

ABSTRACT

STALEY, KATRINA N. Tracing the Development of Understanding Rate of Change: A Case Study of Changes in a Pre-Service Teacher's Pedagogical Content Knowledge. (Under the direction of Sarah B. Berenson).

This investigation adopts an interpretive approach to study the development of understanding rate of change through a pre-service high school teacher's emerging pedagogical content knowledge over a two-year period of time. Shulman's (1986) definition of pedagogical content knowledge and Ma's (1999) definition of profound understanding of fundamental mathematics provides the conceptual framework for the study. The three categories of pedagogical content knowledge identified by Shulman (1986) assert first that knowledge of students' ideas consist of the teacher knowing the areas of student difficulty and the requirements for student learning. Secondly, knowledge of representations includes the symbols, words, graphics/pictures and other representations used by the teacher. And thirdly, knowledge of instructional activities is having a repertoire of activities to use in the classroom. Ma's (1999) profound understanding of fundamental mathematics is also fed into pedagogical content knowledge with subcategories of teachers' having basic mathematical ideas, connectedness and longitudinal coherence. This conceptualization of pedagogical content knowledge was used to analyze the pre-service teacher's construction of knowledge about teaching mathematics.

Lesson plan study, classroom observations, journal reflections and interviews were coordinated to understand what changes took place in order to trace the pre-service

teacher's development of pedagogical content knowledge. By assigning each component to a single construct, pedagogical content knowledge, the components function as parts of a whole. As a result, lack of coherence between components can be problematic in developing and using pedagogical content knowledge, and increased knowledge of a single component may not be sufficient to effect change in practice.

**Tracing the Development of Understanding Rate of Change: A Case Study of
Changes in a Pre-Service Teacher's Pedagogical Content Knowledge**

By

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A dissertation submitted to the Graduate Faculty of

North Carolina State University

in partial fulfillment of

requirements for the Degree of

Doctor of Philosophy

MATHEMATICS EDUCATION

Raleigh

2004

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DEDICATION

To my family.

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ACKNOWLEDGEMENTS

I would like to thank my family for their continued support through all my years of school. First and foremost, I would like to thank my husband, Marcus C. Staley II, without whose sacrifices and love, none of this would be possible. I would like to thank my daughter, Jessica Erin and son, Marcus Charles, for being a most patient and supportive additions to my life.

Special thanks to Dr. Sarah Berenson, who took a chance with me, challenged me to do my best and who guided and encouraged me every step of the way. I would like to thank members of my committee, Dr. Lee V. Stiff, Dr. Hollylynne Stohl, and Dr. Ernest Stitzinger for being part of this process.

I would like to thank all of my family and friends for their prayers and words of encouragement. It was wonderful to know that my mother, Margaret A. Nelson, sister, Kindra. Nelson, niece, Regina Nelson and mother-in-law, Queen Staley supported and believed in me. I would also like to acknowledge Detris Honora-Adelabu, Dr. Gilbert Casterlow, C.E. Davis, Maria Droujkova, Cynthia Dublin, Kimberly Ennett, Clay Gloster, Shirley Hicks, O.C. Holloway, Cheryl Horton, Tracy McCoy, Brenda McDonald, Kelly McMurray, Natalie McQueen, Mr. Robert Olden, Nikita Patterson, Angelia Reid, Annie Mathews, Monique Staley, Linda Steelman, and Denise Terry, for all the talks and times of just being there. Thank you.

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INTRODUCTION

Research Problem

The research problem of this study is to understand the influence of a high school pre-service teacher's subject matter knowledge and pedagogical knowledge of mathematics on his teaching. It is essential to obtain data concerning programs and practices, along with the requirements and characteristics of mathematics teacher education programs. Schoenfeld (1999) agrees with the National Advisory Committee on Mathematics Education (NACOME, 1975) research that teachers should be grounded in both practice and theory. But mathematics education programs vary from state to state and university to university. There is a need for mathematics pre-service teacher programs to be more effective in teaching not only the procedural knowledge but also the conceptual knowledge of K-12 mathematics (National Council of Teachers Mathematics, 2000). Historically, knowledge bases of teacher education focused on the pedagogy of the teacher (Shulman, 1986). According to Ball (2000), the conceptualization and curriculum of teacher education have consistently been structured across a persistent divide between subject matter and pedagogy. The divide that she refers to appears in the prevailing curriculum of teacher education and separates into domains of knowledge: educational psychology, sociology of education, foundations, methods of teaching, and the academic disciplines corresponding to school subjects. In the past assumptions were made that the integration of these domains of knowledge was simple and occurred during the course of teachers' experiences. To the contrary, the integration of these domains of knowledge does not happen easily and frequently does not happen at all (Ball, 2000).

“No matter how committed one is to caring for students, to taking students’ ideas seriously, or to helping students develop robust understandings, none of these tasks of teaching is possible without making use in context of mathematical understanding and insight” (Ball, 2000, p.3). Several researchers (e.g., Ball, 2000; Ball & McDiarmid, 1990; Kennedy, Ball, & McDiarmid, 1993; Wilson, Shulman, & Richert, 1987) rekindled the discussion about the importance of teachers’ content knowledge in learning to teach.

The movement to reform mathematics education is a response to the failure of traditional methods of teaching mathematics and the study of mathematics learning. The mathematical ignorance of United States citizens has stunted our nation’s competitiveness in an increasingly technological marketplace (Battista, 1999). Teachers still teach as how they were taught when they were children (Battista, 1999; Hong, 1995). They tend to show students several examples on the board and have them practice the problems. Students who seem to be doing fine learned how to do algorithms and, therefore are able to perform well on standardized mathematics tests (Howett, 2003). Students’ learning symbolic manipulations become disconnected from their reasoning abilities and they lose sight of mathematics real-world usefulness (Bransford, Brown, & Cocking, 1999). The failure of mathematical reforms to penetrate the core practice of mathematics teaching has continued to preoccupy many scholars (Berman & McLaughlin, 1975; Cohen & Ball, 1999; Cohen, 1989; McLaughlin, 1990; McLaughlin & Marsh, 1978; Sarason, 1971; Tyack & Cuban, 1995).

“Few have looked closely at the practice of elementary or secondary mathematics teaching to consider the nature of its mathematical entailments” (Ball,

Lubienski, & Mewborn, 1994, p.437). The search to define what it is that teachers need to know in order to teach is a challenging one. Ball, Lubienski, and Mewborn (2001) question what knowledge teachers need, how it affects their teaching, and how they can be helped to develop that knowledge. These issues have preoccupied professional developers, policymakers, and researchers alike for more than four decades. The authors examine how research on teaching related to mathematical work was often invisible, leaving questions about critical relationships between teaching and learning unexplored.

Rowan, Chiang, and Miller (1997) provide further confirmation that understanding the use of mathematics during the course of teaching is a critical area, ripe for further examination based on the National Education Longitudinal Study of 1988. This call for more research in pre-service teacher education suggests a need to understand how to integrate knowledge and practice through the use of content specific courses (Wilson, Theule-Lubienski, & Mattson, 1996). Romberg and Carpenter (1986) urge researchers to integrate studies of learning with studies of teaching to “bridge the learning-teaching gap.” Begle (1979) stated, “[t]here are widespread misconceptions, on the part not only of laypeople but also of mathematics educators, about the ways in which teachers influence mathematics learning by their students” (p. 27).

The attempt to understand and reduce the complexity of teaching to enable the study of the teaching process has generated a variety of metaphors and models. Good metaphors make connections between ideas by finding some form of commonality. Good models organize knowledge in new ways, integrate previously disparate findings, suggest explanations, stimulate research and reveal new relationships.

In 1986, Shulman offered a new model and a set of hypothetical domains of teacher knowledge. In reaction to the proliferation of generic educational research, Shulman (1986a) argued that the study of “teachers’ cognitive understanding and the instruction teachers provide students” (p. 25) may be the “missing paradigm” in education research. He went on to differentiate and call for the study of three types of content understandings and their impact on classroom practice: subject matter knowledge, pedagogical knowledge and curricula knowledge. Later model refinements renamed the constructs of content knowledge as subject matter knowledge, curricula knowledge and pedagogical content knowledge (Shulman, 1986b). Of these, pedagogical content knowledge, defined as the “subject matter for teaching” (Shulman, 1986b, p.9), has prompted considerable interest in both arenas of research and practice.

Researchers such as Ball and McDiramid (1990) did substantial investigations in subject matter knowledge. Their results show that teachers who understand the subject matter improve their teaching methods. Research by Borko, Eisenhart, Brown, Underhill, Jones, and Agard (1992) and Simon (1995) focused on teachers’ mathematical understanding. Research on teachers’ subject matter knowledge and its connections with pedagogical content knowledge was done by Even (1993). Baturu and Nason’s (1996) research focused on first-year teacher-education students’ understanding of subject matter knowledge in the area of measurement. Lowery’s (2002) study was to further the understanding of how pre-service teachers construct teacher knowledge and pedagogical content knowledge. Ma’s (1999) research developed a conception of mathematical understanding that explains how teachers can relate mathematical ideas to their students. In addition, Sowder, Phillip, Armstrong, and Schappelle’s (1998) did their research with

five middle school teachers on how their understanding of rational numbers, quantity, and proportional reasoning influences the manner in which they teach.

Two problems need to be addressed to meet the challenge of preparing teachers. The first problem concerns identifying content knowledge needed for teaching. The second problem centers on what it takes to learn to use such knowledge in practice. The following sections describe how to use pedagogical content knowledge and subject matter knowledge as primary categories for the knowledge base of teaching. Last, rate of change will be the content knowledge that will be focused on for teaching.

The Importance of Pedagogical Content Knowledge

Ball (2000) asked what it would take to bring the study of content closer to practice and to prepare teachers to know and be able to use subject matter knowledge effectively in their work as teachers. Rather than view teacher education from the perspective of content or pedagogy, Shulman (1986) believed that teacher education programs should combine these two constituent knowledge domains. Therefore, Shulman (1986) developed a framework for teacher education that included for the first time the concept of pedagogical content knowledge. The use of pedagogical content knowledge as a topic for research and discussion about the nature of an appropriate knowledge base for developing future mathematics teachers steadily increased since its inception (e.g., Shulman, 1986,1987; Wilson, Shulman, & Richert, 1987; Ball, 2000). The discussions of pedagogical content knowledge focused on the knowledge of what is typically difficult for students to learn, the representations that are most useful for teaching a specific subject matter, and ways to develop ideas (Hashweh, 1985; Feiman-

Nemser & Buchmann, 1985; Carpenter, Fennema, Peterson & Carey, 1988; Jones, Buckler, Cooper & Stushein, 1997; Ball, 2000; Stohl-Drier, 2001).

The sharp distinction between knowledge and pedagogy does not represent a tradition dating back centuries, but rather, a more recent development. A century ago the defining characteristic of pedagogical accomplishment was knowledge of content (Shulman, 1986). Today, we assume that most teachers begin with some expertise in the content they teach. Secondary teaching candidates typically have completed a major in their content area or a few classes short of a major in their content area. Shulman's (1986) central question concerns the transition from pre-service student teacher to novice teacher. There is a need to know how to blend properly the content aspects of teaching and the elements of the teaching process. If we do not blend content with the teaching process effectively, then the growth of students' mathematical reasoning and problem-solving skills will be stunted in the classroom (Babttista, 1999; Ball, 2000).

Shulman (1986) stated, "conceptual analysis of knowledge for teachers would necessarily be based on a framework for classifying both the domains and categories of teacher knowledge, and the forms for representing that knowledge" (p.10). As the complexities of teacher understanding and transmission of content knowledge is investigated, the need for a more coherent theoretical framework becomes rapidly apparent (Shulman, 1986).

How might we think about the growth of knowledge in the minds of teachers, with special emphasis on content? Shulman (1986) suggests three categories of content knowledge: a) subject matter content knowledge, b) pedagogical content knowledge, and c) curricula knowledge. Shulman (1986) defined *pedagogical content knowledge* as:

“[t]he most useful forms of [content] representation . . . the most powerful analogies, illustrations, examples, explanation, and demonstrations—in a word, the ways of representing and formulating the subject that makes it comprehensible to others” (p.9).

A teachers’ pedagogical content knowledge is apparent when analogies between the time it takes to ride a bike a certain distance at a constant speed and linear regression are made. Illustrations of data collected and placed on a graph to make conjectures about the populations from which the samples were taken are a useful form of content representation. Another apparent form of teachers’ pedagogical content knowledge is the use of examples of mathematical procedures. Knowing how to construct examples means teachers understand both the mathematical and developmental advantages and disadvantages in the example (NCTM, 1991). The teachers’ knowledge of explanations of mathematical concepts demonstrates a deep understanding of connections between concepts and procedures, connections across mathematical topics, and connections between mathematics and other disciplines (NCTM, 1991). Analogies, illustrations, examples explanations leads to teachers who can respond appropriately to students’ questions, design activities involving an array of mathematical representations, and direct mathematical discourse in the classroom (NCTM, 1991). Demonstrations, such as the relationship of the quadratic formula with a ball bouncing up and down using a graphing calculator, calculator based laboratory, motion detector and a ball, reveal teachers’ knowledge of content representation. Pedagogical content knowledge also includes an understanding of content in a manner that is meaningful to the learner. Teachers should have knowledge of the conceptions their students bring with them to mathematics classrooms (NCTM, 2000). “If those conceptions are misconceptions, which they often

are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners” (Shulman, 1986, p.9). This project will focus on the pedagogical content knowledge of Shulman’s work.

Additional articles provide evolving conceptions of the domains of teacher knowledge and the description of pedagogical content knowledge. In 1987, pedagogical content knowledge was listed by Shulman as one of the seven knowledge bases for teaching, removing it as a subcategory and placing it on equal footing with content knowledge, general pedagogical knowledge, curricula knowledge, learners knowledge, educational contexts knowledge, and philosophical and historical aims of education knowledge. Pedagogical content knowledge was then defined as:

[T]hat special amalgam of content and pedagogy that is uniquely the providence of teachers, their own special form of professional understanding . . . Pedagogical content knowledge . . . identifies the distinctive bodies of knowledge for teaching. It represents the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to diverse interests and abilities of learners, and presented for instruction.

Pedagogical content knowledge is the category most likely to distinguish the understanding of the content specialist from that of the pedagogue.

(Shulman, 1987, p.8)

Grossman (1990) defined “four general areas of teacher knowledge . . . as the cornerstones of the emerging work on professional knowledge for teaching: general pedagogical knowledge, subject matter knowledge, pedagogical content knowledge, and

knowledge of context” (p.5). Of the four knowledge bases, pedagogical content knowledge was anticipated as having the greatest impact on teachers’ classroom actions.

Research in the Third Handbook of Research on Teaching (1990) has exponentially increased within the area of teachers’ understandings of subject matter knowledge. Pedagogical content knowledge is now a commonly accepted construct in the educational community. Books and chapters have been dedicated to the exploration of teachers’ knowledge of subject matter in general (Ball & McDiarmid, 1990; Brophy, 1991). It has been used as a major organizing construct in reviews of the literature on teachers’ knowledge (Borko & Putnam, 1995).

Pedagogical content knowledge has the characteristics of a good model. The study of teacher knowledge in various subject areas such as mathematics, science, social studies and English research has been revitalized. In addition, pedagogical content knowledge provides a new analytical frame for organizing and collecting data on teachers’ cognition. The model has also highlighted the importance of subject matter knowledge and pedagogical knowledge. How pedagogical content knowledge relates to subject matter knowledge is explained in the next section.

The Importance of Subject Matter Knowledge

“Pedagogical content knowledge highlights the interplay of mathematics and pedagogy in teaching. Rooted in content knowledge, it comprises more than understanding the content itself” (Ball, 2000). Teachers’ pedagogical decisions, such as questions they ask of their students and activities they use in the classroom, are based on their subject matter knowledge. Recognizing the importance of subject matter knowledge entails understanding the discipline of mathematics and being able to articulate and

defend a content-specific concept, which are important goals and promote teacher development (Gess-Newsome, 1999).

Research on teachers' content knowledge is not new. Shulman and his colleagues initiated a rich line of research, reframing the definition of subject matter knowledge to include the "nature, form, organization, and content of teacher knowledge" (Grossman, Wilson & Shulman, 1989, p.25-26). This broadened definition of subject matter knowledge helped find the links between the knowledge teachers possess, the instructional actions they employ and the learning, attitudes, and beliefs of the students they teach. Shulman (1986) stated, "teachers must not only be capable of defining for students those accepted truths in a domain, . . . they must also be able to explain why a particular proposition is deemed warranted, why it is worth knowing, and how it relates to other propositions" (p. 9). For example, mathematics teachers need to know how procedural topics and conceptual topics are interwoven. Teachers who have a conceptual understanding of linear equations and intend to promote students' conceptual learning do not have to ignore procedural knowledge. Mathematics teachers need to know how the concept of linear equations fits into the whole mathematical system and its relation with previous mathematical topics.

Teaching involves assisting students to develop their intellectual resources and help them participate in the subject matter that the teachers want them to learn (Morine-Dershimer & Kent, 1999). Research has shown that teachers' subject matter knowledge influences their efforts to help students learn. Teachers' understanding the structure and nature of their discipline and selecting and translating essential content into meaningful

learning activities are essential to their subject matter knowledge (Talbert & McLaughlin, 1993).

Unfortunately, many high school pre-service teachers will not revisit the topics they will teach until they are in their own classrooms. Much of their understanding of these topics is based on their high school experiences, which were likely to have been taught algorithmically with no conceptual understanding (Davis & Hersh, 1981; Goodlad, 1984; Wheeler, 1980). The lack of conceptual understanding of the subject matter directly affected teachers' ability to help their students learn in meaningful ways (Ball, 1990; Ma, 1999; Eisenhart, Borko, Underhill, Brown, Jones, & Agard, 1993).

Ball and McDiramid (1990) stated that in mathematics, a critical dimension of knowledge about the subject is the distinction between conventional and logical construction. Critical knowledge about mathematics includes relationships within and outside of the field: understanding the relationships among mathematical ideas and topics and knowing about the relationship between mathematics and other fields. Knowing the fundamental activities of the field includes looking for patterns, making conjectures, justifying claims, validating solutions, and seeking generalizations, yet another aspect of knowledge about mathematics (Ball & McDiramid, 1990).

Ma (1999) calls subject matter knowledge understanding "the terrain of fundamental mathematics that is deep, broad, and thorough" (p. 120), or a profound understanding of mathematics (PUFM). "Although the term *profound* is often considered to mean intellectual depth, its three connotations, deep, vast, and thorough, are interconnected" (Ma, 1999, p.120). She states that the subject matter knowledge

develops when learning and teaching the subject. Subject matter knowledge deepens when teachers prepare for class, teach the material and reflect on the process.

“Teachers’ subject matter knowledge develops in a cyclic process” (Ma, 1999, p. 145). The cycle starts with schooling and provides a base for solid subject matter knowledge. Future teachers learn some mathematical competencies in their K-12 education, rather than during their teacher preparation programs. The connections between mathematical topics and teaching are begun. It is at this point when subject matter knowledge can be improved. Last, their subject matter knowledge continues to develop during their teaching career only if they are motivated and are provided opportunities to do so, according to Ma (1999). Figure 1 shows how subject matter knowledge cycles through schooling, teaching and teacher preparation.

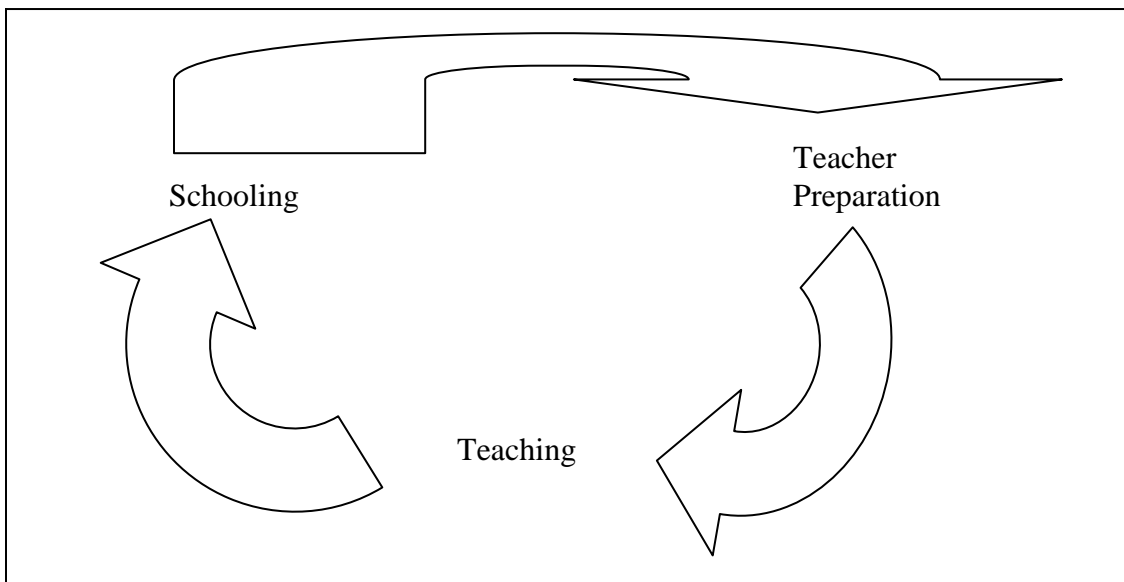


Figure 1. Subject Matter Knowledge Cycle

Subject matter knowledge includes specific information, ideas, and topics that continue to change and grow. This information and these ideas and topics can be subject to disagreement and different interpretations based on competing perspectives within the field (Gess-Newsome, 1999; Ball, 1990; Baturu & Nason, 1996; Even, 1993; Shulman, 1986). This project will focus on pre-service teachers' knowledge about the subject of rate of change.

The Importance of Rate of Change

Fractions, ratios, and proportionality are topics of mathematics that are “mathematically rich, cognitively complicated and difficult to teach” (Smith, 2002, p.3). According to Thompson (1994), there is no conventional distinction between ratio and rate. Therefore, the two terms are frequently used without definition. Lesh, Post and Behr (1988) stated that there is no consensus about the crucial distinctiveness between rates and ratios, and several authors interchange the terminology from one publication to another. However, the two most commonly used definitions of ratio and rate are as follows:

1. A ratio is a comparison between quantities of like nature, for example, pounds versus pounds or yards versus yards. A rate is a comparison of quantities of unlike nature, for example, distance versus time or miles versus gallons.
2. A ratio is a numerical expression of how much there is of one quantity in relation to another quantity. For example, “what is the ratio of each color of M&Ms to the total number of M&Ms in a bag? A rate is a ratio between a quantity and a period of time, for example, distance versus time (Thompson, 1994, p.190).

Thompson (1994) addressed how people come to envision the term rate, which is a constant ratio of two independent quantities. For example, a linear function comparing miles to hours can quantify a rate. The linear function expresses speed of travel in relation to time traveled which produces the distance traveled. Thompson (1994) described the first level of comprehension as a person who learned that rate is a measure of speed; the second level included the visualization of an object in motion.

Thompson's (1994) work showed how students constructed a rate by building-up strategies of a constant ratio. For example, students were asked to find the number of oranges in a bag where the ratio of plums to oranges was 3:4. There were twenty-four plums. Given that there are twenty-four plums, the students determined that there were three plums to every four oranges yielding thirty-two oranges. Then the students realized that the amounts can vary, therefore, they understood that there are three-fourths of an orange for every plum or any proportional part thereof.

Another connection between ratio and rates is identifying sets of values that are related. For example, a problem states for every 200 miles you travel it take 3 hours. The Y axis can be labeled miles and the X axis can be labeled hours in Figure 2 below:

$$\frac{\text{miles}}{\text{hours}} = \frac{200}{3}$$

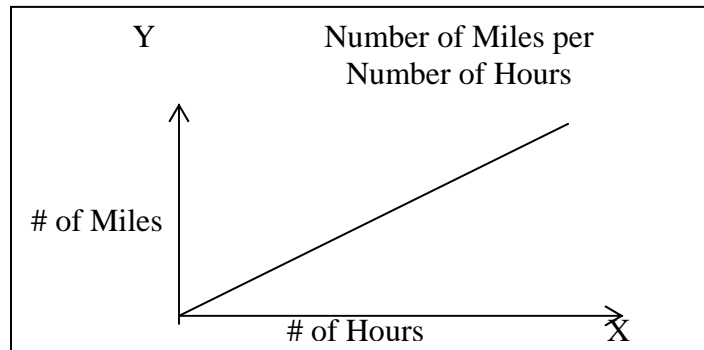


Figure 2. Bats versus Balls

By working from a graph, students can read off the answer to a variety of questions. For example, “How far would you get if you traveled for 5 hours?, What if you traveled 12 hours?” A question could also be asked about miles, for example, “How many miles would you have traveled after 6 hours?”

