THERMOELASTIC STRESS ANALYSIS OF REACTOR SECONDARY CONTAINMENT

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SUMMARY

The authors present an analytical solution to steady-state axisymmetric thermal stress problem for a reactor secondary containment having shell shape and consisting of spherical and cylindrical parts. The stresses in the shell are produced by given temperatures (technological or postaccidental and atmospheric) applied at the internal and external surfaces of the containment.

The problem is solved using the classical theory of thin shells obeying the Kirchhoff-Love hypothesis, and assuming the linear variation of temperature along the thickness of the shell.

The solution is constructed in the form of a sum:

$$(S) = \sum_i (S_i), \qquad i = 0, 1, 2,$$

where: (S_0) is a particular solution corresponding to given temperature field, and (S_1), (S_2) are isothermal residual solutions such that the solution (S) satisfies all equations and boundary conditions of the formulated problem.

In the cylindrical part of the shell the solution (S_0) corresponding to a constant surface temperature is expressed in terms of the radial displacement satisfying a nonhomogeneous ordinary differential equation of the fourth order, while (S_1) and (S_2) are generated by a general solution of this equation involving four unknown constants.

Similarly, for the spherical part of the shell (S_0) is generated by a particular solution of an ordinary differential equation of the fourth order for the shear force, and (S_1) and (S_2) correspond to a general solution of this equation involving two arbitrary constants.

The solution (S) has an elementary form and the six constants are determined from the six conditions: (i) two conditions of the clamped edge at the bottom of the shell, (ii) two conditions of the geometrical continuity, and (iii) two equilibrium conditions at the cup-cylinder joint.

The closed form solution enables us to give a comprehensive discussion of the problem; however, due to the approximate theory the solution should be viewed as a first step in solving the thermal stress problem in a real nuclear reactor containment.

1. Formulation of the problem

We assume that the reactor shield consists of a cylindrical and spherical shells shown in Fig. 1. In the cylindrical part of the shield the temperature field is axisymmetric and given by

$$T(x,z) = T_0(x) + zT_1(x), \quad 0 < x < H, \quad |z| \leq \frac{h}{2}, \tag{1}$$

where

$$\begin{aligned} T_0(x) &= \frac{1}{2} [T^+(x) + T^-(x)], \\ T_1(x) &= \frac{1}{h} [T^+(x) - T^-(x)], \end{aligned} \tag{2}$$

and $T^+(x)$ and $T^-(x)$ denote prescribed external and internal surface temperatures. Similarly, in the spherical part the temperature is also axisymmetric and given by

$$T(\phi,z) = T_0(\phi) + zT_1(\phi), \quad 0 \leq \phi < \phi_0, \quad |z| \leq \frac{h}{2}, \tag{3}$$

where

$$\begin{aligned} T_0(\phi) &= \frac{1}{2} [T^+(\phi) + T^-(\phi)], \\ T_1(\phi) &= \frac{1}{h} [T^+(\phi) - T^-(\phi)], \end{aligned} \tag{4}$$

and $T^+(\phi)$ and $T^-(\phi)$ stand for the external and internal surface temperatures of the spherical cap. Therefore, the temperature $T = T(P)$ of an arbitrary point of the shield is axisymmetric and given by

$$T(P) = \begin{cases} T, & \text{if } P \text{ belongs to the cylindrical part,} \\ T, & \text{if } P \text{ belongs to the spherical part,} \end{cases} \tag{5}$$

where $T(\ldots)$ and $T(\ldots)$ are defined by (1) and (3), respectively.

Our task is to find an axisymmetric state of thermal stresses and displacements, say $[s(P)]$, produced in the shield by the temperature $T(P)$, using the theory given in [1, 2, 3]. The state $[s(P)]$ is determined by the ordered array of functions

$$[S] = [N_x, N_\Theta, Q_x, M_x, M_\Theta; u_x, u_r] \tag{6}$$

defined on the middle surface of the cylindrical shell, and by the set of functions

$$[\tilde{S}] = [\tilde{N}_\phi, \tilde{N}_\Theta, \tilde{Q}_\phi, \tilde{M}_\phi, \tilde{M}_\Theta; \tilde{u}_\phi, \tilde{u}_r] \tag{7}$$

defined on the middle surface of the spherical cap. Thus, $[S]$ depends on x and $[\tilde{S}]$ depends on ϕ only.

