultimate concrete strength with several different planes which the levels are :

$\sigma_1/(\sigma^*)_c$ = 0-0,2-0,25-0,4-0,5-0,6-0,8-1.

(see Fig. 14.)

It must be pointed out :

- The sudden decrease of strength under

tension, along one direction of the specimen, when this specimen is under a lateral compression applied along the perpendicular directions.

- The large area inside which this strength under tension stays equal to zero.
- This area looks like bounded by the curve corresponding to the dislocation limit under biaxial compression for its lower limit, and bounded by the ultimate strength limit under biaxial compression for its upper limit.

This seems to prove that, in such a test (2 compressions + 1 tension), the ultimate state and the dislocation limit (in our meaning) must be very close to each other.

DISCUSSION

C F. K. GARAS, U. K.

The results given in the authors' paper are, like any other experimental work on element study, limited in their application to conditions similar to those in the experiment. The problem facing designers is how to apply experimental results obtained from small element studies to actual structures.

It is well established that experimental results are influenced by testing conditions, size, geometry and curing conditions.

Experimental work on triaxial stresses can be basically split into two groups:

(a) Tests on cubes ranging in size from 2 to 6 inches (5 to 15 cm)

(b) Tests on cylinders ranging in diameter from 1 1/2 inches to 6 inches with a height to diameter ratio equal to 2.

The majority of the available data was obtained from tests carried out on cylindrical specimens when two stresses were equal. Only very little experimental data has been published on triaxially loaded cubes where the three stresses were variable.

We have analysed most of the available data on triaxial stresses and we have found that the authors' results were about 50% higher than have been previously recorded by various investigators, both in the biaxial and triaxial fields of stresses. I think that the author's results have suffered from both the size effect and loading conditions.

Fig. 1 shows the effect of cube size for set values of $\sigma_2/f_c$ on the major principal stress $\sigma_1/f_c$. In this case the minor and intermediate stresses are equal. It can be seen that Launay and Gachons' results from the 2 3/4 inch cubes (7 cm) were more than 50% higher than the results obtained by Bertacchi, who tested 6 inch cubes (16 cm) specimens.

Fig. 2 shows that the size effect is less significant in the case of cylindrical specimens.

Both Figs. 1 and 2 indicate that a decrease in specimen size causes a corresponding increase in the major principal stress for a set value of the lateral stress. Furthermore, it was found that in the stress range $\sigma_2 = \sigma_3$, results obtained from 6 x 12 inch cylinders and 6 inch cubes are comparable.

Fig. 3 shows the effect of different degrees of frictional restraint on the major stress $\sigma_1$, for set values of $\sigma_2$. In this figure the results were plotted with reference to 6 inch cubes. It is clear that the results are influenced by the interface conditions.

In assessing most of the available data on triaxial compression with reference to one single size, that is either 6 x 12 inch cylinders or 6 inch cubes, it was found that :

$$\frac{\sigma_1}{f_c} = 1 + K \frac{\sigma_2}{f_c}$$

Fig. 4 shows the relationship between the major stress $\sigma_1/f_c$ and the biaxial restraint. In this case $\sigma_2 = \sigma_3$. It can be seen that the curve is divided into 3 ranges :

| $\sigma_2/f_c = \sigma_3/f_c$ | K |
|---|---|
| 0.1 - 0.5 | 3.7 |
| 0.6 - 1.0 | 3.2 |
| 1.1 - 1.6 | 2.7 |

Taking the size effect and the loading conditions into consideration, we have develop the failure envelopes shown in Fig. 5. The shape of these envelopes was based on the available results obtained from cubical specimens. The failure criterion is of a paraboloidical nature and may be expressed in the following way :

$$(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 0.75\ (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) - 2.05 f_c (\sigma_1 + \sigma_2 + \sigma_3) + 2.45 f_c^2 = 0$$

With additional tests this expression would be further confirmed or modified.

