

## A CONSTITUTIVE EQUATION FOR CREEP FRACTURE UNDER CONSTANT, VARIABLE OR CYCLIC POSITIVE STRESS

J. D. SNEDDEN

*Creep and Tribology Division, National Engineering Laboratory,  
East Kilbride, Glasgow G75 0QU, Scotland, United Kingdom*

### SUMMARY

Prediction of creep fracture of metals under variable stress is one of the most difficult problems of applied mechanics. At NEL this problem is under investigation using an approach in which creep is represented by two macroscopic components; an anelastic (reversible) component and a plastic (irreversible) component. Under variable loading conditions, the anelastic component's behaviour will be most important and, if an experimental programme is logically planned, the structural processes responsible will be implicit in the resulting constitutive equation describing the material's behaviour.

The present paper deals with the development and application of a constitutive equation for creep fracture of RR58 Aluminium alloy at 180 °C under variable stress and such a constitutive equation can be extrapolated to cover long-time behaviour just as with conventional constant stress creep fracture equations. Constant stress, in fact, is one of the boundary conditions of the general constitutive equation, representing zero prior damage. The other boundary condition is that of "cadence loading" in which the stress is completely removed and then re-applied in a cyclic fashion.

An exploratory paper (SMiRT-3 Paper L3/4) had shown that an important factor affecting residual life was the stage of creep at which a stress change took place. Accordingly, in the present work, stress changes were arranged to take place approximately at two stages during primary creep, one stage corresponding to minimum creep rate and one stage in established tertiary creep. The creep fracture curve for virgin material was also available, thus giving five levels of damage.

A datum stress level was selected and specimens were damaged to each of the above stages of creep, whereupon the stress was changed and the residual life, at a series of new stress levels, determined. Additionally, a few other tests were carried out with initial stress levels other than the datum stress, in order to check the invariance of the damage with respect to the initial stress.

Correlation and regression analyses of the results from 32 tests gave an exponential equation, the constants being dependent upon the degree of damage. This equation predicts the residual life and, of course, the total time to fracture is obtained by summation of the stage times and the residual life. In order to apply the equation to multi-change variable and cyclic tests, the concept of effective damage has to be introduced, effective damage being the difference between the predicted residual life following a stress change and the life from virgin conditions at the new stress level. Acceleration of initial damage is shown to be highest during early primary creep which is when the percentage anelasticity is highest.

A comparison between the predictions of the constitutive equation and those of the life fraction method, for 33 variable and cyclic creep fracture tests, shows the equation to be a significant improvement with an average error of -18.5% as opposed to 62.7%. In effect, this investigation is a practical development of the hypotheses of Goldhoff, ASTM, STP 515 (1972), and Woodford, Joint IME/ASTM/ASME Conf., Sheffield (1974), and it shows how some of the notions of these authors are invalid.

## 1. INTRODUCTION

The predictions of creep deformation and fracture of metals under variable stress and temperature are two of the most difficult problems associated with practical operation of components and structures.

Since creep is a highly structure-sensitive property of a material, due recognition must be made of its metallurgical features even when design is concerned mainly on a phenomenological level with such factors as stress, strain, time to fracture and the ensuing dimensioning of components. Therefore, the development is being pursued at NEL of constitutive equations in which the important structural processes involved on a microscopic scale are implicit. The equations are obtained from correlation and regression analysis of test results within the framework of a logical analysis and specified limitations on the mathematical model.

With this background, creep is represented on a macroscopic level by two components, each of which has structural processes which vary in relative importance with time, stress and temperature. The components are

- (a) the plastic component which is irreversible, and
- (b) the anelastic component which is reversible.

The derivation of constitutive equations involves the use of multiaxial stress criteria and associated anisotropy and, for application to statically indeterminate components and structures, of the representative or reference stress method.

