Usage of Possibilistic Methods in Safety Assessment of VVER-1000 Concrete Containments

Alexander Bausk[1)], Mykola V. Savitsky[2)],

1) Nuclear Engineering Research Lab, PSACEA, Dnipropetrovs'k, Ukraine
2) Prydniprovska State Academy of Civil Engineering and Architecture (PSACEA), Dnipropetrovs'k, Ukraine

ABSTRACT

A serious problem in processing uncertainty which is often neglected is the different nature of aleatory (i.e., dependent on chance) and epistemic uncertainties. Epistemic uncertainties arise frequently in the tasks of NPP structures safety assessment due to lack of statistical data and possible errors in expert evaluations.

Several uncertainty processing methods, including higher-order uncertainty models, are outlined. A possibilistic approach is described in order to allow more relevant processing of uncertainties. This approach is based on fuzzy sets theory and can be more efficient in obtaining uncertain structure responses while keeping computational effort within reasonable limits.

A discussion follows how these methods can be compared and applied to assess the structural safety of WWER-1000 type reinforced concrete containment. The article also shows a comparison of example problems' structural response analysis results.

INTRODUCTION

According to the acting codes, condition evaluation and safety assessment of the safety-critical structures of nuclear power plants are to be conducted on a non-deterministic basis. The international codes describe activities classified as *Probabilistic Safety Assessment* of NPP structures important to safety [1, 2].

In this paper, the possibilistic measure is employed to evaluate uncertainties in the structural analysis of NPP structural components important to safety, particularly to WWER-1000 type concrete containment vessel. Nevertheless, the broad class of non-probabilistic safety assessment methods can generally be applied to other significant procedures in scope of fragility assessment of NPP structural components important to safety.

This paper focuses on developing an uncertainty analysis method that is applicable within the more general scope of probabilistic safety assessment (PSA) procedures that are described in the international IAEA standards [1]. Further, we will discuss a *possibilistic* interpretation of the containment structural analysis using finite element modeling.

The possibilistic safety assessment technique presented here is based on fuzzy structural analysis. In the domain of uncertainties in structural analysis, possibilistic methods in which uncertainties are defined as fuzzy variables form a complimentary to well-developed probabilistic methods such as sampling strategies or first/second order reliability methods.

The principal reason to introduce fuzzy measure in uncertainty analysis is the need to separate aleatory and epistemic uncertainties that arise in the modeling and simulation stages of analysis. As suggested by Ferson and Ginzburg [3] and more recently developed by Helton [4], distinct representation methods are needed to adequately separate random variability (often referred to as "aleatory uncertainty") and imprecision (often referred to as "epistemic uncertainty"). It was also shown [5] that the fuzzy variables provide a relevant mathematical apparatus to propagate epistemic uncertainty in engineering models.

There is a fundamental difference between aleatory and epistemic uncertainty [6]. Aleatory uncertainty is also referred to, depending on discipline, as variability, stochastic uncertainty, or irreducible uncertainty. It arises due to the strictly inherent variation embedded in the physical system, material properties, or the environment under consideration.

Probabilistic methods of structural analysis work well when applied to the systems with aleatory uncertainty. However, it must be stated that the limit of applicability of probabilistic methods is attained when only few or insufficient reliable statistical data are available for describing input values with the aid of probability density functions or probability distribution functions. Moreover, the probability-based methods require that the probability distribution or other statistical data (which are themselves crisp values) be provided for all uncertain input parameters even when such information is not reliable. The voluntary assignment of probabilistic properties that usually takes place in safety assessment practice can lead to serious errors in structural analysis.

This problem is addressed by using more relevant uncertainty analysis methods which can address epistemic uncertainty. This second general type of uncertainty is due to a certain subjective level of ignorance about the behaviour or properties of an engineering system. In [6], the definition of epistemic uncertainty is any lack of knowledge or information in any phase or activity of the modelling process. The key feature in this definition is that its fundamental cause in incomplete information about some characteristic of the system or the environment. It must be stressed that the epistemic uncertainties arise very frequently in the field of the condition assessment of existing structures, and it is of particular interest in nuclear engineering where parts of the structures critical to safety are often completely inaccessible due to radiation or technology issues. Various security-related accessibility problems may also arise if any parts of the safety evaluation process are conducted by a foreign contractor. Other examples of epistemic uncertainty in the safety assessment of nuclear facilities are systematic measurement errors or expert opinions [7].Other terms that refer to epistemic uncertainty are e.g. vagueness or reducible uncertainty.

