FLOOR RESPONSE SPECTRA FOR MULTI-DEGREE-OF-FREEDOM SYSTEMS BY FOURIER TRANSFORM

R.H. SCANLAN

Department of Civil Engineering, Princeton University, Princeton, New Jersey 08540, U.S.A.

K. SACHS

Kraftwerk Union AG, D-6000 Frankfurt am Main, Germany

SUMMARY

The earthquake response spectrum has become a standard tool in earthquake engineering. This is especially true in nuclear engineering where response spectra are extensively used for the analysis of the structure as well as light equipment.

This paper describes a method of generating floor response spectra from a given ground response spectrum. This time-saving approach makes use of Fourier spectrum techniques and the randomness of phase angles.

In matrix form a structure having many degrees-of-freedom is described by the equation of motion with M, C, K as the mass-, damping-, and stiffnes matrices, and $\ddot{z}$ being the acceleration time history of the earthquake and I a direction vector.

If the Fourier spectrum $F\ddot{z}$ of the ground motion is known, then by standard methods the Fourier spectrum $F\ddot{a}$ of the equipment response can be obtained. The assumption of random phase angles for the synthetic time hystory $\ddot{z}$ seems reasonable. The response is then also a superposition of cosine waves with amplitudes C_n and random phase angles γ_n. Then m sets of n random numbers $r_{m,\,n}$ evenly distributed between $[-1, +1]$ are generated. The maximum expected value for the acceleration response of the equipment is

$$\sigma_m = \sum_{i=1}^{n} r_{m\ i} \times c_i$$

where the largest value σ_m as the response is taken. The starting values of $|F\ddot{z}|$ are obtained by a similar iterative scheme.

Good agreement with time history methods is obtained. This method is much faster than time history methods, which are being used in most applications.

I Introduction

The problem of design earthquake loadings and their applications has occupied investigators for some years. In particular,the response spectrum approach has gained a wide acceptance. In this method typical spectra of response are specified,and the structure in question,together with its contents,is designed to withstand the loadings implied by the given spectra.

This spectral approach has numerous attendant problems, two of which will be discussed in the present paper.These are the well-known concerns occasioned by the facts that phasing between structural response modes are not normally defined by the method,and the secondary response spectra of equipment on various levels of the structure are not ordinarily obtained from given spectral data in a direct manner.

The most common recourse in the present state of the art is to turn to time history methods,for which a real or artificial earthquake acceleration time history must be provided.Recourse to such methods does not automatically resolve the ambiguities which are present,however.

The present paper first examines the overall state of the art and then utilizes a Fourier transform method,with time history interpretations,for obtaining secondary re - sponse spectra when a given ground response spectrum has been specified for design.

II Response Spectra and Time Histories

Typically,in the structural analysis of nuclear power plants, the design ground or primary spectra are given. These usually consist of families of smoothed results corresponding to the maximum solutions of equ.(13) - (15) of section III below,for different choices of natural frequency ω , and damping ζ .The smoothing,adjusting,or straightening of curves of such specified families occasionally produces certain internal inconsistencies among the curves drawn for different dampings ζ . These questions are usually not of primary concern.Some questions of internal consistency will be touched on later,however.

When the spectral curves corresponding to the equipment analogs of eqs.(13) - (15) section III are developed, these constitute the "secondary" spectra applicable to equipment design. In the development of such spectra it is common to assume no stiffness or inertial feedback between the equipment motion and the exciting structural motion this is justified for relatively small equipment masses, and it is completely consistent with the basic concepts of secondary spectra. The present paper takes as its main goal the development of such secondary spectra without explicit use of time history methods.

When spectral methods are employed they constitute essentially an assumption about the levels of the anticipated earthquake as distributed over the frequency range. What is ignored therein is any definition of phasing between the responding modal components, such as would be reflected in a time history. Shock response spectra may in fact be said to be only slightly disguised versions of the absolute amplitude of the Fourier transform of the anticipated earthquake, the phasing (due to the ratio of imaginary to real parts of the transform) being considered an unspecified quantity. As is well known, the zero-damping velocity spectrum constitutes a rather close upper bound to the magnitude of the Fourier spectrum of the same quake, and it may, for this reason, be taken as a first approximation to the latter.