Under transient stress and/or temperature conditions, the anelastic component is extremely important since it is reversible. Thus the constitutive equations for predicting creep and fracture under these transient conditions are strongly influenced by the kinetic behaviour of the anelastic component. For creep fracture under variable conditions, the complexity of predicting the deformation behaviour is avoided and again the effects of kinetic anelasticity are implicit in the constitutive equation for these conditions. Extrapolation of such an equation is possible, just as of an equation for constant conditions, although limitations may be necessary since convergence/divergence probably occurs due to more than one level of damage being involved and the residual life being history dependent.

The problem of isothermal creep fracture under variable positive stress conditions is investigated in the present paper and affords a fairly simple test of development of a constitutive equation for transient conditions, since only stress is varied and reversed stress is excluded.

The material used is RR58 Aluminium alloy tested at a temperature of 180°C. This material has been shown to be structurally stable for long times at temperature<sup>[1]</sup> and no damage results from soaking at temperature<sup>[2]</sup>.

## 2. DEVELOPMENT OF CONSTITUTIVE EQUATION

The first step in the development of a constitutive equation for creep fracture under

variable positive stress is to obtain the fracture behaviour under constant stress, which represents the condition of zero prior damage. Tests were carried out in the stress range of 17 to 26 hbar, with fracture times  $t_F$  ranging from 3251.4h to 100.3h and strains varying from 0.5 to 2.6%, the tendency being for the ductility to increase with increasing stress.

The methods of measuring creep damage have been varied, ranging from impact properties, through hardness values, to residual life and ductility. In the author's opinion, only the highly structure-sensitive residual life and ductility values are valid measures, and the former is selected since the determination of fracture strain values is far less reliable than that of time, which in the present series of experiments can be measured to better than 0.1 hour. More importantly, the object of developing a constitutive equation is to predict creep fracture time. Therefore the problem of representing creep damage from this point of view reduces to selecting the most suitable non-dimensional representation of time.

In the original 1971 work of Goldhoff and Woodford<sup>[3]</sup> on Cr-Mo-V steel, these authors decided that the best correlation between residual life and prior damage was that of the product variable  $(\xi_o \cdot t_o)$ , where  $\xi_o$  represented the prior strain and  $t_o$  the prior time. Additionally, the initial damage was accumulated at four temperatures as well as under different stresses. Goldhoff and Woodford<sup>[3]</sup> then combined the product variable  $(\xi_o \cdot t_o)$  with temperature in a conventional parametric representation, and obtained a good correlation with residual life, which was determined at only one stress level and at one of the initial temperatures.

The choice of the product variable  $(\xi_o \cdot t_o)$  was purely empirical, and when damage is considered on a scalar level it is proportional to the components of stress, plastic strain rate and time for a given temperature. However, Goldhoff and Woodford's use of  $(\xi_o \cdot t_o)$  in the correlation with residual life  $t_R$ , led to the inevitable finding that  $t_R$  was dependent on the value of the initial stress, the tendency being that  $t_R$  decreased with decreasing initial stress for a given value of temperature and  $(\xi_o \cdot t_o)$ . If the alternative correlation of  $t/t_F$  is used with  $t_R$ , the same dependence on initial stress for a given value of  $t/t_F$  is found, although the tendency is not very strong.

For the present RR58 Aluminium alloy at 180°C, full details of the anelastic component of creep have been given in reference [4], where it was shown that the anelastic creep  $\gamma_{TR}$ , was given by

$$\gamma_{TR} = 1.918 \times 10^{-6} \sigma^{1.34} t_c^{0.15} \quad (1)$$

where  $t_c$  is the creep period, ie time under load and  $\sigma$  the stress in hbar. The percentage anelasticity was found to range from 100% at low stresses and short creep periods to only a few per cent at high stresses and long creep periods. Therefore, for a given absolute time the percentage anelasticity would increase with decreasing stress. However, if experimental total creep curves are considered and the anelastic strains computed from equation (1), the percentage anelasticity may be calculated and its variation with NON-DIMENSIONAL time,  $t/t_F$ , determined.