2. Solution for cylindrical part of the shell

The solution $[S]$ in (6) is to satisfy the governing equations for the cylindrical shell given in [1, 2, 3]. The equations of equilibrium take the form

$$\begin{aligned} N_\Theta + RQ_{x,x} &= 0, \\ M_{x,x} - Q_x &= 0, \\ N_x &= 0. \end{aligned} \tag{8}$$

The constitutive equations relating the forces and moments to the displacements and temperature read

$$
\begin{aligned}
N_x &= K[u_{x,x} + vR^{-1}u_r - (1+v)\alpha_T T_o],\\
N_\Theta &= K[R^{-1}u_r + vu_{x,x} - (1+v)\alpha_T T_o],\\
M_x &= D[u_{r,xx} + (1+v)\alpha_T T_1],\\
M_\Theta &= D[vu_{r,xx} + (1+v)\alpha_T T_1],
\end{aligned}
\tag{9}
$$

The formulae (8) and (9) subject to suitable boundary conditions for $x = 0$, and $x = H$ determine bending state of the cylinder in a unique way. Next, we show that all functions in (6) describing the bending state can be expressed by u_r only. From $(8)_3$ and $(9)_1$ we obtain

$$u_{x,x} = -vR^{-1}u_r + (1+v)\alpha_T T_o. \tag{10}$$

If we choose $u_x = u_x(x)$ to meet the condition $u_x(0) = 0$, from (10) we get

$$u_x = -vR^{-1}\int_0^x u_r dr + (1+v)\alpha_T \int_0^x T_o dx. \tag{11}$$

Substituting $u_{x,x}$ from (10) into $(9)_2$ we obtain

$$N_\Theta = K(1-v^2)(R^{-1}u_r - \alpha_T T_o). \tag{12}$$

Now, eqs. $(8)_2$ and $(9)_3$ imply

$$Q_x = D[u_{r,xxx} + (1+v)\alpha_T T_{1,x}]. \tag{13}$$

Therefore, substituting (12) and (13) into $(8)_1$ we get

$$u_r'''' + 4k^4 u_r = 4k^4 R\alpha_T T_o - R^2(1+v)\alpha_T T_1'', \qquad (\ldots)' = (\ldots)_{,\xi} \tag{14}$$

where $$k^4 = 3(1-v^2)\left(\frac{R}{h}\right)^2, \qquad \xi = \frac{x}{R}. \tag{15}$$

If T_o = const. and T_1 = const., a solution of (14) takes the form

$$u_r = u_r^o + u_r^1, \tag{16}$$

where $$u_r^o = R\alpha_T T_o, \tag{17}$$

and $$u_r^1 = e^{-k\xi}(C_1^{(T)}\cos k\xi + C_2^{(T)}\sin k\xi) + e^{k\xi}(C_3^{(T)}\cos k\xi + C_4^{(T)}\sin k\xi), \tag{18}$$

Here $C_i^{(T)}$, $(i = 1, 2, 3, 4)$ are arbitrary constants. Therefore, the solution $[S]$ can be expressed in the form

$$[S] = [S^o] + [S^1], \tag{19}$$

where $[S^o]$ and $[S^1]$ are given by

$[S^o]$:

$$N_x^o = 0, \qquad N_\Theta^o = 0, \qquad Q_x^o = 0, \tag{20}$$

$$M_x^o = M_\theta^o = D(1 + v)\alpha_T T_1,$$
$$u_x^o = \alpha_T T_o x, \qquad u_r^o = \alpha_T T_o R, \tag{20}$$
$$\varkappa^o = \frac{1}{R}\, u_r^{o\prime} = 0;$$

and $[S^1]$:

$$u_r^1(\xi) = e^{-k\xi}(C_1^{(T)}\cos k\xi + C_2^{(T)}\sin k\xi) + e^{k\xi}(C_3^{(T)}\cos k\xi + C_4^{(T)}\sin k\xi),$$
$$u_x^1(\xi) = -\frac{v}{2k}\Big\{C_1^{(T)}[1 + e^{-k\xi}(\sin k\xi - \cos k\xi)] +$$
$$+ C_2^{(T)}[1 - e^{-k\xi}(\sin k\xi + \cos k\xi)] -$$
$$- C_3^{(T)}[1 - e^{k\xi}(\sin k\xi + \cos k\xi)] +$$
$$+ C_4^{(T)}[1 + e^{k\xi}(\sin k\xi - \cos k\xi)]\Big\},$$
$$u_r^{1\prime}(\xi) = -ke^{-k\xi}[(C_1^{(T)} - C_2^{(T)})\cos k\xi + (C_1^{(T)} + C_2^{(T)})\sin k\xi] +$$
$$+ ke^{k\xi}[(C_3^{(T)} + C_4^{(T)})\cos k\xi - (C_3^{(T)} - C_4^{(T)})\sin k\xi], \tag{21}$$
$$M_x^1(\xi) = \frac{1}{R^2}\, 2Dk^2\, [e^{-k\xi}(C_1^{(T)}\sin k\xi - C_2^{(T)}\cos k\xi) -$$
$$- e^{k\xi}\,(C_3^{(T)}\sin k\xi - C_4^{(T)}\cos k\xi)],$$
$$Q_x^1(\xi) = \frac{1}{R^3}\, 2Dk^3\Big\{e^{-k\xi}[(C_1^{(T)} + C_2^{(T)})\cos k\xi - (C_1^{(T)} - C_2^{(T)})\sin k\xi] -$$
$$- e^{k\xi}[(C_3^{(T)} - C_4^{(T)})\cos k\xi + (C_3^{(T)} - C_4^{(T)})\sin k\xi]\Big\},$$
$$N_\theta^1(\xi) = \frac{Eh}{R}\, u_r^1(\xi), \qquad M_\theta^1 = vM_x^1, \qquad N_x^1 = 0,$$
$$\varkappa^1(\xi) = \frac{1}{R}\, u_r^{1\prime}(\xi).$$