If now time history methods are also considered, indirect information about the amplitude of Fourier series coefficients is available from the given spectral data, but phasing between these components is not. When a recorded quake is employed as input, both amplitude and phasing are of course known, but when a synthetic time-history is developed, phasing information must be provided randomly (see Ref. [1], for example). This emphazises the fact that infinitely many time histories satisfying the same design spectrum can be developed, and that there remains an unresolvable ambiguity connected with the response to any single quake.

It appears interesting, therefore, to avoid if possible the details of unknowable time histories in design.

III Condensed Review of Problem Statement

In the interest of saving space the material presented below will not have all terms explicitely defined, but the notation generally follows customary forms.

Matrix equation of structural floor displacements q :

$$M \ddot{q} + C \dot{q} + K q = - M \ddot{z}(t) \{I\} \tag{1}$$

Matrix modal transformation:

$$q = \Phi \xi \tag{2}$$

Resulting independent equations:

$$\ddot{\xi}_i + 2 \zeta_i \omega_i \dot{\xi}_i + \omega_i^2 \xi_i = - \alpha_i \ddot{z}(t) \tag{3}$$

Modal participation factor:

$$\alpha_i = (\sum_r m_r \varphi_r^i) / (\sum m_r \varphi_r^{i\,2}) \tag{4}$$

Fourier transform of (3):

$$F_{\xi_i}(\omega) = - \alpha_i H_i(\omega) F_{\ddot{z}}(\omega) \tag{5}$$

Transfer function:

$$H_i(\omega) = (\omega_i^2 - \omega^2 + 2 i \zeta_i \omega_i \omega)^{-1} \tag{6}$$

Transform of modal acceleration:

$$F_{\ddot{\xi}}(\omega) = \omega^2 \alpha_i H_i(\omega) F_{\ddot{z}}(\omega) \tag{7}$$

Transformed relative acceleration $\ddot{q}_r$:

$$F_{\ddot{q}_r}(\omega) = \omega^2 H_{str,r}(\omega) F_{\ddot{z}}(\omega) \tag{8}$$

Structural transfer function, floor r:

$$H_{str,r}(\omega) = \sum_i [\varphi_r^i \alpha_i H_i(\omega)] \tag{9}$$

Transformed absolute acceleration, floor r:

$$F_{\ddot{a}_r}(\omega) = [1 + \omega^2 H_{str,r}(\omega)] F_{\ddot{z}}(\omega) \tag{10}$$

Dynamic equilibrium of equipment relative displacement y_e:

$$\ddot{y}_e + 2 \zeta_e \omega_e \omega \dot{y}_e + \omega_e^2 y_e = - \ddot{a}_r(t) \tag{11}$$

where "e" refers to equipment throughout.

Transformed absolute acceleration of equipment:

$$F_{a_e}(\omega) = [1 + \omega^2 H_e(\omega)] [1 + \omega^2 H_{str,r}(\omega)] F_{\ddot{z}}(\omega) \tag{12}$$

Time history solution of (3):

$$\xi_i = -1/\omega_i \sqrt{1-\zeta_i^2} \cdot \int_0^t e^{-\zeta_i \omega_i (t-\tau)} [\sin(\omega_i \sqrt{1-\zeta_i^2} (t-\tau))] [\alpha_i \ddot{z}(\tau)] d\tau \tag{13}$$

Modal relative velocity:

$$\dot{\xi}_i \cong - \int_0^t e^{-\zeta_i \omega_i (t-\tau)} [\sin \omega_i (t-\tau)] [\alpha_i \ddot{z}(\tau)] d\tau \tag{14}$$

Absolute modal acceleration:

$$\ddot{a}_{\xi_i}(t) \cong \int_0^t \omega_i e^{-\zeta_i \omega_i (t-\tau)} [\sin \omega_i (t-\tau)] [\alpha_i \ddot{z}(\tau)] d\tau \tag{15}$$

Absolute acceleration of floor r:

$$\ddot{a}_r(t) = \sum_i \varphi_r^i \ddot{a}_{\xi_i}(t) \tag{16}$$

Analogous results to (13) - (15) hold for equipment, where the input (16) replaces $\ddot{z}(t)$. The maxima of (14) and (15) constitute the ground relative velocity and absolute acceleration response spectra, as functions of ω_i, respectively. When the input is floor acceleration, the corresponding maxima constitute the applicable floor response spectra.