The other concern about uncertainty propagation methods is their computational feasibility. The modern approaches based on sampling strategies (such as Monte-Carlo simulation of Latin hypercube sampling) usually require either a considerable computational effort or using some kind of a surrogate model, usually considerably less precise. An alternative is to modify the strategy to optimize the amount of operations needed, which often leads to retrieving approximations of the precise solution, sometimes too rough to be of practical use. The estimations of applicability limits of probabilistic methods usually regard 100-150 random variables as very large quantity that requires a huge computational effort. Representing uncertainties in terms of possibility theory considers much greater amount of uncertain parameters, up to thousands. For the large design model of the WWER-1000 containment, such a feature can allow to conduct full uncertainty analysis that takes into account, for instance, independent uncertain variations of elasticity modulus in each finite element, making up several thousands of uncertain interval parameters. The procedure of reducing fuzzy analysis to interval analysis will be described later in this article.

Finally, in the established probabilistic analysis practice, no method exists to propagate the frequent case when a parameter is given in the form of upper and lower bound only. The established practice in the probabilistic procedure is to accept uniform distribution on the bounded interval. However, there is by definition no evidence that the parameter has such kind of distribution. Such assumption often leads to underestimation of probability distribution tails.

This paper presents the application of an interval analysis method to the safety assessment of a WWER-1000 type concrete containment vessel. The proposed method can be easily generalized to include an arbitrary structure critical to safety.

A BRIEF OVERVIEW OF UNCERTAINTY ANALYSIS METHODS

Since the computational methods for structural analysis are now mature and more widely used, the next challenge is to extend classical deterministic analysis to be capable of propagating uncertainty through a simulation model.

The structural analysis of the WWER-1000 pre-stressed concrete containment vessel must yield appropriate results that can be used for probabilistic safety assessment procedures. As follows from [8], the possible failure modes of the WWER-1000 containment are:

- membrane failures of the containment shell;
- failure at the containment wall - base mat junction;
- failure of the containment wall - upper ring junction;
- failure of the dome - upper ring junction;
- failure of the base mat.

These failure modes are heavily affected by uncertainties in the initial data.

Recent developments in the field of processing uncertainties in structural analysis computations and in design optimization procedures yield numerous methods to model uncertainties in structures. The traditional methods described in standards may include semi-probabilistic and stochastic modeling. Yet the various types of uncertainties are treated in the same way. That is not always acceptable in assessment and safety analysis of nuclear engineering structures where it is often simply impossible to collect enough sufficient statistical data.

The probabilistic modeling actually cannot handle cases with incomplete or little information on which to evaluate a probability, or when that information is ambiguous or conflicting. Muhanna, Mullen and Zhang [9] provide a good insight to the generalized models of uncertainty that have been developed to treat such situations. These models begin with classical and highly specialized probability theory, which is followed with increasing generality by possibility theory [10], Dempster-Shafer theory of evidence [11], convex modeling [12], probability bounds analysis [13], recent developments in imprecise probabilities analysis [14], and others. These models can be described well by mathematical apparatus of interval arithmetic.

We will show later how interval arithmetic will be applied as a calculation tool for the generalized fuzzy set uncertainty model.

INTRODUCING UNCERTAINTIES AS FUZZY PARAMETERS

The concept of a fuzzy variable in engineering analysis has been in active development since late 80’s. In this context one can mention the work by Wood, Otto, and Antonsson [15]. Research in applying fuzzy sets and interval computations can be traced back to the publications by Negoiță and Ralescu [16], as well as the fundamental research by

Moore, founder of interval analysis [17]. The theoretic development in this domain now focuses on the development of second-order uncertainty quantification [18, 19]. The second-order uncertainty concept is a very promising direction, yet it is not in the scope of this paper.