It follows from (19) + (21), that $[S]$ can be determined in a unique way if $C_i^{(T)}$, $(i=1,2,3,4)$ are known. The constants $C_3^{(T)}$ and $C_4^{(T)}$ can be expressed in terms of $C_1^{(T)}$ and $C_2^{(T)}$ using the boundary condition that the cylinder is clamped at $x = 0$. This condition takes the form

$$u_r^o + u_r^1(0) = 0, \qquad u_r^{1\prime}(0) = 0. \tag{22}$$

and leads to
$$C_3^{(T)} = -C_1^{(T)} - u_r^o, \qquad C_4^{(T)} = 2C_1^{(T)} - C_2^{(T)} + u_r^o. \tag{23}$$

Substituting $C_3^{(T)}$ and $C_4^{(T)}$ from (23) into (21) we find that $[S^1]$ is determined in a unique way by the constants $C_1^{(T)}$ and $C_2^{(T)}$, and it reads

$[S^1]$:

$$u_r^1(\xi) = u_r^o\, e^{k\xi}\sin k\xi - u_r^o\, e^{k\xi}\cos k\xi +$$
$$+ C_1^{(T)}\,[(e^{-k\xi} - e^{k\xi})\cos k\xi + 2e^{k\xi}\sin k\xi] + \tag{24}$$
$$+ C_2^{(T)}\,[e^{-k\xi} - e^{k\xi}]\sin k\xi,$$

$$
\begin{aligned}
u_r^1(\xi) &= 2ku_r^o\, e^{k\xi}\sin k\xi + \\
&+ C_1^{(T)}k\left[e^{k\xi}(\cos k\xi + 3\sin k\xi) - e^{-k\xi}(\cos k\xi + \sin k\xi)\right] + \\
&+ C_2^{(T)}k\left[e^{-k\xi}(\cos k\xi - \sin k\xi) - e^{k\xi}(\sin k\xi + \cos k\xi)\right], \\
M_x^1(\xi) &= \frac{1}{R^2}\, 2Dk^2\left\{u_r^o\, e^{k\xi}(\cos k\xi + \sin k\xi) + \right. \\
&+ C_1^{(T)}\left[(e^{-k\xi} + e^{k\xi})\sin k\xi + 2e^{k\xi}\cos k\xi\right] - \\
&\left. - C_2^{(T)}(e^{-k\xi} + e^{k\xi})\cos k\xi\right\}, \\
Q_x^1(\xi) &= \frac{1}{R^3}\, 2Dk^3\left\{2u_r^o\, e^{k\xi}\cos k\xi + \right. \\
&+ C_1^{(T)}\left[e^{-k\xi}(\cos k\xi - \sin k\xi) + e^{k\xi}(3\cos k\xi - \sin k\xi)\right] + \\
&\left. + C_2^{(T)}\left[e^{-k\xi}(\cos k\xi + \sin k\xi) - e^{k\xi}(\cos k\xi - \sin k\xi)\right]\right\}.
\end{aligned}
\tag{24}
$$

The constants $C_1^{(T)}$ and $C_2^{(T)}$ in (24) should be determined from suitable joint conditions at the cross section x = H. To obtain an analytical form of these conditions we proceed to find bending state of the spherical cap $[\tilde{S}]$.

3. Solution for spherical part of the shell

The solution $[\tilde{S}]$ describing bending state of the cap should satisfy equilibrium equations and constitutive relations referred to the spherical coordinates (see [1, 2, 3]). The equilibrium equations takes now the form

$$
\begin{aligned}
&(\tilde{N}_\phi \sin\phi)_{,\phi} - \tilde{N}_\Theta \cos\phi - \tilde{Q}_\phi \sin\phi = 0, \\
&(\tilde{Q}_\phi \sin\phi)_{,\phi} + \tilde{N}_\Theta \sin\phi + \tilde{N}_\phi \sin\phi = 0, \\
&(\tilde{M}_\phi \sin\phi)_{,\phi} - \tilde{M}_\Theta \cos\phi - \tilde{R}\tilde{Q}_\phi \sin\phi = 0,
\end{aligned}
\tag{25}
$$