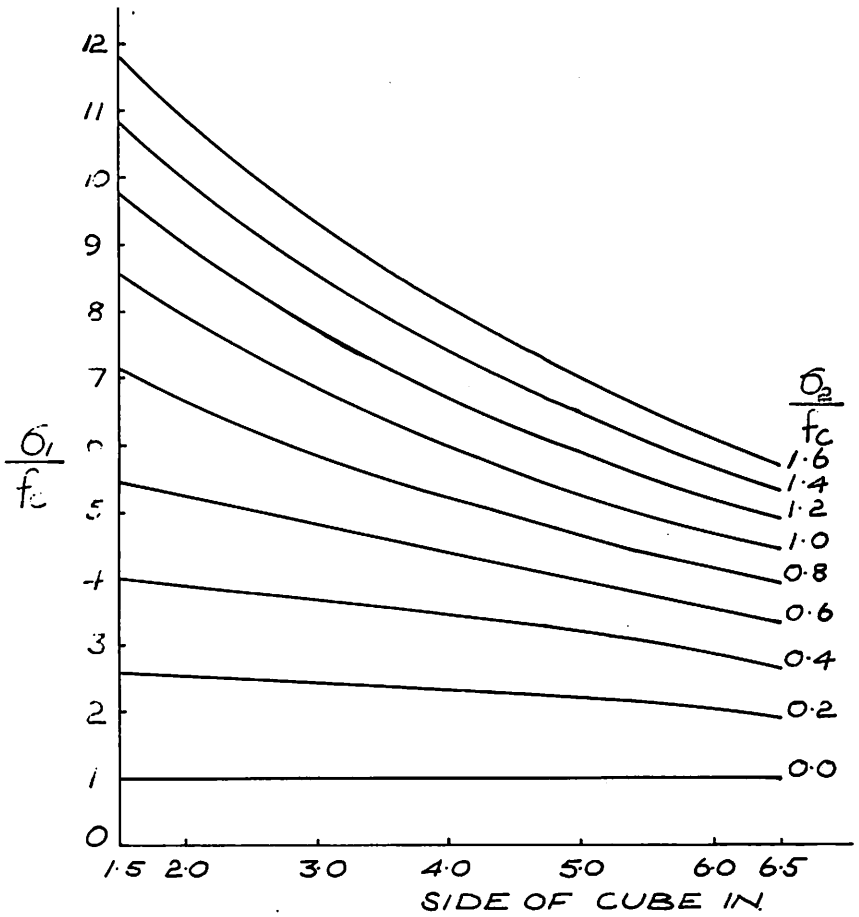



Fig. 1 - $\frac{\sigma_1}{f_c}$ w. r. t. cube size for set values of $\frac{\sigma_2}{f_c} (\sigma_2 = \sigma_3$

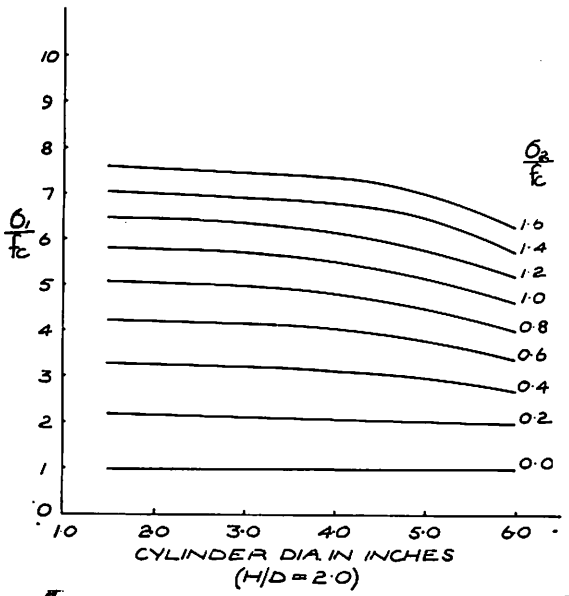



Fig. 2 - $\frac{\sigma_1}{f_c}$ w. r. t. cylinder DIA for different values of $\frac{\sigma_2}{f_c}$