The fuzzy uncertainty measure combines two ideas [5]: firstly, the concept of a confidence interval that defines the variation range of a parameter or variable; secondly, the concept of a degree of membership that is a mathematical description of a subjective assessment of the possibility that the value is within the range associated with the given degree of membership. The measure of the extent to which a value x in the fundamental set X is a member of the subset $A_i \subseteq X$ is described by a degree of membership $\mu_{Ai}(x)$. The set of all confidence intervals that are constructed for every membership degree α_i in $\alpha_i \in [0;1]$ constructs the membership function of the given uncertain variable x_n:

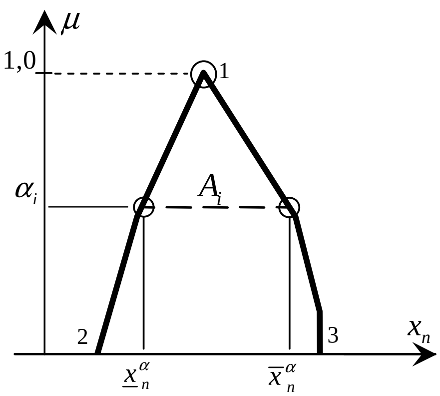

Figure 1. Fuzzy variable x_n.

The fundamental difference between fuzzy measure and probability measure is that the area below the membership function does not necessarily add up to one. Further properties and a rigorous definition of fuzzy measure and fuzzy sets are ubiquitous in the literature [6, 20].

The following examples of uncertainty can be successfully described using the notion of fuzzy parameters, including the case when the interval-valued estimations are the only available data about the parameters:

- Loads and load combinations (the loads are given in the form of 0-max value interval);
- Boundary conditions (rigid, semi-rigid, plastic);
- Concrete creep and shrinkage;
- Anchorage and pre-stressing forces in the concrete containment vessel;
- Factual (i.e. non-probabilistic yet not known precisely) properties of the construction materials.

In practice, the fuzzy response of the structural system is computed through α-level sampling procedure, also known as fuzzification. The fuzzy membership functions of all the parameters are sliced at prescribed degrees of membership $\{\alpha_r\}$. The resulting interval uncertainties are propagated through the deterministic simulation model. The model in our case will be the finite element model of the pre-stressed concrete containment vessel.

A rigorous explanation of the alpha-level cuts concept can be found in the publication by Nakamura [21].

INTERVAL ANALYSIS ALGORITHM FOR THE ASSESSMENT OF CONTAINMENT SAFETY

First of all, we will consider the following methodology of uncertainty propagation through large-scale numerical deterministic models to which belongs virtually every practically feasible modeling of an engineering structure. The classic input-output relationship is employed where the deterministic simulation model $\boldsymbol{M}$ maps the given uncertain inputs $\mathbf{f}$ to a vector of outputs $\mathbf{U}$ [23]:

$$\mathbf{f} \rightarrow \boxed{M(\cdot, X)} \rightarrow \mathbf{U}$$

When using the alpha-level discretization method, a vector of interval input parameters that corresponds to an alpha-level is constructed and used to retrieve interval output for this alpha-level. The fuzzy response vector is built by performing the procedure for multiple alpha-levels:

$$\boldsymbol{X}_\alpha = \begin{Bmatrix} \underline{x}_1^\alpha ; \overline{x}_1^\alpha \\ \underline{x}_2^\alpha ; \overline{x}_2^\alpha \\ \underline{x}_n^\alpha ; \overline{x}_n^\alpha \end{Bmatrix} \Rightarrow M\left(\cdot, \boldsymbol{X}\right) \Rightarrow \boldsymbol{U}_\alpha = \begin{Bmatrix} \underline{u}_1^\alpha ; \overline{u}_1^\alpha \\ \underline{u}_2^\alpha ; \overline{u}_2^\alpha \\ \underline{u}_m^\alpha ; \overline{u}_m^\alpha \end{Bmatrix} \quad (1)$$

In a frequent case of the definition of $M\left(\cdot, \boldsymbol{X}\right)$ as a finite element model which is computed using appropriate deterministic algorithms, the uncertainties in $\boldsymbol{X}_\alpha$ are parametric in the sense that they are included in the equilibrium equations in multiple positions. Due to the so-called dependency problem [9], such a formulation leads to overestimation of the solution hull $\boldsymbol{U}_\alpha$, sometimes unbounded.

This makes explicit interval evaluation of the solution hull practically non-usable. Consequently, implicit methods were developed to deal with the problem.