while the constitutive equations read

$$
\begin{aligned}
\tilde{N}_\phi &= \frac{\tilde{K}}{\tilde{R}}\left[(\tilde{u}_{\phi,\phi} + \tilde{u}_r) + v(\tilde{u}_\phi \mathrm{ctg}\phi + \tilde{u}_r) - \tilde{R}(1 + v)\alpha_T\tilde{T}_o\right], \\
\tilde{N}_\Theta &= \frac{\tilde{K}}{\tilde{R}}\left[(\tilde{u}_\phi \mathrm{ctg}\phi + \tilde{u}_r) + v(\tilde{u}_{\phi,\phi} + \tilde{u}_r) - \tilde{R}(1 + v)\alpha_T\tilde{T}_o\right], \\
\tilde{M}_\phi &= \frac{\tilde{D}}{\tilde{R}}\left[\tilde{\varkappa}_{,\phi} + v\tilde{\varkappa}\mathrm{ctg}\phi + \tilde{R}(1 + v)\alpha_T\tilde{T}_1\right], \\
\tilde{M}_\Theta &= \frac{\tilde{D}}{\tilde{R}}\left[\tilde{\varkappa}\mathrm{ctg}\phi + v\tilde{\varkappa}_{,\phi} + \tilde{R}(1 + v)\alpha_T\tilde{T}_1\right],
\end{aligned}
\tag{26}
$$

where
$$\tilde{\varkappa} = \tilde{R}^{-1}(\tilde{u}_{r,\phi} - \tilde{u}_\phi), \tag{27}$$
and
$$\tilde{K} = \frac{\tilde{E}\tilde{h}}{1 - v^2}, \qquad \tilde{D} = \frac{\tilde{E}\tilde{h}^3}{12(1 - v^2)}. \tag{28}$$

Eqs. (25) + (27) can be reduced to two equations involving $\tilde{\varkappa}$ and $\tilde{Q}_\phi$ only. Thus, if we substitute $(26)_3$ and $(26)_4$ into $(25)_3$, we obtain first of the two equations

$$\tilde{\varkappa}_{,\phi\phi} + \tilde{\varkappa}_{,\phi}\cot\phi - (v + \mathrm{ctg}^2\phi)\tilde{\varkappa} = \frac{\tilde{R}^2}{\tilde{D}}\,\tilde{Q}_\phi - (1 + v)\tilde{R}\alpha_T\tilde{T}_{1,\phi}. \tag{29}$$

To derive the second equation coupling $\tilde{\varkappa}$ and $\tilde{Q}_\phi$ note, that $(26)_1$ and $(26)_2$ can be written in the form

$$\tilde{u}_r + \tilde{u}_{\phi,\phi} = \frac{\tilde{R}}{\tilde{K}(1 - v^2)} (\tilde{N}_\phi - v\tilde{N}_\Theta) + \tilde{R}\alpha_T\tilde{T}_o,$$
$$\tilde{u}_r + \tilde{u}_\phi \text{ctg}\phi = \frac{\tilde{R}}{\tilde{K}(1 - v^2)} (\tilde{N}_\Theta - v\tilde{N}_\phi) + \tilde{R}\alpha_T\tilde{T}_o. \tag{30}$$

Differentiation of $(30)_2$ with respect to ϕ yields

$$\tilde{u}_{\phi,\phi}\text{ctg}\phi - \tilde{u}_\phi \sin^{-2}\phi + \tilde{u}_{r,\phi} = \frac{\tilde{R}}{\tilde{K}(1 - v^2)} (\tilde{N}_\Theta - v\tilde{N}_\phi)_{,\phi} + \tilde{R}\alpha_T\tilde{T}_{o,\phi}. \tag{31}$$

Substracting $(30)_1$ from $(30)_2$ we also obtain

$$\tilde{u}_{\phi,\phi} - \tilde{u}_\phi \text{ctg}\phi = \frac{\tilde{R}}{\tilde{K}(1 - v^2)} (1 + v)(\tilde{N}_\phi - \tilde{N}_\Theta). \tag{32}$$

Finally, elimination of $\tilde{u}_{\phi,\phi}$ from (31) and (32) and formula (27) lead to

$$\tilde{\varkappa} = \frac{1}{\tilde{K}(1 - v^2)} \left[(\tilde{N}_\Theta - v\tilde{N}_\phi)_{,\phi} + (1 + v)(\tilde{N}_\Theta - \tilde{N}_\phi)\text{ctg}\phi \right] + \alpha_T\tilde{T}_{o,\phi}. \tag{33}$$