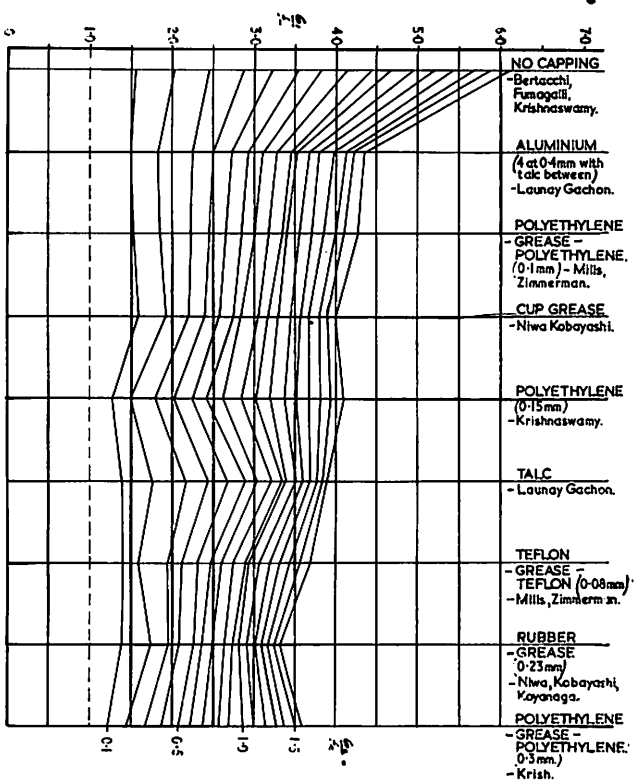



Fig. 3 - Effect of frictional restraint on triaxial compression ( $\frac{\sigma_1}{f_c}$ with reference to a 6" cube)

**Effect of Restraint on Triaxial Compression**

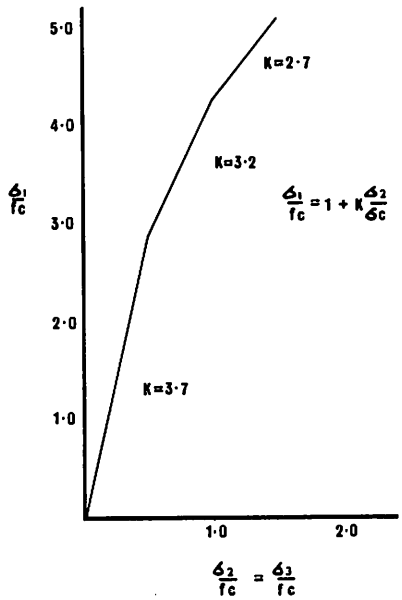



Fig. 4 - Effect of restraint on triaxial compression

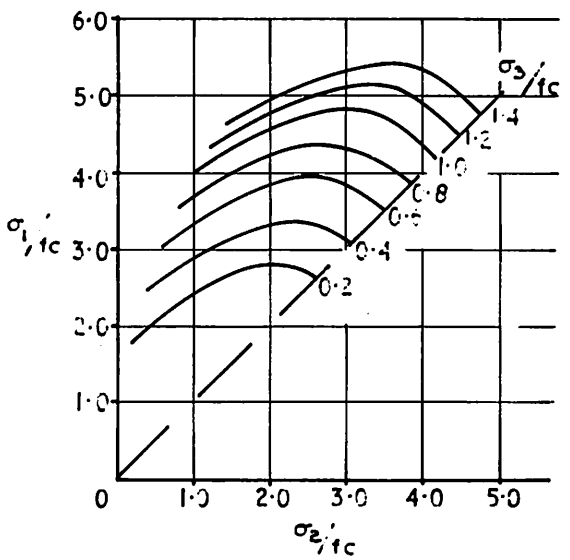



Fig. 5 - Ultimate strength of concrete under triaxial stress

Q D. LANGAN, U. K.

How is it possible to determine whether the surfaces of the specimens are friction free ?

A P. LAUNAY, France

Several uniaxial tests have been carried out using different support systems. We have selected the one which gave results corresponding to the ones obtained with cylindrical specimens $\left(\frac{\text{height}}{\text{diameter}} = 2\right)$, concerning ultimate stresses (failure) as well as strains. The chosen process is a piling of 4 aluminium sheets 0.5 mm thick, each face of which is lubricated with talc.

