CORRELATIONS OF ARTIFICIALLY GENERATED THREE COMPONENT TIME HISTORIES

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SUMMARY

It is a common practice in the industry to consider three component inputs in the seismic resistant design of nuclear power plant facilities. One can either input the three components one by one and combine the peak responses by root sum square approach, or input three components simultaneously and combine the responses algebraically at each time increment. When the latter approach is applied, a question arises as to how should the three components of the ground motion correlate to one another.

Newmark et al. recommended that three components should be independent. Penzien and Watabe proved mathematically that the three components are statistically uncorrelated provided that they are transformed into the principal coordinates of the ground motions. Their proof is based on the assumption that the three components are stationary random processes modified by the same deterministic intensity function to achieve appropriate non-stationarity. Furthermore, by the assumption of ergodic processes, Penzien and Watabe calculated variances and coveriances of strong motion accelerograms recorded at six (6) sites. After proper transformation into principal coordinates, they concluded that the major principal axis is always oriented towards the direction of the reported epicenter and that the minor axis is nearly vertical. Thus, they recommended that the three component statistically uncorrelated time histories, with the minor axis vertical, be applied to the building in the dynamic analysis. However, the three statistically uncorrelated components should be along the principal axis of the ground motion. They also recommended that a variety of the directions as the direction of the major principal axis should be considered in the dynamic analysis of a given structural system.

This recommendation can cause some difficulties in the industrial application. First of all, the time histories used in the industry are generated by an iteration method such that the resulting response spectra match the specified design spectra. During this process of iteration, some correlations are produced among three components. Secondly, the inputs are always applied in the directions of the principal axes of the structure itself. The epicenters of future earthquakes are unpredictable at the present time, and the probability of the principal axes of the structure coinciding with those of the ground motion is small.

In the paper the authors calculate variances and covariances of strong motion accelerograms recorded at 104 sites using the assumption of erogodic process. Examination of the results reveals that majority of the covariances between vertical and horizontal directions are higher than those of the two randomly oriented horizontal directions. This is an indication that the vertical axis is not always one of the principal axis. The contradiction is probably caused by the assumption that all three components have the same intensity function as assumed by Penzien and Watabe. In reality, they are not the same, hence the direction of principal axes of ground motion in general, is a function of time. Thus, the statistically uncorrelated time histories cannot be used as a criterion. Since the industry needs a criterion to define the correlations among the three components, this paper calculates the statistical values of the correlation coefficients from the recorded accelerograms at 104 sites of random orientation and recommends those values as the criterion of correlation among the three components.

1. Introduction

It is a common practice in the U.S. nuclear power industry to consider three component inputs in the seismic resistant design of nuclear power plant facilities. One can either input the three components one by one and combine the peak responses from three individual inputs by the root-sum-square approach or input three components simultaneously and combine the responses algebraically at each time increment. For majority cases, the root-sum-square combination of individual inputs yield satisfactory results. However, in some special cases, e.g., calculating soil bearing pressure underneath the mat [1], etc. it is preferable to input the three components simultaneously. When three components are input simultaneously, a question arises as to how should the three components of the ground motions correlate to one another?

Newmark et. al. [2] recommended that the three components should be independent. Penzien and Watabe [3] showed that an orthegonal set of axes can be defined for earthquake ground motions along which the covariances equal zero. This set of orthogonal axes is called the principal axes of the ground motions. Hence, they concluded that artifically generated components of ground motions need not be correlated statistically provided they are directed along a set of principal axes.

The kind of artificial time histories used by the industry now has to satisfy certain regulatory requirements. That is the response spectra of the artificial time history has to be consistent with the design spectra [2]. This requirement can be satisfied either by local suppressing or amplifying the spectrum of a given time history [4] or by iterating the amplitudes of a series of harmonics with random phase angles where the initial amplitudes can be estimated from the assumed power spectral density function or the undamped pseudo velocity spectrum [5]. During the process of generating this kind of time histories, some correlations are created among the three components. Hence, it is the intention of this paper to determine the acceptable level of correlations among the three components of artificial time histories.