The same relationship can be treated as a series of equivalent fractions:

$$\frac{\text{miles}}{\text{hours}} = \frac{200}{3} = \frac{400}{?} = \frac{?}{12}$$

The subject of rate of change is important in conceptually understanding linear equations so that students come to realize that a change in the independent variable brings about a change in the dependent variable.

Research Questions

This investigation adopted an interpretive approach to study the development of understanding rate of change through a pre-service high school teacher’s emerging pedagogical content knowledge over a two-year period of time. Shulman’s (1986)

definition of pedagogical content knowledge and Ma's (1999) definition of profound understanding of fundamental mathematics provided the conceptual framework for the study. The three subcategories of pedagogical content knowledge identified by Shulman (1986) are defined as follows. First, teachers must know the areas of student difficulty and the requirements for student learning. The second area of knowledge is representations including symbols, words, graphics/pictures and other representations used by the teacher. Third, knowledge of instructional activities is based on having a repertoire of activities to use in the classroom. Ma's (1999) profound understanding of fundamental mathematics is also fed into pedagogical content knowledge with subcategories of teachers having basic mathematical ideas, connectedness and longitudinal coherence. This conceptualization of pedagogical content knowledge was used to analyze pre-service teachers' construction of knowledge about teaching mathematics.

Lesson plan study, classroom observations, journal reflections and interviews were coordinated to understand what changes took place in order to trace pre-service teachers' development of pedagogical content knowledge. By assigning each component to a single construct, which was pedagogical content knowledge, the components functioned as parts of a whole. As a result, lack of coherence between components could be problematic in developing and using pedagogical content knowledge. Additionally, increased knowledge of a single component may not be sufficient to affect change in practice.

The following questions are being considered for this research project:

1. Pre-service Teacher Content Knowledge:
 - a. What themes emerged in a pre-service teacher's understanding when planning and teaching rate of change to an Algebra I class?
 - i. Themes related to content knowledge.
1. Pre-service Teacher Pedagogical Content Knowledge:
 - a. What changes in pre-service teachers' pedagogical content knowledge, both subtle and overt, are exhibited in the way rate of change is treated overtime?

LITERATURE REVIEW

This chapter is designed to orient the reader to the previous work of researchers who studied teacher development. It also outlines the conceptual framework of the research. This chapter is arranged in four parts. First come reviews of research in the areas of pre-service teacher development and preparation, middle grades and high school pre-service teacher development, and in-service teacher development. Second, Shulman's (1989) definition of pedagogical content knowledge is defined. The importance of pedagogical content knowledge is described for each domain including representations, activities and knowledge of student ideas. Third, the research on pre-service and in-service teachers' pedagogical content knowledge of instructional representations, activities, and knowledge of learners is described. Subject matter knowledge is also examined as it is related to proportional reasoning. Last, the conceptual framework is described.

The conceptual framework describes the uniqueness of pedagogical content knowledge and its importance in research. It discusses the effectiveness of the representations teachers use, such as illustrations, examples, models or analogies to increase students' comprehension. It also includes explanations of how activities help students understand specific concepts or relationships of problems, demonstrations, simulations, investigations, or experiments. It states how the knowledge of students' ideas can help the teacher predict a path by which learning might continue in the classroom.

With a focus on pre-service teacher content knowledge and pedagogical content knowledge, this chapter synthesizes the research I used to answer the following questions:

1. Pre-service Teacher Content Knowledge:

- a. What themes emerged in a pre-service teacher's understanding when planning and teaching rate of change to an Algebra I class?

2. Pre-service Teacher Pedagogical Content Knowledge:

- a. What changes in pre-service teachers' pedagogical content knowledge, both subtle and overt, are exhibited in the way rate of change is treated overtime?

The framework of pedagogical content knowledge for mathematics teaching provides a structure for the review of the research literature on understanding the changes of pedagogical knowledge.

Research in Pre-Service Teacher Subject Matter Knowledge

Ball, Lubienski, and Mewborn (2001) asked the questions:

- 1) What knowledge do teachers need to have,
- 2) How does it affect their teaching,
- 3) How can they be helped to develop it, and
- 4) What sides of relationships between teaching and learning are critical to the research community?

These questions represent common concerns of researchers, and are central to describing the concept of pedagogical content knowledge. Researchers need to integrate studies of learning with studies of teaching to "bridge the learning-teaching gap" (Romberg & Carpenter, 1986, p.868).

Research in elementary and secondary pre-service teacher education has focused on subject matter knowledge. Ball and McDiarmid (1990) state that subject matter

knowledge is a central component to what teachers need to know. However, researchers have also focused on “changes in teachers’ role conceptions; their beliefs about their work; their knowledge of students, curriculum and teaching strategies” (p. 437).

Elementary teachers take at least half or more of their courses in the liberal arts college. These range over introductory courses of various disciplines such as history, English, sociology, biology, psychology, and art. Secondary teachers take as few as four or five teacher preparation courses. States such as New Jersey, California, Illinois, Texas, and Virginia have reduced the number of education courses that pre-service teachers have to take.

School mathematics such as similarity, slope and exponents are topics that pre-service teachers will not have revisited since they were in high school. Therefore, their understanding of these topics is based on their mathematical experience prior to college. Their high school instruction may have focused on factual knowledge and procedural proficiency and not conceptual understanding (Ball & McDiarmid, 1990).

Procedural knowledge and conceptual knowledge are pedagogical terms that are commonly used in mathematics research. Procedural knowledge is defined as the computational skills and procedures for recognizing mathematical facts, rules, algorithms and definitions (Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993; Ball, Lubienski & Mewborn, 2001). For example, a student with procedural knowledge of setting up a proportion and solving for an unknown number will know how to use cross multiplication. Finding an unknown in a proportion via procedural knowledge is demonstrated by step-by-step arrangement of rules, algorithms and mnemonics for remembering them .

On the other hand, conceptual understanding is defined as the relationships and interconnections of facts, concepts, principles, and procedures that allows its use in problem solving situations (Eisenhart, Borko, Underhill, Brown, Jones & Agard, 1993; Gess-Newsome, 1999). In the case of finding an unknown in a proportion, conceptual knowledge includes such ideas as the nature of proportions in general and how, when solving proportional reasoning problems, it is important to set up the ratios in a consistent way according to the units associated with the numbers. In general, the following proportions are equivalent, that is, they have the same solution, which can be justified by cross multiplication.

$$\frac{a}{b} = \frac{c}{d} \quad \frac{a}{c} = \frac{b}{d} \quad \frac{b}{a} = \frac{d}{c} \quad \frac{c}{a} = \frac{d}{b}$$

Thus there are several possible correct proportions that can be established when equating ratios with the same four members.

A study conducted at the National Center for Research on Teacher Education reported that pre-service teachers' understandings of explaining certain topics in school mathematics were fragmented (Ball & McDiarmid, 1990). They used rules, tricks and definitions rather than meaningful and connected understandings when explaining mathematical concepts. Students, in general, are able to meet the expectations of satisfactory work without improving their conceptual understanding of the subject matter, which makes it more difficult for the pre-service teachers to teach their student meaningful mathematics.

One potential way to increase subject matter knowledge is teaching in the classroom (Ball & McDiramid, 1990). Wilson, Shulman, and Richert (1987) propose the following model of pedagogical reasoning:

Teachers must critically understand a set of ideas, a piece of content, in terms of both its substantive and syntactic structure. ...Teachers should also understand the relationship between that piece of content and other ideas within the same content as well as ideas in related domains. Math teachers should understand the relationships between fractions and decimals. (p.119)

Such understandings help pre-service teachers learn how to teach subject matter. Pre-service teachers' developing knowledge of their students, context, curriculum, and pedagogy creates a new understanding of content, together forming pedagogical content knowledge (Ball & McDiarmid, 1990; Shulman, 1996).

Research conducted by Simon (1995) focused on teachers' pedagogical decision making with respect to mathematics content and mathematical tasks for classroom learning. Simon (1995) developed the Mathematics Teaching Cycle "as a schematic model of the cyclical interrelationship of aspects of teacher knowledge, thinking, decision making, and activity" (p. 135). The abbreviated Mathematics Teaching Cycle has three components: teacher's knowledge, hypothetical learning trajectory, and assessment of students' knowledge. Within the hypothetical learning trajectory there are the teacher's learning goal, teacher's plan for learning activities, and teacher's hypothesis of learning process. "The model emphasizes the important interplay between the teacher's plans and the teacher and students' collective constitution of classroom activities" (Simon, 1995, p. 141). In other words, the model shows how a teacher's decision making affects the content and activities of a mathematics classroom. There are four themes pertaining to a teacher's decision making that are important in this model: 1) understanding students'

thinking is important in the design of this model, 2) the teacher's pedagogical knowledge grows and the students' mathematical knowledge grows, 3) the continuing changing knowledge of the teacher changes her hypothetical learning trajectory, and 4) planning for instruction includes the goals of the teacher and the importance of the students' learning process (Simon, 1995).

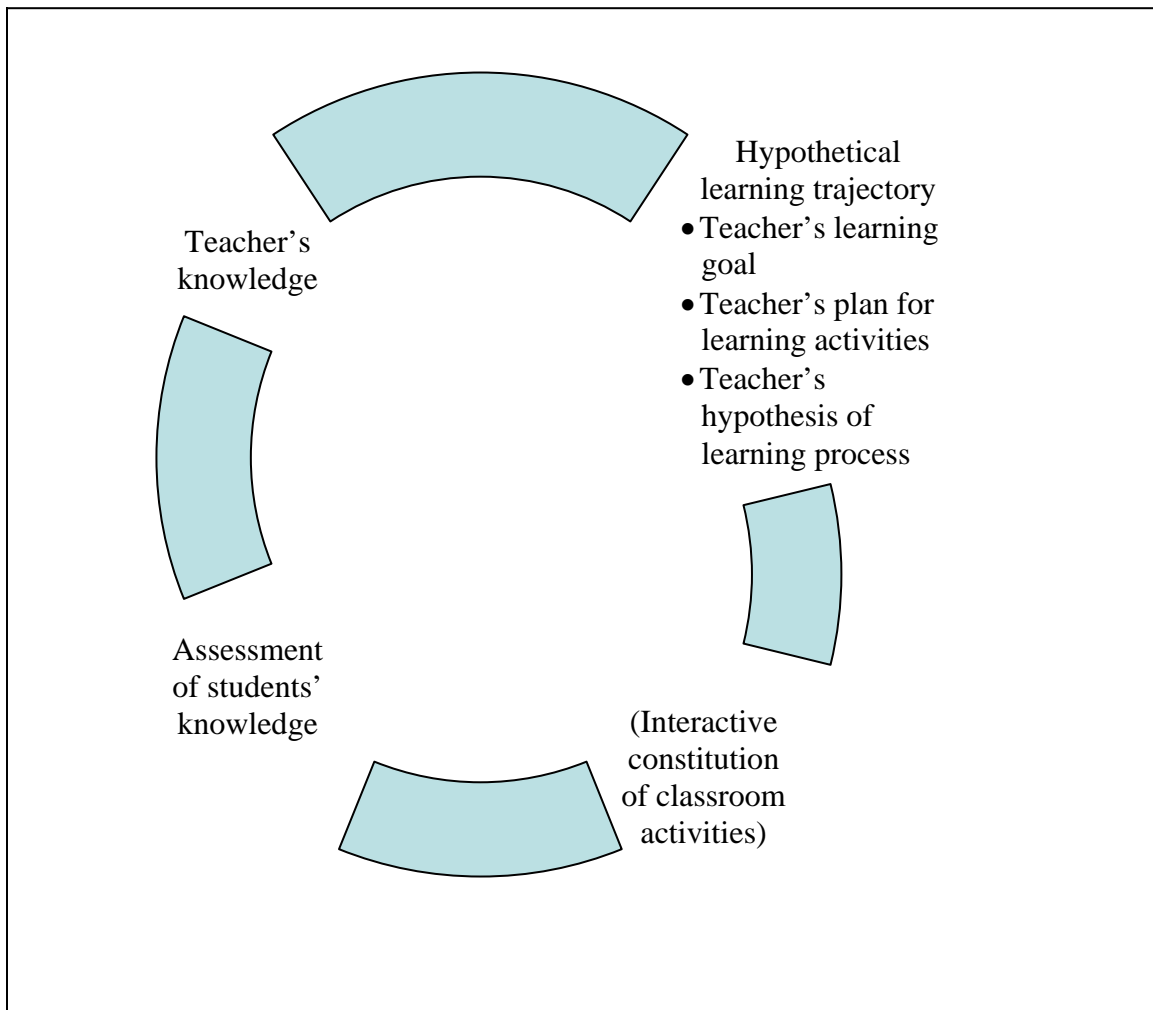


Figure 3. Mathematics teaching cycle (Simon, 1995)

In summary, research in pre-service teacher development and preparation has focused on subject matter knowledge, procedural knowledge and conceptual knowledge

of K-12 mathematics. Understanding how to teach subject matter knowledge helps pre-service teachers make better pedagogical decisions in their classrooms.

Research in Middle Grades and High School Pre-Service Teacher

Development/Preparation

“Few have looked closely at the practice of elementary or secondary mathematics teaching to consider the nature of its mathematical entailments” (Ball, Lubienski, & Mewborn, 2001, p.437). Factors that bring to the forefront the weaknesses in mathematics teachers’ content knowledge include using new curriculum materials, enacting new practices, and teaching new content. For example, implementing new curriculum materials often overwhelms teachers when the focus on skills and procedures does not allow them time to develop the interconnections of the mathematics in the new curriculum. Their own “knowledge and beliefs continue to shape their interpretations and uses of the material” (Ball, Lubienski, & Mewborn, 2001, p.436). Ma (1999) stated that enacting new practices is in opposition to the more traditional instruction of teachers on giving knowledge to students, who are expected to store and remember it. Teaching new content contributed to the weaknesses of mathematics teachers’ content knowledge, because their understanding of fundamental mathematical ideas and relationships was weak, making it difficult to learn new mathematical content (Ma, 1999).

The primary goals of Borko, Eisenhart, Brown, Underhill, Jones, and Agard’s (1992) research were to describe a pre-service teacher’s emergent knowledge, beliefs, thinking, and actions related to teaching mathematics. They wanted to understand the interdependence and mutual influence of these components and learning to teach

mathematics. They also examined the impact of teacher education experiences on the process of learning to teach the elementary and middle school mathematics. The authors focused on a classroom lesson that was unsuccessful: the mathematical content was not taught correctly to the students, and the pre-service teacher did not correct herself in the next day's lesson. The researchers attempted to understand what transpired in that lesson, and to investigate its implications for the pre-service teacher's development as a mathematics teacher. The authors examined the pre-service teacher's own system of knowledge and beliefs related to division of fractions, the mathematical content she taught incorrectly, and the treatment of division of fractions taught in her mathematics methods courses. The conclusions they made on what might have led to the pre-service teacher's not pursuing a conceptual explanation for division of fractions after teaching were that she did not really understand the importance of explaining the division of fractions meaningfully. Second, the demands of the teacher education program required her to think ahead to her next lesson rather than reflect on what she had already done. The implications for teacher education according to Borko, Eishenhart, Brown, Underhill, Jones and Agard (1992) suggested that universities must give pre-service teachers the opportunity to strengthen their subject matter knowledge and their pedagogical content knowledge. This can be done by increasing the number of methods courses required for certification that focus on conceptual development along with methods courses, field experiences will support the learning of pedagogical content knowledge.

The ideas and practices for teaching procedural and conceptual knowledge were major themes of Eisenhart, Borko, Underhill, Brown, Jones and Agard (1993) research, as they worked toward understanding the process of learning to teach middle school

mathematics. The authors' goal was to describe and understand the pre-service teacher's knowledge, beliefs, thinking, and actions related to mathematics teaching. The outcomes they found was that the student teachers, the methods course instructor, and cooperating teachers struggled with how to implement teaching for conceptual understanding, rather than just procedural proficiency. The implications of their research suggest that pre-service teachers must be placed in student teaching environments that support the National Council of Teachers of Mathematics (2000) vision. Also the universities must create ways that allow pre-service teachers and their instructors time to explore and to develop approaches to teaching that foster conceptual understanding. Pre-service teachers need to challenge their own beliefs and mathematical knowledge base about teaching and learning to teach.

Even's (1993) research involved subject matter knowledge and its connections with pedagogical content knowledge of pre-service secondary teachers in the areas related to the concept of function. It appeared that their concept image of a function was an equation with "nice graphs" and that, in their view, was sufficient to define function. Also, none of the pre-service teachers could come up with a reasonable explanation for the need of univalence as a characteristic of a function without prompting. When teachers provide an environment that encourages students to explore and to ask questions, they may have to deal with instances of mathematical concepts that are unfamiliar to teachers themselves, which teachers may be afraid to do. Even (1993) states that their pedagogical decisions are based on their subject matter knowledge. "Therefore, it is important that teachers develop a modern concept image of function" (Even, 1993, p. 113). In conclusion, her study proposed that teachers need to have learning environments

that promote better understanding of subject matter knowledge. Also, teachers need to develop a repertoire of instructional strategies and representations supporting teaching for student understanding.

Baturo and Nason's (1996) research project focused on first-year education students' understanding of subject matter knowledge in the domain of area measurement. This study concentrated on the student teachers' substantive knowledge, that is, knowledge about the nature and discourse of mathematics, mathematics in society, and on the teachers' disposition towards mathematics. The authors found that the first-year teacher education students' had a deficit in subject matter knowledge in the area of measurement. Their lack of knowledge would impede their ability to come up with multiple representations and activities/tasks for their students' different learning styles. Therefore, it would prevent their learners from developing meaningful understandings of mathematical concepts.

Lowery's (2002) goal was to further the understanding of how pre-service teachers construct teacher knowledge and pedagogical content knowledge of elementary mathematics and science. A methods course was designed based on the national mathematics and science standards. The following learning venues for the "construction of knowledge" (Lowery, 2002, p.76) were discovered during the course: learning through collaboration, reflection, exemplary models, and situated context. Lowery (2002) also stated that "these learning venues provided the means for the pre-service teachers to acquire the ends, teacher knowledge and pedagogical content knowledge of elementary mathematics and science" (p. 76). These findings suggest that content-specific school-based experiences give pre-service teachers more opportunities to focus on content and

instructional strategies at the conceptual level. Second, content-specific courses with school-based experiences addressed many anxieties related to teaching in the classroom. Last, they helped pre-service teachers become more confident and competent teachers.

The primary purpose of Lederman and Gess-Newsome's (1999) research was to investigate pre-service science and math teachers' attitudes toward teaching and instructional decision-making. The results of their investigations demonstrate that the translation of teachers' thoughts into action is very complex. The level of complexity of one's subject matter knowledge is a critical factor in transforming it into pedagogical reasoning. The authors suggest that teacher preparation courses provide students with ample opportunities to reflect upon instruction and to experience quality field placements. They also suggest that college content courses along with subject-specific pedagogy courses were part of reforming teacher education.

In summary, research in middle grades and high school pre-service teacher development and preparation has concentrated on how weaknesses in content knowledge make it hard to learn new mathematical content. However, increasing the numbers of methods courses and field base experiences supports pre-service teachers' pedagogical content knowledge. Methods courses would help develop a collection of instructional strategies and representations, while field base experiences would help develop approaches to teaching.

Research in In-Service Teacher Development/Preparation

Ma's (1999) research in in-service elementary teacher development was a comparative study of American and Chinese teachers of mathematics. Ma developed a conception of mathematical understanding that explains the ways teachers can relate

mathematical ideas to their students. The ways include basic ideas, connectedness, multiple representations, and longitudinal coherence. She also found that Chinese teachers continue to learn mathematics throughout their teaching careers, whereas the American teachers are not offered the same opportunities within their school days. The results of her research suggest that changes are needed in mathematics courses in order to foster American teachers' profound understanding of fundamental mathematics.

Sowder, Philipp, Armstrong, and Schappelle's (1998) conducted their research with five middle school teachers. They were interested in how teacher understanding of rational number, quantity, and proportional reasoning influence their manner of teaching. The study concluded that teachers who reached better understanding of mathematical concepts were better able to contribute to their students' conceptual understanding of the mathematics in the area of proportional reasoning.

In summary, research in pre-service and in-service teacher development and preparation stated that teachers who have a better understanding of subject matter knowledge can better relate mathematical ideas to their students.

Shulman's Pedagogical Content Knowledge

Shulman developed a framework for studying teacher education by introducing the concept of pedagogical content knowledge, the "subject matter knowledge for teaching" (Shulman, 1986, p.9). Rather than viewing teacher education from the perspective of either content or pedagogy, Shulman believed that teacher education programs should combine these two knowledge bases to more effectively prepare teachers.

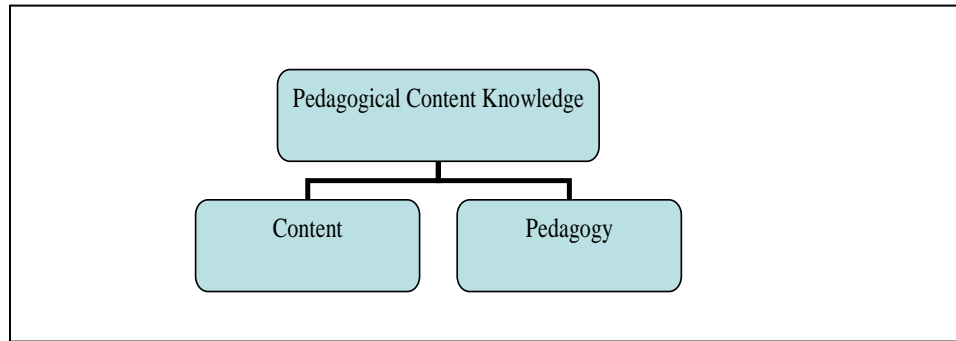


Figure 4. Shulman's Definition of Pedagogical Content Knowledge

Pedagogical content knowledge is described as knowing multiple ways of representing and formulating subject matter. Pedagogical content knowledge allows the focus on making concepts understandable, and on the abilities and interests of learners. It also explains why some topics are easy or difficult to learn (Ball, 1988; Even, 1993; Lampert, 1986; Shulman, 1986; Wilson, Shulman, & Richert, 1987). Shulman (1986) stated that pedagogical content knowledge includes,

...for the most regularly taught topics in one's subject area, the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing the subject that make it comprehensible to others. Since there are no single most powerful forms of representation, the teacher must have at hand a veritable armamentarium of alternative forms of representation, some of which derive from research whereas others originate in the wisdom of practice. ... [It] also includes an understanding of what makes the learning of specific topics easy or difficult; the conceptions and preconceptions that students of different

ages and backgrounds bring with them to learning... If those preconceptions are misconceptions... teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners. (p.9)

Hence teachers' pedagogical content knowledge includes understanding of how to use representations, activities and knowledge of students' ideas. Teachers need to have a repertoire of representations and activities to be effective in the classroom. And for students to gain knowledge, teachers must convey new concepts by restructuring their beliefs of knowledge that may be already present.? rephrase

In the following sections, I elaborate on these three components of pedagogical content knowledge: representations, activities or tasks, and knowledge of student ideas.

Representations Defined

Representations, as stated by Shulman (1986), can be illustrations, examples, metaphors, explanations, simulations, models, or analogies. For example, the areas of square gardens, the altitude of a rocket shot in the air, or the shape of an archway can be represented by functions of the form $f(x)=ax^2+bx+c$, using symbolic representations. An effective teacher must judge whether and when a representation will be useful to support and increase the comprehension of students. The National Council of Teachers of Mathematics (NCTM, 1991) recognizes the role of pedagogical content knowledge in effective teaching. For example, multiple representations are specifically addressed in Standard 3 and Standard 4 of the *Professional Standards for Teaching Mathematics* (NCTM, 1991). Standard 3, "Knowing Students as Learners of Mathematics," proposes

that “the pre-service and continuing education of teachers of mathematics should provide multiple perspectives on students as learners of mathematics...” (p. 144). Standard 4, “Knowing Mathematics Pedagogy,” suggests that “the pre-service and continuing education of teachers of mathematics should develop teachers’ knowledge of and ability to use and evaluate...ways to represent mathematics concepts and procedures...” (p. 151). A multiplicity of representations can be used to illustrate a problem. Also, the teacher must know that some students may more readily understand a particular representation.