These calculations show that the stress dependence is not very strong, when time is considered in this non-dimensional form. Therefore, it seems a reasonable assumption that damage may be represented by  $D = t/t_F$  and that  $D$  is relatively independent of initial stress. However, for established tertiary creep Goldhoff and Woodford's finding on initial stress dependence could attract some support.

Since it is virtually impossible to quantify physical damage processes, such as dislocation refinement and cavitated grain boundaries and triple-point cracking, it is clear that the easily quantified  $t/t_F$  representation of damage is the best compromise. Accordingly, for the development of a general constitutive equation it was decided to allow for the effect of prior damage on residual life by using four levels of  $t/t_F$  as well as the virgin condition. The four levels of prior damage selected were 0.025, 0.0999, 0.1935 and 0.5243, and damage was accumulated under the initial stress,  $\sigma_i$ , to each of these levels, at which time the stress was changed to the final value  $\sigma_f$  and the residual life  $t_R$  determined.

At each of the damage levels, either 6 or 7 tests under various final stresses were carried out, most of the initial damage being accumulated at the stress level of 20 hbar. However, in order to allow for any effects of different initial stress levels as discussed above, either two or three other initial stresses were also used. For the initial stress level of 20 hbar, of the four levels of damage two represent primary creep, one is at approximately the minimum creep rate, and one is in established tertiary creep.

#### 2.1 Analysis of Single-Change Results

A total of 32 tests were carried out at the 5 levels of damage. In order to develop a general constitutive equation, each of the damage levels was considered separately, and their individual residual life equations were determined by conventional correlation and regression analysis. Limitations derived from a logical analysis exclude the use of polynomials, since a fracture curve for a thermally stable material cannot have turning points.

Analysis of the 7 virgin tests, ie zero damage and  $\sigma_i = \sigma_f$ , gave a best correlation coefficient of

$$r = -0.9901$$

for an exponential relationship between the dependent variable, residual life  $t_R$ , and the independent variable, final stress  $\sigma_f$ . The significance of the correlation coefficient was determined by the "t" test where

$$t_t = r \left( \frac{N-2}{1-r^2} \right)^{1/2}$$

$N$  being the number of experimental points. For this level of damage,  $t_t = -15.7749$ , which corresponds to a probability of 99.99%. Overall, the correlation between the variables for a power relationship was slightly inferior.

The regression equation for zero damage was computed to be

$$t_R = 1.4316 \times 10^{(6-0.1587\sigma)} \quad (2)$$

and, for this case,  $t_R = t_F$  since there was zero prior damage. In general

$$t_F = \sum_i t_i + t_R$$

where  $\sum_i t_i$  is the summation of all stage times prior to the final one. The standard error of estimate of the regression equation is computed from

$$S_{\log t_R} \cdot \sigma = S_{\log t_R} (1-r^2)^{\frac{1}{2}}$$

where  $S_{\log t_R}$  is the standard deviation of the values of the transformed dependent variable. The standard error of estimate of equation (2) was

$$S_{\log t_R} \cdot \sigma = \pm 0.0659$$

which corresponded to upper and lower multipliers of 1.1640 and 0.8591 on equation (2). Therefore the scatter on  $t_F$  for virgin conditions is approximately  $\pm 15\%$ . The  $t/t_F$  damage values used were computed from equation (2) and were not derived from the experimental values.

A similar analysis of the 6 tests carried out at the smoothed damage level of 0.025, computed from equation (2), gave a correlation coefficient of

$$r = -0.9969$$

for an exponential equation, with  $t_t = -25.2996$ , thus giving a probability of 99.99%. The regression equation was

$$t_R = 1.6401 \times 10^{(6 - 0.1692\sigma)} \quad (3)$$

with  $S_{\log t_R} \cdot \sigma = \pm 0.0355$ ,  
thus giving multipliers of 1.0853 and 0.9214.

The damage level of 0.0999 yielded a correlation coefficient of

$$r = -0.9881$$

with  $t_t = -14.3842$ , and thus gave a probability of 99.99% again. The regression equation was

$$t_R = 2.8281 \times 10^{(6 - 0.1855\sigma)} \quad (4)$$

with  $S_{\log t_R} \cdot \sigma = \pm 0.0693$   
and multipliers of 1.1731 and 0.8524.