Next, if we put

$$\tilde{N}_\phi = - \tilde{Q}_\phi \text{ctg}\phi, \qquad \tilde{N}_\Theta = - \tilde{Q}_{\phi,\phi}, \tag{34}$$

then the equilibrium equations $(25)_1$, and $(25)_2$ are satisfied identically, and (33) and (34) imply

$$\tilde{Q}_{\phi,\phi\phi} + \tilde{Q}_{\phi,\phi}\text{ctg}\phi - (\text{ctg}^2\phi - v)\tilde{Q}_\phi = - \tilde{K}(1 - v^2)\tilde{\varkappa} + \tilde{K}(1 - v^2)\alpha_T\tilde{T}_{o,\phi}. \tag{35}$$

The eqs. (29) and (35) constitute the set of two equations, we have been looking for. If a solution $(\tilde{\varkappa}, \tilde{Q}_\phi)$ of these equations is found the remaining functions describing the bending state of the cap can be determined in a unique way. Neglecting in (29) and (35) terms involving ctgo we arrive at the approximate set of equations

$$\tilde{\varkappa}_{,\phi\phi} - v\tilde{\varkappa} = \frac{\tilde{R}^2}{\tilde{D}} \tilde{Q}_\phi - (1 + v)\tilde{R}\alpha_T\tilde{T}_{1,\phi},$$
$$\tilde{Q}_{\phi,\phi\phi} + v\tilde{Q}_\phi = - \tilde{K}(1 - v^2)\tilde{\varkappa} + \tilde{K}(1 - v^2)\alpha_T\tilde{T}_{o,\phi}. \tag{36}$$

Elimination of $\tilde{\varkappa}$ from (36) leads to

$$\tilde{Q}_{\phi,\phi\phi\phi\phi} + 4\tilde{k}^4\tilde{Q}_\phi = \tilde{E}\tilde{h} \left[(1 + v)\tilde{R}\alpha_T\tilde{T}_{1,\phi} + \alpha_T(\tilde{T}_{o,\phi\phi} - v\tilde{T}_o)_{,\phi} \right], \tag{37}$$

where

$$\tilde{k}^4 = \frac{\tilde{K}(1 - v^2)\tilde{R}^2}{4\tilde{D}} - \frac{v^2}{4} = 3(1 - v^2)\left(\frac{\tilde{R}}{\tilde{h}}\right)^2. \tag{38}$$

It is easy to express all functions describing the bending state of the shell in terms of $\tilde{Q}_\phi$. Neglecting $v\tilde{Q}_\phi$ in $(36)_2$ we obtain

$$\tilde{\varkappa} = - \frac{1}{\tilde{K}(1 - v^2)} \tilde{Q}_{\phi,\phi\phi} + \alpha_T\tilde{T}_{o,\phi}. \tag{39}$$

The moments $\tilde{M}_\phi$ and $\tilde{M}_\Theta$ are obtained from $(26)_3$, $(26)_4$ and (39), if terms involving ctgϕ are ignored:

$$\tilde{M}_\phi = - \frac{\tilde{R}}{4\tilde{k}^4} \tilde{Q}_{\phi,\phi\phi\phi} + \frac{\tilde{D}}{\tilde{R}} \alpha_T\tilde{T}_{o,\phi\phi} + \tilde{D}(1 + v)\alpha_T\tilde{T}_1, \tag{40}$$

$$\tilde{M}_\Theta = -\frac{\tilde{R}v}{4\tilde{k}^4}\tilde{Q}_{\phi,\phi\phi\phi} + v\frac{\tilde{D}}{\tilde{R}}\alpha_T\tilde{T}_{o,\phi\phi} + \tilde{D}(1+v)\alpha_T\tilde{T}_1. \tag{40}$$

The displacement components $\tilde{u}_r$, $\tilde{u}_\phi$ are given by (30), in which $\tilde{N}_\phi$ and $\tilde{N}_\Theta$ are treated as known functions. If we choose $\tilde{u}_\phi$ in such a way to satisfy the condition $\tilde{u}_\phi(\phi_o) = 0$, then, ignoring $v\tilde{N}_\phi$ in comparison with $\tilde{N}_\Theta$, from $(30)_2$ we get

$$\tilde{u}_r(\phi_o) = \frac{\tilde{R}}{\tilde{E}\tilde{h}}\tilde{N}_\Theta(\phi_o) + \tilde{R}\alpha_T\tilde{T}_o = -\frac{\tilde{R}}{\tilde{E}\tilde{h}}\tilde{Q}_{\phi,\phi}(\phi_o) + \tilde{R}\alpha_T\tilde{T}_o. \tag{41}$$

Therefore, the solution $[\tilde{S}]$ takes the form

$$[\tilde{S}] = [\tilde{S}^o] + [\tilde{S}^1], \tag{42}$$

where $[\tilde{S}^o]$ corresponds to a particular solution of (37), while $[\tilde{S}^1]$ is generated by a general solution of (37) with $\tilde{T}_o = \tilde{T}_1 = 0$.