These methods are divided into two main classes, namely optimization algorithms and various types of sensitivity/worst case analyses

An optimization algorithm should be employed to search for the extrema (max/min) of the system response in the interval parameter domain if the deterministic mapping $M\left(\cdot, \boldsymbol{X}\right)$ is nonlinear with respect to the interval input vector. The optimization approach often encounters practical difficulties. Firstly, it requires sophisticated optimization algorithm, where the objective function is implicit and complicated in almost any formulation of the simulation model, thus often only approximate solution is achievable. Secondly, this approach is computationally expensive. For each response quantity, two optimization problems must be solved to find the extreme lower and the extreme upper bounds. This will be a huge computational effort, especially in the case of practical engineering problems

Recently, sensitivity analysis techniques for the interval finite element analysis have been developed in a number of works. For linear elastic problems, this approach is applicable if linearity assumption is made. The sensitivity analysis approaches decompose the initial system to analyze the independent impacts of uncertainty factors onto the system output.

Fig. 2 shows the actual general algorithm used to construct fuzzy system response. The algorithm yields uncertain system response for all controlled properties of the containment structure, including displacements, stresses, bending or membrane moments etc. It will be discussed later how the uncertain response can be used to quantify structural safety of the containment structure.

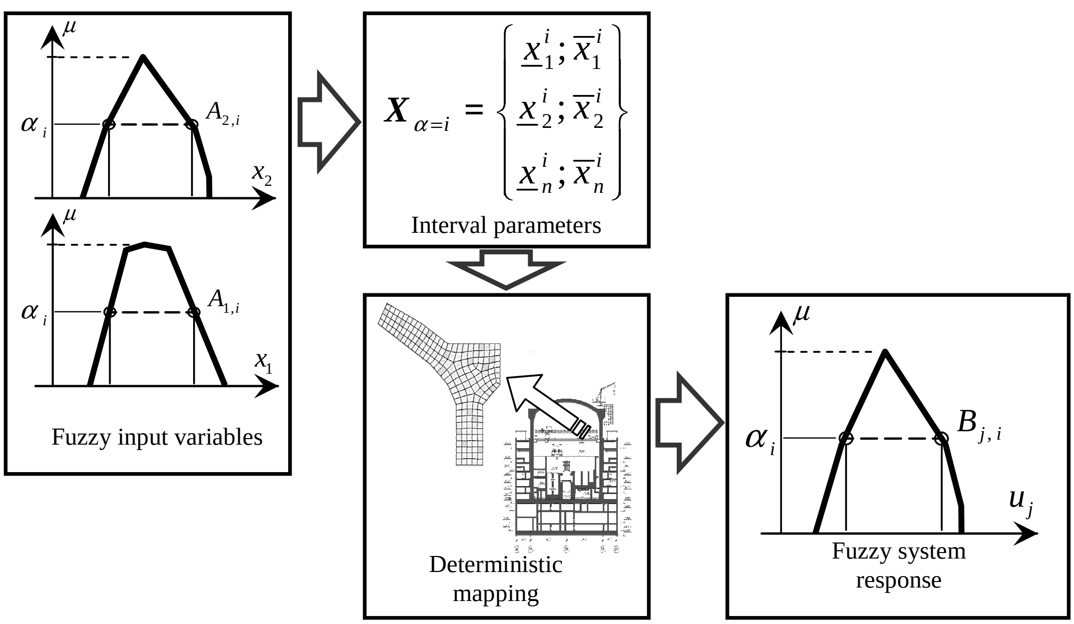

Figure 2. Processing parametric uncertainties to obtain fuzzy system response

The following assumptions are to be accepted for the proposed algorithm:

- linear static structural analysis is considered;
- both the stiffness and matrix and the load vector are monotonic with respect to uncertain input parameters;
- the parametric uncertainty is in sufficiently narrow interval to ensure that the approximate solution is precise.

The term “parametric uncertainty” is used here to indicate that the uncertain parameters are introduced prior to analysis and are propagated through a given simulation model to yield uncertain response of the containment structure.

The proposed algorithm that has been developed for the needs of uncertainty analysis procedure for WWER-1000 type containment uses a modified Neumann Approximate Vertex Solution (NAVS) method [22]. This technique is classified as anti-optimization algorithm that leads to a system of linear equations as if a conventional FEM analysis was used, with the main difference being that the stiffness and load matrices are interval matrices.

