ABSTRACT

CHOI, YOUNG-JIN. International Transmissions of Dynamic Portfolios and Monetary Policy. (Under the direction of Doug Pearce and Nora Traum.)

This dissertation focuses on the analysis of portfolio decisions and exchange rate regimes. Firstly, I show how the degree of flexibility of the nominal exchange rate has an influence on households' asset holdings. As the nominal exchange rate gets less volatile, households move toward holding more riskless bond assets and foreign equities because they are less to currency risk. Secondly, I compare the effect of productivity shocks, monetary policy shocks and investment shocks on optimal portfolios in a two-country, sticky-price DSGE open economy macro model with two different price setting systems - producer-currency pricing (PCP) and local-currency pricing (LCP) - under managed and flexible exchange rate regimes. The results confirm some expected findings that both the PCP and LCP models have similar responses to a foreign monetary policy shock under the managed exchange rate regime. Alternatively, under the flexible exchange rate regime, perfect exchange rate pass-through leads the PCP model to have different real effects from the LCP case. In the symmetric case, where both counties have the same price setting and country size, capital inflows and outflows move together in the same direction under the flexible exchange rate regime in which the equilibrium equity is home biased for each country and real returns on bonds and equities are adjusted by nominal exchange rate fluctuations. Under the managed exchange rate regime, capital inflows and outflows move in the opposite direction caused by the diversification of the equilibrium home and foreign equities for each country and the same rate of increase of real returns on home and foreign equities by low fluctuations of the nominal exchange rate. In the asymmetric case in which the large country exports goods under PCP and the small country exports goods under LCP, responses of real variables to shocks under the flexible exchange rate regime are similar to that in the symmetric case but the effect on dynamic asset and liability holdings is different due to the dollarization in the world market, which affects home equity holdings by the foreign country. Finally, I compare welfare under the flexible and managed exchange rate regimes. When the country experiences a positive productivity shock, the flexible exchange rate regime generates higher welfare than the managed exchange rate regime. On the other hand, the monetary policy shock and the investment shock provide different welfare effects. The welfare effects depend on the benefit of the diversification of financial assets and the cost of the lower fluctuation of the exchange rate.

International Transmissions of Dynamic Portfolios and Monetary Policy

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DEDICATION

To my parents for their endless love and unconditional support.

BIOGRAPHY

Young-Jin Choi was born in Incheon, South Korea. She graduated from Incheon Women's High School. She received the Bachelor of Science majoring in Physics and minoring in Economics from SookMyung Women's University in Seoul, South Korea. When she was a senior, South Korea had an incipient financial crisis that drew people's attention to economic issues. She decided to study economics further in the Master's program at SookMyung Women's University. After working in Seoul for a few years, she came to the United States and started her Ph.D. program in Economics at North Carolina State University. Her fields are international financial and monetary macroeconomics.

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1.1 Introduction

Despite the outstanding growth of the international financial market and the diversification in financial assets in the last decades, investors still prefer a large share of home equities, which is called the "equity home bias puzzle". Table 1.1 shows that countries tend to have large portions of their financial assets in domestic equities. Figure 1.1 shows that emerging markets have higher home bias than developed economies.

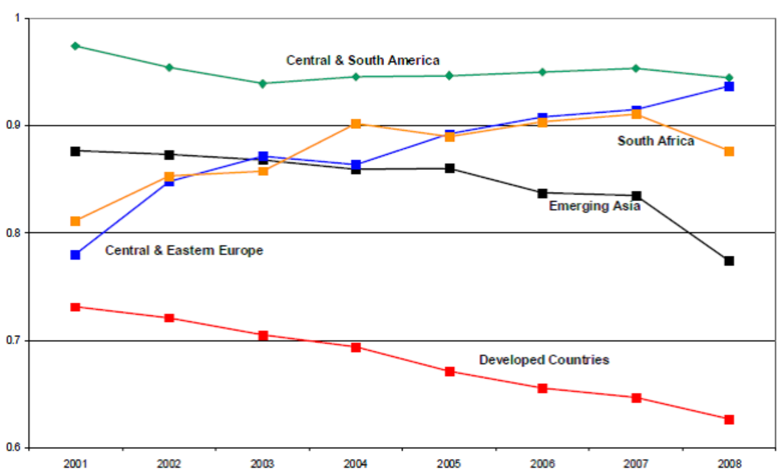


Figure 1.1: Home Bias in Equities Measures across Emerging Countries
Source: Coeurdacier and Rey(2012)