If $\tilde{T}_o = \text{const.} \neq 0$, and $\tilde{T}_1 = \text{const.} \neq 0$, a particular solution of (37) is given by $\tilde{Q}^o_\phi = 0$. In this case $[\tilde{S}^o]$ is described by the formulae

$$\begin{aligned}
&[\tilde{S}^o]:\\
&\tilde{N}^o_\phi = 0, \qquad \tilde{N}^o_\Theta = 0, \qquad \tilde{Q}^o_\phi = 0,\\
&\tilde{M}^o_\phi = \tilde{D}(1+v)\alpha_T\tilde{T}_1, \qquad \tilde{M}^o_\Theta = \tilde{D}(1+v)\alpha_T\tilde{T}_1,\\
&\tilde{u}^o_r(\phi_o) = \tilde{R}\alpha_T\tilde{T}_o,\\
&\tilde{\varkappa}^o = 0,
\end{aligned} \tag{43}$$

and $[\tilde{S}^1]$ takes the form

$$[\tilde{S}^1] = [\tilde{S}^1_1] + [\tilde{S}^1_2], \tag{44}$$

where $[\tilde{S}^1_1]$ and $[\tilde{S}^1_2]$ are given by

$$\begin{aligned}
&[\tilde{S}^1_1]:\\
&\tilde{Q}^1_\phi = X^{(T)}_1\sqrt{2}\sin\phi_o e^{-\tilde{k}w_1}\sin(\tilde{k}w_1 - \frac{\pi}{4}),\\
&\tilde{N}^1_\phi = -\tilde{Q}^1_\phi \operatorname{ctg}\phi,\\
&\tilde{N}^1_\Theta = X^{(T)}_1 2\tilde{k}\sin\phi_o e^{-\tilde{k}w_1}\cos\tilde{k}w_1,\\
&\tilde{M}^1_\phi = X^{(T)}_1\frac{\tilde{R}}{\tilde{k}}\sin\phi_o e^{-\tilde{k}w_1}\sin\tilde{k}w_1,\\
&\tilde{\varkappa}^1 = X^{(T)}_1\frac{2\sqrt{2}}{\tilde{E}\tilde{h}}\tilde{k}^2\sin\phi_o e^{-\tilde{k}w_1}\cos(\tilde{k}w_1 - \frac{\pi}{4});
\end{aligned} \tag{45}$$

$$\begin{aligned}
&\text{and } [\tilde{S}^1_2]:\\
&\tilde{Q}^2_\phi = X^{(T)}_2\frac{2\tilde{k}}{\tilde{R}}e^{-\tilde{k}w_1}\sin\tilde{k}w_1,\\
&\tilde{N}^2_\phi = -\tilde{Q}^2_\phi\operatorname{ctg}\phi,\\
&\tilde{N}^2_\Theta = -X^{(T)}_2\frac{2\sqrt{2}}{\tilde{R}}\tilde{k}^2 e^{-\tilde{k}w_1}\sin(\tilde{k}w_1 - \frac{\pi}{4}),\\
&\tilde{M}^2_\phi = X^{(T)}_2\sqrt{2}\,e^{-\tilde{k}w_1}\sin(\tilde{k}w_1 + \frac{\pi}{4}),\\
&\tilde{\varkappa}^2 = X^{(T)}_2\frac{4\tilde{k}^3}{\tilde{E}\tilde{h}\tilde{R}}e^{-\tilde{k}w_1}\cos\tilde{k}w_1.
\end{aligned} \tag{46}$$

Here $X_1^{(T)}$ and $X_2^{(T)}$ denote unknown horizontal shear force and unknown bending moment acting at the cross section $\phi = \phi_o$, and

$$w_1 = \phi_o - \phi \geq 0, \qquad \tilde{k} = [3(1 - v^2)]^{\frac{1}{4}}(\frac{\tilde{R}}{\tilde{h}})^{\frac{1}{2}}. \tag{47}$$

The total horizontal displacement of the cap $\tilde{\delta}^{(T)}(\phi_o)$, and the resultant rotation angle $\tilde{\varkappa}^{(T)}(\phi_o)$ corresponding to the state $[\tilde{S}]$ are given by

$$\begin{aligned} \tilde{\delta}^{(T)}(\phi_o) &= \tilde{R}\alpha_T\tilde{T}_o\sin\phi_o + X_1^{(T)}\frac{2\tilde{k}\tilde{R}}{\tilde{E}\tilde{h}}\sin^2\phi_o + X_2^{(T)}\frac{2\tilde{k}^2}{\tilde{E}\tilde{h}}\sin\phi_o, \\ \tilde{\varkappa}^{(T)}(\phi_o) &= X_1^{(T)}\frac{2\tilde{k}^2}{\tilde{E}\tilde{h}}\sin\phi_o + X_2^{(T)}\frac{4\tilde{k}^3}{\tilde{E}\tilde{h}\tilde{R}}. \end{aligned} \tag{48}$$