“Emerging views on mathematical learning argue that multiple representations of concepts yield deeper and more flexible understandings” (Keller & Hirsch, 1998, p.1). As technology is more readily available in the classrooms, particular representations of concepts, such as function, emphasize and suppress various aspects of the concept (Piez & Voxman, 1997; Keller & Hersch, 1998; Santos-Trigo, 2002). Interactive geometry software, such as The Geometer’s Sketchpad (Jackiw, 2003) makes a dynamic problem representation possible. If a problem can be presented in multiple representations, the graphical and algebraic solutions can be compared with the geometric solution. Once the problem is solved, variations and extensions can be explored, such as forming conjectures and counterexamples in a process of generalizing results (Santos-Trigo, 2002).

Shulman’s (1987a) description of the model of pedagogical reasoning and action representation includes a teacher building up a representational collection of devices to assist in teaching in the classroom.

Instructional selections occur when the teacher must move from the reformulation of content through representations to the embodiment of

representations in instructional forms or methods. Here the teacher draws upon an instructional repertoire of approaches or strategies of teaching. This repertoire can be quite rich including not only the more conventional alternatives such as lecture, demonstration, recitation, or seatwork, but also a variety of forms of cooperative learning, reciprocal teaching, Socratic dialogue, discovery learning, project methods, and learning outside the classroom setting. (Shulman, 1987a, pp.16-17)

Thus the development of a teacher means becoming able to understand subject matter in new ways, which can be facilitated by use of representations of mathematical concepts. Teachers who are able to use multiple representations are better prepared to address the appropriateness of the representations students bring to the classroom (Gfeffer, Niess, & Lederman, 1999). The possibilities for teachers include using illustrations, examples, metaphors, explanations, simulations, models, or analogies so that students can understand new mathematical concepts.

Activities Defined

Activities or tasks help students comprehend specific concepts or relationships and include problems, demonstrations, simulations, investigations, or experiments (Shulman, 1986). For example, mathematical investigation is a meaningful mathematical exploration with multiple parts that helps to discover new ways of thinking about the mathematics involved, rather than just the solutions to a single problem. “Mathematical investigations demand that students speculate, conjecture, and generalize” (Chapin, 1998).

A teacher's knowledge of the conceptual power of an activity can help clarify a concept for a student. The activity can be used as a platform to provide justification of a mathematical concept (Simon, 1993). And according to the Teaching Principle from the Principles and Standards for School Mathematics (NCTM, 2000), teachers' knowledge base needs to include the ability to select and to use suitable curricular materials, as well as to use appropriate instructional tools and techniques.

By stimulating students with activities and directing them to suitable tasks, teachers must provide learning opportunities for students that engage their higher mental capabilities (Bromme, 1987; Smith 1999). The student can manipulate objects from an activity with the intent of discovering the mathematical properties, interpret feedback from the manipulations, and use the object to create new problems for a mathematical concept (Reed & Jazo, 2002). It is up to the student to learn and to understand within this environment the teacher has provided. Therefore, students' construct their own strategies for solving a problem, and if they make mistakes, it is the teacher's objective to realize that, and to re-teach the student (Bromme, 1987).

Knowledge of Students' Ideas Defined

Steffe (1990) states, "using their own mathematical knowledge, mathematics teachers must interpret the language and actions of their students and then make decisions about possible mathematical knowledge their students might learn" (p. 395). Knowledge of students' ideas can help the teacher predict a path by which learning might continue in the classroom. The Teaching Principle in the Principle and Standards for School Mathematics (NCTM, 2000) states that, "effective teaching involves observing students,

listening carefully to their ideas and explanations, having mathematical goals, and using the information to make instructional decisions” (p.19). Teacher understanding of the prerequisite knowledge that is required for students to learn specific concepts includes knowledge of the abilities and skills that students might need, and of the knowledge students already possess. Teachers’ knowledge of differences in methods of learning includes knowing how students differ in developmental or ability level, or how they may have different learning styles. Students’ needs are different in the classroom, therefore effective teachers are knowledgeable of these differences, and can respond accordingly to their students (Magnusson, Krajcik, & Borko, 1999).

In Bromme’s (1987) research, the most important endeavor in the classroom is that both the teacher and the student convey or gain knowledge and develop understanding. For students to gain knowledge, teachers must convey new concepts by restructuring their preconceptions of knowledge that are already present.

In summary, to help students develop skills and become competent in using multiple representations, activities are used that require numerical, graphical and analytical perspectives. Through these activities, students will gain the flexibility to solve problems using appropriate representations. Therefore the student will gain knowledge from teachers’ knowing the differences in methods of learning.

Research on Pedagogical Content Knowledge of Pre-Service and In-service

Teachers

Shulman (1987) listed pedagogical content knowledge as one of the seven knowledge bases for teaching: content knowledge, general pedagogical knowledge,

curricular knowledge, knowledge of learners, knowledge of educational contexts, and knowledge of the philosophical and historical aims of education. Pedagogical content knowledge was noted as having the greatest impact on teachers' classroom actions. It became a commonly used construct in the educational research community (Grossman, 1990; Shulman, 1987; Ball & McDiarmid, 1990; Borko & Putnam, 1996; Ma, 1999).

An essential integrating feature of pedagogical knowledge is teaching experience. Applying the general pedagogical knowledge in the classroom, where the teacher has to make decisions results with context specific mathematical knowledge that contributes directly to pedagogical content knowledge. As previously stated, there are three separate and interactive aspects of pedagogical knowledge: instructional representations, activities/tasks, and knowledge of learners.

Instructional Representations

Hashweh's (1985) research focused on instructional representations such as textbooks and curriculum. Hashweh (1985) examined the influence of subject matter knowledge on pedagogical reasoning of experienced teachers. He found that their prior subject matter knowledge affected the transformation of the curriculum. Teachers made modifications to the textbook materials and the representations they used to explain concepts and principles based on their subject matter knowledge. Knowledgeable teachers were more likely to notice misleading or poorly articulated themes in curriculum materials. Hashweh (1985) also noted that there were qualitative differences in the representations generated by teachers with higher levels of the subject matter knowledge versus teachers with lower levels of the subject matter knowledge. The analysis showed

that the representations used by the unknowledgeable teachers addressed surface knowledge of the topic, and sometimes were inappropriate for the topic given. The representations used by the knowledgeable teachers showed they had a deeper understanding of the topic, which involved bringing into play basic concepts, principles, themes or conceptual schemes of the topic.

Piez (1997) used graphing calculators in the classrooms, so that students were able to work from a graphical as well as analytical perspective. She found that students preferred one representation to another, and that she had to add activities to the textbook problems to help students develop proficiency in making connections between analytical and graphical representations.

Embse (1998) used interactive geometry software to allow the students to make interconnections among the various modes, including the TI-92 geometry, data graphing, function graphing, and algebraic solution. The software extends the power of visualization to provide multiple approaches to complex problems. Embse (1998) also stated that student's lack of understanding of the problem was a serious obstruction to finding any type of solution strategy, even with technology.

Keller & Hirsch (1998) incorporated multiple representations with technology to determine student preferences for multiple representations of functions. The results indicated that students' preferences between contextualized and non-contextualized settings were more apparent in a graphics calculator-based calculus course versus a traditional calculus course. The use of the graphics calculator removed any constraints perceived by the students on the effortlessness to move between different representations such as graphs, tables or algebra. Keller & Hirsch (1998) also suggested that it was

possible that students' preferences may change from an over-dependence on equations to choosing a balance between equations and other representations, depending on the mathematical task.

Twenty-five first year university students taking calculus for the first time were used in Santos-Trigo's (2002) research. The instructor required the students to use various representations and problem solving strategies to solve assigned problems. The results included the fact that learning a concept was a continuous process, in which students needed to examine the concepts from different viewpoints. Another conclusion was that the use of technology provided students with a way to explore connections among different types representations.

These researchers have argued that the ability to represent the subject matter is an important characteristic of an individual's subject matter knowledge. Teachers must evaluate their own subject matter knowledge, and must be able to generate representations that would foster student understanding. They must have ways of transforming their subject matter knowledge into pedagogical reasoning.

Activities/Tasks

Research that has been done with activities included Stohl-Drier's (2001) study. She stated that spreadsheets could be used as a "conceptual teaching and learning tool" (p. 170). Students can discover mathematical concepts and make connections between mathematical topics using interactive spreadsheets. The teachers' knowledge of how to use and when to use spreadsheets can create a classroom environment where students were actively engaged in learning mathematics.

Jones, Buckler, Cooper and Staushein's (1997) research addressed the integration of content with pedagogy and the development and use of model teaching practices in college courses. They designed activities that a classroom teacher might perform rather than traditional school activities such as a writing assignment. The tasks consisted of concept-teaching activities, chemical demonstrations, and inquiry-based laboratory activities. Students reported that the learning environment was supportive and they felt better prepared to teach.

Steffe and Olive (2002) designed a computer tool for interactive mathematical activity (TIMA) that could be used by children to enact their mathematical operations of unitizing, uniting, fragmenting, segmenting, partitioning, disembedding, iterating and measuring. The children enjoyed placing toys in the simulated playground, and the mathematical extension by the teacher was also pleasurable to the students. Steffe and Olive's goal was to transform the children's cognitive play into mathematical play. A teacher would have to know how to intervene during the cognitive play, and to bring about mathematical play through the use of questions. But these questions must be in the realm of the students' understanding; otherwise the students will become frustrated and make no progress.

Twenty-four undergraduate students participated in Reed and Jazo's (2002) research on using multiple representations to improve conceptions of average speed using an interactive animation program. Average speed was defined as a weighted average where two speeds are weighted by the amount of time spent traveling at that speed. Students improved their understanding of the average speed of two objects by using visual feedback provided by a simulation. Multiple representations of the concept were

used, such as a conceptual explanation and an algebraic explanation. The results showed that the initial estimates of weighted averages by the students were too close to the algebraic average. Once students learned how to find the solution algebraically, they continued to use that procedure to find the solution. Students also found it difficult to make inferences from graphs because of their lack of understanding of the problem.

Activities should be chosen with the specific intention of actively involving learners in seeking understanding of mathematical concepts. The teacher has a clear role in selecting activities that are expected to focus the attention of the learner on constructing the intended learning goals. Therefore, the mathematical activity should be selected or designed to encourage the learner to link between conceptual and procedural ideas. Activities by itself are not enough to promote learning. They need to be carried out with a consideration of aspects of presentation, and of knowledge the students' mental capability.

Knowledge of Learners

Forty first-grade teachers' pedagogical content knowledge of children's solutions of addition and subtraction word problems was investigated by Carpenter, Fennema, Peterson, and Carey (1988). They found that most of the teachers were successful in evaluating their students' problem-solving performance. The teachers' ability to identify their students' success in solving different problems was significantly correlated with the students' achievement. The authors state, "teachers' knowledge of students' success may be related to achievement because instructional decisions are based upon that knowledge, and teachers' knowledge" (Carpenter, Fennema, Peterson, & Carey, 1988, p. 399).

Pedagogical content knowledge is not just multiple representations, or activities, or knowing the knowledge of the learners. Wilson, Shulman and Richert (1988) state, “it (pedagogical content knowledge) is characterized by the way of thinking that facilitates the generation of these transformations, the development of pedagogical reasoning” (p. 115). Feiman-Nemser and Buchmann (1985) explain that teaching is a moral activity that helps people learn. Teaching requires one to think about how to build bridges between your understanding and the students’ understanding. “While teachers cannot directly observe learning, they can learn to detect the signs of understanding and confusion, of feigned interest and genuine absorption. Thus pedagogical thinking is strategic, imaginative, and grounded in knowledge of self, children, and subject matter” (Feiman-Nemser & Buchmann, 1985, p. 2).

In summary, there are three aspects of pedagogical knowledge: instructional representations, activities/tasks, and knowledge of learners. Teachers must evaluate their own subject matter knowledge to be able to generate representations that would foster student understanding. Activities should be chosen with the specific intention of actively involving learners in seeking to understand mathematical concepts. The teachers’ ability to identify their students’ success in solving different problems correlates with the students’ achievement.

Subject Matter Knowledge Related to Proportional Reasoning

Research has established the importance of pedagogical content knowledge in the development of teachers at all levels. This type of professional knowledge provides a framework for collecting and organizing data on teacher development. Within these

studies, the importance of subject matter knowledge emerges. How that knowledge is transformed for teaching remains a relatively unexplored area. Pedagogical content knowledge is acknowledged by a number of researchers as providing an integrated vision of teacher knowledge and classroom practice.

Lamon (1994) stated that ratio and proportion is the foundation of higher mathematics. Her research involved sixth grade student's thinking in the process of solving ratio and proportion problems. It explored how students used unitizing and norming. Unitizing is the ability to make a reference to a unit or whole unit, and then to reinterpret that unit in a situation. Norming takes a given configuration, changes the properties of that configuration, and compares it to some fixed unit or standard. The results of her research stated that children have preconceived understandings of ratio and proportion. Students who were successful in solving the problems used unitizing and norming strategies. This suggested that unitizing and norming may be important for mathematical reasoning. Understanding ratio and proportion depends on one's ability to view a relationship as a single quantity, and then work with the problem.

Ratios and fractions and their roles as tools in proportional reasoning were compared in Clark, Berenson, and Cavey's (2003) research. The authors developed five Venn diagrams to illustrate how teachers think about the relationship between ratios and fractions. Model 1, see Figure 5, shows all ratios are fractions. Fraction was defined as any rational number, and ratio was an expression of such a rational number by division.

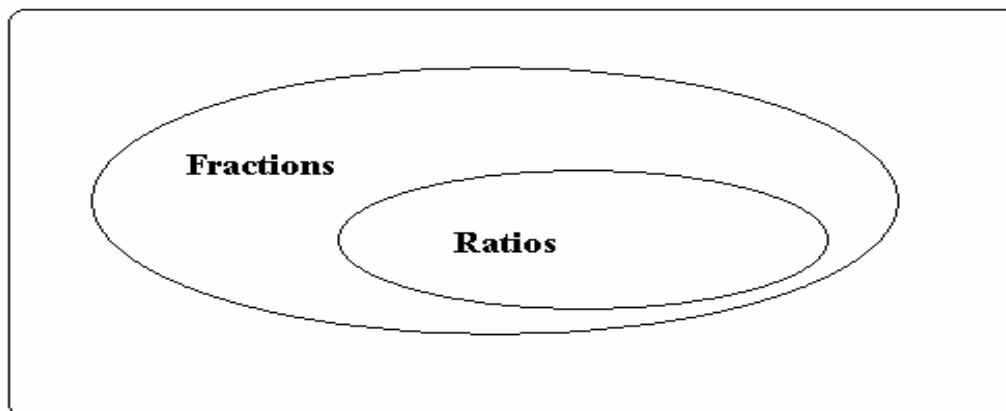


Figure 5. Ratios as a Subset of Fractions (Clark, Berenson, & Cavey, 2003)

Model 2, see Figure 6, displays all fractions are ratios. Fraction was defined as a number of the form a/b where a and b are integers and b is not zero and a ratio was defined as a relative comparison of two numbers that has a division sign, colon, horizontal bar between the numerator and denominator, or the word “to” between the numbers.

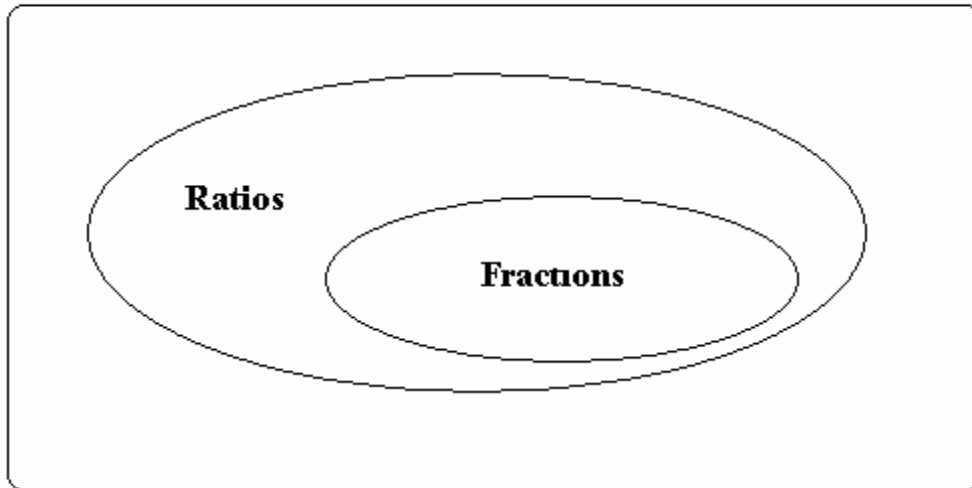


Figure 6. Fractions as a Subset of Ratios (Clark, Berenson, & Cavey, 2003)

Model 3, see Figure 7, suggested that ratios and fractions were two distinct terms.

Fractions represent a part-whole relationship and ratios represent a part-part relationship.

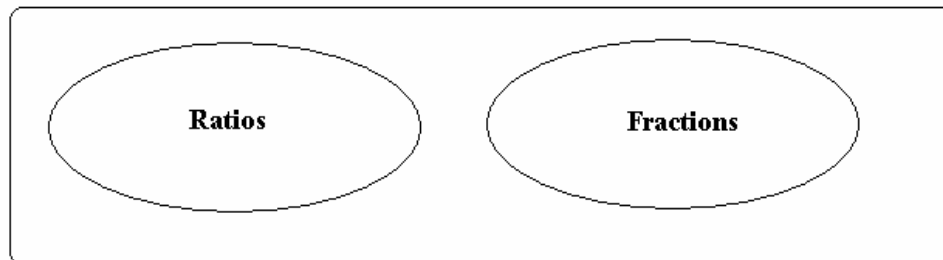


Figure 7. Ratios and Fractions Distinct (Clark, Berenson, & Cavey, 2003)

Model 4, see Figure 8, illustrated overlapping circles where some ratios are fractions and some fractions are ratios. The model included ratios and fractions as two separate terms.

Previous definitions from the other models can be used in describing this model.

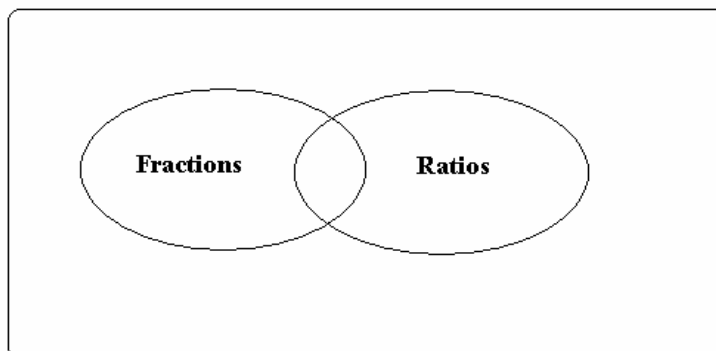


Figure 8. Ratios and Fractions Overlapping (Clark, Berenson, & Cavey, 2003)

In the last model, see Figure 9, ratios and fractions had identical meanings. All ratios were fractions and all fractions were ratios. For example, a fraction was the division of one number by another and a ratio was the first number divided by the second number.

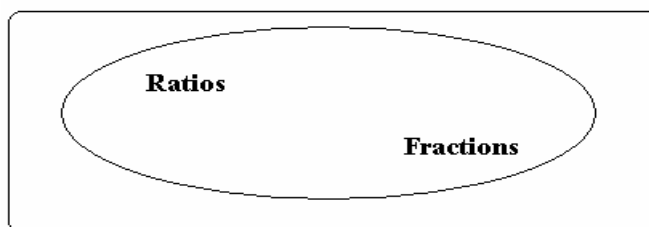


Figure 9. Ratios and Fractions Identical (Clark, Berenson, & Cavey, 2003)

These models helped the researchers analyze students' uses of ratios and fractions in their explanations of proportion related problems.

According to Thompson (1994), there is no conventional distinction between ratio and rate. Therefore, the two terms are frequently used without a definition separating

them. Lesh, Post and Behr (1988) stated that there is no consensus about the crucial distinctiveness between rates and ratios, and that several authors interchange the terminology from one publication to another. However, the two most commonly used definitions of ratio and rate are as follows:

1. A ratio is a comparison between quantities of like nature, for example, pounds versus pounds or yards versus yards. A rate is a comparison of quantities of unlike nature, for example, distance versus time or miles versus gallons.
2. A ratio is a numerical expression of how much there is of one quantity in relation to another quantity. A rate is a ratio between a quantity and a period of time, for example, between distance and time.

Thompson (1994) addressed how people come to envision the term “rate.” At the first level, a person has learned what rate is, for example, learned about speed as a rate. At the second level, a person has comprehended what rate is, for example visualized an object’s motion as a rate.

The following activity from Thompson’s (1994) work shows how students can construct a rate by building-up strategies based on a constant ratio. If a student is asked to find the number of oranges in a bag where the ratio of plums to oranges is 3:4 and there are twenty-four plums, and the student understands that there are three plums to every four oranges, then there are twenty-four plums to thirty-two oranges. Then the student realizes that the amounts can vary, and therefore understands there are $\frac{3}{4}$ of an orange for every plum, or can work with any other proportional part thereof.

Another connection between ratio and rates is identifying sets of values that are related. For example, for every 4 bats you buy, you get 3 free balls. The Y axis can be labeled bats and the X axis can be labeled balls:

$$\frac{\text{Bats bought}}{\text{Free balls}} = \frac{4}{3}$$

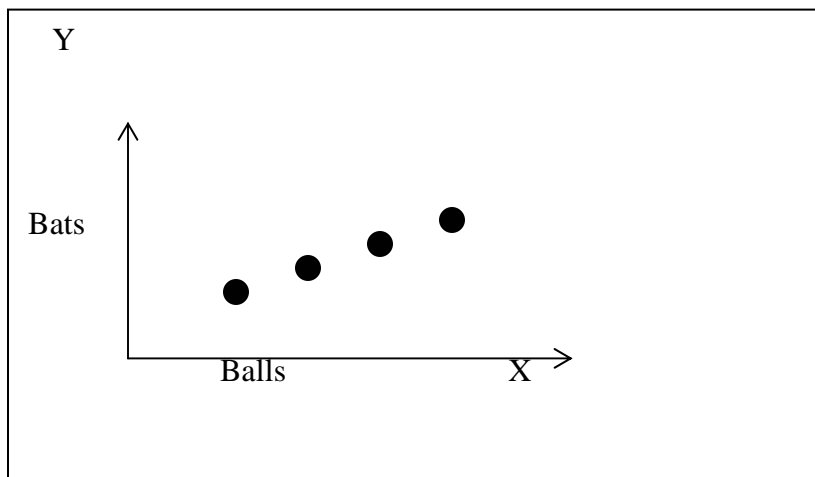


Figure 10. Bats versus Balls

By working from a graph, students can read off the answer to a variety of questions, for example, “How many free balls would you get if you bought 44 bats?” or, “What if you bought 36 bats?” A question could also be asked about bats, for example, “How many bats would you have to buy to get 12 free balls?”

The same relationship can be treated as a series of equivalent fractions:

$$\frac{\text{Bats bought}}{\text{Free balls}} = \frac{4}{3} = \frac{44}{?} = \frac{36}{?} = \frac{?}{12}$$

Alternatively, the ratio can be expressed as the rate of bats to balls. This is a function of y/x .

The rate is the same for all Y or X values.

$$\text{Ball} * \frac{\text{Bats}}{\text{Balls}} = \text{Bats}$$

Conceptual Framework

A conceptual framework is based on previous research and literature as well as current research and literature, according to Eisenhart (1991). She continues her definition of a conceptual framework with the following statement:

A conceptual framework is an argument including different points of view and culminating in a series of reasons for adopting some points - i.e., some ideas or concepts – and not others. The adopted ideas or concepts then serve as guides: to collecting data in a particular study, to ways in which the data from a particular study will be analyzed and explained, or both.