Table 1.1: Home bias in Equities in 2008

Country	Domestic market in % of world market capitalization	Share of portfolio in domestic equity in %	Degree of equity home bias
France	4.3	79.8	0.75
China	7.8	99.2	0.99
Brazil	1.6	98.6	0.99
Canada	2.7	80.2	0.80
Euro Area	13.5	56.7	0.50
South Korea	1.4	88.5	0.88
Japan	8.9	73.5	0.71
Switzerland	2.3	50.9	0.50
UK	8.1	77.0	0.69
US	32.6	77.2	0.66

Source: Coeurdacier and Rey (2012)

In the early papers, attempts to explain the home bias puzzle had not been successful. Baxter and Jermann (1997) find that the positive correlation between labor incomes and profits reduces the home bias in equity and nominal bond holdings in the flexible price case. On the other hand, Engel and Matsumoto(2005) investigate the equity home bias with the international traded equities and forward contracts in a complete market model with nominal price rigidity and money supply shocks. They find that the human capital return and the equity returns are negatively correlated and in turn, the home bias in equity can be explained by hedging the non-tradable labor income risks. Rahbari(2009) extends Engel and Matsumoto(2005) adding endogenous capital accumulation, sticky wage setting, country specific investment shocks and endogenous monetary policy. He finds that the correlation between the returns on human capital and on equity is conditionally negative on bond returns and unconditionally positive. The differences from Engel and Matsumoto (2005) result from the endogenous capital accumulation. Bui(2009) compares home bias in the complete market and the incomplete market but provides no analytical explanations. He shows that the sensitivity to the relative risk aversion, the elasticity of substitution, and the share of imports is similar between the complete market and the incomplete market cases while the sensitivity to the nominal rigidity and the labor share is different between the cases.

During the past decade, a number of papers have developed different methods to solve the technical difficulty for the indeterminacy problem of the equilibrium portfolio. All financial assets are perfect substitutes with the same rate of return. In the deterministic steady state, the optimal portfolio is not uniquely pinned down with no risk introduced. In the stochastic case,

different sources of risk differentiate between financial assets and the steady state portfolio is determined by the functions of the variance-covariances of the shocks. However, all financial assets have the same expected rate of return up to the first order approximation and the assets are still perfect substitutes. The higher order approximation solves this problem. Devereux and Sutherland(2010) develop the general method with this logic. Heathcote and Perri(2013) develop the numerical iteration method with the logic that households pay less portfolio adjustment cost as they are closer to the steady state portfolio. There are several contributions that analyze the equity home bias using these methods. None of the papers, however, analysis of the relation between home bias in equity and the exchange rate regime. The home bias in equity is very sensitive to the degree of the fluctuations of the nominal exchange rate. The degree of equity home bias gradually changes with changes in the other key parameters, the degree of nominal rigidity, the persistence of the productivity shock or the degree of the capital share, etc. Households pay the cost to reduce the fluctuation of the nominal exchange rate and obtain the benefit generated by the diversification of the portfolio. The question of whether the managed exchange rate regime is the optimal monetary policy is left for future work.
This chapter is organized as follows. Section 2 describes the model in detail. The benchmark calibration is described in section 3. Section 4 states the solution methods. The sensitivity of the equilibrium portfolios are shown in section 5. Section 6 concludes the chapter.

1.2 Model

The world economy consists of two countries of different sizes with a continuum of infinitely-lived households. Households are denoted by the unit interval $j \in [0, 1]$ and reside in one of two countries, Home (H) and Foreign (F). The interval $j \in [0, n]$ denotes home country households and the interval $j \in [n, 1]$ denotes foreign country households. When the home country is defined as the small country, the parameter n that measures the relative size of the country is close to zero and the small country's policy decisions have negligible impacts on the large country. Both countries have monopolistically competitive firms producing differentiated intermediate goods and perfectly competitive firms producing final goods. The firms are represented by the interval $i \in [0, 1]$. The home and foreign firms are indexed by the interval $i \in [0, n]$ and $i \in [n, 1]$, respectively. The intermediate goods firms set nominal prices in a staggered fashion, as in Rotemberg (1982). The price setting problems for the export market have different assumptions for each country. For the asymmetric case, small country exporters set prices using the importing market's currency under Local-Currency Pricing (LCP). The large foreign country exporters set their prices in their own currency under Producer-Currency Pricing (PCP).