IV Overview of Methods for Obtaining Floor Response Spectra

The most accepted method is the time-history technique, wherein either a known recorded earthquake acceleration time-history is provided as input, or an artificial substitute is designed. Actual recorded earthquakes usually do not possess ground response spectra compatible with a given design spectrum. Hence the development of artificial substitutes has taken place. Ref. 1 exhibits one method wherein the earthquake acceleration z(t) is conceived of in the form

$$\ddot{z}(t) = \sum_{n=1}^{N} z_n \cos\left(\frac{2\pi n t}{T} + \varphi_n\right) \qquad (17)$$

where the N phase angles φ_n are random over $(0, 2\pi)$, the z_n being at first unknown, but by an iterative process described in [1] are eventually determined so as to be compatible with a given design response spectrum. This basic iterative process involves, in principle, development of the full time history response (15) for an assumed function $\ddot{z}(t)$ as in (17) and the selection of maxima for the definition of the acceleration spectrum Sa. Improved coefficients z_n are then determined by the iterative procedure:

$$z_n' = \frac{S_a^T}{S_a} \cdot z_n \qquad (18)$$

where S_a^T is the target spectrum. This method has already proved to be very effective.

Thus, the basic portion of the problem of spectrum compatibility can be considered resolved when the coefficients Zn are known through any process. The next part of the problem, that of determining floor response spectra, is in principle straight forward, when time-history methods are employed, but the objection encountered in practice is the great cost in analysis and computer time required for detailed development of the fairly large number of such spectra that may be required.

Many methods of short cutting this process have been developed, some utilizing intuitive or other quasi-empirical approaches. A number of these require some cross-check such as time-history. The more intuitive approaches are now being abandoned. Approaches based on stochastic methods have also been employed. These have the advantage of sound mathematical treatment. An important consideration, however, remains the definition of a representative power spectral density function for the earthquake. Because an earthquake is not a stationary stochastic process, some care has to be employed, in these stochastic approaches, to maintain properties consistent with recorded earthquake information.

V Floor Response Spectra by Fourier Transform Methods

The present paper will employ a method in some ways similar to the stochastic methods alluded to,but based on the Fourier transform tech - nique already outlined in eqs.(5) - (12) of section III.This method may be said to be inspired by the time-history method with the added desire of avoiding the expenditures of analysis and computer time inherent in the latter.For this reason,time histories will be alluded to as theo - retical background,but avoided in their final details where possible.

With $\ddot{Z}(t)$ assumed to have the form (17) the time-history of acceleration response of a structure with damping value ζ_s and natural frequency n_s may be assumed to have a similar form:

$$\ddot{a}(n_s,t) = \sum_{n=1}^{N} A_n(n_s) \cos\left(\frac{2n\pi t}{T} + \varphi_n\right) \quad (19)$$

Given then the target response spectrum $\overset{T}{Sa}(n_s)$ with which $\ddot{a}(n_s,t)$ is to be made compatible,the problem becomes one of either determining $\ddot{Z}(t)$ for direct application to this purpose,or of creating another function capable of reproducing $\overset{T}{Sa}(n_s)$.

Note first the absolute value of (10), for a single-degree structural mode of natural frequency n_s:

$$|F_{\ddot{a}}(\omega)| = |1 + \omega^2 H(\omega_s,\omega)| \cdot |F_{\ddot{z}}(\omega)| \quad (20)$$

which permits writing

$$A_n(n_s) = \left|1 + \left(\frac{2\pi n}{T}\right)^2 H(n_s,n)\right| Z_n \quad (21)$$

where

$$H(n_s,n) = 1/\left[\left(\frac{2\pi n_s}{T}\right)^2 - \left(\frac{2\pi n}{T}\right)^2 + 2i\zeta\left(\frac{2\pi n_s}{T}\right)\left(\frac{2\pi n}{T}\right)\right] \quad (22)$$

An assumption is introduced at this point,namely that Sa (ω_s) is pro - portional to $\sqrt{\sum_n A_n^2(n_s)}$. This is equivalent to estimating that maximum points on the spectral response target curve are located at some mul - tiple of σ, the standard deviation of the acceleration response.

The portion in question will in general be frequency dependant, i.e. a function of the structural natural frequency n_s so that Sa (n_s) may be written:

$$\overset{T}{S}_a(n_s) = K(n_s)\sqrt{\sum_n [A_n(n_s)]^2} \quad (23)$$

An iterative procedure is then set up to determine a set of coefficients of the form (21) such that (23) is satisfied,K(n_s) being determined in the process. To this end,let first Zn (n=1,2 ...N) be any set of initial approximations to the correct Fourier coefficients for $\ddot{Z}(t)$.