The correlation coefficient for  $D = 0.1935$  was

$$r = -0.9871$$

with  $t_t = -12.3495$  and a probability of 99.98%. The regression equation for the 6 tests at this level of damage was

$$t_R = 2.3759 \times 10^{(6 - 0.1823\sigma)} \quad (5)$$

with  $S_{\log t_R} \cdot \sigma = \pm 0.0660$ ,  
thus giving upper and lower multipliers of 1.1642 and 0.8590 respectively.

Finally at the damage level of 0.5243, which represents well established tertiary creep for all the initial stress levels, the correlation coefficient was

$$r = -0.9401$$

with  $t_t = -5.5142$  and the resulting probability level of 99.47%. The regression equation was  

$$t_R = 7.0639 \times 10^{(7 - 0.3008\sigma)} \quad (6)$$

and the associated standard error of estimate was computed to be

$$S_{\log t_R, \sigma} = \pm 0.3062,$$

which implied upper and lower multipliers of 2.0237 and 0.4941 respectively on equation (6).

The fact that the probability level of the correlation coefficients of equations (2)-(6) hardly decreases, even at the damage level corresponding to established tertiary creep, lends confidence to the general method of approach and also to the degree of development and regularity of the present material.

In general, therefore, the residual life equations are of the form

$$t_R = A10^{b\sigma} \quad (7)$$

where A and b are dependent on the value of the damage D.

Analysis of the A values showed that an exponential relationship between A and D was again the best. The actual correlation coefficient for an exponential relationship was

$$r = 0.9682,$$

while for a power relationship it was only

$$r = 0.3688.$$

The regression equation was

$$A = 1.1748 \times 10^{(6 + 3.2007D)} \quad (8)$$

with  $S_{\log A, D} = \pm 0.1574$ .

Analysis of the relationship between b and D showed that again an exponential equation was best, although a simple linear relationship was only marginally inferior. For the exponential relationship

$$r = 0.9783,$$

whereas for the linear relationship,  $r = 0.9732$ .

The regression equation was computed to be

$$-b = 0.1592 \times 10^{(0.5069D)} \quad (9)$$

with  $S_{\log b, D} = \pm 0.0204$ .

Therefore, the general constitutive equation for creep fracture is obtained from equations (7)-(9), and it is of interest that when  $D = 0$ , i.e. virgin conditions and constant stress, the general equation reduces to

$$t_R = 1.1748 \times 10^{(6 - 0.1592\sigma)} \quad (10)$$

which should be compared with the individual virgin equation (2).

The general constitutive equation shows convergence of residual life lines with decreasing final stress value. Therefore, when extrapolation is taken to relatively low stresses and long times to fracture, it is inevitable that the residual life lines will cross over, and hence it will be predicted that prior damage will lead to longer residual lives than those in the virgin condition. Whether this is valid remains to be established by experimental investigation. In many prior investigations<sup>[5]</sup> it has been claimed that this phenomenon can happen and in any case there was convergence, at low stresses and long times to fracture, towards a common curve.

As a matter of interest, the stress level,  $\sigma_{EX}$ , can be calculated at which any two lines of residual life converge. From equation (7), for two levels of damage to give the same residual life

$$A_1 10^{b_1 \sigma} = A_2 10^{b_2 \sigma}$$

and  $\sigma_{EX} = \frac{\log A_2 - \log A_1}{(b_1 - b_2)}$  (11)

In general, a line of residual life for a given damage level will cut the line of lowest damage level first, and then the next lowest, and so on. Therefore, if the possibility is excluded of damaged material being stronger than virgin material, the limit of extrapolation will be given from equation (11), the virgin condition being one of the damage levels.

Clearly, extensive extrapolation is possible when the damage is significant.

