Experimental and Analytical Approach to Non-Linear Vibration Due to Gaps

A. Sakurai, Y. Masuko
Central Research Institute of Electric Power Industry, 1646 Abiko, Abiko-shi, Chiba 270-11, Japan
Y. Sonoda, M. Madokoro, Y. Sato
Hitachi Works, Hitachi, Ltd., 3-1-1, Saiwai-cho, Hitachi-shi, Ibaraki 317, Japan

Abstract

Non-linear seismic response of structures having a gap in its supports has been studied experimentally and analytically. Among the structures of this type, the reactor structure is a typical one when it is equipped with radial keys for the amelioration of the seismic resistance. The purpose of this study is to set up a simplified analytical method for an estimation of the non-linear seismic response and to verify the applicability of a numerical method for a time domain non-linear response analysis.

The paper consists of three parts 1) vibration test of a simplified reactor vessel and radial keys 2) set up of an method for the estimation of non-linear seismic response 3) introduction and verification of a numerical method for a time domain non-linear response analysis of structures with gap.

The following results have been obtained. 1) For the seismic motion whose level is lower than Ac, the response is elastic. (Ac is the lowest level of seismic motion to cause a contact at the gap.) 2) For the seismic motion whose level is between 1 to a few times Ac, maximum displacement and contact force of the non-linear response can be estimated by using a linear response spectrum. 3) For the seismic motion whose level is higher than ten teimes Ac, the non-linear response is roughly identical to that obtained by neglecting the gap. 4) For an intermediate seismic level (between a few times to ten times Ac), response analysis requires a fully non-linear time domain calculation. In this case, a modal analysis based on two series of eigen values, that of the connected and disconnected states of the structure at the anticipated contact points, was found to be effective and cost saving.

1. Introduction

Under Japanese stringent seismic design conditions, the reactor vessel of FBR which is ordinarily suspended from the roof deck may need additional seismic supports at its lower parts. In this case, the seismic supports are required to have enough stiffness against the seismic load and to allow the thermal expansion. Radial keys are one of the design solutions to satisfy the above-mentioned requirements.

An example of radial keys is shown in the Fig. 1. The reactor vessel which is suspended from the roof deck has radial keys around the meridional circumference at the intermediate level. The radial keys are received by as many keyways which are, in this case, attached to the guard vessel. There is a gap of the order of several milimeters between the radial keys and the keyways. When the horizontal relative displacement exceeds the gap during earthquakes, the two vessels contact. While the excitation level is relatively low, the reactor vessel and the guard vessel vibrate independently within the gap. For a higher excitation level, the two vessels repeat contacts while vibrating during the earthquake. This is a kind of a non-linear vibration problem with abrupt changements of the stiffness and the mass which generally requires an elaborating numerical calculation.

From the viewpoint of a conceptual design of reactor structures, two methodologies of different exactness are useful to treat this non-linear behavior, a simple estimation method of the earthquake response and a more sophisticated time domain direct non-linear analysis. The purpose of the present study is to verify the applicability of a simple method for the estimation of the non-linear seismic response and to verify the effectivity of a numerical calculation method for the time domain non-linear response analysis.

2. Experiment

Fig. 2 and Fig. 3 shows the test model representing the reactor vessel and the guard vessel. The model is schematized by two inverted pendulums whose weight is in the range of hundreds of kilograms and have a gap of the order of several milimeters between them. The structures are excited by sine waves and seismic waves by means of a shaking table. The acceleration and the displacement of each pendulum have been measured. Fig. 4 shows an example of the resonance curve obtained from a sine sweep test. As is commonly observed in a non-linear vibration due to gaps, the resonance curve splits into two lines neighboring the peak corresponding to the sweep up and the sweep down. The magnification factor and the apparent natural frequency (the frequency corresponding to the peak resonance) depend on the excitation level and the sweep up or down.

The apparent natural frequency is plotted against the excitation level in the Fig. 5. For a small excitation level, the observed natural frequency is that of the pendulum vibrating independently. For higher excitation levels, the apparent natural frequency increases as a function of the excitation level and converges to the natural frequency of rigidly connected pendulumns. Here also, the line splits between the excitation level Ac and 10 times Ac. (Ac is the lowest excitation level where the splitting is observed, that is, the lowest excitation level necessary to make the two pendulumns contact) Conversion to the natual frequency of the rigidly connected pendulums implicates that the non-linear response to higher excitation can be roughly estimated by neglecting the gap.