4. Final solution for the reactor shield (containment)

An analysis of the states $[S]$ (see formulae (19), (20), and (24)), and $[\tilde{S}]$ (see (42), (43) ÷ (46)) leads to the conclusion that $[S]$ is determined in a unique way if $(C_1^{(T)}, C_2^{(T)})$ are known, while $[\tilde{S}]$ is unique for a unique pair $(X_1^{(T)}, X_2^{(T)})$. These four constants should be determined from two geometric conditions and two equilibrium equations which must be satisfied at the joint cross-section $\phi = \phi_o$. These conditions take the form

$$\begin{aligned} &u_r^o + u_r^1(w) = \tilde{\delta}^{(T)}(\phi_o), \qquad \frac{1}{R}u_r^{1\prime}(w) = \tilde{\varkappa}^{(T)}(\phi_o), \\ &M^o + M_x^1(w) = \tilde{M}^o + X_2^{(T)}, \qquad Q_x^1(w) = -X_1^{(T)}, \qquad (w = \tfrac{H}{R}), \end{aligned} \tag{49}$$

where $$M^o = D(1 + v)\alpha_T T_1, \qquad \tilde{M}^o = \tilde{D}(1 + v)\alpha_T\tilde{T}_1. \tag{50}$$

The conditions $(49)_1$ and $(49)_2$ represent continuity of the horizontal displacement and the rotation angle of the shield at $\phi = \phi_o$, while $(49)_3$ and $(49)_4$ express equilibrium of the moments and horizontal shear forces at $\phi = \phi_o$. These conditions lead to the four linear nonhomogeneous algebraic equations for $X_1^{(T)}$, $X_2^{(T)}$, $C_1^{(T)}$, and $C_2^{(T)}$:

$$\begin{aligned} X_1^{(T)}\delta_{11} + X_2^{(T)}\delta_{12} + C_1^{(T)}\delta_{13} + C_2^{(T)}\delta_{14} &= f_1^{(T)}, \\ X_1^{(T)}\delta_{21} + X_2^{(T)}\delta_{22} + C_1^{(T)}\delta_{23} + C_2^{(T)}\delta_{24} &= f_2^{(T)}, \\ X_2^{(T)}\delta_{32} + C_1^{(T)}\delta_{33} + C_2^{(T)}\delta_{34} &= f_3^{(T)}, \\ X_1^{(T)}\delta_{41} + C_1^{(T)}\delta_{43} + C_2^{(T)}\delta_{44} &= f_4^{(T)}, \end{aligned} \tag{51}$$

where δ_{ik} is given by

$$\begin{aligned} &\delta_{11} = \frac{2\tilde{k}\tilde{R}}{\tilde{E}\tilde{h}}\sin^2\phi_o, \qquad \delta_{12} = \frac{2\tilde{k}^2}{\tilde{E}\tilde{h}}\sin\phi_o, \\ &\delta_{13} = e^{kw}(\cos kw - 2\sin kw) - e^{-kw}\cos kw, \\ &\delta_{14} = (e^{kw} - e^{-kw})\sin kw, \\ &\delta_{21} = \delta_{12}, \qquad \delta_{22} = \frac{4\tilde{k}^3}{\tilde{E}\tilde{h}\tilde{R}}, \end{aligned} \tag{52}$$

$$\delta_{23} = \frac{k}{R}\left[e^{kw}(\cos kw + 3\sin kw) - e^{-kw}(\cos kw + \sin kw)\right],$$

$$\delta_{24} = -\frac{k}{R}\left[e^{kw}(\cos kw + \sin kw) - e^{-kw}(\cos kw - \sin kw)\right],$$

$$\delta_{31} = 0, \qquad \delta_{32} = \frac{R^2}{2Dk^2},$$

$$\delta_{33} = -e^{kw}(\sin kw + 2\cos kw) - e^{-kw}\sin kw, \tag{52}$$

$$\delta_{34} = (e^{kw} + e^{-kw})\cos kw,$$

$$\delta_{41} = \frac{R^3}{2Dk^3}, \qquad \delta_{42} = 0,$$

$$\delta_{43} = e^{kw}(3\cos kw - \sin kw) + e^{-kw}(\cos kw - \sin kw),$$

$$\delta_{44} = -e^{kw}(\cos kw - \sin kw) + e^{-kw}(\cos kw + \sin kw);$$

and $f_i^{(T)}$, $(i=1,2,3,4)$, $(\alpha = kw)$ are defined through

$$f_1^{(T)} = u_r^o - \tilde{u}_r^o + u_r^o e^{\alpha}(\sin\alpha - \cos\alpha),$$

$$f_2^{(T)} = 2\,\frac{k}{R}\,u_r^o e^{\alpha}\sin\alpha,$$

$$f_3^{(T)} = u_r^o e^{\alpha}(\cos\alpha + \sin\alpha) + (M^o - \tilde{M}^o)\delta_{32}, \tag{53}$$

$$f_4^{(T)} = -2u_r^o e^{\alpha}\cos\alpha, \qquad \tilde{u}_r^o = \tilde{R}\alpha_T\tilde{T}_o\sin\phi_o, \qquad u_r^o = R\alpha_T T_o.$$

Having found the constants $X_1^{(T)}$, $X_2^{(T)}$, $C_1^{(T)}$, $C_2^{(T)}$ satisfying (51), we can determine the bending state of the cap from (42) and the bending state of the cylinder from (19).