2. The Principal Axes of Ground Motions [3]

Penzien & Watabe [3] suggested that the three dimensional ground motions be simulated as

$$
\begin{aligned}
a_x(t) &= \zeta(t)\ b_x(t) \\
a_y(t) &= \zeta(t)\ b_y(t) \qquad (1) \\
a_z(t) &= \zeta(t)\ b_z(t)
\end{aligned}
$$

Where $b_x(t)$, $b_y(t)$, and $b_z(t)$ are stationary random processes and $\zeta(t)$ is a deterministic intensity function giving appropriate nonstationarity to the ground motion processes. The intensity function should be established by the statistical study of the recorded accelerograms, and the power spectral density functions or corresponding autocorrelation functions should also be determined by the statistical study in order to properly simulate processes $b_x(t)$, $b_y(t)$ and $b_z(t)$.

The same ground motions can be represented by components $a_u(t)$, $a_v(t)$, and $a_w(t)$ along a set of orthogonal axes uvw. The transformation can be represented by the transformation.

$$\begin{Bmatrix} a_u(t) \\ a_v(t) \\ a_w(t) \end{Bmatrix} = \begin{bmatrix} a_{ux} & a_{uy} & a_{uz} \\ a_{vx} & a_{vy} & a_{vz} \\ a_{wx} & a_{wy} & a_{wz} \end{bmatrix} \begin{Bmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{Bmatrix} \tag{2}$$

Where the transformation matrix $\underline{a}$ consists of the direction cosines among xyz and uvw axes, and it is orthogonal.

Assuming $a_x(t)$, $a_y(t)$, and $a_z(t)$ are considered as zero-mean nonstationery random process as defined in Eq. (1), the covariance functions are

$$E\ [a_i(t)\ a_j(t+t)] = \zeta(t)\ \zeta(t+t)\ E\ [b_i(t)\ b_j(t+\tau)] \qquad i,j = x,y,z \tag{3}$$

Where E denotes ensemble average. If the process is Gaussian, these covariance functions completely characterize the ground motion processes in a probabilistic sense. Since real earthquake accelerograms show a very rapid loss in correlation with increase values of $|\tau|$ the influence of coordinate directions on the covariance functions can be investigated by considering the relations

$$E\ [a_i(t)\ a_j(t)] = \zeta(t)\ E\ [b_i(t)\ b_j(t)] \qquad i,j = x,y,z \tag{4}$$

Equation (4) can be expressed in the form

$$\underline{\mu}(t) = \zeta(t)^2\ \underline{\beta} \tag{5}$$

Where the covariance matrices $\underline{\mu}(t)$ and $\underline{\beta}$ are

$$\underline{\mu}(t) = \begin{bmatrix} \mu_{xx}(t) & \mu_{xy}(t) & \mu_{xz}(t) \\ \mu_{yx}(t) & \mu_{yy}(t) & \mu_{yz}(t) \\ \mu_{zx}(t) & \mu_{zy}(t) & \mu_{zz}(t) \end{bmatrix} ; \qquad \beta = \begin{bmatrix} \beta_{xx} & \beta_{xy} & \beta_{xz} \\ \beta_{yx} & \beta_{yy} & \beta_{yz} \\ \beta_{zx} & \beta_{zy} & \beta_{zz} \end{bmatrix} \tag{6}$$

and $\mu_{ij}(t) = E\,[a_i(t)\ a_j(t)]$; $\beta_{ij} = E\,[b_i(t)\ f_j(t)]$ $\quad i,j = x,y,z$ (7)

Equation (5) can also be expressed in the uvw coordinate system as

$$\underline{\mu}'(t) = \zeta(t)^2 \underline{\beta} \tag{8}$$

$$\text{where } \underline{\mu}'(t) = \begin{bmatrix} \mu_{uu}(t) & \mu_{uv}(t) & \mu_{uw}(t) \\ \mu_{vu}(t) & \mu_{vv}(t) & \mu_{vw}(t) \\ \mu_{wu}(t) & \mu_{wv}(t) & \mu_{ww}(t) \end{bmatrix} \; ; \quad \underline{\beta}' = \begin{bmatrix} \beta_{uu} & \beta_{uv} & \beta_{uw} \\ \beta_{vu} & \beta_{vv} & \beta_{vw} \\ \beta_{wu} & \beta_{wv} & \beta_{ww} \end{bmatrix} \tag{9}$$

Using the coordinate transformation matrix $\underline{a}$, we have

$$\underline{\beta}' = \underline{a}^t\ \underline{\beta}\ \underline{a} \tag{10}$$

Hence, Eq. (8) can be rewritten as

$$\mu'(t) = \zeta(t)^2\ \underline{a}^t\ \underline{\beta}\ \underline{a} \tag{11}$$

This covariance matrix transformation is analogous to the stress matri transormation. Thus, one can transform the matrix to the principal axes of the ground motions such the the off diagonal terms of the covariance matrix equal zero. Thus Penzien & Watabe recommended that Eq. (1) can be used to simulate three dimensional ground motions and the components need not be correlated statistically provided that they are in principal axes of the ground motion.