### 3. APPLICATION OF CONSTITUTIVE EQUATION

The general constitutive equation for creep fracture developed in the previous section is limited to positive stress only. Thus fatigue, caused by reversed stressing, is excluded; hence the boundary conditions are

- (a) constant stress, ie zero prior damage, as given by equation (10), and
- (b) cadence loading, wherein the stress is completely removed and then re-applied to the same level after creep recovery has exhausted itself.

Before considering the application of the constitutive equation to multi-change conditions, the effective damage,  $D_f$ , immediately following a stress change must be investigated as a first step.

For a given value of  $D = t/t_F$  at the initial stress  $\sigma_1$ , the residual life at the new stress level  $\sigma_f$  may be calculated from equations (7)-(9). Therefore, the effective damage immediately following the stress change is given by

$$D_f = \left( \frac{t_F - t_R}{t_F} \right) = 1 - \frac{t_R}{t_F} \quad (12)$$

The value of  $1 - \frac{t_R}{t_F}$  is dependent upon the initial damage  $D$  and the final stress  $\sigma_f$ . As a result of the nature of the dependence of  $t_R$  upon  $D$  and  $\sigma_f$ , as given in equations (7)-(9), the value of  $D_f$  increases with increasing  $\sigma_f$  for a given value of  $D$ .

For the life fraction law, the effective damage immediately following a stress change is exactly as before the stress change. For the present RR58 Aluminium alloy, the life fraction law is optimistic.

Since the importance of the role of the anelastic component of creep has been emphasized, it is logical to investigate the dependence of  $D_f$  upon  $D$ , noting again that as  $D$  increases the percentage anelasticity decreases.

When the acceleration of  $D_f$  is represented by  $(D_f - D)/D$ , and  $D$  is accumulated at a given stress level for which the value of  $D$  corresponding to the minimum creep rate is known, the effect of anelasticity is revealed. Damage acceleration is very high during early primary creep, when the percentage anelasticity is high, but decreases rapidly as tertiary creep is approached, thus proving the importance of kinetic anelasticity during variable stress creep fracture.

RR58 Aluminium alloy has been shown<sup>[6]</sup> to be a material which is highly resistant to intercrystalline cracking and is therefore for purely tensile loading a non-cracking material. The damage acceleration behaviour for other types of materials may, however, be substantially different.

In multichange applications the effective damage following the first change is given by equation (12), after which additional damage is accumulated at the new stress level for a time  $t_1$ . Thus the total damage when the stress is next changed is

$$D = \frac{t_{F1} - t_{R1} + t_1}{t_{F1}} \tag{13}$$

When the value of relation (13) is substituted in equations (7)-(9), the value of  $t_R$  at the new stress level is calculated.

From continuing work on kinetic anelasticity it is suggested that the predictions of the constitutive equation should tend, on average, to be conservative for multichange conditions, while for single-change tests, the equation will be quite accurate. The greater the number of changes, the more conservative should be the prediction of the constitutive equation.

Woodford<sup>[7]</sup> developed the earlier ideas of Goldhoff and himself<sup>[3]</sup> to include the case where a range of final stresses was involved, and for a given initial stress hypothesized that if the stress was increased after a given amount of damage, expressed simply as  $t/t_F$ , and the residual life measured at the new stress level, or decreased and the residual life again measured, then the three residual lives (ie including the constant stress case) would be on a common line representing a line of constant damage. Woodford called all these lines, for various values of initial damage, constant damage lines in terms of remaining life. It appears that he intended this in terms of the remaining life at the initial datum stress, since he commented upon the convergence of these residual life lines at low stress.

Woodford<sup>[7]</sup> then stated that all such lines were independent of the point of entry on them, and referred to this property as reversibility. In the present author's opinion, it



appeared unjustifiable that the residual life at the datum stress and without a dynamic change should lie on a common line with the residual lives, following a dynamic change. The results of a few exploratory tests supported this opinion.

