TRIAXIAL STATE OF STRESS OF CONCRETE STRUCTURES

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ABSTRACT

The assessment of the state of stress and the safety margin against failure of triaxially stressed concrete structures is based on the surface of actual concrete strengths. According to the test results its form is not rotary. For analytical purposes a simplified form is introduced suggested by Reimann [13] which is compared with the form deduced from the tests carried out by Launay, Gachon et Poitevin [10] . The application of Mohr's general theory according to Filonenko-Borodich [5] makes it further possible, in the meaning of Mohr's failure hypothesis, to discern whether concrete under the given type of stress fails in shear (similarly as in the case of plain compression) or in tension (similarly as in the case of plain tension). In the case of the failure in tension reinforcement is designed to transfer the tension. In this process a graduation of the permissible safety margin values is recommended in accordance with whether the structure is of plain, reinforced or prestressed concrete. The method makes it also possible to follow theoretically the states of overloading the structure to the point of failure.

1. INTRODUCTION

The methods enabling the determination of the safety margin of a concrete structure under triaxial state of stress, incl. the assessment of the stress of concrete reinforcement, have not been elaborated in great detail so far. The works dealing recently with this very problem have been stimulated particularly by the use of prestressed concrete for the structures of reactor containment vessels. Simpler approximate methods, among which particularly the Mohr-Caquot method [3] , [4] is used very often, can be made more accurate primarily on the basis of the results of extensive laboratory works whose pupose is to follow the deformations, cracking and manners of failurs of concrete in the triaxial state of stress, and finally the deduction of the shape of the actual concrete strengths surface. These works have not been by far completed; however, their results make it possible to draw certain conclusions which can form the basis of design methods.

The design methods based on the actual concrete strengths surface only, do not make it possible to determine the manner of concrete failure. Since this circumstance is of great importance particularly for the improvement of the accuracy of reinforcement design, it is desirable to supplement the actual strengths area with the hypothesis of the physical causes of failure. Such suitable hypothesis is the Mohré hypothesis in the general form deduced by Filonenko-Borodich [5] .

2. ACTUAL STRENGTHS SURFACE AND ITS RELATION TO MOHR'S GENERAL THEORY

The condition of the strength (or plasticity) of concrete can by expressed analytically by the function

$$\psi(z, r, \delta) = 0 \tag{1}$$

where

$$z = \frac{1}{\sqrt{3}} (\sigma_1 + \sigma_2 + \sigma_3) \tag{2}$$

$$r = \frac{1}{\sqrt{3}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \tag{3}$$

$$\cos \delta = \frac{\sigma_1 + \sigma_2 - 2\sigma_3}{\sqrt{2}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}} \tag{4}$$

which determines the actual concrete strengths surface in the system of ordinates z, r, δ. In this process

$$\sigma_1 \geqq \sigma_2 \geqq \sigma_3 \tag{5}$$

means the principal stresses.

From the theoretical analysis confirmed by the results of laboratory tests it follows that the axis of the actual concrete strengths surface lies in the axis z, the shapes of its sections in the planes perpendicular to this axis are very near triangles with rounded sides and corners, its meridians being almost parabolic. The surface opens in the direction of the negative direction of the axis z, its apex having a positive coordinate z. The axis z deviates equally from the directions of the principal stresses σ_1, σ_2, σ_3. The shape of this surface is shown axonometrically in Fig. 1.

The strength condition (1) can be written, in accordance with the Mohr's general theory according to Filonenko-Borodich [5] in the form expressing the relation of the shear octahedric stress τ_{oct}, the normal octahedric stress σ_{oct} and the relative excentricity of Mohr's circles α , viz.

$$\tau_{oct} = \omega(\sigma_{oct}, \alpha) \tag{6}$$

where

$$\tau_{oct} = \frac{1}{\sqrt{3}} r, \tag{7}$$

$$\sigma_{oct} = \frac{1}{\sqrt{3}} z, \tag{8}$$

$$\alpha = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \tag{9}$$

According to Mohr's representation of the state of stress by means of circles in the plane σ, τ (Fig.2) the position of the inner two circles is fully determined either by the ratio

$$\alpha = \frac{c}{b} = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$

or the ratio

$$\beta = \frac{c'}{b'} = \frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2 - 2\sigma_3} = \sqrt{3} \cdot \mathrm{tg}\, \delta \tag{10}$$

To a certain angle δ there belongs a certain value of the ratio α or β. When using the symbols for the principal stresses according to (5) the angle δ can attain the values varying from 0 to $\frac{\pi}{3}$, the relative excentricity α the values varying from + 1 to - 1, and the relative excentricity β the values varying from 0 to 3.

In the geometric representation a certain meridian of the actual strengths surface corresponds with every value of the angle δ (or every value of the relative excentricity α or β). Of importance are the meridians passing through the points a, b, c (see Figs. 1, 4, 5)representing the tensile, compressive and shearing character of the state of stress on which there are the points representing the stresses corresponding with plain compressive, plain shearing and plain tensile strengths. The respective values δ, α, β of these meridians are shown in Table I.