Now, let us consider the structural system of a structure to be described by a system of linear interval equations yielded by finite element method:

$$K(\mathbf{X})\times U = Q(\mathbf{X}) \tag{2}$$

where $K(\mathbf{X})\in IR^{n\times n}$ is a stiffness matrix, $\mathbf{X}\in IR^{m}$ is an interval parameters vector, $U\in IR^{n}$ is a displacement vector, and $Q(\mathbf{X})\in IR^{n}$ is a load vector dependent on the interval parameters. Here we will consider without loss of generality only the stiffness matrix uncertainties. The direct solution of this system of linear interval equations naturally leads to interval solution U.

The simplest implicit technique to solve Eq.2 is vertex method, also referred to as combinatorial method. It is also frequently used in many other formulations of worst case search problems. The vertex method needs to perform 2^m solutions to find every possible variation of upper and lower bounds of every parameter in $\mathbf{X}$.

This method is clearly inapplicable to problems such as WWER-1000 containment analysis since the amount of uncertain parameters can be too large to be computed in a reasonable time frame. The following expression of the variations of stiffness matrix can be used to construct approximate solution [22]:

$$K = \left[\underline{K};\overline{K}\right] = \tilde{K} + \sum_{i=1}^{m}\mu_i \Delta K_i\left(x_i\right) \tag{3}$$

where $\underline{K},\ \overline{K}\in R^{n\times n}$ is lower and upper bounds of stiffness matrix intervals, $\tilde{K}$ is mean value of the stiffness matrix, ΔK_i is the stiffness matrix perturbation inflicted by *i*-th interval parameter in the input vector. $\mu_i\in\{-1;1\}$ corresponds to searching either lower of upper bounds of K.

Braibant, Delcroix et al. [22] have developed the efficient NAVS (Neumann approximate vertex solution) method that selects the signs in μ so as to minimize or maximize a prescribed design criterion *c* to determine one component of the response vector U:

$$u_j^{(k+1)} = \tilde{u}_j + \sum_{i=1}^{n}\mu_i \tilde{K}^{-1}\Delta K_i u_j^{(k)} \tag{4}$$

A modified NAVS algorithm have been developed in [24] to satisfy the need of software implementation simplicity and to process all components of the displacement vector in a single analysis procedure. The main idea of the *diagonal section method* (NAVS-DS) is a simplified procedure of selecting signs μ_i, which is computed for the entire vector U. The perturbation of the stiffness matrix is sought and the rectangular matrix $M\in R^{n\times m}$ of signs μ_i is constructed:

$$d_i = \tilde{K}^{-1}\Delta K_i \tilde{u};\ \ M_i = \operatorname{sign}\left(d_i\right); \tag{5}$$

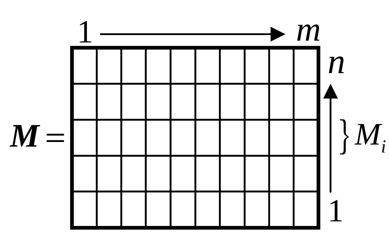

Figure 3. Graphical representation of the construction of matrix ***M***.

In such formulation, there is no need to search for signs μ_i through the minimization/maximization of design criteria. Now, using expressions in Eq.5, we can re-formulate the iteration equation Eq.4 so that the interval solution vector $\boldsymbol{U}_{(j)}$ defined by the *j*-th column of matrix $\boldsymbol{M}$ will contain the most precise solution that corresponds to *j*-th degree of freedom or displacement vector component (hence the name of the method):

$$\boldsymbol{U}_{(j)} = [\underline{\boldsymbol{u}}, \overline{\boldsymbol{u}}] = \begin{cases} \underline{\boldsymbol{u}}_{(j)}^{(k+1)} = \tilde{u} - \sum_{i=1}^{n} \boldsymbol{M}_{i,j} \tilde{K}^{-1} \Delta K_i \underline{u}^{(k)} \\ \overline{\boldsymbol{u}}_{(j)}^{(k+1)} = \tilde{u} + \sum_{i=1}^{n} \boldsymbol{M}_{i,j} \tilde{K}^{-1} \Delta K_i \overline{u}^{(k)} \end{cases} \quad (6)$$

and, consequently, the solution hull for the displacement vector is defined by selecting the *j*-th component of the set of vectors $\{\boldsymbol{U}_{(m)}\}$:

$$\boldsymbol{U}^* = \begin{Bmatrix} \underline{u}_{(1),1}; & \overline{u}_{(1),1} \\ \underline{u}_{(j),j}; & \overline{u}_{(j),j} \\ \ldots & \ldots \\ \underline{u}_{(m),m}; & \overline{u}_{(m),m} \end{Bmatrix} \quad (7)$$