1.2.1 Consumers

The representative household j maximizes the lifetime utility function

$$U\left(C,H\right)=\frac{C_t^{1-\gamma}}{1-\gamma}-\frac{H_t^{1+\omega}}{1+\omega} \tag{1.1}$$

where C_t^j is a composite consumption index and H_t^j denotes hours of labor. The index C_t^j is defined by the CES function

$$C_t^j=\left[(1-\eta)^{\frac{1}{\mu}}\left(C_{H,t}^j\right)^{\frac{\mu-1}{\mu}}+\eta^{\frac{1}{\mu}}\left(C_{F,t}^j\right)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{1.2}$$

where parameter $\eta\in[0,1]$ inversely represents a degree of home bias in consumption and the parameter $\mu>0$ measures the elasticity of substitution between domestic and foreign goods. $C_{H,t}^j$ is defined by the CES composite consumption index of domestic goods

$$C_{H,t}^j=\left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}}\left(\int_0^n C_{H,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right)\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.3}$$

where $i\in[0,n]$ denotes the variety of the domestic goods. $C_{F,t}^j$ is defined by the CES composite consumption index of goods imported and consumed by domestic households.

$$C_{F,t}^j=\left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}}\left(\int_n^1 C_{F,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}}di\right)\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{1.4}$$

where $\epsilon>1$ measures the elasticity of substitution between varieties produced in the same country. The optimal $C_{H,t}^j(i)$ and $C_{F,t}^j(i)$ of any given levels of $C_{H,t}^j$ and $C_{F,t}^j$ are respectively

$$C_{H,t}^j(i)=\left(\frac{1}{n}\right)\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}C_{H,t}^j \tag{1.5}$$

$$C_{F,t}^j(i)=\left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon}C_{F,t}^j \tag{1.6}$$

where $P_{H,t}$ is the domestic currency price index of domestically produced goods and $P_{F,t}$ is the import price index from the foreign country. The corresponding price index is

$$P_t = \left[(1-\eta)\, P_{H,t}^{1-\mu} + \eta P_{F,t}^{1-\mu} \right]^{\frac{1}{1-\mu}} \tag{1.7}$$

With

$$P_{H,t} = \left[\frac{1}{n} \left(\int_0^n P_{H,t}^j(i)^{1-\varepsilon} di \right) \right]^{\frac{1}{1-\varepsilon}} \tag{1.8}$$

$$P_{F,t} = \left[\frac{1}{1-n} \left(\int_n^1 P_{F,t}^j(i)^{1-\varepsilon} di \right) \right]^{\frac{1}{1-\varepsilon}} \tag{1.9}$$

where P_t is the Consumer Price Index (CPI) in the model. It follows that $\int_0^n P_{H,t}(i)\, C_{H,t}^j(i)\, di = P_{H,t} C_{H,t}^j$ and $\int_n^1 P_{F,t}(i)\, C_{F,t}^j(i)\, di = P_{F,t} C_{F,t}^j$. The household j's optimal allocation between home and foreign goods is represented by

$$C_{H,t}^j = (1-\eta) \left(\frac{P_{H,t}(i)}{P_t} \right)^{-\mu} C_t^j \tag{1.10}$$

$$C_{F,t}^j = \eta \left(\frac{P_{F,t}(i)}{P_t} \right)^{-\mu} C_t^j \tag{1.11}$$

The aggregate demand functions for the home and foreign good i are represented by

$$C_{H,t}(i) = \frac{1}{n} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \tag{1.12}$$

$$C_{F,t}(i) = \left(\frac{1}{1-n} \right) \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \tag{1.13}$$