(p. 209)

The framework may contain different viewpoints on ideas, different theories, or researchers' knowledge that can provide a guide for collecting data for a study.

According to Borko and Putnam (1996), "...the willingness of psychologists to hypothesize mental structures and mental events as meaningful objects of study" (p. 674) has paved the way for researchers to describe how knowledge is constructed. Ma's (1999) and Shulman's (1986) interest in teachers' knowledge, especially their pedagogical content knowledge of the content they teach, is the foundation for the framework of this study .

In 1986, Lee Shulman created a new conceptual framework of teacher knowledge. He argued that the study of teachers' subject matter knowledge and pedagogy is the

missing factor in educational research. He then defined three types of content understandings and its impact in the classroom: subject matter knowledge, curricular knowledge, and pedagogical content knowledge (Shulman, 1986). Of these, pedagogical content knowledge, which is categorized as representations, activities and/or tasks, and knowledge of student ideas has influenced research and practice.

Recently, Ma (1999) developed a conception of mathematical understanding that addresses those characteristics of knowledge that will most likely contribute to a teacher's ability to explain important mathematical ideas to students. Thus, her requirements for teachers with profound understanding of fundamental mathematics include the following four properties: 1) *basic ideas*, 2) *connectedness*, 3) *multiple representations*, and 4) *longitudinal coherence*.

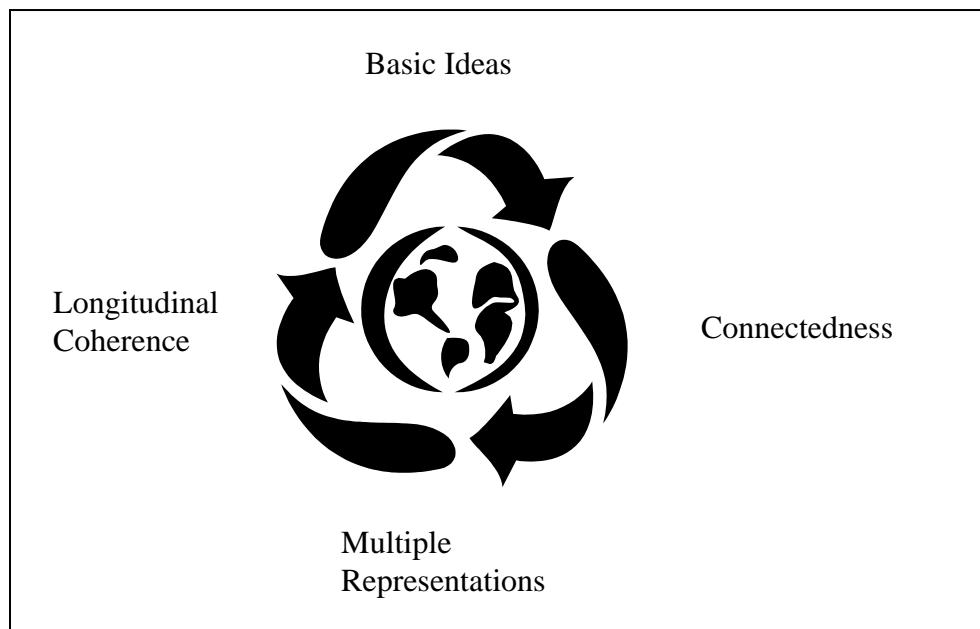


Figure 11. Ma's (1999) Definition of Pedagogical Content Knowledge for Mathematics Teaching

Teachers with profound understanding of fundamental mathematics display deep understanding of basic ideas by their ability to know “simple but powerful basic concepts and principles of mathematics (i.e., the idea of an equation)” (Ma, 1999, p. 122). They focus on *basic ideas* by revisiting and reinforcing problems while guiding students through a mathematical activity. Having *connectedness* involves making connections within a mathematical topic to a previous learned topic, or between procedural knowledge and conceptual knowledge, or between a mathematical topic and an applicable real world problem. “Mathematics makes more sense and is easier to remember and to apply when students connect new knowledge to existing knowledge in meaningful ways” (NCTM, 2000, p. 20). *Multiple representations* entail various ways of solving a problem and being able to provide mathematical explanations of the advantages and disadvantages of the method of the solution. Finally, *longitudinal coherence* includes knowing the mathematical curriculum beyond the grade level that is currently taught. For example, a teacher with profound understanding of fundamental mathematics will know what concepts the students learned prior to their class, and what they will learn after leaving their class, which is necessary to “lay the proper foundation” (Ma, 1999, p.122).

The ideas of pedagogical content knowledge can significantly improve our understanding of the knowledge required for teaching. The concepts involve teachers knowing their content procedurally and conceptually, understanding how to represent mathematical ideas using various examples and making connections, and paying attention to the common student difficulties with specific mathematical concepts. This emergent

framework for understanding the mathematical content needed for teachers to recognize and instruct the thinking of their students' serves as the foci for my current work.

Figure 12 graphically displays the conception of pedagogical content knowledge. The figure is an interpretation of the three categories of pedagogical content knowledge identified by Shulman (1986). Knowledge of students' ideas consists of the teacher knowing the areas of student difficulty and the requirements for student learning. Knowledge of representations includes the symbols, words, graphics/pictures and other representations used by the teacher to communicate mathematical ideas to students. Knowledge of instructional activities is having a repertoire of activities to use in the classroom. Ma's (1999) profound understanding of fundamental mathematics is also included into pedagogical content knowledge, with subcategories of teachers having basic mathematical ideas, connectedness, and longitudinal coherence. This conceptualization of pedagogical content knowledge will serve to organize the analysis of my data.

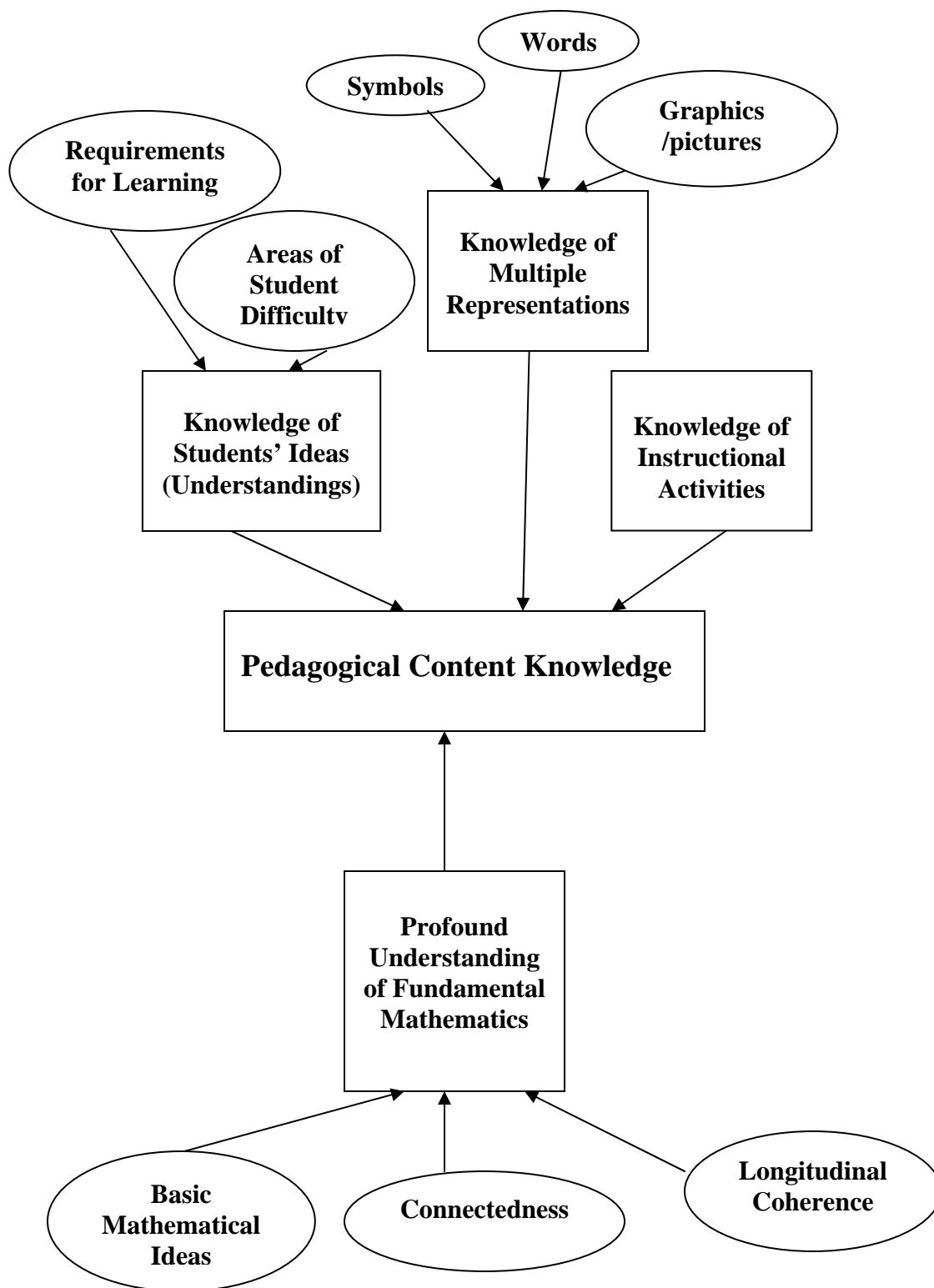


Figure12. Components of pedagogical content knowledge for mathematics teaching

METHODOLOGY

This chapter describes data collection and analysis activities of this study. They include the selection process, the content knowledge assessment, the semi-structured interviews, and the observations. The research analyzes one pre-service teacher's understanding of ratio and proportion concepts, and his pedagogical content knowledge with respect to rate of change, as it developed over a two-year span in his teacher preparation. Further, I discuss the method of inquiry and the analysis of the data.

Merriam (1988) suggests that generalizing case study research is difficult if not impossible. However, the researcher can improve the generalizability of the case study findings by providing well-documented and richly described events so that the reader can compare the case with other cases to come up with his or her own conclusions (Sowder, Phillip, Armstrong, & Schappelle, 1998).

The researcher has five years of classroom experience as a mathematics teacher/instructor, one year as an elementary teacher, two years as a high school teacher and two years as a college lecturer. For this project, the researcher was interested in the participant's perspectives, in order to understand how pedagogical content knowledge works. Therefore, the following strategies have been implemented to strengthen the validity of this qualitative study: (Merriam, 1988):

1. Triangulation (see Table 1).
 - a. Three video taped interviews
 - b. Five observations (1 was done with another observer)
 - c. Lesson plans from the pre-service teacher
 - d. Reflections

Data Source	Year 1 – First Methods Course	Year 3 – Student Teaching
Observation		2 Observations in Week 1 2 Observations in Week 2 1 Observation in Week 4
Interview	Pre-Rate of Change Lesson Plan Study Interview Post-Rate of Change Lesson Plan Study Interview	Pre-Interview Post-Interview
Lesson Plan	2 Rate of Change Lesson Plans	2 Lesson Plans
Reflection	Week 1 through Week 9	Week 1 through Week 6

Table 1. Data Collection Timeline for Each Semester

Ethical concerns have been addressed in this research study in several ways. The pre-service teacher signed a consent form giving permission to videotape and to observe him in the classroom. He was asked if he wanted his name used and he said it did not matter to him. However, a pseudonym was chosen for his name and the school he taught at to maintain confidentiality. During class observations, considerations were taken so as not to disrupt the normal activities of the class. During his student teaching, there were other occasions where the vice principal, the pre-service teacher's college instructor, and/or the cooperating teacher observed Chris. Therefore, the Chris was familiar with having outside people in his classroom during instruction.

In my pursuit of being accountable for decisions regarding ethical issues, there were attempts to use nonjudgmental words in the analyses of Chris' descriptions. Other mathematics educators were asked to proofread this case study. I sent a letter informing the participant that the monograph is finished and that he was invited to read his case study and discuss the study with the researcher afterward. Diener and Crandall (1978) stated that ethical decisions are made by responsible people who realize the value of their implications depending on their choices. The ethical researcher is concerned about the participants' welfare, and about the future uses of the knowledge gained from the research.

Data Analysis

Data analysis in qualitative research is a systematic process of taking something apart to look for patterns. According to Stakes (1995), "with intrinsic case study, the caseworker sequences the action, categorizes properties, and makes tallies in some intuitive aggregation" (p.74). A case study is carried out in order to explore the interpretative and subjective dimensions of educational phenomena (Cohen & Manion, 1992). Since the lesson plan study of rate of change is a subjective and multifaceted idea, the investigation can be done effectively through case study methods (Burns, 2000). Furthermore, such cases can contribute to the extension of a theory by supporting an existing principle, or by challenging it through the outcomes of case studies (Burns, 2000; Merriam, 1988). My primary task is to understand the case by concentrating on relationships identified in the research questions.

While looking for evidence of the pre-service teachers' change of pedagogical content knowledge, episodes from the transcripts and written artifacts were analyzed in search of patterns (Bogdan & Biklen, 1992). To find patterns, I asked myself, "What did that mean?" I also continued to reflect, to triangulate, and to be skeptical about first impressions and simple meanings from the videotapes and audio recordings of observations and interviews (Stakes, 1995).

In the analyses of Chris' descriptions, I used Brownlee, Purdie, & Lewis' (2003) definition of changed views to describe Chris' changes in the content of rate of change and pedagogical content knowledge. Individuals with changed views are transformative. Information undergoing "a process of construction or transformation in relation to an individual's prior knowledge" is transformative (Brownlee, Purdie, & Lewis, 2003, p.111).

Figure 13 graphically displays how I coded and sorted the data. Year 1 analysis starts with the initial coding the data by using acronyms for each component of the pedagogical content model, such as knowledge of multiple representations – graphs would be kmrg. The data was then sorted by knowledge of student ideas, knowledge of representations, knowledge of instructional activities and profound understanding of fundamental mathematics. Next, I sorted the knowledge of student ideas by areas of student difficulty and by requirements for learning. Knowledge of representations data was sorted into the categories of symbols, words, and graphs/pictures. Finally, profound understanding of fundamental mathematics data was arranged in accordance with basic mathematical ideas, connectedness and longitudinal coherence.

Year 3 data was coded and sorted exactly as Year 1 data. Additionally, I compared Year 1 data in the areas of student difficulty, requirements for learning, symbols, words, graphs/pictures, activities, basic mathematical ideas, connectedness, and longitudinal coherence with Year 3 data of the same categories.

After coding and sorting the data, I created an initial list of themes. The list included themes such as representations and activities, packing and unpacking, content knowledge of slope and representations of rate of change. However, I began to focus on the overt things Chris stated in the data that were important to him such as graphs. This led me to ratio versus fraction, independent versus dependent variables and continuous versus discrete variables themes.

The Investigation

Lesson plan study, adapted from the Japanese Lesson Study method of professional development, was used in this investigation. A team of four educators who actively studied and organized the lesson study drew upon each other's strengths, which included mathematics education expertise at different grade levels. The participants were pre-service teachers in their first secondary mathematics methods course. Their task was to think about how they would teach rate of change. During a five-week period, the pre-service teachers had several occasions to exchange ideas with their classmates, and to reflect on these ideas. The team of educators was interested in the participants' knowledge of mathematical concepts and in their understanding of connections among concepts and procedures. Interviews were conducted to assess pre-service teachers' subject matter knowledge and pedagogical content knowledge. The team also analyzed

group presentations of rate of change for evidence of the students' conceptual and procedural knowledge and appropriate representations for teaching procedures and concepts. Thus this research examines some of the mathematical and pedagogical results of a pre-service teachers' lesson plan study.

The investigation also included the subject's student teaching for ten weeks in Year3. During this time, the pre-service teacher was also taking a technology methods course and mathematics methods course. The subject was not monitored during Year2 of his program. During Year2, the subject was involved in another research study program.

The Participant

This is a case study with one participant, a pre-service teacher. Hereafter, the participant is referred to as Chris (pseudonym). Chris was chosen because of his previous involvement in an educational experiment in the fall of 2000 within the Center of Research of Mathematics and Science Education (CRMSE) at North Carolina State University.

The pre-service teacher was a mathematics education major at North Carolina State University. Chris is 22 years old, single white American male who attended a local high school. He was an academic mentor at North Carolina State University, a member of a fraternity, philanthropy chairman, and on the Dean's List. His extra-curricular involvement included volunteering at Food Bank of North Carolina and tutoring with the Adopt-A-School Program. He also worked as a manager of a pool and a lifeguard during the summers. Chris enjoys sports such as basketball and golf and traveling. His goal

after graduation is to teach at a high school and coach the basketball or golf team. He would also consider pursuing a master's degree in mathematics education.

I chose this pre-service teacher because he is enthusiastic about teaching, highly verbal in expressing and explaining his ideas, and a willing participant. He told me that he would be happy to help me in whatever I needed for my class and/or dissertation. The data later confirmed that we had a rapport and mutual respect.

Sources of Data

Triangulation is a method of establishing internal validity of a qualitative research. It may be defined as the use of two or more methods of data collection in the study of some aspect of human behavior (Burns, 2000; Cohen & Manion, 1992). To establish triangulation in determining answers to the research questions, multiple data sources were considered (Glesne & Peshkin, 1992). These methods included videotaped interviews, audio taped classroom observations, and artifacts. These data were collected in Fall 2000 and Fall 2002 semesters.

Year 1 Interview of the Pre-service Teacher

According to Fontana and Frey (1994), individual, face-to-face verbal interviewing is one of the most common ways to try to understand people. They suggest that semi-structured interviews provide greater breadth of understanding a phenomenon.

Chris was interviewed on six different occasions. During Chris's first methods course at North Carolina State University, there were two interviews. Each of these interviews had three parts: 1) a preliminary interview, 2) a lesson planning task, and 3) a

post-planning interview. The first part of the first interview started with a videotaped session where an interviewer asked Chris what he remembered about learning ratio and proportion, and how he would define rate of change. The protocol that guided this interview is included in Appendix A, Interview Protocol for Ratio and Proportion Lesson-Planning Activity. The pre-service teacher was encouraged to tell the story of his secondary and undergraduate education, highlighting the courses and experiences that had the greatest influence, in his memory. The following part of the interview involved Chris in developing a lesson plan that would introduce the concept of rate of change to an Algebra I class, and connecting the lesson to ratio and proportion concepts. The pre-service teacher was provided with an Algebra I textbook, the North Carolina Algebra I curriculum, manipulatives, and a graphing calculator. During the next part of the interview, Chris explained his lesson, which was also videotaped. The interview design allowed for some departure and follow-up on the written protocol questions to clarify any comments.

Year 1 Lesson Plan

Chris created one lesson plan during his methods course based on rate of change. This was after consulting with a small group of peers from his class. The rubric for the lesson plan is included in Appendix B, Lesson Plan Score Sheet. The revised and final version of his rate of change lesson plan is in Appendix C, Lesson Plan.

Year 1 Reflections

Reflection involves active, persistent, and thoughtful consideration of behavior or practice (Clark & Yinger, 1987). It is a way to respond to past behaviors or situations. Reflection is a deliberate and responsive intellectual action. Thus, there were nine reflective writings Chris turned in for his first methods course. The reflections were based on his classroom observations of an Algebra I class in a public school in North Carolina.

Year 3 Interview of the Pre-service Teacher

During Chris's student teaching block at the high school, he was assigned to teach an Enhanced Algebra Course. There were two videotaped interviews conducted on the campus of North Carolina State University. Both interviews focused on the topic of linear equations that he taught during student teaching. This was to check for any changes that might have occurred during his student teaching. The protocol that guided these interviews is included in Appendix D, Interview Protocol for Student Teaching.

Year 3 Observations

Spradley (1980) defines passive participant as someone who "is present at the scene of action but does not participate or interact with other people to any great extent" (p.59). The observer was a passive participant who sat in the back of the class observing and recording what went on in the classroom during Chris's student teaching block at the high school in the Enhanced Algebra Course.

Chris was observed five times in the classroom. Each observation of the 50-minute class period was audiotaped. During the observation I either took field notes, or completed an observation form (see Appendix E). I also collected his lesson plans for all the lessons he taught for the topic of linear equations. The observations were dependent on whether or not he was teaching content relevant to the study. He has also given me all his lesson plans for the topic of linear equations in the Enhanced Algebra class.

Year 3 Reflections

There were six reflective writings Chris turned in during his student teaching. The reflections were based on his experience of student teaching in a public school in North Carolina. Chris's reflections describe practical deliberations of him processing a situation that took place in the classroom. They give insight on how his future actions differed from his past actions.

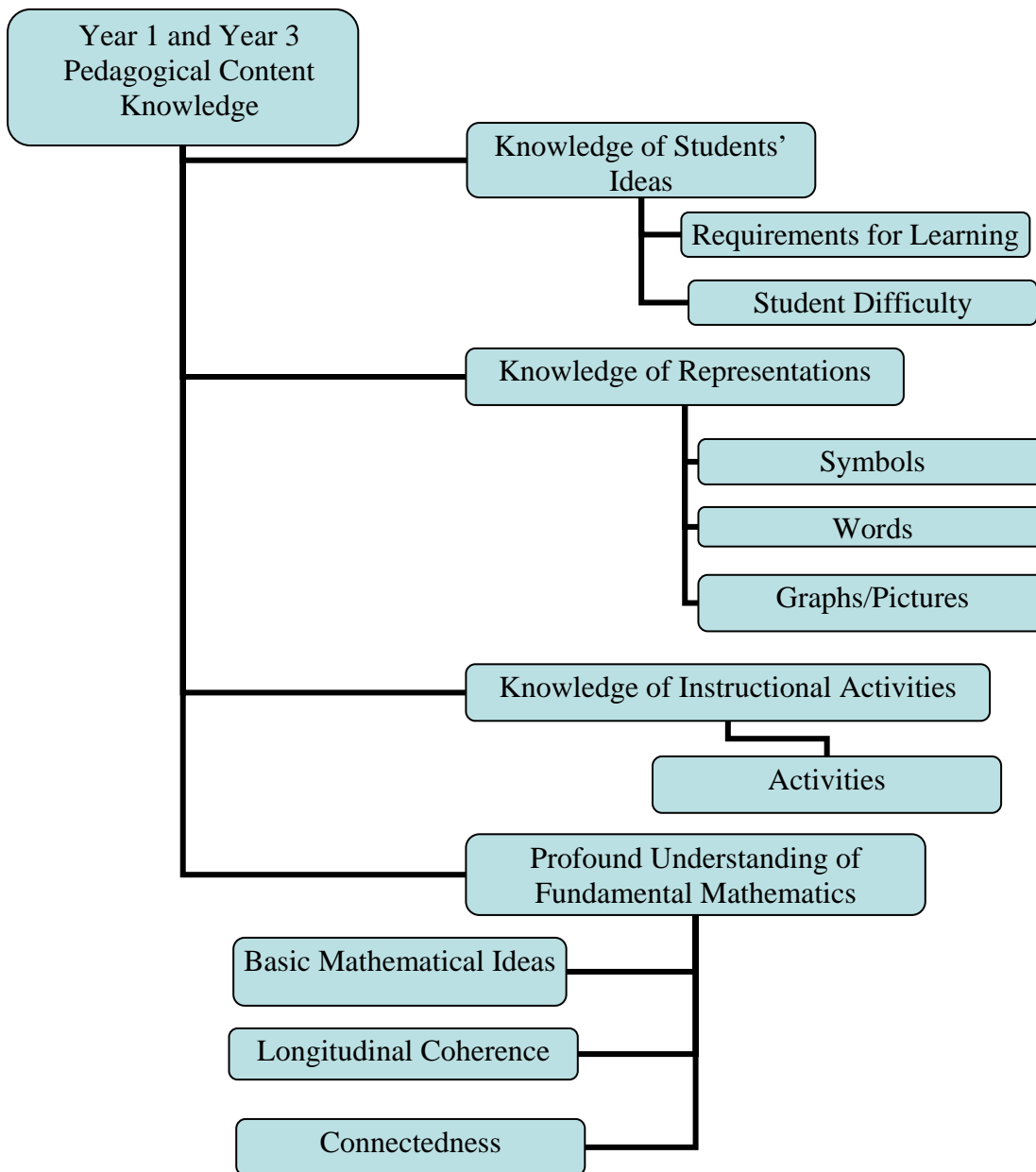


Figure 13. Data Analysis of Pedagogical Content Knowledge

Summary

During the two year time period of examining changes in attitudes and conceptual understanding, I hoped to understand how a pre-service teacher's pedagogical knowledge develops with respect to his understanding of ratio and proportion, as it relates to rate of change, using the semi-structured interviews, observations and written artifacts.