APPLICATIONS OF THE PROPOSED METHOD

The NAVS-DS interval analysis method is considered here as a calculation tool for more general descriptions of uncertainty. In this paper, we will show an example that will be analyzed using the proposed method. The fragment of a thick shell structure is an appropriate case study found in literature and it can provide us with a clear insight into how the method works and how data about uncertainties in complex systems, such as the WWER-1000 containment, can be processed.

The structure possesses 25 uncertain interval parameters – a number that a direct combinatorial method strictly cannot handle in such models. The master degrees of freedom count is 180 due to the use of three-dimensional shell (SHELL181 type). For the problem of uncertainty propagation, it is envisaged to evaluate the effect of 10% uncertainty in the value of structure's elasticity modulus provided that the only information given is the interval in which the parameters may vary. It is worth noting that in the case of probabilistic analysis the variances in all 25 elements would be processed together, and thus the information is lost about how the system behaves when the parameters are not uniform.

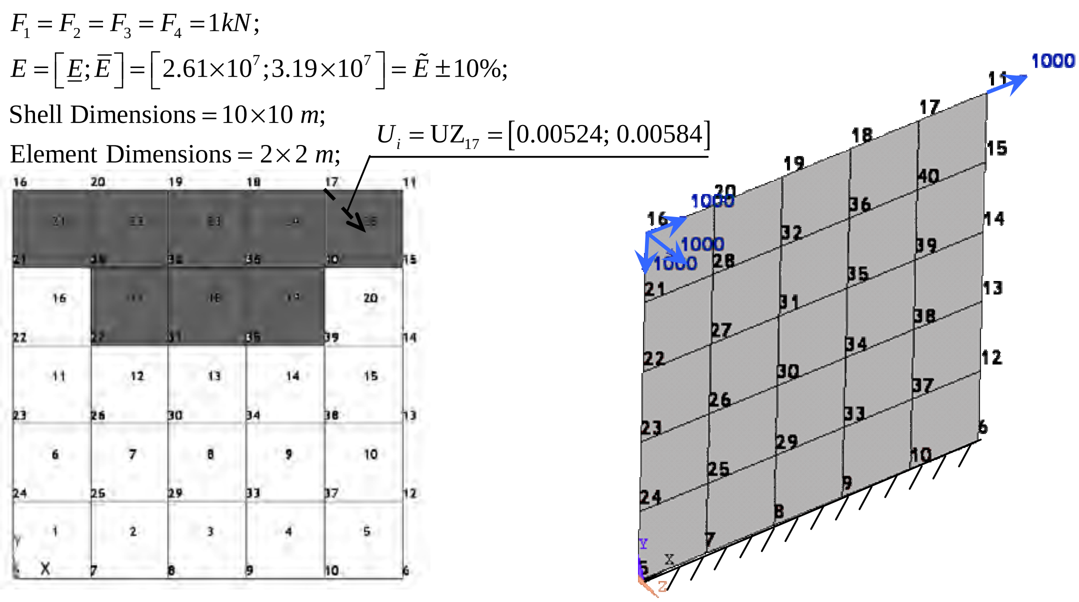

Figure 4. Right: Finite element model of shell fragment. Left: example mapping of uncertain parameters for a component of displacement UZ in node 17.

The resulting matrix of perturbations signs $\boldsymbol{M}$ can be checked if necessary by conducting separate manual analyses as shown in Fig.4. The NAVS-DS method allowed to efficiently estimate all displacements intervals for the 180 master degrees of freedom in 30 nodes. Fig.4 shows that, in the case of $\overline{UZ}_{17}$, the uncertain parameters in the elements No. 17-19 and 21-25 take value $\underline{E}_i$ and all remaining parameters take value $\overline{E}_i$. Similar data can be retrieved for any of the 180 DOFs.