Table I

Character of stress	δ	$\cos\delta$	$\mathrm{tg}\,\delta$	α	β
Tensile $\sigma_2 = \sigma_3$, $\sigma_1 > \sigma_2$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\sqrt{3}$	-1	3
Shearing $\sigma_2 = \frac{\sigma_1 + \sigma_3}{2}$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	0	1
Compressive $\sigma_1 = \sigma_2$, $\sigma_3 < \sigma_1$	0	1	0	1	0

The apex of the actual strengths surface corresponds with the triaxial uniform (hydrostatic) tensile strength.

With every point of the actual strengths surface there correspond three Mohr's circles in the plane σ, τ. With every meridian Γ of the actual strengths surface (Fig. 3) there corresponds a set of triples of Mohr's circles of a constant excentricity α (or β).

Let us investigate now, whether the failure is decisively influenced by the shearing stress τ_m in combination with the normal stress σ_m (Fig.2) in the meaning of Mohr's hypothesis, or the maximum tensile stress σ_1. Since the points corresponding with the maximum shearing stress τ_m are situated on the largest, the so-called main Mohr's circle, it is possible to work further only with the set of main Mohr's circles. This method simultaneously introduces the assumption that the shear failure occurs in the planes passing through the axis of stresses σ_3.

On the meridian Γ it is possible to find the point U which divides the meridian into two parts. The points on the left hand side from the point U (in the negative direction of the axis z) correspond with the main Mohr's circles whose envelope ϕ is real. In the meaning of Mohr's theory the stress applied to the failure planes is determined by the coordinates σ_m (normal stress) and τ_m (tangential stress) of the point of contact T of the main Mohr's circle and its envelope ϕ . This stress is applied to the failure plane deviating from the direction of the principal stress σ_1 by the angle ε and passing through the axis of stresses σ_3 (Fig. 2).

The points on the right hand side from the point U (in the positive direction of the axis z) correspond with the main Mohr's circles which have no real envelope, being situated inside one another. It can be assumed that the failure in this region is due to the fracture in the planes perpendicular to the direction of the maximum tensile stress, i. e. the direction of the stress σ_1. This assumption is quite logical, but must be yet thoroughly verified experimentally.

The coordinate of the point U is determined by the gradient of the tangent to the meridian Γ and is

$$\left| \frac{d\ \tau_{oct}}{d\ \sigma_{oct}} \right| = \frac{\sqrt{6 + 2\alpha^2}}{3 - \alpha} = k_o \tag{11}$$

The dependence of the form of the meridian Γ and the position of the point U on the relative excentricity α introduces into the classical Mohr's hypothesis the influence of the mean principal stress σ_2 and limits its validity to a certain region of the state of stress only.

3. SHAPE OF THE ACTUAL STRENGTHS SURFACE

So far nobody succeeded in expressing mathematically the shape of the actual strengths surface which would correspond with the results of the respective tests. The respective function Ψ (see eq.(1)) should comprize a number of constants expressing the strengths of the material under basic types of stresses, such as the plain compressive strength R_D, plain tensile strength R_Z, biaxial uniform compressive strength, triaxial uniform tensile strength, etc. Their number determines the accuracy of the analytical definition of the form of the meridians under the compressive and tensile character of the stresses.

In the past four years the properties of concrete under triaxial stress were investigated in detail by Launay, Gachon and Poitevin [10] who also deduced the shape of the actual strengths surface. Its analytical expression was devised in the form of

$$\varrho^2 \left\{ \frac{\cos^2 \frac{3}{2}\delta}{[\varrho_D(\zeta)]^2} + \frac{\sin^2 \frac{3}{2}\delta}{[\varrho_Z(\zeta)]^2} \right\} = 1, \tag{12}$$

where

$$\varrho = \frac{r}{R_D}, \qquad \zeta = \frac{z}{R_D}, \tag{13}$$

R_D being plain compressive strength of concrete. The functions $\varrho_D(\zeta)$, $\varrho_Z(\zeta)$ express the form of the meridians Γ_D and Γ_Z under compressive and tensile characters of the state of stress. However, so far the authors have been unable to formulate analytically the functions $\varrho_D(\zeta)$ and $\varrho_Z(\zeta)$.

An approximate analytical expression for the shape of the actual strengths surface was suggested by Reimann [13] on the basis of an analysis of the results of tests carried out by Balmer [2] , Richart, Brandzaeg and Brown [14] and Weigler and Becker [16] . To attain a certain simplification, he introduced an approximate assumption that between the meridians Γ_D and Γ_Z (with the compressive and tensile character of the state of stress) there holds an affinity relation with an affinity coefficient λ , viz.

$$\varrho_Z(\zeta) = \lambda \varrho_D(\zeta) \tag{14}$$

For the basic meridian Γ_D he introduced a quadratic curve

$$\varrho_D(\zeta) = \sqrt{29,06 - 11,93\zeta} - 5,18 \tag{15}$$

Between points a and b they replaced the actual form of the cross sections in the planes perpendicular to the axis by a part of the circle about point <u>a</u> and its tangent (Fig. 5, dashed lines), thus attaining a discontinuous, but simple function $\varkappa = \varphi(\delta)$ whose form is

$$\text{for} \quad \cos\delta \leqq \lambda \qquad \varkappa = \lambda \tag{16}$$

for $1 > \cos\delta > \lambda$, $\quad \varkappa = \frac{1}{\cos\delta + \sqrt{(\frac{1}{\lambda^2} - 1)(1 - \cos^2\delta)}}$ (17)

The authors recommend to consider the coefficient λ with the value of λ = = 0,635.