For example,one may choose Zn = 1 (all n) or find Zn from the approxi - mation to $F_{\ddot{Z}}(\omega)$ which is the undamped velocity response spectrum for $\ddot{Z}$ (t),if this quantity happens to be available.One then calculates An (n_s) (for all n) from (21) for a range of structural natural frequen- cies n_s :

The sum $\sqrt{\sum [A_n(n_s)]^2}$ (24) is then calculated for each value of n_s . This is then compared to $\overset{T}{Sa}(n_s)$ by (23):

$$\frac{\overset{T}{S}_a(n_s)}{\sqrt{\sum_n [A_n(n_s)]^2}} = K_{ns} \quad (25)$$

At this point it may be observed how far off,throughout the structural frequency range of interest, (usually $0 \leq n_s \leq 30$ Hz) the initial set Z_n

has caused the response spectrum to be estimated.

The points on the response spectrum of $\ddot{a}(t)$ will usually be strongly influenced by those components of $\ddot{z}(t)$ which have frequencies close to the structural natural frequency n_s in question. Hence an iteration formula in which n_s is taken as n for K (ns): $Z_n^{i+1} = K_n^i \cdot Z_n^i$ can be intuitively justified. The set Z_n^{i+1} is thus calculated, replacing the set Z_n^i in (24), and a new set A_n is obtained. Again the comparison criterion:

$$\frac{S_a^T(n_s)}{\sqrt{\sum_n [A_n(n_s)]^2}} = K(n_s) \qquad (27)$$

is applied to find an adjusted function K(ns). The process is repeated until the function K(ns) stabilizes, typically to unity for all ns .

VI Examples

The multi-degree reactor building model pictured in fig. 1 is taken as an example, for which the 2 percent target design acceleration spectrum given in fig. 2 is specified (upper curve with smoothed portions). The iteration process described above converges on a set of coefficients A_n for which with practical identity (curves indistinguishable): $\sqrt{\sum [A_n(n_s)]^2} = S_a^T(n_s)$ (28)

The matching of the same target spectrum by purely time-history methods (see Ref. [1]) gives the jagged envelope shown adjacent to the target curve.

The method may also be employed, with the final set to a new value of damping ζ. This gives rise to the two other smooth curves of fig. 2 , for ζ = 0,04 and ζ = 0,06 respectively, which are shown together with their time-history counter parts.

It should of course be noted that the coefficients do not define an actual earthquake level, but rather an envelope level. This is illustrated in fig. 3 where the upper curve is a plot of $\frac{T}{2} Z_n$ whereas the lower, jagged curve is the actual value $|F_{\ddot{z}}(\omega)|$ for the same case developed by time-history methods. The smooth curve envelopes the peaks of $F_{\ddot{z}}(\omega)$ and lies approximately a factor 1.6 to 1.7 higher than the mean of the jagged curve.

The method achieves its greatest effectiveness, however, when used to calculate secondary spectra directly. This is shown in figs. 4,5 and 6, where the Z_n are employed to calculate a "pseudo" transform for use in the absolute value of eqn. (12). The results are plotted against typical secondary spectra obtained for the same case by time-history methods.

VII Discussion

The method based upon the three concepts: basic Fourier transform theory,the assumption that the maxima of amplitude spectra are some multiples of the standard deviation of response and the reasoning that spectral response maxima are most influenced by exciting frequencies lying close to natural structural frequencies,is demonstrated by examples to produce results lying very close to time-history results.

The method has the advantage of avoiding some difficulties found with time-history methods,wherein,for example,it may become necessary to change the input earthquake to better match target spectra when new damping values are introduced, or specific details of a single time-history may give erratic results, or time-history calculations are time-consuming and costly. The time savings in this respect may be of the order of a factor 4 or 5 .

The method appears to give values which fall somewhat low (as compared to time-history results) when portions of spectral curves away from the peaks are being estimated. This is undoubtedly a result of using the highly oversimplified iteration device (26),which,nontheless,appears to give good overall results,especially in the important peak regions.

It should be remarked too that the method tends to give more reliable smooth results than any single time-history.