We summarize this example with an observation that the intuitively plausible solution that for $\overline{U}_j$ we have to assign $\underline{E}_i$ to all uncertain parameters and vice versa is correct only to several observed system responses; for example, it is evident that it is so for $UZ_{16} = [0.01048;\ 0.01175]$. For the majority of real engineering practice cases, such assumption leads to a serious underestimation of the solution hull $\boldsymbol{U}^*$, thus exposing the safety-critical system to unnecessary hazards.

It is worth to be noted that the presented method yield rigorous results only if the stiffness matrix is monotone. An approximate solution is obtained when the stiffness matrix is slightly nonlinear, and errors are accumulated as nonlinearity grows. If the problem is nonlinear, then another technique should be proposed that uses higher-order expansion.

CONCLUSIONS

In this paper, a possibilistic formulation of uncertainties in structural analysis is presented along with the interval analysis method that serves as a tool for fuzzy set based safety assessment.

It has been shown that the non-probabilistic methods can propagate epistemic and aleatory uncertainties through very large deterministic simulation models relatively effectively. Such techniques are to be considered as complimentary to the probabilistic analysis techniques already in use. Moreover, since a non-deterministic analysis is rarely conducted simply to evaluate uncertainties in the system response. The proposed technique can easily be used to provide sensitivity analysis of the structure to determine what uncertainty sources contribute to the system uncertain response.

Nonetheless, a considerable amount of effort is to be done before the fuzzy set based methods can be used in safety-critical applications with enough ease. The future research should be directed to the development of “black-box” algorithms and computational codes to propagate uncertainties of different nature through a deterministic computational model, and methods to retrieve fuzzy reliability are to be developed in order to provide a relevant tool for safety assessment of nuclear engineering structures.

NOMENCLATURE

A_i	=	subset of *X* defined by *i*-th alpha-cut
$B_{j,i}$	=	solution subset of response u_j representing solution obtained on *i*-th alpha-cut
μ	=	membership function of a fuzzy set
$\mu_{Ai}(x)$	=	degree of membership of subset A_i
α_i	=	value of the degree of membership corresponding to *i*-th alpha-cut
x_n	=	an uncertain variable
$\mathbf{f}$	=	model input data
M	=	deterministic model
$\boldsymbol{X}$	=	vector of uncertain parameters
$\mathbf{U}$	=	model output vector
$\boldsymbol{X}_\alpha, \boldsymbol{U}_\alpha$	=	interval vectors of model input and output corresponding to an alpha-cut
$K(\mathbf{X})$	=	interval stiffness matrix depending on interval input vector
$Q(\mathbf{X})$	=	interval load matrix depending on interval input vector
U	=	interval solution (displacements) vector
$\underline{K};\overline{K}$	=	upper and lower bounds of stiffness matrix
$\tilde{K}$	=	mean of stiffness matrix
μ_i	=	sign of *i*-th perturbation (depending on whether upper or lower bound is sought)
ΔK_i	=	perturbation of stiffness matrix that corresponds to *i*-th input parameter

$u_j^{(k)}$ = k-th iteration of j-th output displacement

M_i = matrix of signs of perturbations $d_i = \tilde{K}^{-1}\Delta K_i \tilde{u}$ corresponding to i-th input parameter

$\boldsymbol{M}$ = aggregate matrix of signs

$\boldsymbol{U}_{(j)}, \boldsymbol{U}^*$ = solution hull vector of to local optimum of j-th response and to global solution hull