Thus the simplified function of the actual strengths surface according to Reimann has a general form of

$$\varrho = \varkappa(\sqrt{29,06 - 11,93\,\xi} - 5,18) \qquad (18)$$

Let us compare this simplified form of the actual strengths surface with the form deduced from the test results of Launay, Gachon and Poitevin [10] .

The graphic comparison of the form of the meridians Γ_D for the compressive character of the state of stress (Fig. 4) reveals very good accordance within the region of up to about $\xi = 4$; for higher values of ξ the actual strengths according Reimann are much higher than those computed on the basis of the results of the French tests. In the case of the meridian Γ_Z for the tensile character of the state of stress very good accordance can be found in the region to up to about $\xi = 2$; for higher values of ξ the values computed on the basis of the simplified characteristics are on the safe side. The approximate character of the assumption of the affinity of the course of the meridians is obvious from a comparison of the ratios ϱ_Z/ϱ_D shown in Fig. 4. Instead of the constant value of $\lambda = 0,635$ introduced in the simplified version of the surface form this ratio actually varies between about 0,8 and 0,48. The introduced constant value, consequently, nears the average. The differences between the idealized form of cross sections (shown in dashed lines) of the actual strengths surface and the form deduced from the French test results(shown in solid lines) are shown in Fig. 5.

Thus it is possible to sum up that the introduction of Reimann's simplified form of the actual strengths surface is always on the safe side in the region of stresses currently occuring in structures. Only in the case of higher stresses limited, however, to a narrow region in the vicinity of the compressive character of the state of stress, this idealization results in a minor overrating of the actual strength. In general it is possible to conclude that in the present state of laboratory research of the actual strength of concrete under triaxial stresses the idealization of the actual strengths surface proposed by Reimann is well acceptable.

4. PRINCIPAL PROCEDURE OF PRACTICAL CALCULATION

Let us assume that in the given point in which the state of stress should be assessed the stress $\sigma_1 \geqq \sigma_2 \geqq \sigma_3$ is applied. The relations (2) , (3), toggther with the introduction of the relations (13), determine the coordinates ξ_{A^+}, ϱ_{A^+} of the point A^+ (Fig. 6) on the meridian Γ_{red} determined by the angle δ according to the relation (4).

If we reduce the coordinates ξ, ϱ of the meridian Γ of the actual strengths surface by a safety margin m, we obtain the meridian Γ_d of the permissible stresses surface, the value of the angle δ and, consequently, that of the coefficient α remaining constant. When applying the statistical method of the limit state design, the formal aspect of the procedure remains the same.

Consequently, between the coordinates ξ_A, ϱ_A of point A on the curve Γ and those $\xi_{A'}$, $\varrho_{A'}$ of point A′ on the curve Γ_d there holds the relation

$$m = \frac{\xi_A}{\xi_{A'}} = \frac{\varrho_A}{\varrho_{A'}} \tag{19}$$

and, similarly, between the coordinates $\xi_{A'}$, $\varrho_{A'}$ and those ξ_{A^+}, ϱ_{A^+} of point A^+ on the curve Γ_{red}, the relation

$$\mu' = \frac{\xi_{A'}}{\xi_{A^+}} = \frac{\varrho_{A'}}{\varrho_{A^+}} \tag{20}$$

The overall safety margin is determined by the product

$$\mu = m\,\mu' \tag{21}$$

For the actual strengths surface according to (17) the coordinate $\xi_{A'}$ is computed as one of the roots of the quadratic equation

$$\xi^2 + (10{,}36\,\frac{j}{\varkappa} + 11{,}93)\,\frac{\varkappa^2}{j^2 \cdot m}\,\xi - 2{,}23 \cdot \frac{\varkappa^2}{m^2\, j^2} = 0 \tag{22}$$

where

$$j = \frac{\varrho_{A^+}}{\xi_{A^+}} \left(= \frac{\varrho_{A'}}{\xi_{A'}}\right)$$

Physically valid is the root with the same sign as that of the value j, as the ordinate $\varrho_{A'}$ must be always positive. With the introduction of the relation (11) and the substitution of the values α according to (9) we obtain for the actual strengths surface according to (18) the coordinates

$$\xi_U = -\,2{,}9825\,\frac{\varkappa^2}{k_o^2} + 2{,}4358 \tag{23}$$

4.1 ZONE OF COMPRESSIVE FAILURE

This zone is characterized by the real envelope of Mohr′s circles. The state of stress lies in the zone of compressive failure, if

$$\xi_{A^+} < \frac{1}{m\,\mu'}\,\xi_U \tag{24}$$

The failure is due - in the meaning of Mohr′s hypothesis - to shear. The shearing plane of failure as well as the normal stress σ_m and the tangential stress τ_m existing at the moment of failure are determined by the coordinates of the point of contact T of the Mohr's circle and the envelope ϕ in the plane σ, τ.