Furthermore, if the stationary processes $b_x(t)$, $b_y(t)$ and $b_z(t)$ are assumed to be ergodic, the variances and covariances can be obtained by the temporal average. Thus, B_{ij} of Eq. (7) can be expressed as

$$\beta_{ij} = \langle b_i(t)\ b_j(t) \rangle \qquad i,j = x,y,z \tag{12}$$

Where the triangular brackets denote temporal average. Penzien & Watabe calculated the variances and covariances of recorded acceleregrams at six different sites using the equation

$$\mu_{ij} = \langle [a_i(t) - \bar{a}_i]\ [a_j(t) - \bar{a}_j] \rangle_{t_1}^{t_2} \qquad i,j = x,y,z \tag{13}$$

where $\bar{a}_i$ and $\bar{a}_j$ are the mean values over the entire duration of motion. After transformation into the principal axes, they found that the major principal axis direction is very close to the epicenter direction and minor principal axis is nearly vertical.

3. Acceptable Level of Correlations among Components of Artifically Generated Time Histories

In the dynamic analysis, the ground input is always applied in the direction of apparent principal axes of the structure itself. If the concept of the principal axes of ground motions is applied, one has to assume that the apparent principal axes of the structure coincide with the principal axes of the ground motions. Furthermore, the artifical time histories used in the industry has to satisfy certain regulatory requirements such that the response spectra of artificial time history are consistent with the design spectra [2]. In order to generate this kind of spectra compatible time history, the motions described in Eq. (1) has to be modified [4, 5]. During the process of modification, some correlations are generated among the components. The question arises as to what is the acceptable level of correlations? In order to answer this question, the correlation coefficients between components (H_1, V), (H_2, V), and (H_1, H_2) are calculated for recorded accelerograms in 104 events [6].

The correlation coefficients are calculated by the relation

$$\rho_{ij} = \frac{< [a_i(t) - \bar{a}_i] \, [a_j(t) - \bar{a}_j] >}{\sigma_i \, \sigma_j} \qquad i,j = x,y,z \quad i = j \qquad (14)$$

where the triangular brackets denote the temporal average, and $\bar{a}_i$, and σ_i are temporal mean and temporal standard deviation respectively. Among all the correlation coefficients calculated, no consistent trend of (H_1, V) and (H_2, V) components less the (H_1, H_2) component could be determined. After the correlation coefficients are calculated, their statistical properties are also computed and shown in the table below.

Table I - Statistical Properties of Correlation Coefficients for Strong Motion Accelerograms Recorded in 104 Events

Components	True Mean	Deviation	Absolute Mean	Absolute Maximum	Absolute Minimum
(H_1, H_2)	0.0029	0.2116	0.1632	0.6801	0.0014
(H_1, V)	0.0187	0.1774	0.1387	0.4957	0.0004
(H_2, V)	0.0055	0.1841	0.1321	0.7430	0.0005

As shown in the Table, if we use the absolute mean value as the criterion, the acceptable level of absolute correlation coefficients is equal to or less than 0.16. However, if we use mean plus one standard deviation as the criterion, the acceptable level of absolute correlaton coefficients is equal to or less than 0.2.

4. Conclusions

When three components of artificial time histories applied simultaneously in the dynamic analysis, Newmark et. al. [2] recommended that they should be independent. Penzien & Watabe [3] suggested that the three components need not be correlated statistically provided that they are in the principal axes of the ground motions.

Since the three components of artificial time histories used in the industry have to satisfy the regulatory requirement of being consistent with the design spectra, some correlations exist among the component. In this paper, the statistical values of the correlation coefficients of strong motion accelerograms recorded in 104 events are calculated. Based on this study, it is recommended that the acceptable level of correlation among any two of the three components can be 0.16 or 0.2 depending on whether the absolute mean or mean plus one standard deviation is used as the criterion.

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