PRESENTATION OF FINDINGS AND INTERPRETATIONS

In this chapter, I will examine the data collected from the pre-service teacher's first methods course, six weeks of lesson planning and student teaching experience over the course of a two-year period. The data included the pre-service teacher's interactions and responses during the interviews, classroom observations and reflections through transcriptions and analysis. The description of this transformation was focused on the mathematical concept of rate of change over a time period that allows for examination of conceptual understanding. Pre-service teachers need to understand the fundamental principles that underlie school mathematics such as rate of change, so that they can teach it to diverse groups of students' which emphasize the interconnections among theory, procedures and applications.

Tracing the Development of Rate of Change

Part of the model for pedagogical content knowledge was used to organize the development of mathematical understanding by examining the pre-service teacher's pedagogical content knowledge over the course of a two-year period, see Figure 14. Figure 4 graphically displays the conception of pedagogical content knowledge that I was able to analyze from the data collected. The graphical representation of pedagogical content knowledge was revised to look at what Shulman (1986) categorized as knowledge of representations, knowledge of instructional activities, and Ma's (1999) basic mathematical ideas which was a subcategory of the profound understanding of fundamental mathematical ideas. Knowledge of representations includes the symbols,

words, graphics/pictures and other representations used by the teacher to communicate mathematical ideas to students. Knowledge of instructional activities is having a repertoire of activities to use in the classroom. Having basic mathematical ideas of rate of change was revisiting and reinforcing problems while guiding students through a mathematical activity.

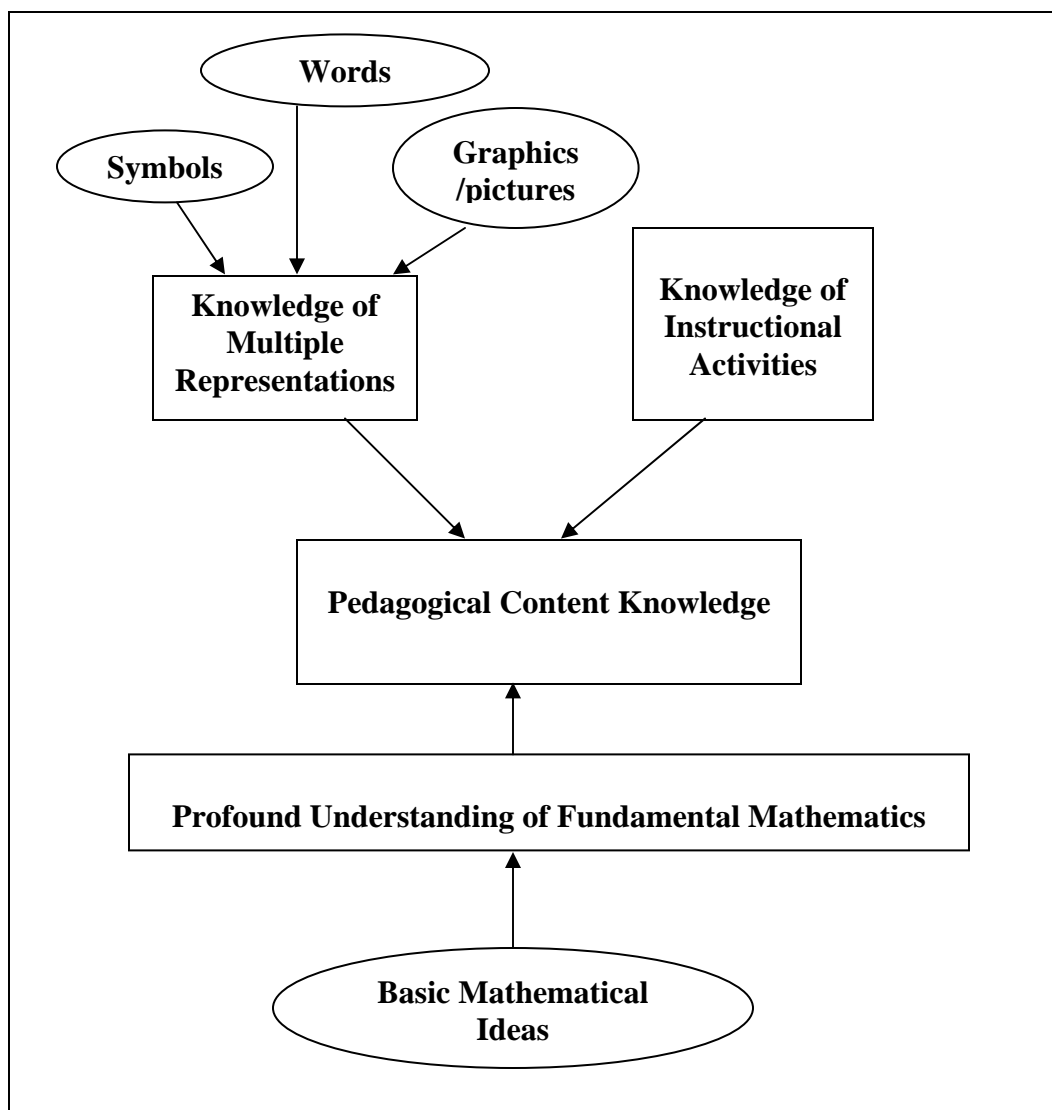


Figure 14. Revised Pedagogical Content Knowledge Model

Knowledge of multiple representations, knowledge of student activities and basic mathematical ideas are used to trace the pre-service teacher's concept of rate of change. The rest of this chapter is divided into three themes: 1) Ratio versus Fractions, 2) Discrete versus Continuous and 3) Dependent versus Independent. Each section used the pre-service teacher's knowledge of multiple representations, knowledge of student activities and his basic mathematical ideas of the concept of rate of change. A brief description of ratio, fraction, discrete, continuous, dependent and independent was given at the beginning of each section. The descriptions of the terms were followed by an explanation of the data that was used from the lesson plan study and student teaching experience. There was an interpretation of the data that was used after the presentation of the data.

Ratios versus Fractions

The word ratio means different things to different people. Different mathematics texts have various meanings attached to the word ratio. Clark, Berenson, and Cavey (2003) stated that ratios can come in three forms. Ratios can be written in a non-fractional form such as 2:3, 70 miles per hour or 2 cups of sugar to three cups of flour. Second, ratios include part-part fractional representations such as 30 red marbles to 40 blue marbles. And third, ratios can be written as a part-whole fractional representation such as two-thirds or $\frac{2}{3}$.

The word fraction also has diverse meanings according to different people and mathematics textbooks. A fraction can also be written as part-part and part-whole representations like ratios according to Clark, Berenson, and Cavey (2003). The researchers also stated that fractions differ from ratios when fractions are used as decimal numbers, integers, counting numbers, rational numbers and real numbers.

Data taken from the lesson plan study described what ratio and fraction meant to Chris. His basic mathematical idea of ratio reminds him of various representations. These images included the phrase “green to blue” which represents a part-part relationship where an amount is compared to another amount. Last, he extends his definition of “rise over run” to “rise over run would be a fraction.”

I: What do you remember about learning ratio and proportion when you were in school?

CHRIS: I guess when I think about ratio, two things come to my mind.

Rise over run type of stuff. Then stuff as far as, you have forty marbles. Thirty are green and ten are blue. The ratio of green to blue, things like that.

I: Is there any notation that you would use? Any ways that you would represent that marbles problem?

CHRIS: Yeah, like A and B. I guess that's what I remember, writing them like that. Or rise over run would be the fraction of them.

In Chris' statement above, he talked about having forty marbles, thirty were green and ten were blue. As the interview continued, he explained how he would set the marbles up as a ratio using a colon in between the numbers and without the units after the number.

I: Ok, the colon form [of writing a ratio]?

CHRIS: Yes, if it said green to blue, you would write the green first.

I: What's an example in terms of number?

CHRIS: If you had forty marbles where thirty were green and ten were blue, the ratio would be three to one, green to blue. That's what I'm thinking as far as that.

Chris' representation of his ratio was "green : blue" and "3 : 1" in colon form. A decontextualized ratio occurs when the representation of a ratio is detached from the meaning of the problem (Clark, Berenson, & Cavey, 2003). Chris decontextualized his ratio when he detached the meaning of the thirty green marbles and ten blue marbles and when he did not use the units after the numbers. However, he admitted that he use to get

confused when setting up a ratio when he was first introduced to ratios during his K-12 experience.

I: What problems, if any, did you have learning about ratio and proportion when you were in school?

CHRIS: I think my biggest problem used to be the order of those [green and blue marbles]. That's something little, but I use to get that confused.

I: So that means something different if you say green to blue marbles versus blue to green marbles?

CHRIS: Certainly.

I: Did you ever write down the units with those to help you keep track of them?

CHRIS: No, I think that would have been a great idea. I never even thought about that. I never had a teacher mention that because it has always been numbers – never [with units] - but I think that would've helped me out a lot.

After analyzing the data from the lesson plan study, Chris also decontextualized the pills versus minutes word problem. The first word problem introduced in his lesson plan was a pills versus minutes problem, see Figure 15.

You just found a pill that allows you to fly! You have found that if you take two pills, you can fly for exactly 9 minutes. You want to fly to your best friend's house to show him your amazing discovery. He lives 63 minutes away. How many pills must you take to complete the journey?

Figure 15. Pills versus Minutes Word Problem

Chris explained in his lesson plan that he would show his students how to solve the problem by setting up the numbers as equivalent ratios, see Figure 16.

$$\frac{2}{9} = \frac{x}{63} \quad \leftarrow \begin{array}{c} \text{No Units} \\ \text{were used} \end{array} \quad \rightarrow \quad \frac{9}{2} = \frac{63}{x}$$

$$9x = 126 \quad \quad \quad 126 = 9x$$

$$x = 14 \quad \quad \quad x = 14$$

Figure 16 . Pills versus Minutes Problem with No Units

Chris did not use the units attached to the numbers as noted, 2 pills to 9 minutes is equivalent to x pills to 63 minutes. However, he stated in the interview above that he thought it “would have been a great idea” to have the units with the numbers so that it would not be confusing. In other part of the lesson plan study during Chris’ Year1 Methods course, he contextualized the ratio examples he created, see Figure 17. The soda and pizza examples were created during the lesson plan study interview. And the miles per hour and the miles per gallons ratio were created in his lesson plan. Each example is explained in detail in the following sections of this chapter.

Soda Ratio:	$\frac{1 \text{ soda}}{2 \text{ dollars}}$	$\frac{x \text{ soda}}{4 \text{ dollars}}$
Pizza Ratio:	$\frac{3 \text{ eaten}}{4 \text{ total}}$	$\frac{? \text{ eaten}}{8 \text{ total}}$
Miles and Hour Ratio:	$\frac{4 \text{ miles}}{1 \text{ hr}}$	$\frac{x \text{ miles}}{3.5 \text{ hr}}$
Miles and Gallons Ratio:	$\frac{30 \text{ miles}}{1 \text{ gallon}}$	$\frac{270 \text{ miles}}{x \text{ gallons}}$

Figure 17. Examples of Contextualized Ratios

The individual that helped Chris to understand ratios that kept its meaning in a word problem was his high school teacher. He used an airplane example to help him remember “rise over run” with the basic mathematical idea of slope. The airplane example is explained in detail in the dependent versus independent section.

CHRIS: Rise over run was actually the term she [his teacher] was using with the airplane.

I: That seems to be a term that almost everybody mentions. It seems to be one of those things that has been repeated over and over in school until people associate that with the word slope. When I say slope is that what you are immediately thinking?

CHRIS: Definitely.

Chris was then asked during the lesson plan study interview how he would describe the relationship between ratio and fraction. “All fractions are ratios [but] it is not correct to say that all ratios are fractions,” as stated by Van de Walle (1994, p.275) was Chris’ understanding of ratios and fractions. Model 2: fractions as a subset of ratios from Clark, Berenson, and Cavey (2003) best represented Chris’ explanation of ratios and fractions, see Figure 18.

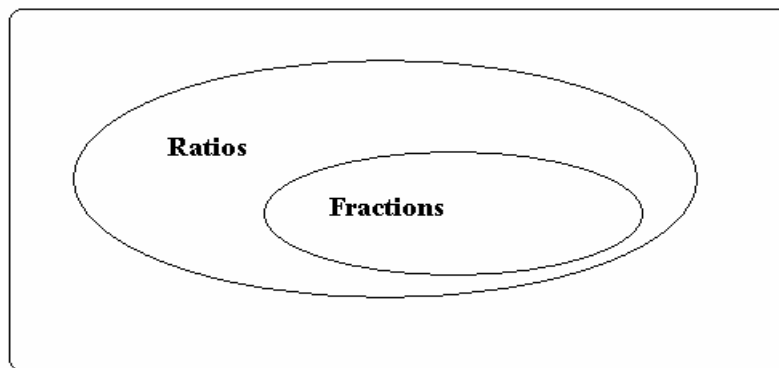


Figure 18. Model 2: Fractions as a Subset of Ratios (Clark, Berenson, & Cavey, 2003)

- I: How would you describe the relationship between ratio and fraction?
- CHRIS: I think I would explain that every fraction is a ratio. I would explain that whatever fraction you had: two to eight or sixteen to twenty-one, whatever. That’s in a ratio of parts to whole. We have sixteen pieces of a whole, and the whole is twenty-one. And explain that every fraction is a ratio.
- I: So fractions are types of ratios?
- CHRIS: Right, but not necessarily can ratios be written as fractions. Then

maybe give the example of maybe ten to zero. You couldn't write the ratio as a fraction, and give an example why it's not.

Somebody might ask, does that mean every ratio can be written as a fraction? So you would have to explain that example.

I: So you could write ten colon zero, but not ten over zero?

CHRIS: Right.

I: All fractions are ratios, but not all ratios are fractions?

CHRIS: I believe so.

The problem with Model 2: fractions as a subset of ratios, was that the term ratio loses its power of discrimination when compared to a fraction (Clark, Berenson, & Cavey, 2003). The researchers defined ratios in non-fractional form that include 4:9, 70 miles per hour, or 2 cups of sugar for every 3 cups of flour. And fractions that are in number related context only include $\frac{2}{3}$ as a number, as a point on a number line, or as in the inequality $\frac{2}{3} > \frac{1}{2}$.

In Year 3 of Chris' student teaching, he was never observed connecting ratio to slope in his Algebra class. He related slope with the formula and focused on the procedures.

CHRIS: Here are the graphs of five lines [five lines are drawn on a coordinate plane on the board]. Which lines have the same slope? Find the slope of each line.

Chris used flying pills versus minutes, sodas versus money, and pizza versus money as discrete variables to represent a ratio in this section. He was able to exhibit the ability to operate within the ratio and proportion process to solve problems without a

graph. In the discrete versus continuous section, Chris' discrete variable poses a problem when you have to interpret fractional parts of a pill, soda, pizza or money. This lack of understanding about discrete and continuous variables may have contributed to Chris' confusion when he chooses a different representation such as a graph.

Discrete versus Continuous Variables

The visualization of data can be presented as a graph or a chart (table). Choosing the type of graph or a chart depends whether the variables are discrete or continuous. Discrete variables are a “set of numbers that is countable (possibly countably infinite) (Horowitz, 1981).” In particular, discrete variables are usually but not necessarily countable numbers of distinct values or integers such as 0, 1, 2, 3 . . . Examples of discrete variables include the number of children in a family, the states of a coin toss (either heads or tails), the number of cracked eggs in an egg carton or the number of credit hours completed for a degree. A scatter plot graph is one way to display discrete variables.

Continuous variables can assume any numerical values over a range of values. A function is continuous on an interval if and only if it is continuous at every point of the interval (Finney, Demana, Waits, & Kennedy, 1999). Real numbers measure continuous variables. Examples of continuous variables also include weight that can be expressed in pounds (e.g., 180lbs.), tenths of pounds (e.g., 180.5lbs.), hundredths (e.g., 180.52lbs.), thousandths (e.g., 180.521lbs), and so on. Other examples of continuous variables are temperature, height, or the time to take to run a mile. Continuous graphs are lines or curved lines without breaks in them.

The data that was used in Chris’ lesson plan study interview from YEAR1 methods course to analyze how he planned to teach the mathematical idea of rate of change included an example using discrete variables on a continuous graph. He started his lesson with an example that he had written on the board that his students would do at their seats. He used rectangles to represent the cans of sodas on a separate sheet of paper,

see Figure 19. He would then write the definition of ratio on the board before going over the example.

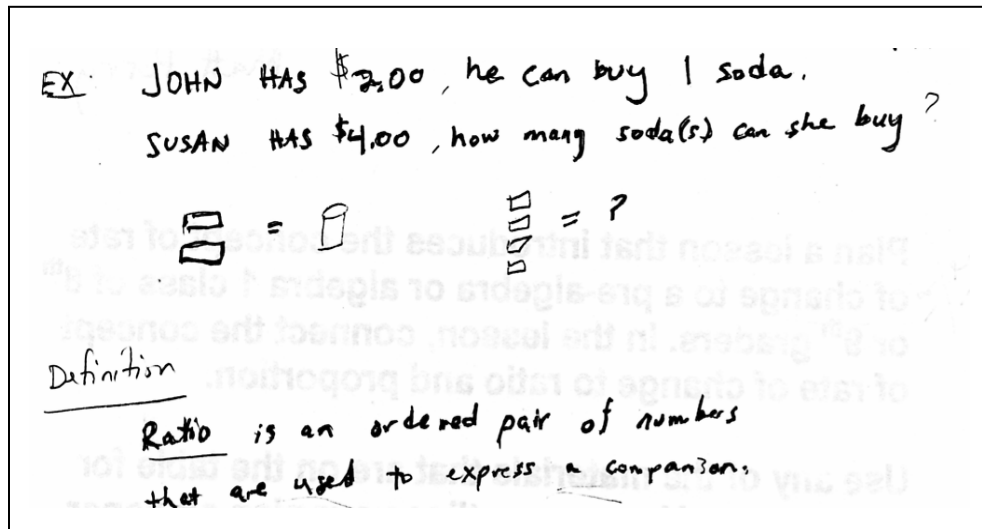


Figure 19. Chris' soda example

CHRIS: What I was going to do was start off having a problem on the board when they walked in just so they would get in the frame of mind of math. So I was going to have this simple . . . John has two dollars, he buys one can of soda; Susan has four dollars, how many can she buy?

The interview continued with Chris changing his pictorial representation to numbers, measurements, symbols, and fractions, see Figure 20.

EX: JOHN HAS \$2.00, he can buy 1 soda.
 SUSAN HAS \$4.00, how many soda(s) can she buy?

$\frac{1}{2} = \frac{x}{4}$ $\frac{1}{2} = \frac{x}{4}$

Definition
 Ratio is an ordered pair of numbers
 that are used to express a comparison.

in EX: $\frac{1 \text{ soda}}{2 \text{ dollars}}$ $\frac{x \text{ soda}}{4 \text{ dollars}}$

$\frac{1}{2} = \frac{x}{4}$ $4(1) = 2x$
 $\frac{4}{2} = x$
 $x = 2$

Figure 20. Chris' soda example using measurements and numbers

CHRIS: I wanted to start it with, he had one soda for two dollars and you weren't sure how many sodas she had, but she had four dollars. So for every one soda, he had two dollars, and I tried to write it down in equation form. Then you'd solve for x.

The next time Chris referred to this problem during the interview, he related the soda and dollar problem to a graph. He plotted the point (1, 2) with two being the number of dollars and one representing the number of sodas and drew a line through the points, see Figure 21.

CHRIS: Then I was going to try to relate that to the graph more just to show them that we can relate the same idea in different form. So we have our simple graph, and over here we have the price, one dollar, two dollars, three dollars. Then on this side we have the number of sodas. I probably wouldn't have done this with a half if

I would have gone back, but I had already done it just because that might be confusing with blocks. I wanted to show that you could relate it to, here's one soda to two dollars, then we had two sodas to four dollars. If we drew the line it connects those dots, so we know that these are proportional to either one of these.

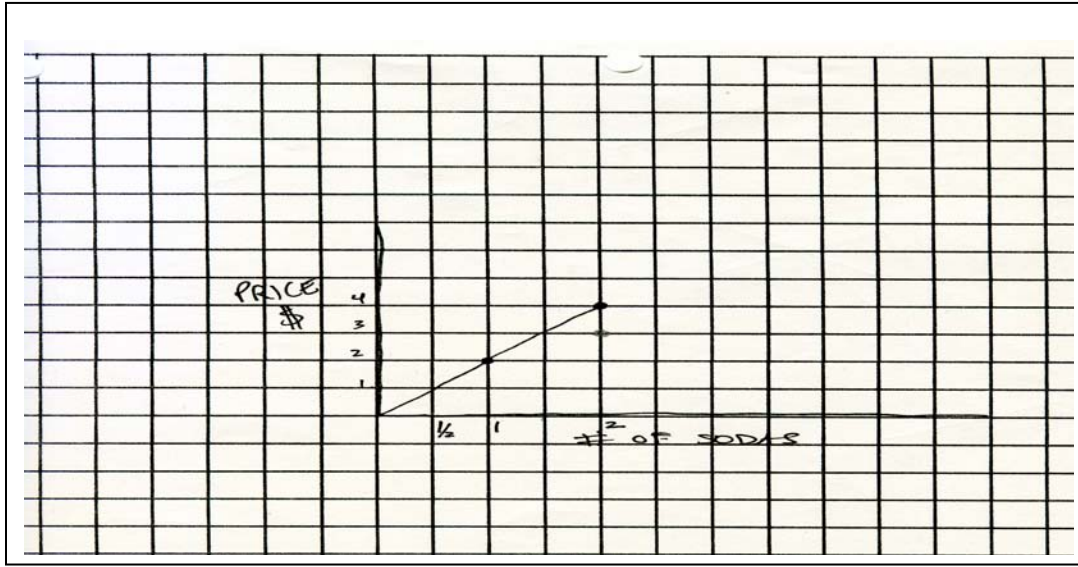


Figure 21. Chris' soda example using a graph

Sodas and dollars are both discrete variables although Chris drew a line through the points. Drawing a line through the points change the meaning of the graph from discrete to continuous except you can not have three-fourths of a dollar or half a soda. Chris realized that he probably should not have used this problem after talking with the interviewer but he would try to explain to his students how they could use the graph to understand proportions.

- I: OK, you mentioned showing values at fractional parts of sodas.
Would students have questions about that?

CHRIS: They might ask, you can't have a half a soda, or what if you had three dollars, what would you do. I could explain that in our example you might not be able to buy half a soda, but if you relate it to proportion and you had three dollars that would be equivalent to one and a half sodas, or one and a half whatever you had. Then I could go back to the pizza example and say, if there was two dollars to every piece of pizza. Then if you had three dollars you could get one and a half pieces of pizza because you could split a pizza. Maybe you could do that. Just show them that just because in my example. A half doesn't work doesn't mean that if it falls anywhere on that line [referring to the line in Figure 12].

Chris would have his students' just work with the proportion to solve for the number of sodas given the number of dollars instead of using the graph as a way to solve the problem. Part of Chris' lesson that he created earlier in the lesson plan interview was comparing fractions using the soda example explained above and a pizza example. His fraction for the pizza example was the number of pizza slices eaten over the total amount of pizza slices which was explained in the ratio versus fraction piece, see Figure 22. In Chris' statement above, he also tried to correct his soda example by using his pizza example. He saw the pizza as a continuous variable. He recognized that you cannot have a half of a soda but you can have half of a pizza.