The equation of Mohr's circles

$$(\sigma - \sigma_{oct} + \frac{\alpha}{n} \tau_{oct})^2 + \tau^2 = \frac{9}{n^2} \tau_{oct}^2 \tag{25}$$

where

$$n = \sqrt{6 + 2\alpha^2}$$

is changed by the introduction of coordinates ξ, ρ into the form of

$$(\sigma - \frac{R_D}{\sqrt{3}} \xi + \frac{R_D}{\sqrt{3}} \frac{\alpha}{n} \rho)^2 + \tau^2 = \frac{3R_D^2}{n^2} \rho^2 \tag{26}$$

After the substitution from eq. (18) for ξ, derivation according to ρ and substitution $\rho = \rho_{A+}$ we obtain an equation for the unknown coordinate σ_m of the point of contact T of Mohr's circle which yields

$$\sigma_m = \frac{A_2 - 2B_2 \rho_{A+} - 3C \rho_{A+}^2 - 4D \rho_{A+}^3}{A_1 + B_1 \rho_{A+}}, \tag{27}$$

where

$$A_1 = \frac{\sqrt{3}}{R_D \mu} \left(\frac{1{,}7366}{\varkappa} + \frac{2\alpha}{n} \right), \qquad A_2 = \frac{0{,}3244}{\varkappa} + \frac{0{,}3736\alpha}{n},$$

$$B_1 = \frac{0{,}33528}{\varkappa^2} \frac{\sqrt{3}}{R_D \mu}, \qquad B_2 = \frac{0{,}72263}{\varkappa^2} + \frac{1{,}7366\alpha}{\varkappa n} + \frac{\alpha^2 - 9}{n^2}$$

$$C = \frac{0{,}14556}{\varkappa^3} + \frac{0{,}16764}{\varkappa^2 n} \alpha, \qquad D = \frac{0{,}007026}{\varkappa^4}$$

The respective tangential stress is determined by the formula

$$\tau_m = \frac{R_D \mu}{\sqrt{3}} \sqrt{\frac{9}{n^2} \rho_{A+}^2 - \left(\frac{\sqrt{3}}{R_D \mu} \sigma_m - \xi_{A+} + \frac{\alpha}{n} \rho_{A+} \right)^2} \tag{28}$$

and the angle ε by

$$\operatorname{tg} \varepsilon = \frac{\tau_m}{\sigma_1 - \sigma_m} \tag{29}$$

The calculated safety margin μ must exceed its required design value m, i. e.

$$\mu \geqq m \qquad \text{or} \qquad \mu' \geqq 1 \tag{30}$$

The safety cannot be substantially increased by the reinforcement, as the decisive influence for the failure is the shearing strength of concrete τ_m under simultaneous affect of σ_m. This manner of failure is characteristic of the failure due to plain compression.

4.2 ZONE OF TENSILE FAILURE

If

$$\zeta_{A^+} \geqq \frac{1}{m\,\mu'}\,\zeta_U \qquad (31)$$

the failure - according to the applied hypothesis - can be due to the tension σ_1. This manner of failure is characteristic of plain tensile failure. If the condition (30) has not been complied with, the safety of the structure can be possibly increased by the application of reinforcement which will transfer the respective tension. In this zone the calculated safety margin not only characterizes the margin of safety against the origin of cracks, but can also be used as a criterion for the tensile reinforcement design. When departing from the requirement that approximately the same safety margin should be maintained as that required in the majority of design codes for uniaxial state of stress, it is possible to recommend for the individual cases approximately the following values of the calculated safety margin μ :

When we reguire that concrete alone should be able to transfer tensile stresses without the cooperation of reinforcement, it is necessary, with regard to the uncertainty of the tensile strength of concrete due to the influence of secondary stresses, to increase the safety margin about 2,5 times, i. e. to introduce the condition

$$\mu \geqq 2{,}5\ m \qquad (32)$$

If this condition has not been complied with, it is necessary to sustain the whole tension by means of suitably designed reinforcement. Moreover, it is possible to propose a graduation of the coefficient μ in accordance with whether the structure is of fully or partly prestressed or merely reinforced concrete.

For the structures of fully prestressed concrete it is possible to recommend that the coefficient μ may vary approximately within the limits of

$$2{,}5\ m \geqq \mu \geqq 1{,}8\ m, \qquad (33)$$

For the structures of partly prestressed concrete, where the disturbance of the tensile zones of concrete by hair cracks is permissible, it is possible to recommend approximately the limits of

$$2{,}5\ m \geqq \mu \geqq 0{,}7\ m, \qquad (34)$$

Similarly for reinforced concrete structures it is possible to permit the limits of about

$$2{,}5\ m \geqq \mu \geqq 0{,}5 \quad \text{to} \quad 0{,}3 \qquad (35)$$

4.3 ARRANGEMENT AND DESIGN OF REINFORCEMENT

Since the state of stress in a triaxially stressed concrete body changes from one place to another, from the viewpoint of the crack prevention and that of the increase of safety against the failure of the structure as a whole the most effective arrangement of reinforcement is a rectangular spacial skeleton.