References:

[1] PARKUS, H., MELAN, E., Wärmespannungen infolge stationärer temperaturfelder, Springer-Verlag, Wien, 1955.

[2] HAMPE, E., Statik rotationssymmetrischer flächentragwerke, B. 1, Veb Verlag für Bauwesen, Berlin, 1969.

[3] BACZYŃSKI, Z. F., IGNACZAK, J., and others, Analysis of reactor secondary containment for the First Polish Nuclear Power Station, Research Report, Polish Academy of Sciences, Warsaw, 1974.

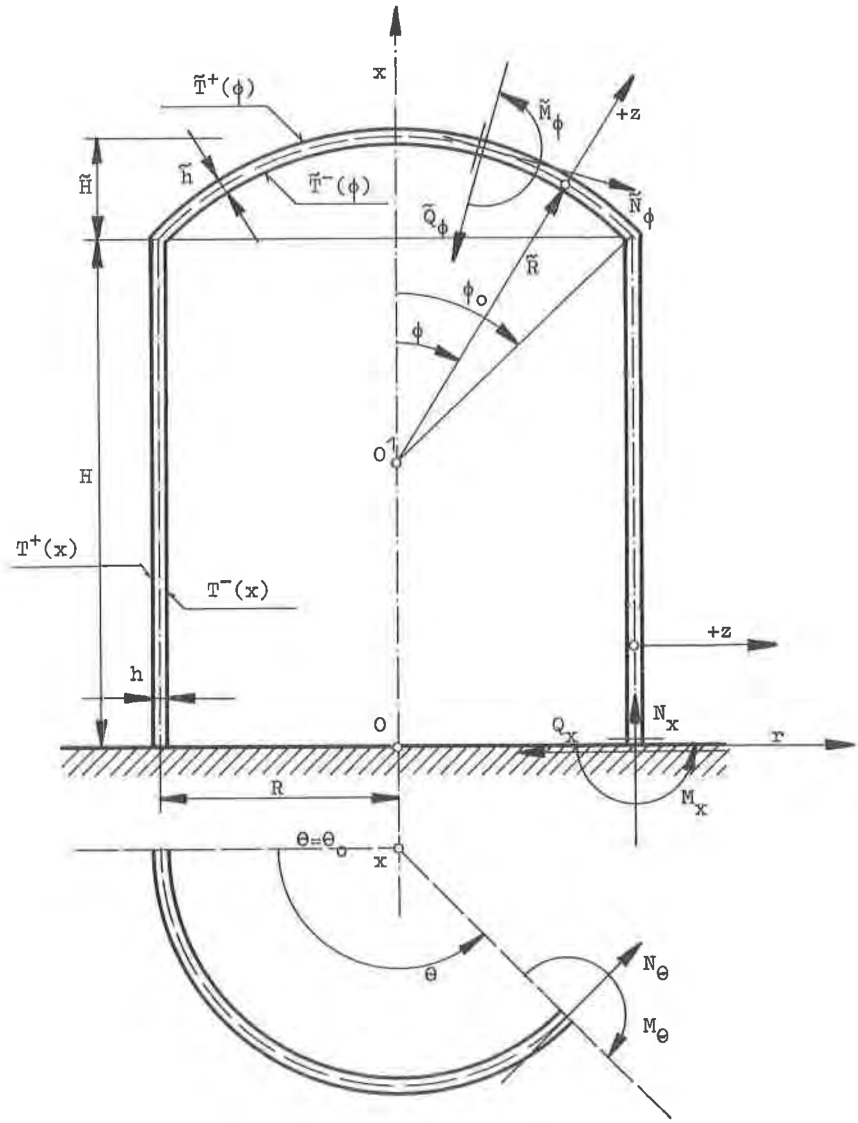
x
$\tilde{T}^+(\phi)$
$\tilde{M}_\phi$
+z
$\tilde{H}$
$\tilde{h}$
$\tilde{T}^-(\phi)$
$\tilde{N}_\phi$
$\tilde{Q}_\phi$
$\tilde{R}$
ϕ_o
ϕ
O'
H
$T^+(x)$
$T^-(x)$
+z
h
N_x
O
Q_x
r
M_x
R
$\Theta=\Theta_o$
x
Θ
N_Θ
M_Θ

Fig. 1