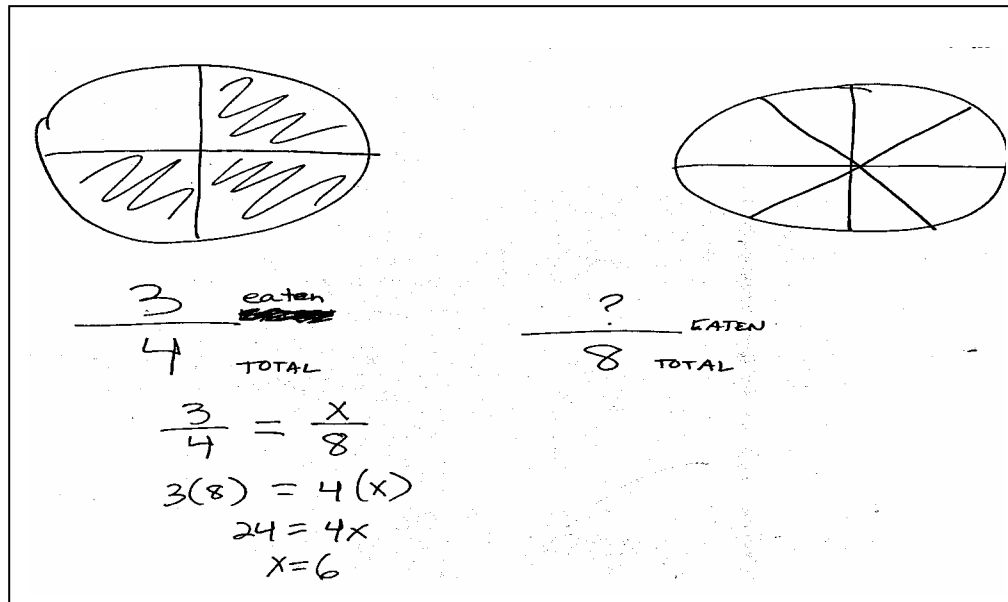


Figure 22. Chris' Pizza Example

Chris also used discrete variables in his lesson plan during his methods course for the pills versus minutes word problem, see Appendix E. The pills were the discrete variable because you can not take a fraction of a pill accurately and the minutes were the continuous variable. Before he drew the graph, he had his class to find the ordered pairs from the flying pills word problem, see Figure 23.

Next I would ask if we could write these in a different form (ordered pairs). If I do not get this, then I will offer this to the class. We will show, in this case we would have (2, 9) and (14, 63). We could then graph these two to start to draw the relation between the word problem and the graphic representation (Re-emphasizing graphing given two points).

These two points give us this line. Once we have two points on a line, we can find the slope.

Slope= m

Figure 23. Plotting Two Points on the Graph Pills versus Minutes

He then used the points from the ordered pairs to draw a line and from the line he had his class to find the slope of the line, see Figure 24. The line through the points would suggest that both variables were continuous but since the pills were the discrete variable it would be difficult to extrapolate some information from the graph such as how many pills you need to fly for 12 minutes.

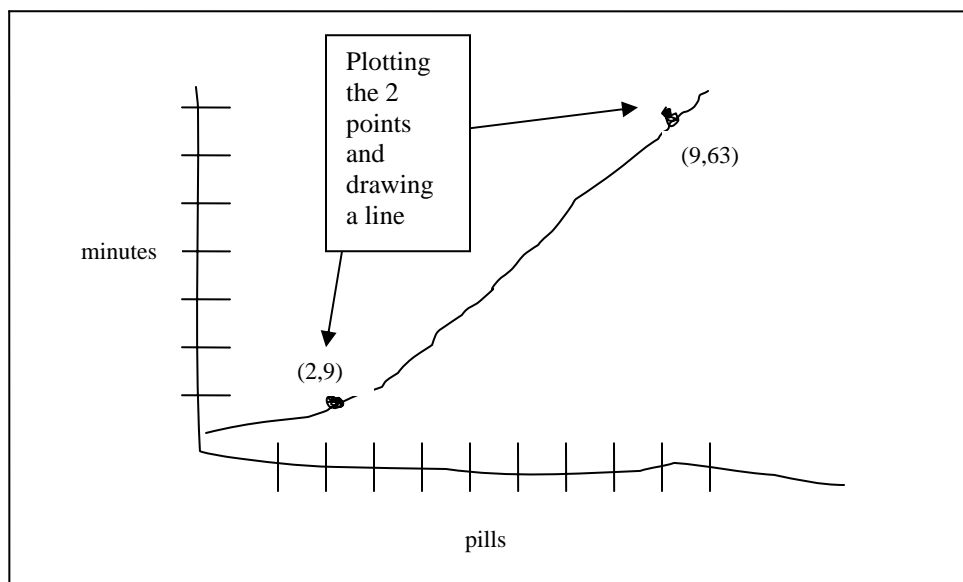


Figure 24. Pills vs Minutes Graph

After the graph was drawn, Chris used a fourth type of representation to describe the data, a chart. The chart was another way to solve the pills versus minutes word problem, see Figure 25. The chart was an appropriate form of representation since the pills were a discrete variable.

X(pills)	Y(minutes)
0	0
1	4.5
2	9
3	13.5
.	.
.	.
.	.
14	63

Figure 25. Pills vs. Minutes Chart

In the second half of Chris' lesson plan during his YEAR1 METHODS COURSE class, he used variables that were both continuous on the x-axis which is the independent variable and y-axis which is the dependent variable, see Figure 26 The continuous variables that he had chosen were miles, gas in gallons, and time (hours) for his graphs would be consistent with plotting points and drawing a line through the points to help solve questions. The problem with Chris' graphs was how the x- and y-axis were labeled.

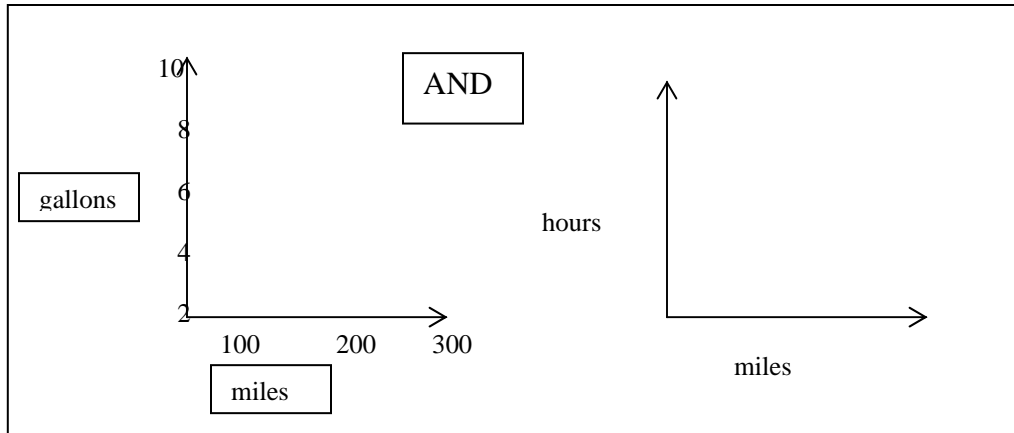


Figure 26. Graphs of Continuous Variables

However, Chris mentioned in his first interview of the lesson plan study that being able to visualize a problem helps him understand how to solve a problem.

I: What helps you learn about ratio related concepts, either now or in your math experience?

CHRIS: Helps me learn about them?

I: Yes.

CHRIS: I think the visual is the biggest thing for me. And the reason I was trying to do real-life examples, is sometimes I think, not just in ratios, but in math, I get to the point where it's just numbers on a piece of paper, and you don't really understand what you are solving. You might know how to do it by formulas, or by repetition through examples, but I think it helps me to explain, even if it takes five minutes, what exactly we're doing. That if we have a function and we're doing rate of change, explain that

runner's distance, his time is changing or whatever we talked about earlier. And just explain exactly what we're finding.

Chris stated how important it was to represent new mathematical ideas through real life examples. The graphical representations of Chris' data, also stated in the dependent versus independent section, would be difficult for a student to answer questions from Chris' real life word problems. Even though a graph was appropriate for his continuous variables, the graph was not useful to extrapolate information for further use. The graph was a visual that Chris used but it appeared that the graph was a separate object to be seen but not used to predict any future or past behavior of the data collected.

As a senior during Chris' student teaching, he used continuous variables on a graph with his calculator based laboratory using the bouncing ball activity, see Appendix G. The calculator based laboratory bouncing ball activity is explained in the independent versus dependent section. The graph that was produced by the program in the graphing calculator automatically labeled the x-axis, time and the y-axis, height. The time and height variables were both continuous variables. He then had his students' recreate the graph on a separate sheet of graph paper and fill in the chart on the worksheet by following the procedure, see Figure 27.

PROCEDURE

1. Practice ball drop and measurement from first height.
2. Drop ball three times at each height (12", 24", 36", 48", 60") then record the consistent measurement at each height.
3. **Using your data, plot the ordered pairs on the graph paper (be certain to properly label your graph).**
4. **Draw the best fit line.**
5. **Write the equation for the line.**
6. **Use the equation to predict how high the ball would bounce if dropped from 72".**
7. Next predict how another ball used in class will compare to the linear equation you found (are their lines steeper, flatter, or the same?).

X	Y
12"	
24"	
36"	
48"	
60"	

Figure 27. Procedure and Chart for Bouncing Ball Activity

CHRIS: Alright you should be graphing your 5 points. Everybody should be graphing. [As he goes around the class collecting balls].

CHRIS: Alright, once you get your points graphed, draw in your line of best fit.

At this point, the graph would have been the correct representation for the data the students collected from the calculator based laboratory activity, see Figure28.



Figure 28. Graphing Calculator Screen, Height vs Time

In spite of this, Chris created a procedure that his students should have followed to obtain the data from the activity, see Figure 28. He had his students use the equation generated by the graphing calculator instead of using the graph on the graphing calculator screen to predict how high the ball would bounce if dropped from 72". The students could have used the cursor on the graphing calculator to locate the height of the ball after it was dropped from 72". Chris went from collecting data to a graph and then to solving the problems using an equation.

In the data collected on Chris during the lesson plan study, his methods course, and his student teaching experience, Chris used words, symbols, graphs and charts as forms of representations in his work. The pill versus minutes graph was labeled correctly and you could use the graph for whole numbers but it would have been difficult to use the graph if your given information was a fraction of a pill, see Figure 16. The bouncing ball activity graph was also labeled correctly and functional to analyze data but Chris did not use the graph to answer the questions he presented on his worksheet, see Figure 29.

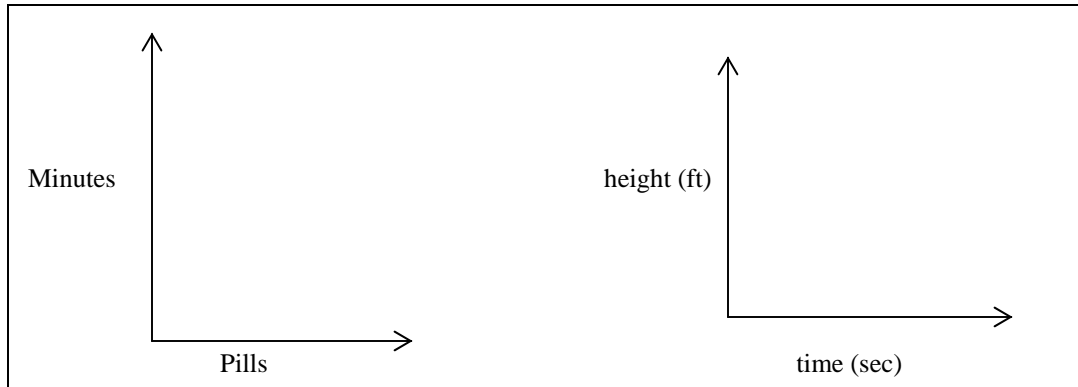


Figure 29. Graphs Labeled Correctly

Chris' graphs such as his soda versus money, used only discrete variables and his gallons versus miles and hours versus miles graphs used continuous variables but were labeled incorrectly. Chris' inability to translate between algebraic and graphical representations made his graphs problematic to answer questions related to the graph. The fundamental properties and functions of graphs in representing relationships among variables must be meaningful to help solve problems.

Independent versus Dependent Variables

Data should be presented in the manner that is most meaningful to the situation. There is no fast rule on dependent versus independent variables for either the x- or y-axis. The most common expression in the United States of labeling the axes in a coordinate plane is assigning the dependent variable to the vertical or y-axis and the independent variable to the horizontal or x-axis. In a cause-effect problem, the expected cause is on the x-axis and the effect is on the y-axis (Sullivan, 2002). Consider the linear graph of position versus time. The slope of the line gives the velocity for the interpreters of the graph from the United States.

The data that was taken from the lesson plan study included the pre-service teacher writing a lesson plan using the concept of rate of change. The objectives in Chris' rate of change lesson plan were 1) to find the slope of a line given two points, 2) given two ratios, convert them to ordered pairs and find the slope of the line formed, 3) translate ratio word problems into graph form and find the slope of the line, and 4) be able to work basic rate of change problems, see Appendix E.

Chris started his lesson with a word problem for the class to solve. According to his problem, you need to take two pills to fly nine minutes, see Figure 30.

You just found a pill that allows you to fly!! You have found that if you take two pills, you can fly for exactly 9 minutes. You want to fly to your best friend's house to show him your amazing discovery. He lives 63 minutes away. How many pills must you take to complete the journey?

Figure30. Flying Pills Word Problem

Chris then set up the problem using proportions and then solved for x algebraically in his lesson plan, see Figure31.

$\frac{2}{9} = \frac{x}{63}$	$\frac{9}{2} = \frac{63}{x}$
$9x = 126$	$126 = 9x$
$x = 14$	$x = 14$

Figure31. Flying Pills Problem Set Up As Proportions

If the students had problems understanding how he set up and solved the proportions then his contingent plan was to use a table, see Figure 32. Chris explained that as the “x values are changing, so are our y values (Appendix E).” Chris knew “that the x values and the y values were directly proportional (Appendix E).”

Lets look back at our original problem. As our X values are changing, so are our Y values. We had the ordered pairs.

X	Y
4	0
5	4.5
6	9
7	13.5
.	.
.	.
.	.
14	63

Figure 32. Chart Representation of the Rate of Change Word Problem

He graphed the ordered pairs he found from solving the problem, (2, 9) and (14, 63) and labeled the y-axis, minutes and the x-axis, pills, see Figure 33. These points gave him a straight line. Therefore, when you set up a proportion then you get a straight line.

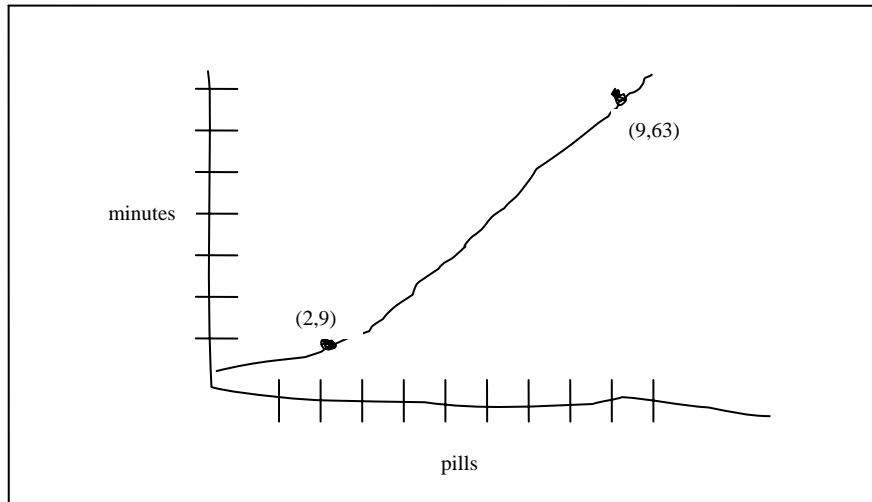


Figure 33. Graphical Representation of the Rate of Change Word Problem

From this graph, Chris wanted his students to find the slope of the line. He stated, “this leads us to the term Rate of Change (Appendix E).” He defined rate of change and applied it to the flying pills word problem, see Figure 34. The slope of the line for this problem “would be 1 pill / 4.5 minutes (Appendix E).”

This leads us to the term Rate of Change
 What comes to mind when you hear rate of change. (Lead discussion building off student’s ideas to get a definition of rate of change.
 -the rate at which one variable changes with respect to another variable
 Therefore, as we look back at our problem on the board, we have X changing with respect to Y, so we have a rate of change.

Figure 34. Chris’ Lesson Plan Defining Rate of Change

In this example Chris has shown that the two pills were the cause of flying. After taking the pills, flying for nine minutes were the effect. He then set up his proportions and graphed the points. The x-axis, the independent variable, was labeled pills and the y-axis, the dependent variable, was labeled minutes. This satisfies the United States standard way of graphing independent versus dependent variables.

The problem with Chris' example arises after he defines rate of change. He states, "we have x changing with respect to y," see Figure 21. This effects the second half of his lesson plan of anticipated problem and guided practice, see Appendix E. In the anticipated problem section of his lesson plan he gives a word problem, sets up the proportion and graphs the solution, see Figure 35.

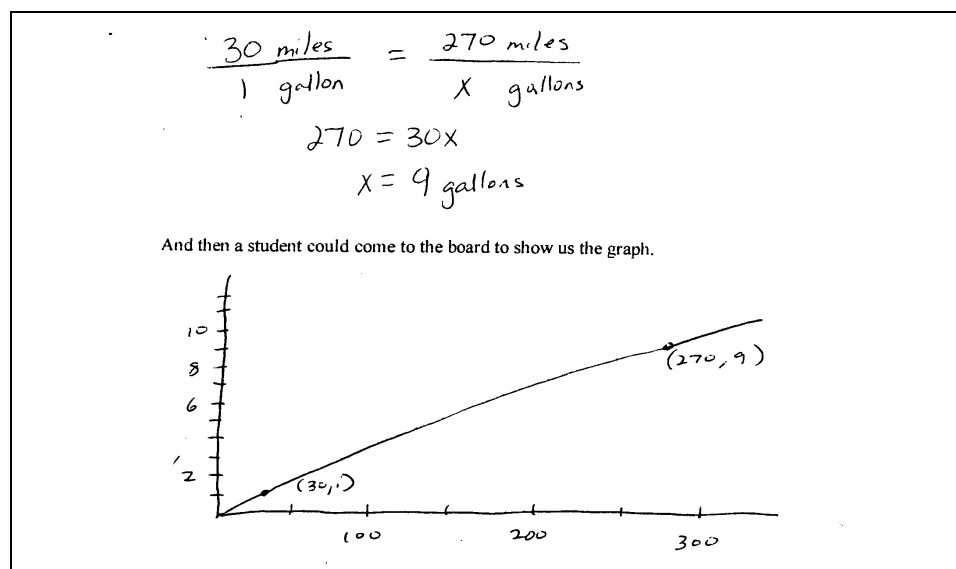


Figure 35. Second Word Problem in Lesson Plan

And in the guided practice part of his lesson plan he gives another word problem and he sets up a proportion and graphs the results, see Figure 36.

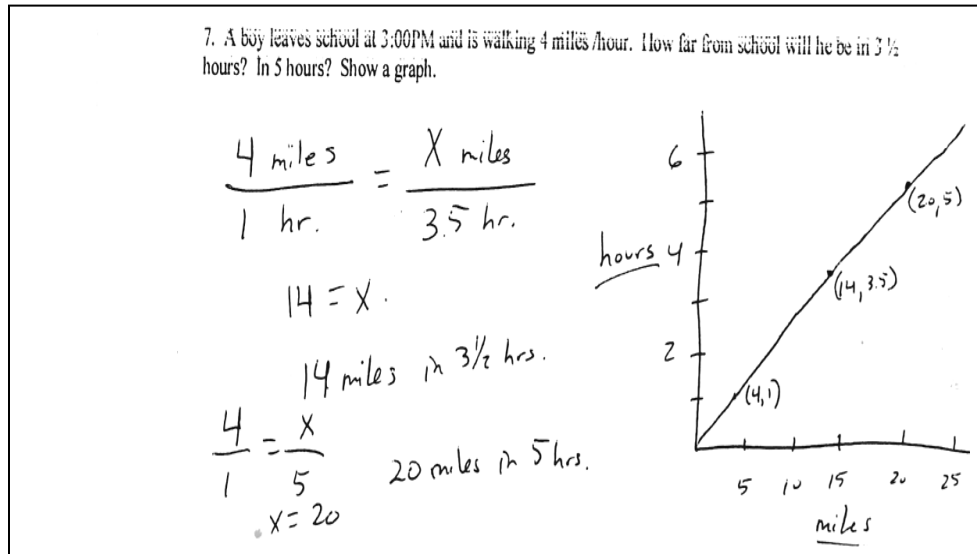


Figure 36. Third Word Problem in Lesson Plan

Chris solved these problems algebraically first. He knew from the beginning of his lesson plan that proportionality was equivalent to a straight line. As a result, it does not matter what set of units would be the independent variable or the dependent variable. Therefore, he placed his results, an ordered pair of coordinates on the graph. It would be problematic to solve the problem in Figure I or H if you were just to look at the graph. The boy's speed in the third word problem can not be determined by the slope of the line given in Figure 23.

Chris realized in his first interview of the lesson plan study that he had difficulties remembering the correct way to write the slope formula. It was difficult for Chris to recall exactly how his algebra teacher taught slope when he was in the 8th grade. In a preliminary interview in his first week of YEAR1 METHODS COURSE, Chris recalled a representation that his teacher used in teaching slope, that of an airplane taking off, see Figure 37.

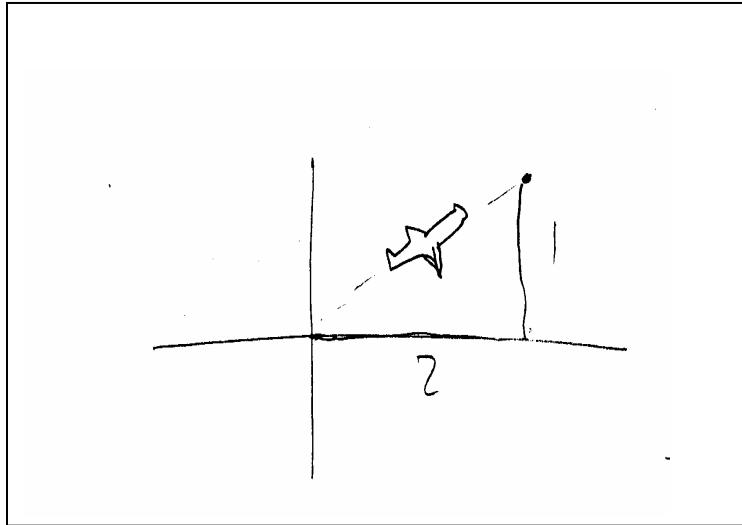


Figure 37. Chris' Airplane Example

I: Do you remember your teacher ever using visual representations to help you?

CHRIS: Yes, I remember at the very first I had trouble with rise over run. She drew a picture of an airplane. Just seeing that airplane helped me a lot better than a graph would.

I: Can you make a sketch of the airplane, I've never seen anything like that before.

CHRIS: She did the sketch so you could relate it to the graph. She drew a plane and then said that if he was going here he has got to take one way, and then take off. Like if this is his pathway, he has to start off going here before he takes off. For some reason that helped me see it.

I: Once the plane takes off it is going in a straight line?

CHRIS: Yes, it goes down the runway and then it goes . . .

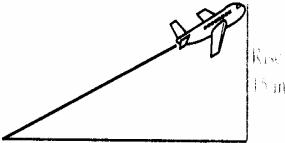
The airplane example can also be found in a high school Algebra book (Brown, Dolciani, Sorgenfrey, & Coe, 2000), see Figure 38. The authors use the airplane to describe slope. The airplane example that Chris used can be found in a typical Algebra I textbook.

Linear Equations

8-3 Slope of a Line

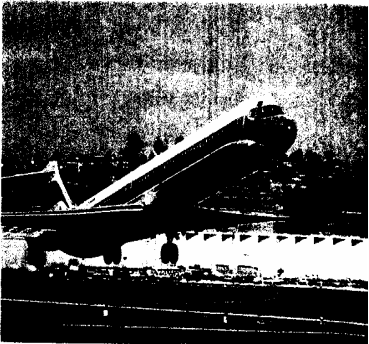
Objective. To find the slope of a line.

You can describe the steepness, or *slope*, of an airplane's flight path shortly after takeoff by giving the ratio of its vertical *rise* to its horizontal *run*, as shown below.



Run: 100 m

$$\frac{\text{rise}}{\text{run}} = \frac{15}{100} = \frac{3}{20}$$



To describe the slope of a straight line, first choose any two points on the line. Then count the units in the rise and the units in the run from one point to the other. The ratio of the rise to the run is the slope of the line.