Finally, the optimal allocation of expenditures between domestic and foreign goods is given by

$$C_{H,t} = (1-\eta) \left(\frac{P_{H,t}(i)}{P_t} \right)^{-\mu} C_t \tag{1.14}$$

$$C_{F,t} = \eta \left(\frac{P_{F,t}(i)}{P_t} \right)^{-\mu} C_t \tag{1.15}$$

Therefore, the home country's total consumption expenditure is $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$, where $P_{H,t}C_{H,t} = \int_0^n P_{H,t}(i)\,C_{H,t}(i)\,di$ and $P_{F,t}C_{F,t} = \int_n^1 P_{F,t}(i)\,C_{F,t}(i)\,di$. Accordingly, the household maximization of the utility function

$$\mathrm{U}(C,H) = E_t\sum_{t=0}^{\infty}\beta_t\left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{H_t^{1+\omega}}{1+\omega}\right) \tag{1.16}$$

is subject to a real budget constraint given by

$$\begin{aligned} b_{H,t} + e_tb_{F,t} + j_tx_{H,t} + e_tj_t^*x_{F,t} &= w_th_t + r_t^kk_t - c_t - \frac{p_{i,t}}{p_t}I_t \\ &+ r_{H,t}b_{H,t-1} + e_tr_{F,t}b_{F,t-1} + (j_t + d_t)x_{H,t-1} + e_t(j_t^* + d_t^*)x_{F,t-1} \end{aligned}$$

The discount factor β_t is endogenously determined by the function of the aggregate home consumption as follows

$$\beta_{t+1} = \beta_t\beta(C_t)\,,\beta_0 = 1 \tag{1.17}$$

where $0 < \beta(C_t) < 1$, $\beta'(C_t) < 0$, and C_t is the aggregate home consumption. Schmitt-Grohe and Uribe (2003) consider the five different specifications for the stationarity in incomplete asset market models. In this paper, I employ the endogenous discount factor to eliminate the unit root and solve the non-stationarity problem with the functional form of $\beta(C_t)$

$$\beta(C_t) = \tilde{\beta}C_t^{-\nu} \tag{1.18}$$

where $0 \le \nu < \gamma$ and $0 < \beta C_t^{-\nu} < 1$. Home agents hold two bonds, b_H and b_F, respectively denominated in home and foreign currency with real returns of $r_{H,t}^b$ and $r_{F,t}^b$ and two equities, x_H and x_F, respectively denominated in home and foreign currency with real returns of $r_{H,t}^e$ and $r_{F,t}^e$ The real exchange rate e_t is defined as the price of the foreign good in terms of the home good as following $e_t = S_t\frac{P_t^*}{P_t}$. S_t is the nominal exchange rate. Real exchange rate appreciation corresponds to a decrease in e_t, while depreciation corresponds to an increase in e_t. By defining net wealth W_t as the differences between gross assets and gross liabilities, i.e. $NFA_t = e_tj_t^*x_{F,t} + e_tb_{F,t} - j_tx_{H,t}^* - b_{H,t}^*$, the budget constraints can be rewritten as

$$\begin{aligned} NFA_t &= r_{H,t}^bNFA_{t-1} + w_th_t + r_t^kk_t - c_t + d_t - \frac{p_{i,t}}{p_t}I_t \\ &+ \alpha_{F,t-1}^b(r_{F,t}^b - r_{H,t}^b) - \alpha_{H,t-1}^e(r_{H,t}^e - r_{H,t}^b) + \alpha_{F,t-1}^e(r_{F,t}^e - r_{H,t}^b) \end{aligned}$$

where $r_{x,t}$ is a real excess return on bonds which is defined by

$$r_{x,t} = \left[(r^b_{F,t} - r^b_{H,t}) \quad (r^e_{H,t} - r^b_{H,t}) \quad (r^e_{F,t} - r^b_{H,t}) \right] \tag{1.19}$$

The real returns on bonds and equities are defined by

$$r^b_{H,t} = R_{H,t-1}\frac{P_{t-1}}{P_t} \qquad r^e_{H,t} = \frac{J_t + D_t}{J_{t-1}}\frac{P_t}{P_{t-1}}$$
$$r^b_{F,t} = R_{F,t-1}\frac{P^*_{t-1}}{P^*_t}\frac{e_t}{e_{t-1}} \qquad r^e_{F,t} = \frac{J^*_t + D^*_t}{J^*_{t-1}}\frac{P^*_t}{P^*_{t-1}}\frac{e_t}{e_{t-1}}$$

The real holdings of bonds and equities are defined by

$$\alpha^b_{H,t} = \frac{B_{H,t}}{R_{H,t-1}P_t} \qquad \alpha^e_{H,t} = \frac{J_t x_{H,t}}{P_t}$$
$$\alpha^b_{F,t} = e_t\frac{B_{F,t}}{R_{F,t-1}P^*_t} \qquad \alpha^e_{F,t} = e_t\frac{J^*_t x_{F,t}}{P^*_t}$$