For the deduction of formulas for the forces transferred by reinforcement we shall introduce simplifying assumptions which are generally in accordance with the assumptions introduced into the calculation of reinforced concrete structures. We shall assume that concrete is not capable of transferring tensile stresses in any direction and that reinforcement transfers tension in the direction of the longitudinal bar axis only. The dowel effect in the shear transfer is neglected. Another assumption is that the shearing stresses τ_{xy}, τ_{yz}, τ_{zx}, applied in concrete in the planes perpendicular to the directions x, y, z of the longitudinal axes of the bars of the reinforcement skeleton, are decomposed into additional tensile stresses σ_{xy}, σ_{yz}, σ_{zx} in the directions of the longitudinal axes of the bars, and additional compressive stresses ν_{xy}, ν_{yz}, ν_{zx} of concrete deviating from these planes at the angles of φ_{xy}, φ_{yz} and φ_{zx} (Fig. 7). These angles can be selected arbitrarily. The suitability of the selection must be assessed, however, from the viewpoint of the most economic reinforcement design and from that of the ability of concrete to transfer additional compressive stress in the given direction. Similar assumptions were introduced for the reinforcement design of slab structures, which is most frequently used at present, for example by Kuyt [9] or by Hilleborg [6] and others.

According to Fig. 7 the additional compressive stresses of concrete ν_{xy}, ν_{xz}, ν_{zx} are determined by the formulas

$$\nu_{xy} = \nu_{yx} = \frac{\tau_{xy}}{\sin \varphi_{xy} \cos \varphi_{xy}} \tag{36}$$

$$\nu_{yx} = \nu_{zy} = \frac{\tau_{yz}}{\sin \varphi_{yz} \cos \varphi_{yz}} \tag{37}$$

$$\nu_{zx} = \nu_{xz} = \frac{\tau_{zx}}{\sin \varphi_{zx} \cos \varphi_{zx}} \tag{38}$$

The forces p_x, p_y, p_z transferred by the reinforcement from unit area perpendicular to the directions of the bar axes x, y, z respectively, are determined by the formulas

$$p_x = \sigma_x + \tau_{xy} \operatorname{tg} \varphi_{xy} + \tau_{zx} \operatorname{cotg} \varphi_{zx} \tag{39}$$

$$p_y = \sigma_y + \tau_{xy} \operatorname{cotg} \varphi_{xy} + \tau_{yz} \operatorname{tg} \varphi_{yz} \tag{40}$$

$$p_z = \sigma_z + \tau_{zx} \operatorname{tg} \varphi_{zx} + \tau_{yz} \operatorname{cotg} \varphi_{yz} \qquad (41)$$

where $\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$ are the stresses of concrete in the planes xy, yz, zx respectively.

5. FOLLOW-UP OF STATES DURING OVERLOADING OF STRUCTURES TO THE POINT OF FAILURE

The limit below which it is possible, with satisfactory accuracy, to calculate the stresses of a concrete body on the basis of the assumption that the concrete body is elastic and homogenous can be considered approximately with the condition (35) introduced for reinforced concrete structures. For the calculations of the states of the overloaded structure it is more consistant to introduce as a criterion of the exclusion of concrete from the work in the tensile zone the condition

$$\mu \leqq 1 \qquad (42)$$

If this conditions has been complied with, it is necessary to introduce into the structural design the assumption that all tensile stresses at the given point are transferred by reinforcement, and repeat the calculation. To advantage it is possible to use the finite element method (such as Rashid [12]) The beginning of the plastic behaviour of the material can be approximately considered at the moment when steel attains the yield point and the compressed zone of concrete about 90 % of the average actual strength of concrete.

In the calculation it is useful to determine the magnitude of the increased live load $p_1 = p \cdot s_1$ (where p is the operational live load) in which the cracks originate in the structure permissible in reinforced concrete structures and all tensile stresses are transferred by reinforcement. In this loading state the condition (35) should be complied with in the tensile zones of the structure and the condition (30) in the compressive zones, which could be possibly reduced to as much as $\mu \geqq 1,8$ corresponding approximately with the limit of discontinuity and decohesion ascertain by the tests by Launay, Gachon and Poitevin [10] . This state of overloading of the structure corresponds with the state when the structure is fully capable of use after the removal of the load.

The attainment of the beginning of the plastic behaviour of the structure corresponds with the live load of $p_2 = p \cdot s_2$. In the tensile zones the condition (35) is not complied with, but the stress of reinforcement is limited by its yield limit. In compressive zones it is possible to introduce the condition $\mu \geqq 1,1$.

The calculated coefficients s_1 and s_2 are not connected in any way with the attained or the stipulated safety margins μ or m respectively.

The calculation can be also used for the theoretical follow up of the development of the cracks during overloading of the structure, which will originate in the zones of tensile failure, according to the applied hypothesis, in the planes perpendicular to the direction of principal stresses σ_1 and in the zones of compressive failure in the planes whose normal is inclined at the angle of ε determined by the formula (29) from the positive direction of the stress σ_1.