Figure 38. Textbook Illustration of Slope

After analyzing Chris' airplane example, one conclusion would be that he saw the graph as representing the physical path of motion to describe rate of change. The position-time graph, see Figure 39, shows that the slope is both constant (meaning a

constant velocity) and positive (meaning a positive velocity). The velocity-time graph shows a horizontal line with zero slope (meaning that there is zero acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph shows a horizontal line at the zero mark (meaning zero acceleration).

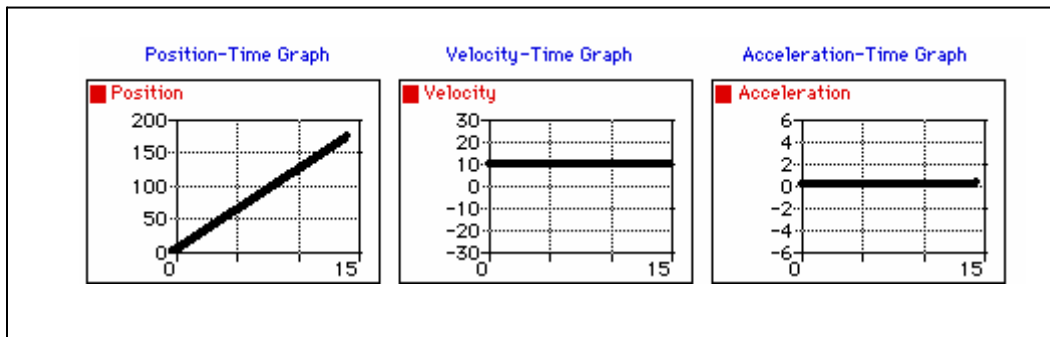


Figure 39. Researcher's Graphs of the Physical Path of an Airplane Taking Off

The physical path of the airplane that was drawn on the paper by Chris did not explicitly refer to the airplane's acceleration when taking off from the runway. Therefore, the position-time graph, see Figure 40, would show that the slope is changing (meaning a changing velocity) and positive (meaning a positive velocity). The velocity-time graph would show a line with a positive (upward) slope (meaning that there is a positive acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph would show a horizontal line in the positive region of the graph (meaning a positive acceleration).

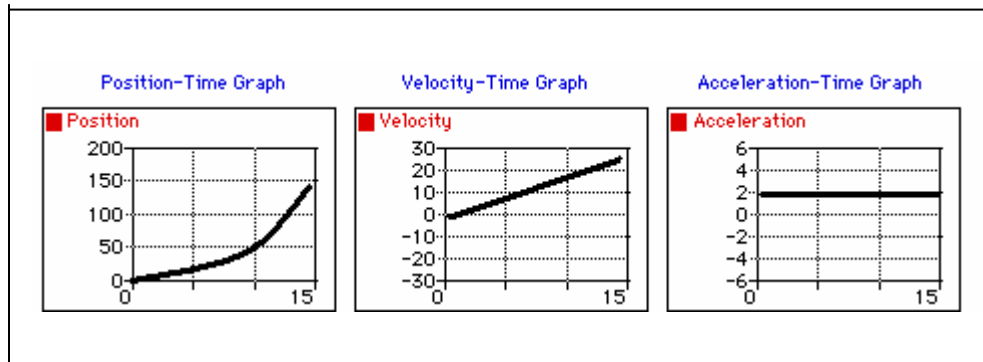


Figure 40. Researcher's Graphs of an Airplane Taking Off

Later in the interview Chris explained how he used the airplane analogy to remember how to derive slope if he was trying to solve a word problem.

I: OK, so the word problem might have been thirty green to ten blue, but when you actually represent it on paper it was thirty colon ten, or three colon one without any mention of what those numbers represent?

CHRIS: Right, and sometimes reading the question I wasn't sure which one to put. I would figure out the ratio but I wasn't sure which way to write it in this form. So, yes, that would have been helpful if we had learned to write thirty green to ten blue. I think that would have helped me visualize it.

I: You also mentioned the problem initially with slope, but the airplane diagram helped.

CHRIS: Definitely I'm trying to think why that helped so much. I think I was having trouble knowing which way he goes as far as what's you're A and what's your B.

I: How to figure out the slope, whether it was the change in which distance over the change in which distance.

CHRIS: Yes, like if you had two over one, which way you wrote that.

I: Rise over run versus run over rise.

CHRIS: Rise over run was actually the term she was using with the airplane.

Chris used the physical path of the airplane where the airplane rises over the run way at an airport to remember which variable or number goes in the numerator of a fraction and which variable or number goes in the denominator of a fraction.

As a sophomore, Chris was unclear concerning ideas related to independent and dependent variables. There was little indication that he associated covariation with a linear function. Two years later, this lack of content knowledge was associated with difficulties while teaching slope to his Enhanced Algebra class. Chris' students were initially confused as to how to use "standard form", $Ax+By=C$, and "point-slope", $y-y_1=m(x-x_1)$, formulas to find the numerical value of slope.

Chris was having trouble engaging his students in the mathematics he was teaching and he sought assistance from his university supervisor. The university supervisor suggested that he fulfill his technology requirement and engage his students by teaching a calculator based laboratory activity. The use of a calculator based laboratory allows students to collect real-time physical data such as temperature, light, sound or

motion. Then these data can be transferred into a graphing calculator to be studied through a variety of representations such as graphs or tables (charts).

Chris chose a bouncing ball calculator based laboratory activity that he created, see Appendix G. The first objective of this activity included collecting distance data as a ball bounces up and down. Second, analyze the distance versus time graph generated by the calculator. And third, determine the equation of the line of best fit for the distance versus time graph.

The concept of the bouncing ball calculator based activity is when a ball bounces up and down on a flat surface, the maximum height it reaches decreases from bounce to bounce, see Figure 41.

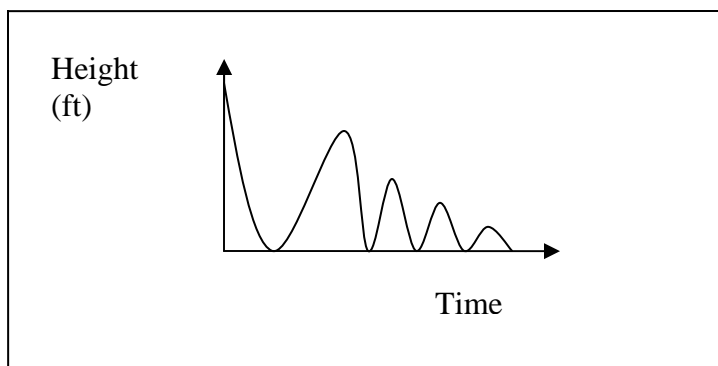


Figure 41. Calculator's View Screen of a Ball Bounce Graph

There was a common pattern as the ball decreases with different types of balls. All the balls were bounced at the same initial height. The graph produced would look like the graph in Figure A except with slight variations due to air resistance, inflation of the ball and ball size. The mathematical relationship is between the maximum height attained by the ball on a given bounce and number of bounces that have occurred. The equation of

this mathematical relationship is of the form $y=hp^x$ where the y represents the rebound height after the initial release of the ball, x is the number of bounces, h is the release height of the ball, and p is a constant that depends on the physical characteristics of the ball used. The equation, $y=hp^x$, can be considered as a linear equation by dividing h and taking the natural logarithm of both sides which gives:

$$y = hp^x$$

$$\frac{y}{h} = p^x$$

$$\ln \frac{y}{h} = \ln p^x$$

Simplifying using logarithm rules and algebra gives:

$$\ln \frac{y}{h} = x \ln p$$

$$\ln y * \ln h = x \ln p$$

$$\ln y = (\ln p)x + \ln h$$

A graph of $\ln y$ versus x is linear with a slope of $\ln p$ and a y -intercept of $\ln h$. The data collected during the calculator based laboratory activity can make a linear graph by plotting the ordered pair of the dependent variable which was the maximum height at each bounce and the independent variable which was the time it took the ball to return to the air after the bounce, see Figure 42.

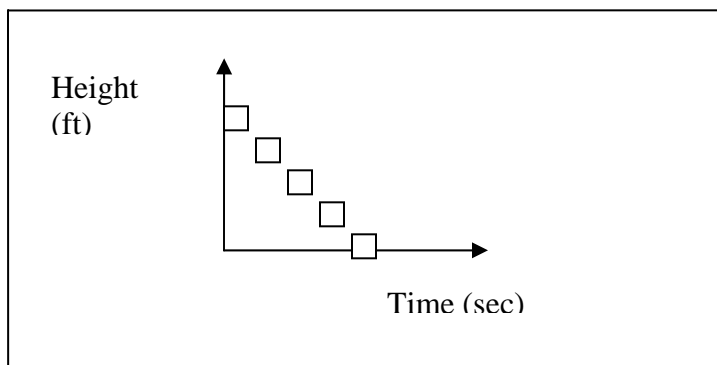


Figure 42. Researcher's Ball Bounce Linear Graph

The materials needed for the activity included one calculator based laboratory unit, one TI-83 graphing calculator, one calculator based laboratory motion detector or ranger, one yardstick and a ball. The calculator based laboratory, TI-83 graphing calculator and the motion detector were all connected to collect the data. The graphing calculator used a program that converted the data collected with the motion detector to a distance versus time graph to the viewing screen of the calculator. The graph was pre-labeled height (ft) on the y-axis and time (sec) on the x-axis on the calculator, see Figure 42. The calculator computed the linear equation with a built-in feature that allows it to compute the line of best fit from the data set.

Chris was observed during the bouncing ball calculator based activity. He put the students in groups of three or four. Each group received a different ball. The different trials had a different height for the ball. Every group used the same drop height for each trial. The students collected their materials for the activity at the beginning of class with a worksheet to record their data. Chris had the students involved in collecting data that entailed translating a non-linear relationship into a linear relationship. But Chris did not

focus on the logarithm and algebra, the characteristics of the graph and the general ideas of the slope of the linear equation involved in the activity.

CHRIS: We're going to collect some data . . . when we're looking at the line of best fit we're trying to predict what y is going to be . . . can be explained by the change in y . . . we're going to use that data plot it for now we'll do that to the best of your ability . . .

From this dialogue I noted Chris' vagueness about the graphing activity and particularly as to the relationship between height and time variables. He did not deal with covariation of the $\ln y$ with respect to x in the activity. The slope of the graph that entailed how the relationship of the $\ln y$ versus x was unchanged as the ball's height decreases as time elapsed was not addressed. Unfortunately, he thought the graph generated by this activity was a scatter plot and the calculator was just fitting a line through the points on the graph. In his interview during his student teaching he stated, "we're doing graphing [with the calculator based laboratory activity] and they're going to be looking at scatter plots." Consequently, he focused on implementing the activity and giving directions on how to use the tools by following instructions.

CHRIS: (in front of classroom with CT) All right, let's walk through this real fast...So plug your calculator into your CBL on . . .

S: where?

CHRIS: on the side

CHRIS: Alright, (overhead calculator and CBL on in front of class) go to program, go down to Ranger and hit enter . . Is everybody there?

S: yeah (one student)

- CHRIS: Go down to Application #3 . . . Everybody there
- S: yeah (one student)
- CHRIS: (Demonstrates the CBL and Ball activity) We're only measuring the 1st bounce . . . Plug it back in and hit enter on the calculator . . . What does that mean (refer to graph on overhead) hit trace . . . I want to know how much it bounced up on the first bounce.
- S: How does . . . if the CBL is a foot in a half away?
- CHRIS: It needs to be a certain amount of distance to measure the ball . . . Once you've done with that . . . Just hit enter again. (re-do trail) . . . Remember CBL needs to be a foot and a half away from the ball. Just make sure it's above it.

Students were working in groups, bouncing their balls with repeated trials at different heights during the class. The worksheet the students followed also showed Chris' ambiguity about the relationship of the dependent variable and independent variable, see Appendix E. He thought the independent variable was the height of the ball but it was the time the ball took to rebound after the ball hit the ground. And he referred to the dependent variable as the bounce of the ball which could be interpreted as the height of the ball after each bounce or after the ball hits the ground, see Figure 43.

In this experiment, X is the independent variable (the height the ball is dropped from), and Y is the dependent variable (the bounce of the ball). Begin by practicing measuring the ball bounce before you begin the collection of data.

Figure 43 . X is the height of the ball and Y is the bounce of the ball

After about fifteen minutes, Chris interrupts the class. On the worksheet Chris handed out was a procedure of things to do for the activity, see Figure 44.

<p>PROCEDURE</p> <ol style="list-style-type: none">1. Practice ball drop and measurement from first height.2. Drop ball three times at each height (12", 24", 36", 48", 60") then record the consistent measurement at each height.3. Using your data, plot the ordered pairs on the graph paper (be certain to properly label your graph).4. Draw the best fit line.5. Write the equation for the line.6. Use the equation to predict how high the ball would bounce if dropped from 72".7. Next predict how another ball used in class will compare to the linear equation you found (are their lines steeper, flatter, or the same?).
--

Figure 44. Procedure for Bouncing Ball Experiment

He was focused on whether the students collected and recorded their data properly.

CHRIS: All right you should have measurements . . .

All right you should be about finishing up on 5 feet [height of the drop]. After that you want to use your graph paper and draw in your line of best fit so when you're graphing what's going to be on the x-axis?

Chris wanted the students to record their data on the worksheet and their graph on graph paper. The students could have used their data to create their graph or they could have copied the graph from the graphing calculator screen. Students were busy wrapping up their activity and did not address Chris' question.

Later in the class Chris mentioned the x as the independent variable but not in conjunction with the dependent variable, y .

- CHRIS: yes, x is your independent variable . . .
- Which one is your independent variable, which one is determining
 . . .
- S: (responds)
- CHRIS: OK, what axis is going to be the inches . . . that it bounces
 . . . All right you should be graphing your 5 points. Everybody
 should be graphing . . .
- All right, once you get your points graphed, draw in your line of
 best fit, best estimate . . .
- Once you get your line of best fit, find another group and compare
 (said 2 times)

Once the data were collected and recorded, Chris had each group of students compare their graphs with other groups in the class. He also asked questions on the worksheet he handed out, see Figure 45.

Equation of your line:
How high do you predict the ball would bounce if dropped from 72”? (Show how you got your answer)
How did your linear equation compare with another groups? Compare the slopes of each equation and give a possible explanation of why the equations are different?

Figure 45. Questions from the Bouncing Ball Activity

At this point, the graph would have been the correct representation to answer questions from the worksheet pertaining to the graph. In spite of this, Chris had his students use the equation generated by the calculator instead of using the graph on the graphing calculator screen to answer the questions.

The class ended before any generalizations about the different bouncing balls and graphical disparity could be established. Chris was only able to meet two out of three objectives, collecting the data as a ball bounces up and down. The emphasis of his lesson was to follow the procedures he had written for the calculator based laboratory. Second, he stressed obtaining graphs from the graphing calculator and having the graphing calculator produce the line of best fit through the data. Third, Chris had the students transfer their graphs to paper.

The slope of the line of best fit was not mentioned in Chris' calculator based laboratory activity which was a major part of one of his objectives. Analyzing the height versus time graph generated by the calculator would have addressed the concept of slope in the real world problem he chose for this activity. Since the height of the ball was a constant in each group the p in the equation, $y=hp^x$, decreased because the rebound heights decreased. When a straight line was drawn through the data points, the slope of the line was close to the accepted value of the acceleration due to gravity of -9.8m/s^2 .

Chris used the calculator based laboratory activity because he believed mathematics is learned best if the teacher uses real life examples. In his first interview in the lesson plan study he stated, "I like real life examples because I think it helps them [students] visualize what we're [teachers] talking about." After his classroom observation from one of his university supervisor, he stated that the bouncing ball activity

showed examples to help make the concept of linear equations and slope more comprehensible to his students. Chris also thought the calculator based laboratory activity was used in context that was meaningful to his students. However Chris admitted in an interview prior to him doing this activity in class that he did not spend much time planning, “we’re doing a lab tomorrow and so I haven’t gotten into any heavy planning.” His action of not planning affected the potential this calculator based laboratory could have done therefore Chris got lost in using the tools of the activity. He never returned to help students focus on slope as the idea for the comparison of the graphs. He never interpreted the bouncing ball graphs for the students nor explained how the line of best fit was derived.

Summary of the Changes in Pedagogical Content Knowledge

Chris collected different representations and activities during the Year1 methods course. Using books, memories of his past teachers, and ideas of his fellow students Chris packed up a number of representations and activities for teaching slope . “A goal of teacher preparation is to pack more understanding of how to teach into the [pre-service] teachers’ primitive knowledge” (Berenson, Cavey, Clark, & Staley, 2002, p.7). Figure 46 illustrates a pre-service teacher collecting representations and activities from YEAR1 METHODS COURSE and then packs away the collection for teaching.

confirming evidence that pedagogical content knowledge is strongly tied to an individual's knowledge of the content.

DISCUSSION AND CONCLUSION

The purpose of this research was to investigate a pre-service teacher's pedagogical content knowledge of rate of change over time. This chapter is organized into two major parts. First, a brief summary of the study is followed by conclusions based on qualitative results reported in chapter four in relation to the research questions. Second, a discussion of recommendations for mathematics teacher preparation programs and future research is proposed.

For this study, a framework derived from the definitions proposed by Shulman (1986) and Ma (1999) was developed to help analyze changes in a pre-service teachers' understanding of rate of change and his pedagogical content knowledge of rate of change. The participant of the study was a college student enrolled as a secondary mathematics education major at a four-year university in the south-eastern part of the United States. The qualitative data sources used in this investigation of the pre-service teacher were individual interviews, individual lesson plans, reflections, observations and other written artifacts.

The following is a summary of what was found in the analysis of the data. The data were analyzed using the conceptual framework and the three themes were looked at for consistency in determining the aspects of rate of change and pedagogical content knowledge of rate of change.

Summary of the Study

Evidence was sought to assess the following two interrelated research questions:

Research Question 1: Pre-service Teacher Content Knowledge

What themes related to content knowledge emerged in a pre-service teacher's understanding when planning and teaching rate of change to an Algebra I class?

Researchers have discussed the disjointed content knowledge of pre-service elementary teachers (Ball, 1988; Ball, 1990; Baturu & Nason, 1996; Ma, 1999; Simon & Blume, 1994; Van Dooren, Verschaffel., & Onghena, 2002). These studies substantiate identifying pre-service secondary teachers who do not have a knowledge base for interrelated ideas of content knowledge (Ball, 1998). The subject struggled with some of the basic mathematical ideas of rate of change. His knowledge of rate of change was limited to the terms of rise over run, slope, and associated rate of change with time. The participant started out with two problems demonstrating rate of change, graphically and making connections with other mathematical concepts. The airplane analogy that he used to remember rate of change was connected to rise over run and slope.

Even (1993) and Ma (1999) suggested that teachers' ability to recognize the strength in a concept such as rate of change was reflective of their basic mathematical content knowledge. In each theme discussed in chapter four the subject showed some weaknesses in his content knowledge. His explanation that all fractions were ratios was evidence that he did not have a clear understanding between the distinction of ratios and fractions.

Graphs were a major form of representation the participant used throughout the data that were analyzed. Hashweh (1985) stated that there were qualitative differences in the

representations generated by teachers who knew their subject matter knowledge versus teachers who were weaker in their subject matter knowledge. His examples of graphs for each theme gave some insight on his subject matter knowledge of rate of change. The lesson plan the subject created for his first methods course labeled graphs incorrectly. For example, the sodas versus dollars graph where dollars were on the dependent axes and sodas were on the independent axes that gave a ratio of dollars per soda but the question referred to the number of sodas per dollar amount. Another example was the miles versus hours graph where the miles were on the independent axes and the hours were on the dependent axes that gave a ratio of hours per miles but the question referred to miles per hour. These graphs followed an algebraic solution to the problem therefore the graphs were not used as a way to solve the problem. The graphs represented more of a pictorial representation rather than a source of problem solving.

Graphs were also a form of representation in the participant's calculator based laboratory activity. The bouncing ball calculator based laboratory had the ability to meet his goals of keeping the students interested and create a conceptual representation of rate of change. The graphs produced by the TI-83 graphing calculator were labeled correctly and had the ability of answering the questions that were posed in this activity.

Alternatively the subject chose to use the equation also generated by the TI-83 graphing calculator to answer the questions. Past studies have suggested that students continued to rely on algebraic representations even when another form of representation existed which would have made the problem easier to solve (Knuth, 2000; Thompson, 1994).

According to Reed and Jazo's (2002) research, students (the pre-service teacher is the student) found it difficult to make inferences from graphs because of their lack of

understanding the problem. The subject stated in his interview during his student teaching that he did not spend a lot of time planning for this bouncing ball calculator based laboratory activity hence he did not have the mathematical knowledge that was involved in this activity. He focused on implementing the activity and giving directions on how to use the tools. Embse (1998) stated that student's lack of understanding of the problem was a serious obstruction to finding any type of solution strategy, even with technology.

The participant overemphasized procedural knowledge without concern for the limited conceptual student understanding. He could successfully select and use algorithms without understanding the underlying mathematical concepts of rate of change. His reliance on mathematical procedures and rules tend to make mathematics efficient and easy but these procedures also tend to hide the pedagogy needed for conceptual mathematical reasoning which is detrimental to the students.

Three themes were evident after analyzing the data for essential features of the pre-service teacher's pedagogical content knowledge of rate of change. The themes were ratio versus fraction, discrete versus continuous variables and dependent versus independent variables. The first theme, ratio versus fraction, dealt with the pre-service teacher's ideas of ratios and fractions. The subject was able to manipulate the ratios and fractions to solve the problems procedurally. However, as the pre-service teacher struggled with differentiating between a ratio and a fraction, it effected his graphical representations. The graphs were labeled with discrete and continuous variables with a line drawn through the points on the graph which led to the second theme, ideas of discrete versus continuous variables. For example, the subject had difficulty explaining

how to obtain fractional parts of a discrete variable. The use of discrete variables versus continuous variables focused attention of the variables that were used on the independent and dependent axes. The independent variable was sometimes exchanged for the dependent variable on the graph. For example, instead of the relationship y to x expressed as a ratio for a graphical representation, the subject chose to represent the relationship x to y as a ratio. This led me to identify the independent versus dependent theme.

Research Question 2: Pre-service Teacher Pedagogical Content Knowledge

What changes in pre-service teachers' pedagogical content knowledge, both subtle and overt, are exhibited in the way rate of change is treated over time?

Students, in general, are able to meet the expectations of satisfactory work without improving their conceptual understanding of the subject matter which hinders pre-service teachers to teach their student meaningful mathematics (Ball & McDiarmid, 1990). The change in the subject's pedagogical content knowledge of rate of change was apparent. Changes in his instruction were influenced by changes in his understanding of content knowledge and his comfort level with that content.

The lesson plan study data that was analyzed showed a change in the subject's pedagogical content knowledge. The participant was explaining his lesson plan on rate of change where he used sodas and dollars in a word problem. As the interviewer probed how the pre-service teacher's problem would work if you only had enough money to buy a half of a soda, the subject switched his illustrations from sodas to pizza. He saw the

pizza as an image that his students can divide into fractional parts depending on the number of dollars that was given in a problem.

One of the pre-service teacher's changes in his pedagogical content knowledge occurred in his lesson plan during his first methods course. In the flying pills word problem he introduced to his students in his lesson plan, the subject solved the problem algebraically and then went to a graphical representation. The algebraic procedures were correct when solving the problem and the graph was labeled correctly for his graphical representation. The problem with the graph arose when trying to solve the problem with using the graph without the algebraic procedure. However, he went from the graph to a chart which was an appropriate representation for solving the flying pills word problem without the algebra.

During the subject's student teaching, he was motivated to connect the mathematics he was teaching to his students' lives during his student teaching. He sought the assistance of his university supervisor to help him make a connection of student's everyday life to mathematics. He knew he was having trouble keeping his students' interest in the mathematics. Ma (1999) also stated that a pre-service teacher's inability to devise meaningful activities reflected his/her limited substantive and syntactic structures. However, with the suggestion of the university supervisor, Chris adapted a calculator based laboratory activity to help keep the students' interest and tried to relate the activity to the concept of slope.

Chris' willingness to take guidance from experienced instructors along with teaching experience will help him develop a better grasp of pedagogical content knowledge. As Struyk (1991) asserts, "Many teachers are willing to make necessary changes in their

teaching provided they have access to data indicating where they should begin. Through self-evaluation, teachers can observe and analyze their teaching and then make decisions based on the analysis” (p. 18).