Reference

1 Scanlan,R.H. and K.Sachs, " Earthquake Time Histories and Response Spectra " Jnl. Eng.Mech. Div. ASCE, Vol. 100 EM 4, Aug. 1974, pp 635 - 655

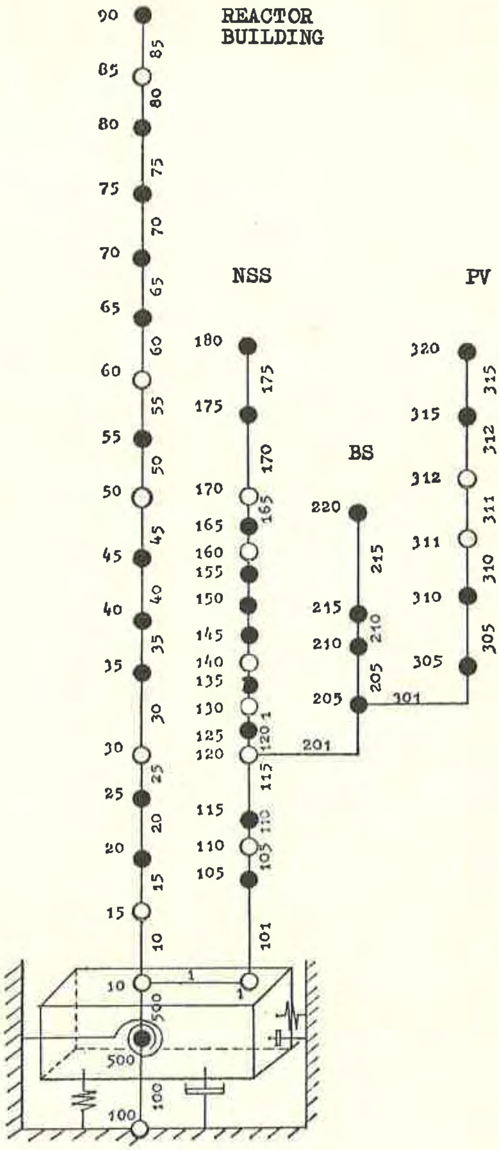

Fig. 1 Reactor Building Model

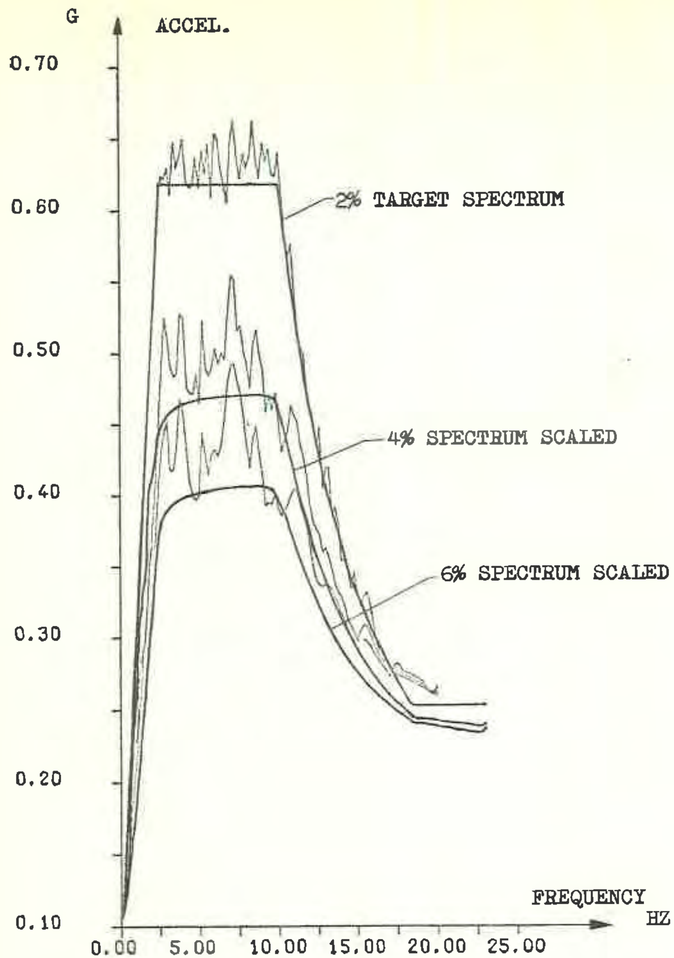

Fig. 2 Ground Response Spectra