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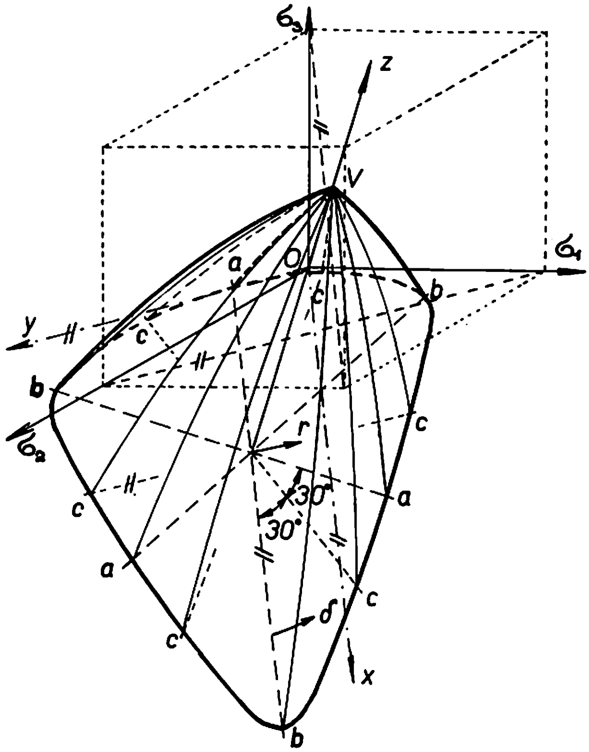

Fig. 1 Axonometric representation of the actual concrete strengths surface

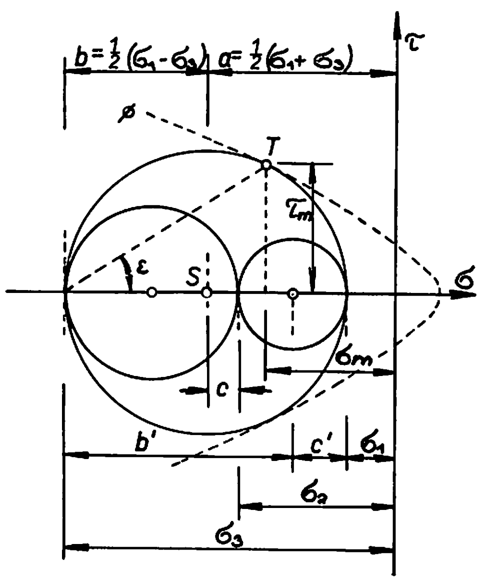

Fig. 2 Mohr's representation of the state of stress in the system of coordinates

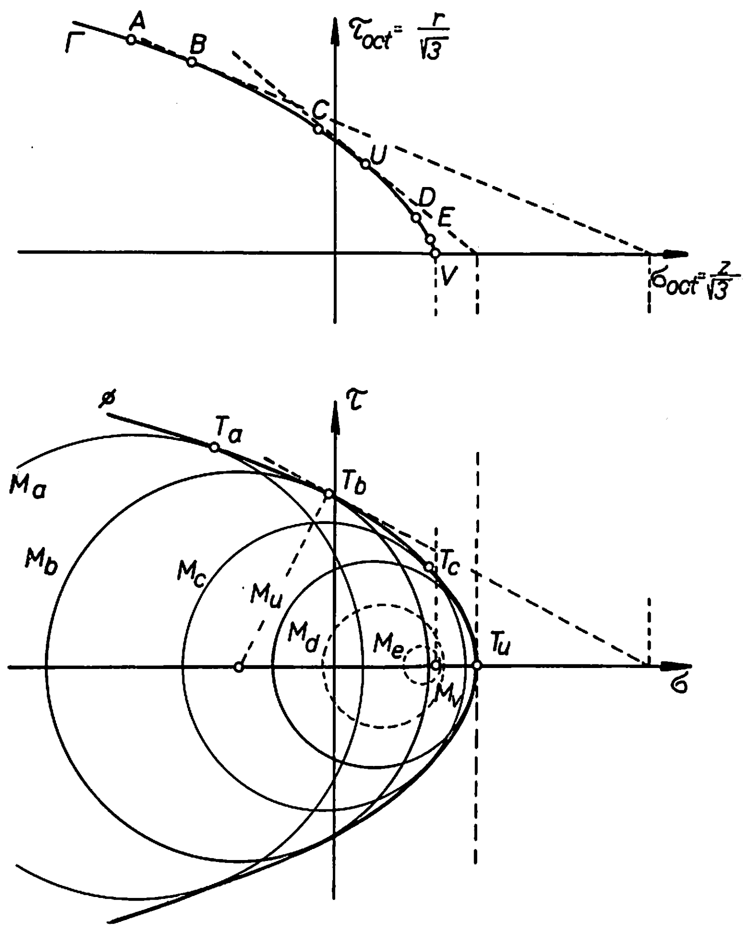

Fig. 3 Mohr's circles corresponding with the meridian Γ of the actual strengths surface