The first order conditions for bond assets imply

$$C_t^{\nu-\gamma} = \beta E_t C_{t+1}^{-\gamma} r_{H,t+1} \tag{1.20}$$

$$\beta E_t \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \left(r_{H,t+1} - r_{F,t+1} \right) = 0 \tag{1.21}$$

1.2.2 Firms

In each country, a continuum of monopolistically competitive firms produces the differentiated intermediate goods using capital and labor. A standard Cobb-Douglas production function is

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} \tag{1.22}$$

where A_t is a technology shock that follows the AR(1) process

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t \tag{1.23}$$

The law of capital evolution is

$$k_{t+1} = (1-\delta)\,k_t + I_t - \phi\left(I_t, I_{t-1}\right)_t \tag{1.24}$$

where δ is the depreciation rate of capital, I_t is investment and $\phi(I_t, I_{t-1})$ is a measure of investment adjustment costs. I assume a specific functional form for the investment adjustment costs.

$$\phi\left(I_t, I_{t-1}\right) = \frac{\kappa}{2} I_t \left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{1.25}$$

where κ is the investment adjustment cost parameter. I assume Rotemberg style sluggish price adjustment so that a firm sets the price with a quadratic adjustment cost in the inflation rate of the goods. Each firm faces two separate pricing problems, one for the domestic market and one for the export market and the export market has two different pricing scenarios. The optimal price-setting strategy for the domestic market can be approximated by the typical New Keynesian Phillips curve for domestic inflation. The home firm maximizes the present value of expected profits by resetting price $\overline{p}_{H,t}$ for the domestic market. When the exporting goods price is determined by the PCP, the home firms set their domestic goods prices for the domestic market. The home firm's nominal profit is

$$\begin{aligned} D_t\left(i\right) &= P_{H,t}\left(i\right) Y_t\left(i\right) - w_t h_t\left(i\right) - R_t^k k_t\left(i\right) \\ &- \frac{\varphi_H}{2}\left[\frac{P_{H,t}\left(i\right)}{\bar{\pi}_H P_{H,t-1}\left(i\right)} - 1\right]^2 P_{H,t} Y_t \end{aligned} \tag{1.26}$$

The firms real profit maximization can be written as

$$\begin{aligned} \max_{H_t, K_t, P_{H,t}, P^*_{H,t}} E_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ &\frac{P_{H,t}\left(i\right)}{P_t}\left(\frac{P_{H,t}\left(i\right)}{P_{H,t}}\right)^{-\varepsilon} Y_t - \frac{w_t h_t\left(i\right) - R_t^k k_t\left(i\right)}{P_t} \\ &- \frac{\varphi_H}{2}\left[\frac{P_{H,t}\left(i\right)}{\bar{\pi}_H P_{H,t-1}\left(i\right)} - 1\right]^2 \frac{P_{H,t} Y_t}{P_t} \end{aligned}$$

Subject to $Y_t\left(i\right) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t$, firm is output equals its total demands. The foreign firms also set their foreign goods prices for the foreign market. The expected profits of the foreign firm in the foreign market can be written as

$$\begin{aligned} \max_{H^*_t, K^*_t, P^*_{F,t}} E_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ &\frac{P^*_{F,t}\left(i\right)}{P^*_t}\left(\frac{P^*_{F,t}\left(i\right)}{P^*_{F,t}}\right)^{-\varepsilon} Y^*_t - \frac{w^*_t h^*_t\left(i\right) - R_t^{k^*} k^*_t\left(i\right)}{P^*_t} \\ &- \frac{\varphi^*_F}{2}\left[\frac{P^*_{F,t}\left(i\right)}{\bar{\pi}^*_F P^*_{F,t-1}\left(i\right)} - 1\right]^2 \frac{P^*_{F,t} Y^*_t}{P^*_t} \} \end{aligned}$$

subject to the production function,$y^*_t\left(i\right) = \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}}\right))^{-\varepsilon} Y^*_t$.