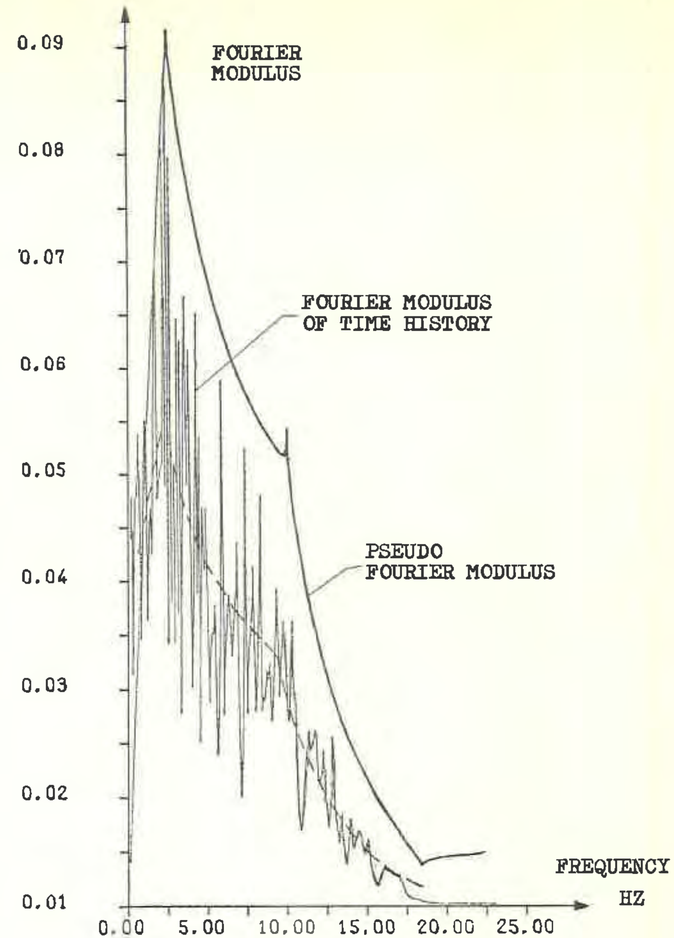

Fig. 3 Fourier Moduli

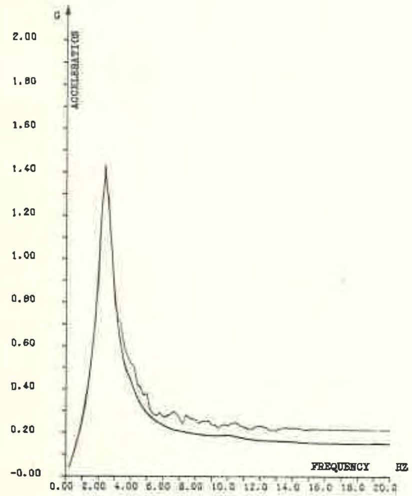

Fig. 4 2% Secondary Response Spectrum Point 55

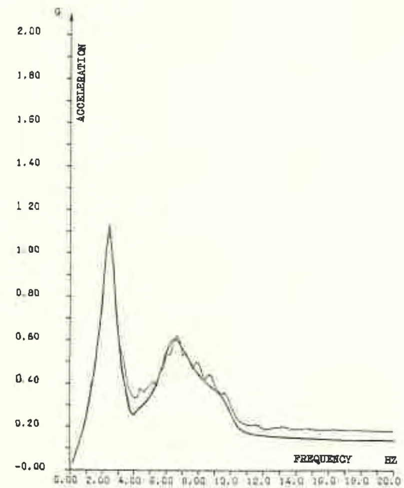

Fig. 5 2% Secondary Response Spectrum Point 115

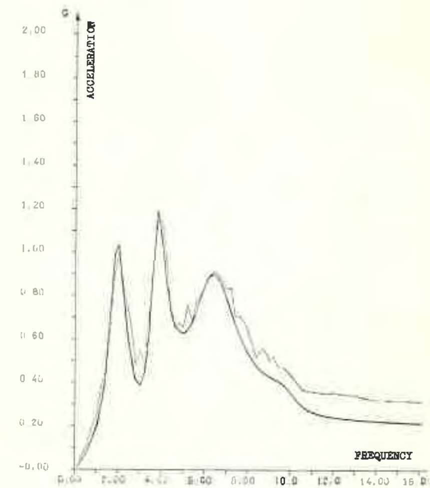

Fig. 6 3% Secondary Response Spectrum Point 310