The pre-service teacher experienced changes in his pedagogical content knowledge whether he was cognizant or incognizant of the changes. The concept of rate of change did not happen in one step. Vinner and Dreyfus (1989) stated that for an individual to understand a concept, it will take many experiences and understandings to create a stronger image to complete the realization of a concept such as rate of change. The subject matter knowledge of rate of change became more familiar with the participant over time and through discourse with university personnel. The familiarity with rate of change helped the subject with his pedagogical content knowledge.

Recommendations

Implications for Mathematics Teacher Preparation

There is still much to learn about each component of pedagogical content knowledge depicted in Figure 12. Shulman’s concept of pedagogical content knowledge and Ma’s concept of profound understanding of fundamental mathematics are unique contributions to research on teaching and teacher education. The two sets of recommendations for practice presented here would benefit from further research to identify significant results. I propose two conjectures as I use my pedagogical content knowledge model to build on Shulman’s and Ma’s research.

1. Addressing the relationship between content and pedagogical content knowledge.

2. Using a model of components of pedagogical content knowledge to guide learning to teach experiences.

Conjecture 1: Content and Pedagogical Content Knowledge

Pre-service high school mathematics teachers are required to take far higher levels of university mathematics than they will teach in the schools. They are also expected to use their knowledge to explain real world problems and conceptual mathematics. However, they may not be adequately prepared for the task. Many pre-service teachers may not have had opportunities to develop investigations to answer questions or construct explanations from classroom discussions on mathematical ideas at the conceptual level. Such experiences can help pre-service teachers develop the content needed for developing pedagogical content knowledge. Researchers have suggested that universities must give pre-service teachers the opportunity to strengthen their subject matter knowledge and pedagogical content knowledge (Jones & Agard, 1992; Ma, 1999; NCTM, 1991).

Teacher education programs can feature pairing mathematics content courses with mathematics methods courses focused on teaching the same content. Then the methods course can also provide a learning environment for pre-service teachers to experience the instruction on a conceptual level and in meaningful and supportive contexts that they are being prepared to teach. Content-specific and school-based experiences will give pre-service teachers more opportunities to focus on content and instructional strategies at the conceptual level.

Pre-service teachers benefit from learning environments that promote better understanding of subject matter knowledge and help develop a repertoire of instructional strategies and representations in teaching for student understanding. While the pre-service teachers are in their first methods course, it is not surprising that they have a limited repertoire of instructional representations for mathematical concepts (Ball, 1990; Borko et al., 1992; Eisenhart et al., 1993). The limitations in pre-service teachers' representations imply that their students might not get a profound understanding of the mathematics these pre-service teachers have to teach.

Consideration must be given to teacher education on the concepts of Ratio vs Fraction, Discrete vs Continuous variables and Independent vs Dependent variables. Ball (1998) stated that institutions of higher learning should teach the subject matter knowledge that pre-service teachers are expected to teach in the classroom. Ratios, fraction, discrete, continuous, independent and dependent were concepts that affected this pre-service teacher's answers to mathematical problems that was posed to him during interviews or that he posed to his students during student teaching.

The conversations (discourse) that took place during the lesson plan study interview helped the pre-service teacher construct his ideas of rate of change on a conceptual level. The subject realized that he probably should not have used the soda versus dollar problem to explain fractional parts of a soda but through discourse with the interviewer he changed the variable of soda to pizza. He saw pizza as a continuous variable where he could explain fractional parts of a pizza. The interviews provided an opportunity to make connections from procedural knowledge to conceptual knowledge. A discourse of

pedagogical content knowledge in the participant's methods course was developed after the interviews.

Conjecture 2: Guidance Provided by a Model of Components of Pedagogical Content Knowledge

Pedagogical content knowledge is acknowledged by a number of researchers as providing an integrated vision of teacher knowledge and classroom practice (Ball, 1998; Even, 1993; Lampert, 1986; Shulman, 1986). Providing guidance to pre-service teachers using a model of components of pedagogical content knowledge also suggested. The figure can serve as a map for specifying desired knowledge outcomes of a teacher education program. From this study, there was a change in pedagogical content knowledge as a result of teaching. But pre-service teachers will only acquire a portion of the pedagogical content knowledge needed to be effective given that pedagogical content knowledge is transformed after student teaching. One must develop pedagogical content knowledge for each topic that is taught. Therefore, pedagogical content knowledge must be the focus of work with in-service teachers.

Conclusion

The components of the pedagogical content knowledge framework are a way for teacher educators and teacher candidates to talk about what to teach that goes beyond the usual list of topics for different grade levels. Within each component, teachers have specific knowledge that they need to develop with respect to all of the aspects of pedagogical content knowledge. By assigning each component to a single construct, pedagogical content knowledge, the components functions as parts of a whole. As a

result, lack of coherence between components can be problematic in developing and using pedagogical content knowledge and increased knowledge of a single component may not be sufficient to effect change in practice. Stofflett and Stoddard (1994) advise that “if teachers are to use conceptual teaching methods in their own classrooms, they need to learn content and pedagogy through the same conceptually based methods” (p.18).

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APPENDIX

Appendix A

Interview Protocol for Ratio and Proportion Lesson-Planning Activity with YEAR1 METHODS COURSE Students (Individual Task)

Name(s) of Interviewer(s):

Name of Participant:

Date:

I. Pre-Task Interview

1. Explain the purpose of the study
 - a) To understand how preservice teachers would teach ratio and proportion
 - b) To understand how preservice teachers learn about ratio and proportion concepts and how they learn about teaching ratio and proportion concepts. (Let them know that I will ask them about this at the end of the interview.) I'd like you to think about what you're learning and how you learn.
2. Explain the process for this part of the study
 - a) Pre-task interview
 - b) Lesson-planning activity
 - c) Post-task interview
3. What do you remember about learning ratio and proportion in school?
4. What do you remember about what your teacher showed you on paper, on the board, or on the overhead about ratio and proportion? Ask the student to sketch this on paper.

5. What problems, if any, did you have learning ratio and proportion in school? Ask the student to sketch this on paper.
6. Make a list of other topics in school mathematics that depend on an understanding of ratio and proportion. You can take this list with you and add to it as you think about your lesson.
7. What does “rate of change” mean to you? Show me on this piece of paper.
8. Could you plan a lesson to introduce the concept of “rate of change” to a pre-algebra or algebra I class? Do you have any questions you want to ask me?
9. Use any of these materials on this table for your lesson. You may want to make notes about the lesson so that you can explain your lesson to me.

II. Lesson Planning Activity

Plan a lesson that introduces the concept of rate of change to a pre-algebra or algebra I class of 8th or 9th graders. In the lesson, connect the concept of rate of change to ratio and proportion.

Use any of the materials that are on the table for your lesson. You may outline your plan on paper to refer to later in the interview.

III. Post-Task Interview

1. Explain your lesson plan. (Or, So what did you come up with?) Probe for explanations, rich in detail, concerning the representations they would use and their justification for the lesson plan.

2. How would you describe ratio to a middle grades student?
3. How would you describe the relationship between ratio and fraction?
 - a) How are they similar?
 - b) How are they different?
4. What helps you learn about ratio-related concepts?
5. What helps you learn about teaching ratio-related concepts?

Appendix B

Lesson Plan Score Sheet

I. Focus and Review:

- A. Included a rationale ___
- B. Listed prerequisite skills ___

II. Statement of Objective(s) (in terms the students will understand) ___**III. Teacher Input**

- A. Introduction of the lesson ___
- B. Lesson development, including problems to be posed and range of possible solutions/answers and/or specific examples to be used and possible solutions with work shown and explanations given) ___
- C. Logical sequence to problems/examples, progressing from easier to more difficult ___
- D. Included variety in the problems/examples ___
- E. Problems/examples “match” the objectives of the lesson ___
- F. Includes things you will point out to the students ___
- G. Noted areas where you think students may have difficulty and told how you plan to address these areas ___
- H. Included the questions you plan to ask, as well as a range of possible answers ___
- I. Used good questioning techniques ___

IV. Guided Practice

- A. Stated problems clearly, including directions ___

- B. Contains solutions to all problems, with work shown ___
 - C. “Matches” the objectives of the lesson ___
 - D. Included variety of types of problems ___
 - E. Described how the entire class would receive feedback ___
- V. Independent Practice**
- A. Stated problems clearly, including directions ___
 - B. Contains solutions to all problems, with work shown ___
 - C. “Matches” the objectives of the lesson ___
 - D. Goes beyond “drill & practice” ___
 - E. Included variety of types of problems ___
- VI. Closure:** Included the students in summarizing major concepts of the lesson ___
- * Students are actively involved in the lesson ___

Appendix C – Lesson Plan

Focus and Review(4 min.)

- Graphing a line given two points
- finding x and y intercepts
- finding ratios and proportions

Statement and Objectives(1 min.)

After today's lesson, the students should be able to:

- find the slope of a line, given two points
- given two ratios, convert them to ordered pairs and find the slope of the line formed
- translate ratio word problems into graph form and find the slope of the line
- be able to work basic rate of change problems

Teacher Input(30 min.)

We will begin with an opening problem on the board and as the students walk in they will be given a few minutes to work at their desk. This will get them focused on math and lead into the lesson. Here we would discuss briefly the many uses in the real world that rate of change has including stock car racing, baseball, rowing, to breathing. A brief discussion of how these relate would give good basis for application to real world experiences. The opening problem will be appear like this:

You just found a pill that allows you to fly!! You have found that if you take two pills, you can fly for exactly 9 minutes. You want to fly to your best friend's house to show him your amazing discovery. He lives 63 minutes away. How many pills must you take to complete the journey?
(As the class is working, I would walk around and see if students were working the problem in different ways. Then, I would call on two students who did the problem differently, to come to the board and work it.

$$\textcircled{1} \quad \frac{2}{9} = \frac{x}{63}$$

$$9x = 126$$

$$\boxed{x = 14}$$

$$\textcircled{2} \quad \frac{9}{2} = \frac{63}{x}$$

$$126 = 9x$$

$$\boxed{x = 14}$$

Using this example we could review ratios and show we have two equal ratios.

Next I would ask if we could write these in a different form(ordered pairs). If I do not get this, then I will offer this to the class. We will show, in this case we would have (2,9) and (14, 63). We could then graph these two to start to draw the relation between the word problem and the graphic representation(Re-emphasizing graphing given two points). The graph would look like this:

We want to find the slope of this line. What is one way we can do this?

(Find two points and use slope formula.)

So we have the points (3,1) and (9,3).

Again we assign variables (again reinforcing that it is irrelevant which) and plug into slope equation.

$$\begin{array}{cc} (3, 1) & (9, 3) \\ \uparrow \uparrow & \uparrow \uparrow \\ x_2, y_2 & x_1, y_1 \end{array} \quad m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 1}{9 - 3} = \frac{2}{6} = \frac{1}{3}$$

We get out slope $m = 1/3$

Anticipated Problem- (the students will want to know which ordered pair should be (X_2, Y_2) and which should be (X_1, Y_1)). Therefore, I will assign them into groups of four at this point. As we go through some guided practice, we will have half the class working it with one X_2, Y_2 and the other working with the other. By both groups getting the same answer, this will reinforce that either will do. Another anticipated problem is the students getting the inverse by putting the X values on the top and the Y values on the bottom. Therefore, by cautioning them here, I will try to avoid them making that mistake.

Lets look back at our original problem. As our X values are changing, so are our Y values. We had the ordered pairs,

X	Y
0	0
1	4.5
2	9
3	13.5
⋮	⋮
149	63

So we can see that the X values and the Y values are directly proportional. —

This leads us to the term Rate of Change.

What comes to mind when you hear rate of change. (Lead discussion building off student's ideas to get a definition of rate of change.)

-the rate at which one variable changes with respect to another variable.

Therefore, as we look back at our problem on the board, we have X changing with respect to Y, so we have a rate of change. In our case it would be (pill / 4.5 minutes).

Anticipated Problem- (the students could have trouble understanding the / sign as per. I would have to make sure that was clear for those who had not seen that notation before.)

Lets take a look at another example.

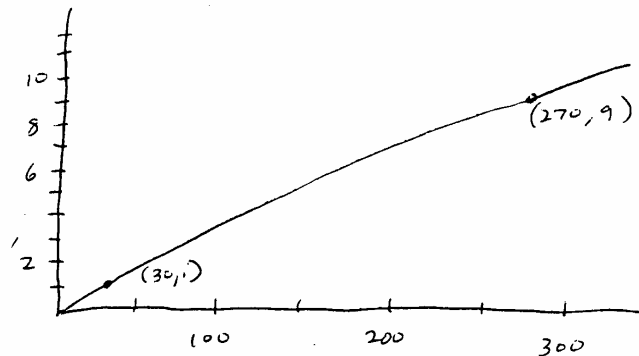
An economy car gets 30-miles/ gallon. How many gallons will it take to go 270 miles? We can make this look just like our ratio problems and we can show it graphically. How would we set up our ratio?

$$\frac{30 \text{ miles}}{1 \text{ gallon}} = \frac{270 \text{ miles}}{X \text{ gallons}}$$

$$270 = 30X$$

$$X = 9 \text{ gallons}$$

And then a student could come to the board to show us the graph.



Guided Practice (10 min.)

In our groups, we will work on some practice problems.

Find the slope of the line that passes through these points.

1. (2,4) (5,1)

$$x_2, y_2 \quad x_1, y_1$$

$$\frac{4-1}{2-5} = \frac{3}{-3} = -1$$

$$m = -1$$

2. (8, 23) (14, 32)

$$x_1, y_1 \quad x_2, y_2$$

$$\frac{32-23}{14-8} = \frac{9}{6}$$

$$m = \frac{3}{2}$$

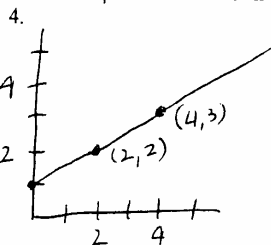
3. (-12, 17) (-4, -18)

$$x_2, y_2 \quad x_1, y_1$$

$$\frac{17-(-18)}{-12-(-4)} = \frac{35}{-8}$$

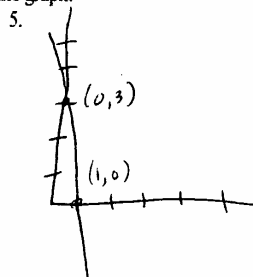
$$m = -\frac{35}{8}$$

Find the slope of the line shown on the graph.



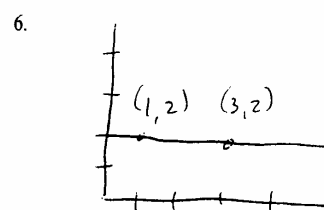
$$\frac{3-2}{4-2} = \frac{1}{2}$$

$$m = \frac{1}{2}$$



$$\frac{3-0}{0-1} = \frac{3}{-1}$$

$$m = -3$$



$$\frac{2-2}{3-1} = \frac{0}{2} = 0$$

$$m = 0$$

7. A boy leaves school at 3:00PM and is walking 4 miles/hour. How far from school will he be in $3\frac{1}{2}$ hours? In 5 hours? Show a graph.

$$\frac{4 \text{ miles}}{1 \text{ hr.}} = \frac{X \text{ miles}}{3.5 \text{ hr.}}$$

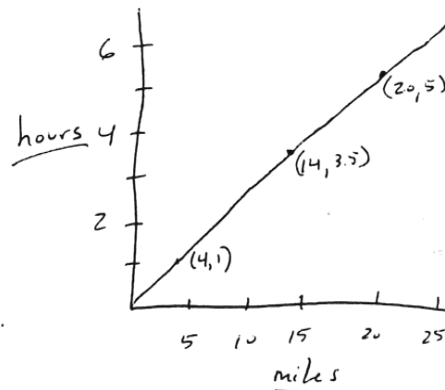
$$14 = X.$$

14 miles in $3\frac{1}{2}$ hrs.

$$\frac{4}{1} = \frac{X}{5}$$

20 miles in 5 hrs.

$$X = 20$$



Independent Practice (p. 245 4-9 and #1, 2, and 3)

4. (1,1) (3,3)

5. (0,0) (2,4)

6. (-1,2) (3,-6)

$$\frac{3-1}{3-1} = \frac{2}{2} = 1$$

$$m = 1$$

$$\frac{4-0}{2-0} = \frac{4}{2}$$

$$m = 2$$

$$\frac{-6-2}{3-(-1)} = \frac{-8}{4}$$

$$m = -2$$

7. (5,0) (0,10)

8. (-4,-1) (4,7)

9. (-2,0) (-6,-1)

$$\frac{10-0}{0-5} = \frac{10}{-5}$$

$$m = -2$$

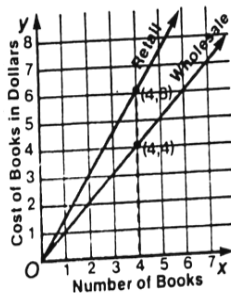
$$\frac{7-(-1)}{4-(-4)} = \frac{8}{8} = 1$$

$$m = 1$$

$$\frac{-1-0}{-6-(-2)} = \frac{-1}{-4}$$

$$m = 1/4$$

1. Find the Slopes of these two lines



Retail - Pts (4,6) (2,3)

$$\frac{6-3}{4-2} = \frac{3}{2} \quad m = 3/2$$

Wholesale Pts. (4,4) (2,2)

$$\frac{4-2}{4-2} = \frac{2}{2} = 1 \quad m = 1$$

Find the slope of the line

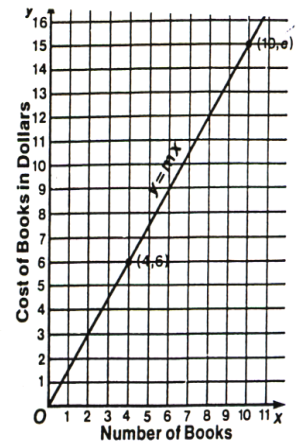
pts. $(4, 6)$ & $(2, 3)$

x_1, y_1 x_2, y_2

2.

$$\frac{3-6}{2-4} = \frac{-3}{-2} = \frac{3}{2}$$

$$m = \frac{3}{2}$$



3. A plane takes off and flies 3000 miles. How fast is the plane flying if it takes 12 hours to complete the trip?

$$\frac{3000 \text{ miles}}{12 \text{ hours}} = \frac{x \text{ miles}}{1 \text{ hr}}$$

$$12x = 3000$$

$$x = 250 \text{ miles / 1 hr.}$$

Closure (2 min.)

Now let's say our principal walked in right now and asked us what we learned today. What would we tell her/him. (Leading the discussion I would draw out the main points and objectives).

Appendix D

Interview Protocol for Student Teaching

1. What were your original goals for this lesson?
2. How did you plan on accomplishing these goals?
3. Do you think you accomplished your goals?
4. How did your plan compare with what you actually did?
5. Explain some of the decisions that you made that accounted for the differences.
6. Describe the mathematical thinking the tasks required students to engage in.
7. Give examples of how you communicated with students in a way that was nonjudgmental and encouraged the participation of each student.
8. Did you require students to give full explanations and justifications or demonstrations orally and/or in writing? Give examples.
9. Did you pose questions that were of different levels and types and use appropriate wait time to elicit, engage, and challenge student thinking?

Appendix E

North Carolina State University Mathematics Student Teaching Observation Rubric

Student Teacher:

Date Observed:

Observer:

Class Period Observed:

Lesson Content:

Competencies	Rating	Comments
<p>Mathematical Content Knowledge</p> <ul style="list-style-type: none"> • Knowledge of mathematical content is accurate • Uses mathematical language and notation correctly • Demonstrates understanding of connections between mathematical concepts, topics and real world applications 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	
<p>Professional Attributes</p> <ul style="list-style-type: none"> • Exemplary attendance and always on time • Takes initiative, needs minimal supervision • Sensitive to others' feelings and opinions • Always maintains professional appearance 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	
<p>Classroom Learning Environment</p> <ul style="list-style-type: none"> • Demonstrates warmth and caring towards all students • Demonstrates an awareness and respect for diversity (e.g., ethnic, gender, religion, age, exceptionality, sexual orientation, social class) • Treats all students in an equitable manner • Communicates enthusiasm for mathematics 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	
<p>Management of Student Behavior</p> <ul style="list-style-type: none"> • Establishes and consistently implements rules and procedures for student behavior • Demonstrates awareness of student 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	

<p>activities</p> <ul style="list-style-type: none"> • Frequently monitors behavior of all students • Stops inappropriate behavior promptly and consistently, maintaining dignity of student 		
<p>Management of Instructional Time</p> <ul style="list-style-type: none"> • Maintains high level of student time on task • Has materials, supplies, and equipment ready at beginning of lesson • Addresses interruptions with minimal interference with instruction • Develops lessons to maximize available classroom time 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	
<p>Use of Materials for Teaching and Learning</p> <ul style="list-style-type: none"> • Chooses and effectively uses appropriate tools and materials to enhance student learning • Tools and materials are aligned with objectives and used in mathematically meaningful ways • [During semester]Uses a variety of tools and materials, including manipulatives, technology, etc. 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	
<p>Use of Instructional Strategies</p> <ul style="list-style-type: none"> • Facilitates classroom discourse that focuses on important mathematical ideas • Chooses and effectively uses instructional strategies that promote the development of students' conceptual understanding of mathematics • Chooses and effectively uses instructional strategies that promote proficiency with mathematical skills • Uses multiple representations to convey mathematical ideas (e.g., tables, graphs, symbols, etc.) • [During semester]Uses a variety of 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	

instructional strategies, including cooperative learning, discovery, etc.		
<p>Use of Questioning Techniques</p> <ul style="list-style-type: none"> • Uses questioning techniques that promote the development of students' conceptual understanding of mathematics (e.g., higher level questions, focus on concept development, etc.) • Uses a variety of questions in written and oral formats that promote mathematical thinking • Asks questions of all students and allows appropriate wait time responses • Poses questions clearly and succinctly 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	
<p>Monitoring Student Performance and Providing Instructional Feedback</p> <ul style="list-style-type: none"> • Monitors students' understanding and uses that information to make instructional decisions during a lesson • Continually uses student work products to assess progress • Provides meaningful and timely feedback to students • [During semester] Uses a variety of formative and summative assessment techniques (e.g., journals, portfolios, conversations with individual students, projects, tests, quizzes, etc.) that are aligned with instructional objectives 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	
<p>Teaching of Mathematics is Aligned with State Guidelines and National Perspectives</p> <ul style="list-style-type: none"> • Instructional objectives supportive of NCDPI Standard Course of Study • Teaching is consistent with vision portrayed in NCTM Principles and Standards 	<ul style="list-style-type: none"> <input type="checkbox"/> Above Average (4) <input type="checkbox"/> Acceptable (3) <input type="checkbox"/> Needs Improvement (2) <input type="checkbox"/> Not Acceptable (1) 	

Appendix F

Bouncing Ball Experiment Scatterplots and Best-Fit Lines

In this experiment you will predict the height of a ball drop based upon the linear function of ball drop trials and the corresponding bounce. In addition, each group will have a different type of ball, thus generating different sets of data. Once you have found your linear function, you will discuss and compare it with the linear function of a group who used a different type of ball.

In this experiment, X is the independent variable (the height the ball is dropped from), and Y is the dependent variable (the bounce of the ball). Begin by practicing measuring the ball bounce before you begin the collection of data. Once you have practiced, use three drops from each height and take the most consistent measurement and record the bounce height.

PROCEDURE

1. Practice ball drop and measurement from first height.
2. Drop ball three times at each height (12", 24", 36", 48", 60") then record the consistent measurement at each height.
3. Using your data, plot the ordered pairs on the graph paper (be certain to properly label your graph).
4. Draw the best-fit line.
5. Write the equation for the line.
6. Use the equation to predict how high the ball would bounce if dropped from 72".
7. Next predict how another ball used in class will compare to the linear equation you found (Are their lines steeper, flatter, or the same?). Get with another group and compare your linear equation. When comparing, discuss the slope of each other's linear equation.

X	Y	Equation of your line:
12"		
24"		How high do you predict the ball would bounce if dropped from 72"?(Show how you got your answer)
36"		
48"		
60"		
		How did your linear equation compare with another groups? Compare the slopes of each equation and give a possible explanation of why the equations are different.