Under PCP, the export price is determined by the domestic goods price and the exchange rate under the law of one price (LOP).
The first order conditions for this problem are given by

$$\lambda_t w_t = (1-\alpha)\,\zeta_t A_t k_t^{\alpha} h_t^{-\alpha} \tag{1.27}$$

$$\lambda_t r_t^k = \alpha \zeta_t A_t k_t^{\alpha-1} h_t^{1-\alpha} \tag{1.28}$$

$$\zeta_t = \frac{1}{A_t} w_t^{1-\alpha} \left(r_t^k\right)^{\alpha} \left[\alpha^{-1}(1-\alpha)^{-(1-\alpha)}\right] \lambda_t \tag{1.29}$$

where $\varepsilon/(\varepsilon-1)$ is the constant gross mark-up over marginal cost. With the assumed technology, the firms real marginal cost is

$$mc_t = \frac{1}{A_t} w_t^{1-\alpha} \left(r_t^k\right)^{\alpha} \left[\alpha^{-1}(1-\alpha)^{-(1-\alpha)}\right] \tag{1.30}$$

The optimal rule for setting prices of home firm can be written as

$$\begin{aligned} \varphi_H \left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right] \frac{\pi_{H,t}}{\bar{\pi}_H} &= (1-\varepsilon) + \varepsilon\Big[(1-\eta) + \eta\tau_t^{1-\mu}\Big]^{\frac{1}{1-\mu}} mc_t \\ &+ \tilde{\beta} c_t^{-\nu} \varphi_H E_t \left[\frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right] \frac{\pi_{H,t+1}^2}{\bar{\pi}_H} \frac{y_{t+1}}{y_t} \frac{1}{\pi_{t+1}}\right]^2 \end{aligned}$$

The optimal rule for setting prices of foreign firm can be written as

$$\begin{aligned} \varphi_F \left[\frac{\pi_{F,t}^*}{\bar{\pi}_F^*} - 1\right] \frac{\pi_{F,t}^*}{\bar{\pi}_F^*} &= (1-\varepsilon) + \varepsilon\Big[(1-\eta) + \eta\tau_t^{1-\mu}*\Big]^{\frac{1}{1-\mu}} mc_t^* \\ &+ \tilde{\beta} c_t^* {-\nu} \varphi_F E_t \left[\frac{c_{t+1}^* {-\gamma}}{c_t^* {-\gamma}} \left[\frac{\pi_{F,t}^*}{\bar{\pi}_H} - 1\right] \frac{\pi_{F,t+1}^2 *}{\bar{\pi}_F^*} \frac{y_{t+1}^*}{y_t^*} \frac{1}{\pi_{t+1}^*}\right]^2 \end{aligned}$$

Home firm's exporting goods prices are determined by

$$P_{H,t}^* = \frac{P_{H,t}}{S_t} \tag{1.31}$$

Foreign firm's exporting goods prices are determined by

$$P_{F,t} = P_{F,t}^* S_t \tag{1.32}$$

The nominal export prices in a producer currency moves one-to-one with the fluctuation of nominal exchange rate, S_t.

1.2.3 Monetary Policy and the exchange rate regime

Monetary policy is assumed to follow an extended form of a Taylor-type interest rate rule in which deviations of inflation, output and the nominal exchange rate from the long run targets have a feed back on short run movements of the nominal interest rate. The log linear approximated form is

$$\widehat{R}_{H,t} = \rho_H \widehat{R}_{H,t-1} + (1-\rho_H)\,\phi_\pi \widehat{\pi}_t + (1-\rho_H)\,\phi_y \widehat{y}_t + (1-\rho_H)\,\phi_s \widehat{s}_t + \zeta_{m,t} \tag{1.33}$$

where $\zeta_{m,t}$ is monetary policy shock. The rule describes an economy in which monetary policy is constrained by a managed or fixed exchange rate regime. The parameter ρ_H allows the monetary authority to smooth changes in the interest rate. The parameter $(1-\rho_H)\,\phi_\pi$ and $(1-\rho_H)\,\phi_y$ reflect the responses to the changes in the CPI inflation rate and real GDP, respectively. The parameter $(1-\rho_H)\,\phi_s$ reflects the degree of the managed exchange rate between fixed and floating regimes. A floating exchange rate regime implies $\phi_s = 0$ while a fixed exchange rate implies $\phi_s = \infty$.