However, when a residual life line is determined from a single datum initial stress, a point corresponding to the datum stress level does exist on the residual life line. As mentioned earlier, one of the boundary conditions of the constitutive equation was the case of cadence loading. It therefore appears logical that, since kinetic anelasticity is the important element of the dynamic acceleration of damage, the point at the datum stress level actually represents the residual life when the stress is first removed then re-applied following the exhaustion of creep recovery. Bearing in mind the expected scatter of at least  $\pm 15\%$  on  $t_F$ , this cadence loading theory was found to be quite accurate when an investigation was carried out at the datum stress level of 20 hbar.

### 3.1 Predictions of the Constitutive Equation

The predictions of the constitutive equation have been computed for 33 tests, the details of which are given in Table I. Tests 36 to 61 have previously been reported<sup>[8]</sup>, when they were used to investigate the range of life fraction summations under varied conditions. Tests 62 to 68 were carried out additionally, to extend the range of conditions and thus give a total of 33 multi-change experimental results.

The range of the experimental tests of Table I is divided into individual areas, as follows. Tests 36-41 are examples of initial stress-relaxation and are examples of continuously reducing stress history during purely primary creep. Tests 42-55 are tests representing both increasing and decreasing stress histories during primary creep, with either two or three changes of stress. On the other hand, tests 56-61 are examples of variable stress confined to changes during tertiary creep. Finally, tests 62-68 combine the stages of creep, to give continuously cycling stress changes throughout the life.

Table I gives the computed values of  $t_F$  from the constitutive equation as well as from the life fraction law. The percentage errors of the two methods of prediction relative to the experimental values are also tabulated.

If the six tests having initial stress-relaxation, as given by tests 36-41, are considered first, the average error of the predictions from the constitutive equation is 18.1%, while that of the life fraction law is 47.7%. The errors of the equation range from 3.4% to 29.5%, and for the life fraction law they range from 32% to 59.2%. All the predictions are therefore optimistic, and in each individual case the predictions of the equation are better and safer than those of the life fraction law.

The method of obtaining an equivalent stress for stress-relaxation has previously been discussed<sup>[8]</sup>, and simply involves substituting the appropriate values of creep strain and time in the primary creep equation and solving for the equivalent constant stress to give the same strain in the same time. The percentage anelasticity is practically identical using this

method.

Consideration of the fourteen tests having two or three stress changes during purely primary creep, as represented by tests 42-55, reveals that the average error of the equation is -14.3%, whereas for the life fraction law it is 58.9%. The errors range from -39.8% to 29.2% for the equation, and from 15.4% to 138.6% for the life fraction law. Only two of the fourteen predictions of the equation are optimistic, and twelve of the fourteen predictions of the equation are more accurate than those of the life fraction law.

Tests 56-61 show the life fraction law in its best light, since the changes are confined to tertiary creep, the region where the percentage anelasticity is lowest. The average error of the equation is -44.6%, while for the life fraction law it is only 5.7%. The life fraction predictions range from -13.6% to 20.2% but again all the predictions of the equation are conservative, ranging from -84.2% to -9.9%. In five of the six tests, the life fraction predictions are more accurate, and for test 57 the one and only conservative prediction of the life fraction law appears. For this test the life fraction summation was 1.0705, and this is the only summation value greater than unity for the sixty-one tests having stress changes. This exception is, however, insignificant with respect to the expected scatter of  $t_F$  values as given by the standard errors of estimate.

Finally the seven tests having a cyclic stress history are represented by tests 62-68. For these tests the average prediction error of the equation is -36.2%, and for the life fraction law it is 132.0%. Again the life fraction predictions are always optimistic, in some cases very dangerously so. Only in one case is the equation's prediction optimistic, although the errors range from -85.4% to 96.5%. Tests 65, 66 and 68 had some minor deviations in the cyclic times at weekends, and in the computations the actual times were used.