3. Analytical Solution

A method for an estimation of the apparent natural frequency of a non-linear vibration has been derived and compared with the experimental results. Because the weight of the guard vessel is small compared to that of the reactor structure (order of 1/30 for example), neglecting the weight of the guard vessel, the reactor vessel can be simplified to an oscillator with an external spring having a gap between them (Fig. 6). The force displacement relation of the oscillator is shown in the Fig. 7. The oscillator has a typically non-linear hardenning type restoring force characteristics.

Apparent natural frequency (or the free oscillation frequency) of the system is expressed as a function of the amplitude as following.

Suppose that the oscillator has an initial displacement $x = \delta_I$ and a null initial velocity at time t=0 The time increment t_1 so as the oscillator passes from the initial point to the point $x = \delta_c$ is expressed;

$$t_1 = \omega_1^{-1} \cos^{-1} \frac{\delta_c}{\delta_I} \tag{1}$$

$$\dot{x}_{t=t_1} = - \omega_1 \sqrt{\delta_I^2 - \delta_c^2} \tag{2}$$

$$\text{where} \quad \omega_1^2 = \frac{k1}{m} \tag{3}$$

The time increment t_2 so as the oscillator passes from the point $x = \delta_c$ with the initial velocity $\dot{x}_{t=t_1}$ to the neutral point is expressed;

$$t_2 = \left(\frac{\pi}{2} - \varphi\right) \frac{1}{\omega_2} \tag{4}$$

$$\text{where } \omega_2^2 = \frac{k2}{m} \tag{5}$$

$$\varphi = \tan^{-1}\left[\frac{\omega_1}{\omega_2} \sqrt{\left(\frac{\delta_I}{\delta_c}\right)^2 - 1}\right] \tag{6}$$

Apparent natural frequency of the system for the amplitude δ_I is

$$f_{ap} = \frac{1}{4\,(t_1 + t_2)} \tag{7}$$

The maximum kinetic energy E_{kmax} at the displacement $x = \delta_I$ is expressed;

$$E_{kmax} = \frac{1}{2}\, k2\, (\delta_I - \delta_c)^2 + \frac{1}{2}\, k2 \cdot \delta_I^2 \tag{8}$$

The dotted line in the Fig. 5 represents the apparent natural frequency calculated according to the eq. (7) by assuming the total energy at the resonance be proportional to the square of the excitation level. The analytical result and the observed data have a good accordance but the analytical result converges more rapidly than the other one. The difference between them may be attributed to the neglection of the local deformation in the analytical solution. For a rough estimation, the both results show that the gap can be neglected for the excitation whose level is higher than 10 Ac.

4. Numerical calculation

According to the modal analysis, the response x(t, u) of the structure is expressed by a summation of the products of time functions and mode shapes.

$$x(t, u) = \sum_i x_i(t)\, \phi_i(u) \tag{9}$$

Suppose the structure hits another structure at time t=tc and the mode shape during they are in contact be expressed by ψ(u), the response Y(t, u) of the structure while they are in contact is expressed;

$$Y(t, u) = \sum_i y_i(t)\, \psi_i (u) \tag{10}$$

The conditions to be satisfied between the two states of vibration at the moment of contract are following,

1. identity of the deformation

$$x (t_C, u) = Y (t_C, u) \tag{11}$$

2. identity of the quantity of movement

$$\int_V m(u)\dot{X}(t_C, u)du = \int_V n(u)\, \dot{Y}\, (t_C, u)\, du \tag{12}$$

m(u): mass distribution before contact

n(u): mass distribution while in contact

Substituting the eq. (9) and (10) and using a matrix expression, the eq. (11) becomes,

$$\sum_i \alpha_i\, \Phi_i = \sum_i \beta_i\, \Psi_i \tag{13}$$

where $\alpha_i = x_i (t_C)$

$\beta_i = y_i (t_C)$

Φ_i and Ψ_i are eigen vector of the structure before contact and while in contact respectively.