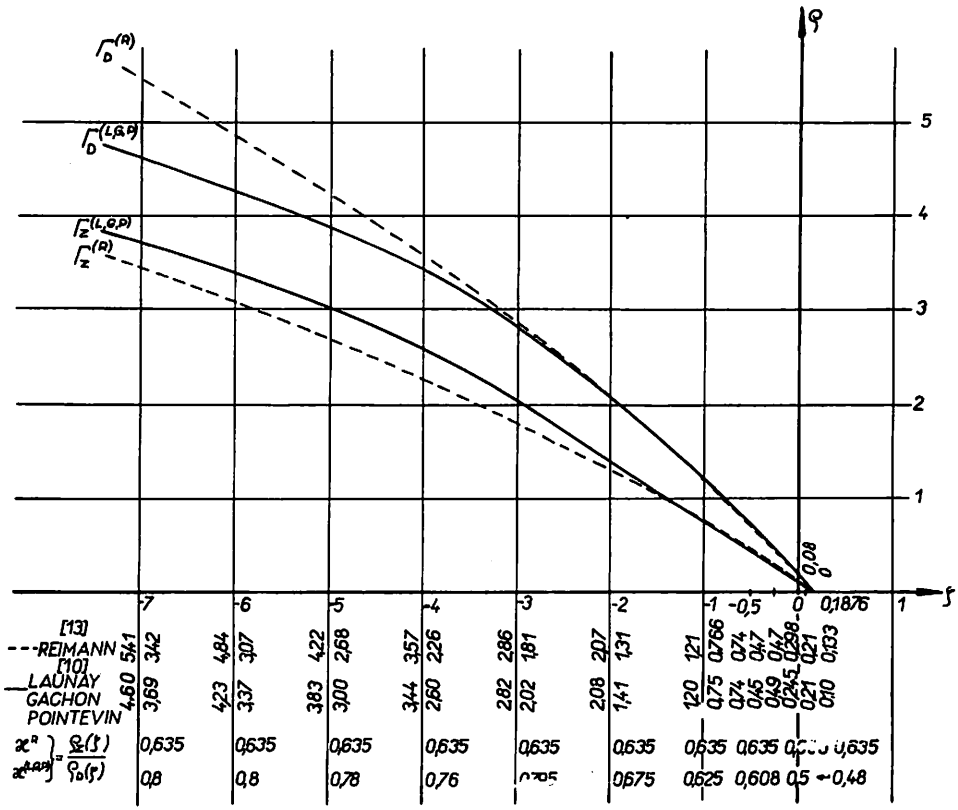

Fig. 4 Comparison of meridian forms for the compressive (Γ_D) and the tensile (Γ_Z) characters of the state of stress according to Launay, Gachon and Poitevin [10] (solid lines) and according to Reimann [13] (dashed lines)

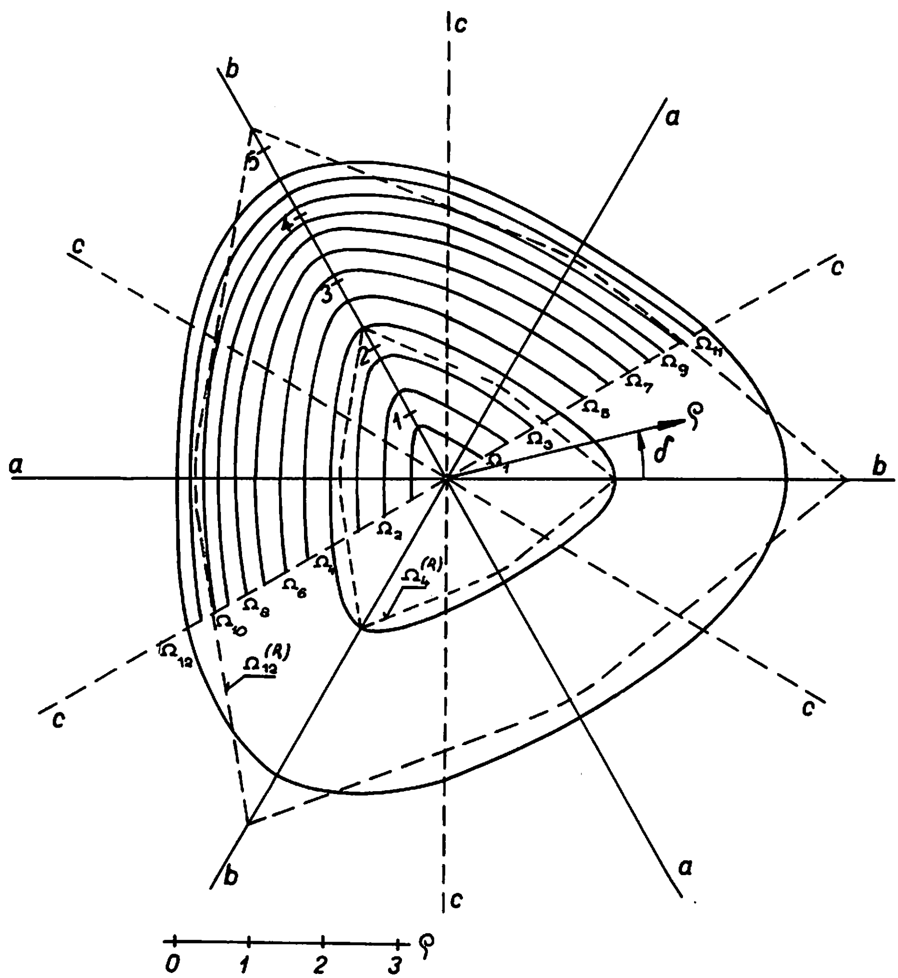

Fig. 5 Comparison of the form of cross sections Ω_1 and Ω_2 of the actual strengths surface in the planes of $\zeta_1 = \frac{1}{\sqrt{3}}$ to $\zeta_{12} = \frac{12}{\sqrt{3}}$ according to [10] (solid lines) and [13] (dashed lines)