REFERENCES

1. IAEA. "Procedures for Conducting Probabilistic Safety Assessments in Nuclear Power Plants (Level 2): Accident Progression, Containment Analysis and Estimation of Accident Source Terms", Safety Series No. 50-P-8, IAEA, Vienna, Austria, 1995.
2. IAEA. "Applications of Probabilistic Safety Assessment (PSA) for Nuclear Power Plants", IAEA-TECDOC-1200, IAEA, Vienna, Austria, 2001.
3. Ferson, S. and L.R. Ginzburg. "Different Methods Are Needed To Propagate Ignorance and Variability," Reliability Engineering and System Safety. Vol. 54, pp. 133-144, 1996.
4. Helton, J.C. "Uncertainty and Sensitivity Analysis in the Presence of Stochastic and Subjective Uncertainty," Journal of Statistical Computation and Simulation, Vol. 57, pp. 3-76, 1997.
5. B. Moller, M. Beer. "Application of Fuzzy Modeling in Civil Engineering", Proc. of the 2nd International ICSC Simposium on Fuzzy Logic and Applications, ISFL'97, ETH Zurich, Switzerland, February 1997, pp. 345-351.
6. W.L. Oberkampf, J.C. Helton, K. Sentz. "Mathematical Representation of Uncertainty," AIAA 2001-1645, American Institute of Aeronautics and Astronautics, Reston, USA, 2001.
7. C. Baudrit, D. Dubois. "Practical representations of incomplete probabilistic knowledge", Computational Statistics and Data Analysis, Elsevier, V. 51, p. 86-108, 2006.
8. IAEA. "Overview of Level 2 PSA", IAEA Workshop, IAEA, Vienna, Austria, 2002.
9. R. Muhanna, R. Mullen, H. Zhang. "Interval Finite Element as a Basis for Generalized Models of Uncertainty in Engineering Mechanics", Proc. of the NSF Workshop on Reliable Engineering Computing, Savannah, Georgia, USA, 2004.
10. D. Dubois, L. Foulloy, G. Mauris, H. Prade. "Probability-possibility transformations, triangular fuzzy sets, and probabilistic inequalities", Reliable Computing, 10, pp. 273-297, 2004.
11. G. Shafer. "A Mathematical Theory of Evidence", Princeton University Press, 1976.
12. C. Pantelides, S. Ganzerli. "Comparison of Fuzzy Set and Convex Model Theories in Structural Design", Mechanical Systems and Signal Processing, Volume 15, pp. 499-511, 2001.
13. S. Ferson, J. Hajagos, W.T. Tucker . "Probability Bounds Analysis is a Global Sensitivity Analysis", Workshop on Sensitivity Analysis of Model Output. Los Alamos National Laboratory, USA, 2005.
14. D. Dubois, H. Prade. "Interval-valued Fuzzy Sets, Possibility Theory and Imprecise Probability", Proc.of Int. Conf. in Fuzzy Logic and Technology (EUSFLAT'05), Barcelona, 2005, pp.314-319, 2005.
15. K.L. Wood, K.N. Otto, E.K. Antonsson. "Engineering Design Calculations with Fuzzy Parameters", California Institute of Technology, Pasadena, USA, 1992.
16. C.V. Negoiţă, D.A. Ralescu. "Applications of Fuzzy Sets to Systems Analysis", Halsted Press, New York, 1975.
17. R. E. Moore. "Methods and Applications of Interval Analysis", Society for Industrial and Applied Mathematics, Philadelphia, USA, 1979.
18. C. Baudrit, I. Couso, D. Dubois. "Probabilities of Events Induced by Fuzzy Random Variables", Proc.of Int. Conf. in Fuzzy Logic and Technology (EUSFLAT'05), Barcelona, 2005, 2005.
19. H.T. Nguyen, V. Kreinovich, L. Longpré. "Second-Order Uncertainty as a Bridge Between Probabilistic and Fuzzy Approaches", Proc. 2nd Conf. European Society for Fuzzy Logic and Technology (EUSFLAT'01), Leicester, England, pp. 410-413, 2001.
20. T.J. Ross. "Fuzzy Logic With Engineering Applications", McGraw-Hill Inc., New York, USA, 1995.
21. Y. Nakamura. "Extension of algebraic calculus on fuzzy numbers using alpha-level sets," Fuzzy Information Processing Symposium (FIP-84), Kauai, Hawaii, USA, 1984.
22. V. Braibant, A. Oudshoorn, C. Boyer, F. Delcroix. "Non-Deterministic Possibilistic Approaches for Structural Analysis and Optimal Design", AIAA-98-4750, American Institute of Aeronautics and Astronautics, Reston, USA, 1998.
23. S.F. Wojtkiewicz, M. S. Eldred, R.V. Field, Jr., A. Urbina, J.R. Red-Horse. "Uncertainty Quantification in Large Computational Engineering Models", AIAA-01-1455, American Institute for Aeronautics and Astronautics, Reston, USA, 2001.
24. A. Bausk, M.V. Savitsky. "NAVS-DS: an uncertainty analysis method for large-scale structures", Theoretical Foundations of Civil Engineering – Proc. of XV Polish-Ukrainian seminar, Warsaw, Poland, 2007, to appear.