Multiplying the both sides by $\Psi_j^T M$, the eq. (13) becomes as following, thanks to the orthogonality of eigen vectors,

$$\Psi_j^T M \sum_i \alpha_i\, \Phi_i = \Psi_j^T M \sum_i \beta_i\, \Psi_i \tag{14}$$

where M is a mass matrix.

$$\sum_i \alpha_i\, \Psi_j^T M\, \Phi_i = \beta_j\, \Psi_j^T M\, \Psi_j \tag{15}$$

$$\beta_j = \sum_i \alpha_i\, \zeta_{ij} \tag{16}$$

where $\zeta_{ij} = \dfrac{\Psi_j^T M\, \Phi_i}{\Psi_j^T M\, \Psi_j}$

By the same operation, the eq. (12) becomes

$$\dot{\beta}_j = \sum_i \dot{\alpha}_i\, \zeta_{ij} \tag{17}$$

where $\dot{\alpha}_i = \dot{x}_i (t_C)$

$$\dot{\beta}_i = \dot{y}_i(t_c)$$

Thus the initial condition for the response after the contact, that is β_i and $\dot{\beta}_i$, are obtained by the eq. (16) and (17). The term ζ_{ij} represents a kind of sensibility of the j^{th} mode after contact against the i^{th} mode before contact. It has a maximum value of 1.0 for a couple of exactly identical Ψ_{ij} and Φ_i. Following a similar operation, initial conditions at the moment of separation are expressed as follow,

$$\alpha_j = \sum_i \beta_i \xi_{ij} \tag{18}$$

$$\dot{\alpha}_j = \sum_i \dot{\beta}_i \xi_{ij} \tag{19}$$

where $$\xi_{ij} = \frac{\Phi_j^T M \Psi_i}{\Phi_j^T M \Phi_j} = \zeta_{ij}^{-1}$$

Consequently, the time domain non-linear response analysis results in a succession of modal analyses with a re-set of initial conditions at each contact or separation.

The advantage of this method lies in its rapid numerical operation. Once the matrix ζ and ξ are obtained, the initial conditions are obtained by $2xN^2$ operations, where N is the number of modes to be considered. On the other hand, the calculation becomes elaborating when the number of contact points increases because an eigen analysis is needed for each combination of the contact points.

Because the modal amplitudes are transposed on the new series of vibration modes at the contact, if the series of modes is truncated at the N^{th} mode, the modal amplitudes (consequently the energy) to be transfered to the modes higher than N^{th} mode are lost. That is the apparent damping effect caused by the modal truncation. In that reason, the number of modes N has to be set as high as possible in order to assertain the mathematical exactness. But in actual problems, the number of modes can be determined so as the apparent damping by truncation be equivalent to the actual energy loss at each contact.

Fig. 8 shows a comparaison of the maximum displacement observed in the test and that by non-linear calculation. Both results have a good accordance and have a common feature in the relation between the input level and the maximum displacement.

5. Concluding remarks

Vibration tests have been performed in order to study the seismic behavior of a reactor structure equipped with radial keys. The following results have been obtained. 1) For the seismic motion whose level is lower than Ac, the response is elastic. (Ac is the lowest level of seismic motion to cause a contact at the gap.) 2) For the seismic motion whose level is between 1 to a few times Ac, maximum displacement and contact force of the non-linear response can be estimated by using a linear response spectrum. 3) For the seismic motion whose level is higher than ten teimes Ac, the non-linear response is roughly identical to that obtained by neglecting the gap. 4) For an intermediate seismic level (between a few times to ten times Ac), response analysis requires a fully non-linear time domain calculation. In this case, a modal analysis based on two series of eigen values, that of the connected and disconnected state of the structure at the anticipated contact points, was found to be effective and cost saving.

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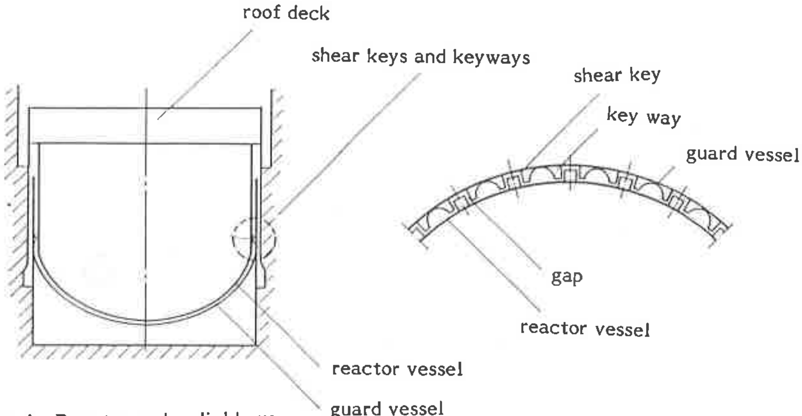