The overall average error for the thirty-three test results given in Table I was -18.5% for the constitutive equation's predictions, and 62.7% for the life fraction predictions. The equation's errors ranged from -85.4% to 96.5%, while for the life fraction law the errors ranged from -13.6% to 298.1%. Taken relative to the material's minimum expected scatter of  $\pm 15\%$  on times to fracture under constant stress conditions, the average error of the developed constitutive equation can be regarded only as exceptionally good, particularly since it is on the safe side. Even so, the average error of the life fraction predictions is also good, although the predictions always tend to be optimistic, ie dangerous.

#### 4. CONCLUSIONS

- (1) The developed constitutive equation for creep fracture under variable or cyclic stress leads to significantly better predictions of life than those of the conventional life fraction law.
- (2) The constitutive equation has an inbuilt tendency to give conservative, ie safe, predictions of creep fracture time under multi-change conditions.
- (3) The RR58 Aluminium alloy at 180°C is not prone to intercrystalline cracking or cavitation. Neither is it significantly damaged by soaking unstressed at

temperature. The principal damaging process therefore appears to be due to dislocation movement, and under variable stress conditions kinetic anelasticity is the factor responsible for damage acceleration.

- (4) It is suggested that the compromise reached in this investigation in using five levels of damage, including the virgin condition, is justified by the resulting increase in accuracy of prediction. The total number of tests required to develop the constitutive equation was thirty-two, compared with only seven required for the life fraction method. If more levels of damage were included and additional tests carried out at each damage level, the resulting constitutive equation should have a higher degree of confidence, although the added complexity might not be justified since the accuracy of prediction of the simple life fraction method is quite good.

#### ACKNOWLEDGEMENT

This paper is published by permission of the Director of the National Engineering Laboratory, Department of Industry, and is reproduced by permission of the controller of HMSO.

#### REFERENCES

- [1] DOYLE, W. M., "The Development of Aluminium-RR58 Aluminium Alloy," Aircraft Eng., 1969, November, pp. 11-14.
- [2] BERRY, D. A. and ANSTEE, R. F. W., "A Preliminary Study of the Creep and Recovery Behaviour of DTD 5070A Aluminium Alloy," Conf. on Recent Advances in Stress Analysis: New Concepts and Techniques and Their Practical Application. Roy. Aero. Soc., 1968.
- [3] GOLDHOFF, R. M. and WOODFORD, D.A., "The Evaluation of Creep Damage in a Cr-Mo-V Steel," ASTM STP 515, Testing for Prediction of Material Performance in Structures and Components, 1972, pp. 89-106.
- [4] SNEDDEN, J. D., "Primary Creep of Anisotropic RR58 Aluminium Alloy at 180°C," NEL Rpt. No. 531, 1972.
- [5] FREEMAN, J. W. and VOORHEES, H. R., "Literature Survey on Creep Damage in Metals," ASTM STP 391, 1965.
- [6] SNEDDEN, J.D., "Creep and Fracture Prediction of a Simple Component," NEL Rpt. No. 533, 1972.
- [7] WOODFORD, D. A., "A Critical Assessment of the Life Fraction Rule for Creep Rupture Under Nonsteady Stress or Temperature," Joint Int. Conf. on Creep and Fatigue in Elevated Temperature Applications, Philadelphia/Sheffield, 1973/74. Instn mech. Engrs, 1975, Vol. I.
- [8] SNEDDEN, J. D., "Effects of Prior Stress History on Minimum Creep Rate and Creep Fracture," Third SMIRT Conf., London, 1975.