C J. LINK, Germany

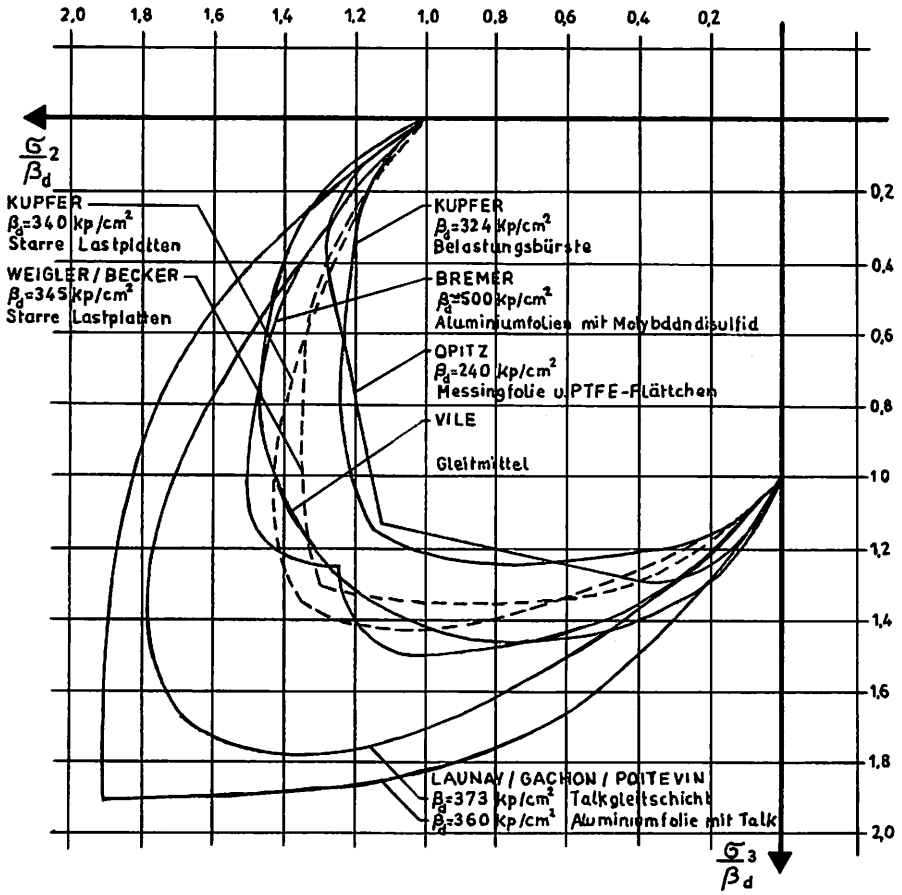
The authors had published their test results of concrete under triaxial stress elsewhere (1). Out of these results the biaxial strength, related to the uniaxial strength, is plotted in a figure (2) in comparison with the results of other investigators. Most of the other tests including the author's were carried out with special kinds of capping, others with rigid bearing-plates as shown in the figure by dash-lined curves. In the figure is visible, that the increase of strength in the biaxial state of stress is seldom more than 50 percent even with rigid bearing-plates, except the results of the authors with nearly 90 percent. The writer is quite concerned about the possibility, that the results presented by the authors may lead to an over-estimation of the multiaxial strength as long as the real reason for the great differences is unknown.

Since the investigations of multiaxial concrete strength and other material properties of concrete, e. g. multiaxial deformation, creep, shrinkage, temperature behaviour, are necessarily still going on, the designing engineer is constrained to choose materials data out of the great band-range of available test results. It would be very interesting to investigate in further studies the influence of concrete properties varying within the range of test results on the computational results of a simplified concrete structure, e. g. an axially symmetrical pressure vessel, under actual load conditions. Thereby information would be found about the different influences of concrete properties and their band-range on the design-stresses and strains and about the urgent advances to be made in testing special concrete properties. In this concern work is going to be induced by the German Arbeitskreis Förderungsprogramm Spannbeton-Reaktordruckbehälter in the Deutscher Ausschuss für Stahlbeton.

References:

(1) - P. Launay and H. Gachon "Strain and ultimate strength of concrete under triaxial stress", ACI Seminar on Concrete for Nuclear Reactors, Berlin (October 1970).

(2) - Arbeitskreis Förderungsprogramm Spannbeton-Reaktordruckbehälter im Deutschen Ausschuss für Stahlbeton, Bericht zur mehraxialen Festigkeit, Darmstadt (January 1971, unpublished).




Betonfestigkeit bei zweiaxialer Belastung - Vergleich mit und ohne Reibungsbehinderung

$\beta_d$ : einaxiale Festigkeit