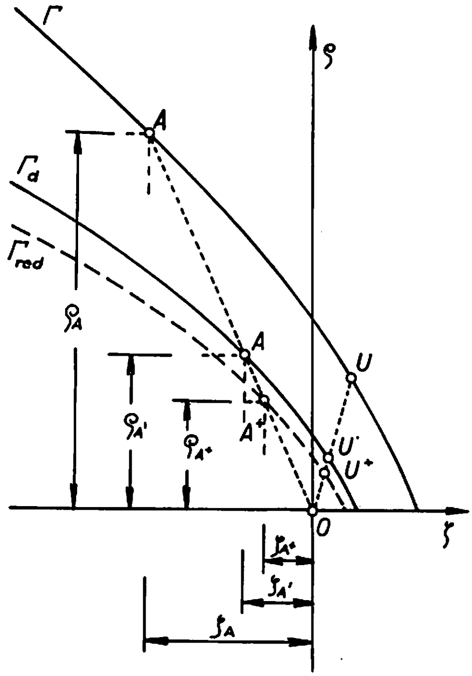

Fig. 6 Determination of the safety margin for the given state of stress

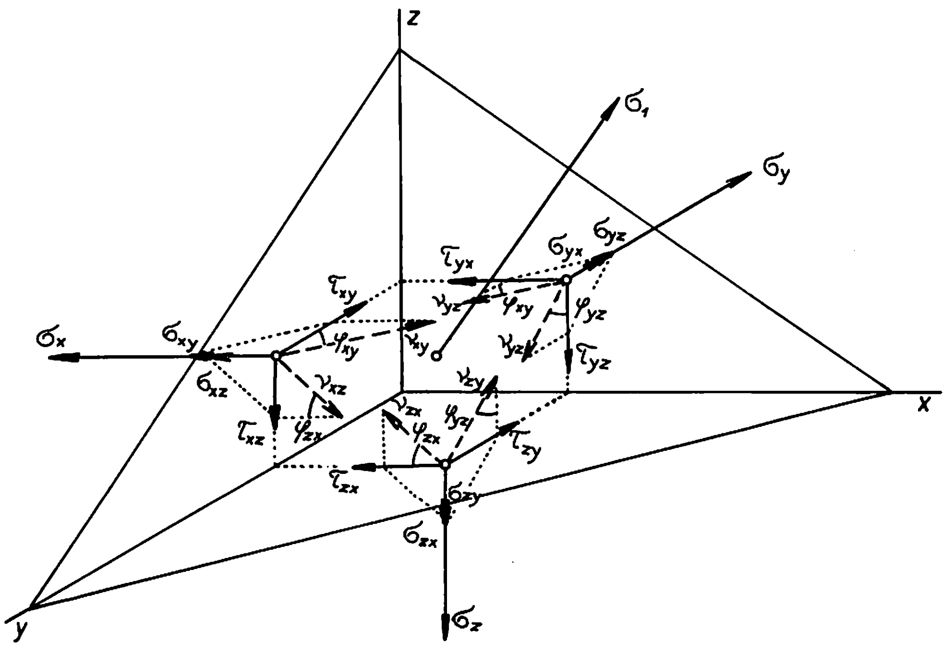

Fig. 7 Spatial decomposition of forces when tensile and shearing stresses are sustained by reinforcement in the directions x, y, z and additional compressive stresses by concrete.

DISCUSSION

C

Z. P. BAZANT, U. S. A.

The Mohr's theory of failure which you utilized assumes shear failure under pure uniaxial compression. However, the mode of failure in this case, as observed on long specimens in which friction at the ends is removed as much as possible, is axial cleavage fracture (splitting). This may possibly require some refinements of the theory. Furthermore, it is also possible that strains, in addition to stresses, should also appear in the failure criterion.

C

F. C. WEILER, U. S. A.

Z. P. Bazant commented on using strain instead of stress for failure theories. I would like to say a few words about this concept.. I work a lot with graphites and ceramics, much more so than concrete, and we use a multiaxial strain failure surface (envelope) because we can measure strain. We have to calculate stress from the strain and elastic constants of the material. If we don't know these elastic constants exactly, then we introduce some error into the constants necessary for describing the stress failure envelope. Hence, we use a strain failure envelope instead of a stress failure envelope.

C

B. SAUGY, Switzerland

The different modes of rupture of a specimen tested under uniaxial compression are apparently in contradiction with the use of a surface of rupture in which one point represents all these modes.

However, by considering the specimen as a three-dimensional structure, for example in a finite element model and by taking into account the mode of application of load, it is possible to explain this phenomenon if instead of working with shear stress one takes into consideration the deviatoric stress.