Fig. 1 Reactor and radial keys

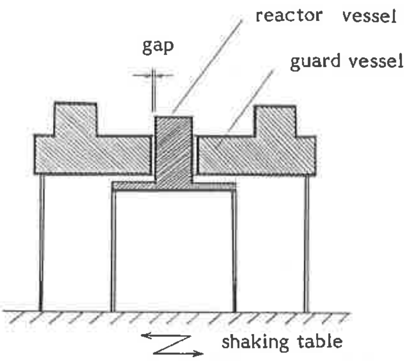

Fig. 2 Test model

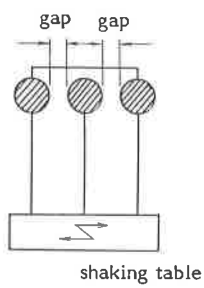

Fig. 3 Schematization of test model

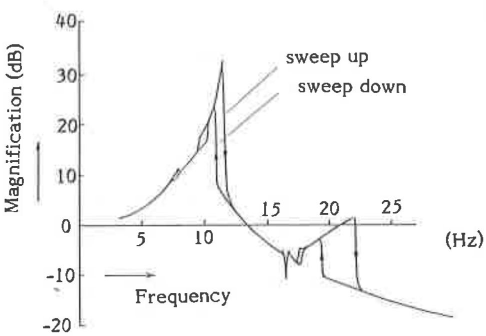

Fig. 4 Resonance curve obtained from since sweep test

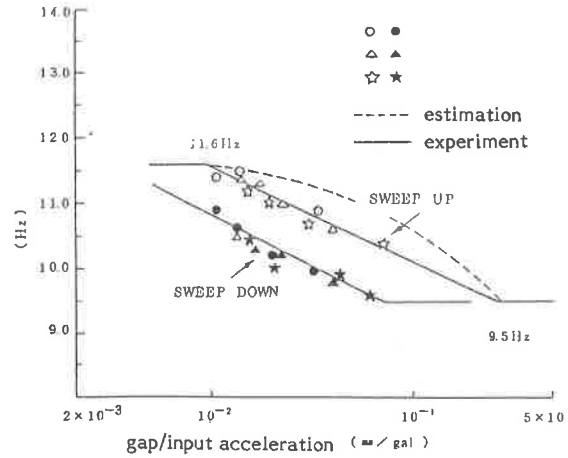

Fig. 5 Apparent natural frequency versus excitation level (sinusoidal excitation)

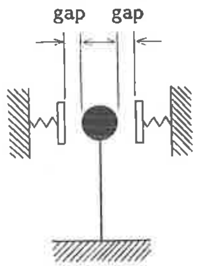

Fig. 6 Simplified analysis model

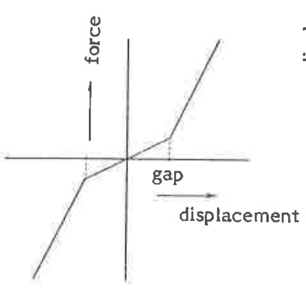

Fig. 7 Force-displacement relation of ascillator

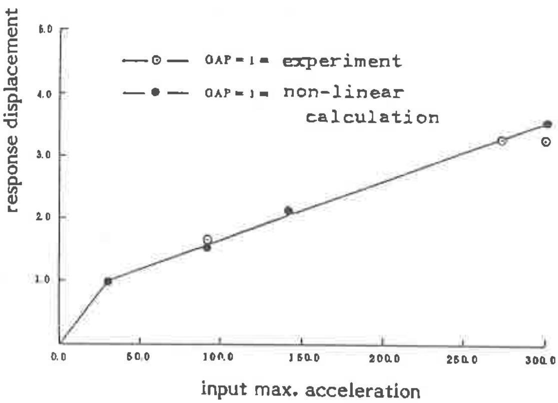

Fig. 8 Apparent natural frequency versus excitation level (white noise)