TABLE I COMPARISON OF EXPERIMENTAL AND PREDICTED RESULTS FROM MULTI-CHANGE TESTS

Test No.	Stress History (hbar)	Stage Times (h)	Total $t_F$ (h)	Total $\epsilon_F$ (%)	Equation Prediction of $t_F$ (h)	Life Fraction Prediction of $t_F$ (h) <sup>F</sup>	Equation %age Error	Life Fraction %age Error
36	19.85*, 18.5	47, 985	1032	0.611	1336.2	1632.6	29.5	58.2
37	18.17*, 18.5	23.5, 1082	1105.5	0.814	1348.9	1665.3	22.0	50.6
38	22.39*, 20	23.3, 560	583.3	0.789	730.5	928.7	25.2	59.2
39	20.2*, 20	22.5, 612	634.5	1.051	768.1	959.5	21.1	51.2
40	20.88*, 21.5	6.4, 406	412.4	1.128	443.6	557.0	7.6	35.1
41	21.76*, 21.5	15, 405	420.0	1.115	434.3	554.2	3.4	32.0
42	14, 15.5, 17	474.8, 218.4, 2153.4	2846.6	0.64	2604.3	3284.0	-8.5	15.4
43	20, 18.5, 17	47.8, 95.5, 1781.7	1925.0	0.63	1761.8	2711.1	-8.5	40.8
44	12.5, 15.5, 18.5	96.6, 47.4, 1068.0	1212.0	0.86	1125.3	1780.0	-7.2	46.9
45	15.5, 17, 18.5	96.3, 47.9, 819.5	963.7	0.65	1094.3	1746.8	13.6	81.3
46	24.5, 21.5, 18.5	1, 24, 648.5	673.5	0.65	870.5	1606.8	29.2	138.6
47	21.5, 20, 18.5	24, 24, 1214.0	1262.0	0.75	839.7	1597.2	-33.5	26.6
48	14, 17, 20	24.5, 48, 496.9	569.4	1.18	542.4	1015.0	-4.7	78.3
49	17, 18.5, 20	24, 24, 605.0	653.0	1.07	512.4	987.4	-21.5	51.2
50	26, 23, 20	0.5, 2, 576.8	579.3	1.20	465.0	953.2	-19.7	64.5
51	23, 21.5, 20	5.1, 23.7, 492.2	521.0	0.96	408.2	933.8	-21.7	79.2
52	18.5, 20, 21.5	24, 24, 374.9	422.9	1.67	254.6	581.8	-39.8	37.6
53	24.5, 23, 21.5	1, 6, 343.0	350.0	1.20	220.9	549.3	-36.9	56.9
54	18.5, 20, 21.5, 23	162.1, 24.2, 23.9, 94.5	304.7	1.78	210.3	478.2	-30.9	56.9
55	18.5, 21.5, 24, 26	114.8, 24.3, 2.3, 14.0	156.2	3.80	141.4	235.6	-9.5	50.8
56	23, 21.5, 20, 18.5	76.5, 120.8, 263.2, 393.5	854.0	1.11	199.0	902.9	-76.7	5.7
57	26, 23, 20	41.2, 120.6, 353.2	497.0	1.17	78.3	429.3	-84.2	-13.6
58	23, 21.5, 20	142, 175, 182.0	499.0	1.68	198.5	550.6	-60.2	10.3
59	18.5, 20, 21.5, 23	412.3, 192.5, 119.1, 85.8	809.7	1.77	606.1	823.3	-25.2	2.8
60	20, 21.5, 23	310.5, 167.9, 19.9	498.3	1.80	440.8	598.8	-11.5	20.2
61	20, 23, 26	310.1, 95.4, 4.2	409.7	4.00	368.7	446.4	-9.9	8.9
62	20 $\rightarrow$ 0	24, 144	2543.8	1.21	672.3	6721.2	-73.6	164.2
63	26 $\rightarrow$ 0	24, 192	219.9	3.00	432.0	875.4	96.5	298.1
64	26 $\rightarrow$ 17	24, 192	222.4	3.40	220.8	661.9	-0.7	197.6
65	18.5 $\rightarrow$ 20.08	168*, 168*	815.7	1.05	538.6	1232.3	-33.9	51.1
66	21.62 $\rightarrow$ 20.08	24*, 24*	471.5	0.70	79.1	752.7	-83.2	59.6
67	17 $\rightarrow$ 21.5	24, 24	411.2	1.85	111.7	942.9	-72.8	129.3
68	20.08 $\rightarrow$ 21.62	18*, 6*	650.0	0.80	95.0	808.1	-85.4	24.3

\* Equivalent stress for relaxation

+ Some variations at weekends