1.2.4 Market clearing conditions

Goods market clearing conditions for the home country is

$$Y_t^* = C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^* - \frac{\varphi_H}{2}\left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right]^2 Y_t \tag{1.34}$$

Goods market clearing conditions for the foreign country is

$$Y_t^* = C_{F,t}^* + C_{F,t} + I_{F,t}^* + I_{F,t} - \frac{\varphi_F^*}{2}\left[\frac{\pi_{F,t}^*}{\bar{\pi}_F^*} - 1\right]^2 Y_t^* \tag{1.35}$$

Bond market clearing conditions are

$$b_{H,t} + b_{H,t}^* = 0 \tag{1.36}$$

$$b_{F,t}^* + b_{F,t} = 0 \tag{1.37}$$

The bond assets $b_{H,t}$ and $b_{F,t}$ are issued in the home and foreign currencies, respectively. The bond holdings by foreign agents are denoted by an asterisk (*).
Equity market clearing conditions are

$$x_{H,t} + x_{H,t}^{*} = 1 \tag{1.38}$$

$$x_{F,t}^{*} + x_{F,t} = 1 \tag{1.39}$$

The equity assets $x_{H,t}$ and $x_{F,t}$ are issued in the home and foreign currencies, respectively. The equity holdings by foreign agents are denoted by an asterisk (*).

1.3 Calibrations

In this paper, for the symmetric case, I calibrate the benchmark model using empirical estimates based on US quarterly data. The endogenous discount factor, β is calibrated as 0.99 in the steady state so that the annual nominal interest rate is 4 percent. The time-varying discount factor depends on the aggregate consumption such as $\tilde{\beta}C_t^{-\nu}$. In the steady state, $\tilde{\beta}$ is calibrated assuming $\nu = 0.01$ and $\tilde{\beta}C^{-\nu} = \beta$. The Frisch labor supply elasticity $1/\omega$ is set equal to 0.33. The consumption home bias $\eta = 0.85$, implying a 15 percent import to GDP ratio and the elasticity of substitution $\mu = 0.9$ following Heathcote and Perri (2002). The capital share in the production function α is set at 0.28. The depreciation rate of capital is 0.025 which is commonly used for US quarterly data, implying a 10 percent annual rate of depreciation. The Rotemberg price adjustment parameter ϕ and the markup ε are 58 and 6, equivalent to setting the Calvo price parameter equal to 0.75 which is about 4 quarters for the average duration of the price adjustment. The capital adjustment parameter is set equal to 4.5. For the monetary policy rule, I adopt the benchmark Taylor rule with partially adjustment calibrated by Canzoneri et. al. (2007) as follows

$$\widehat{R}_{H,t} = \rho_H \widehat{R}_{H,t-1} + (1-\rho_H)\,\phi_\pi \widehat{\pi}_t + (1-\rho_H)\,\phi_y \widehat{y}_t + (1-\rho_H)\,\phi_s \widehat{s}_t + \zeta_{m,t} \tag{1.40}$$

For the foreign country's monetary policy rule, the degree of the response to the nominal exchange rate is negative due to the definition of the exchange rate.

$$\widehat{R}_{F,t} = \rho_F \widehat{R}_{F,t-1} + (1-\rho_F)\,\phi_\pi^{*} \widehat{\pi}_t^{*} + (1-\rho_F)\,\phi_y^{*} \widehat{y}_t^{*} - (1-\rho_H)\,\phi_s^{*} \widehat{s}_t + \zeta_{m,t}^{*} \tag{1.41}$$

For the asymmetric case, the size of home country changes between interval [0.2, 0.83]. In the

Table 1.2: Calibration

Parameter	Interpretation	Value
β	Discount factor	0.99
γ	Risk aversion	2
ω	1/ Elasticity of Labor supply	0.33
η	Consumption home bias	0.85
η^i	Investment home bias	0.75
μ	Subst. Elasticity for Consumption Home and Foreign	0.9
μ^i	Subst. Elasticity for Investment Home and Foreign	1.2
α	capital share in production	0.28
δ	Depreciation Rate	0.025
φ	Price Adjustment cost	58
κ	Investment Adjustment cost	4.5
ε	Subst. Elasticity for Domestic Varieties	6
ρ_r	Persistence of Interest rate	0.824
ρ_a	Persistence of Productivity shock	0.923
ρ_m	Persistence of Monetary shock	0
ρ_i	Persistence of Investment shock	0.85
ϕ_y	Response to Output gap	0.184
ϕ_π	Response to Inflation	2.02
ϕ_s	Response to Exchange rate	0
σ_A	Std of Productivity shock	0.000861
σ_M	Std of Monetary shock	0.000245
σ_I	Std of Investment shock	0.00036
n	Country Size	0.5

Table 1.3: Country Size

Country size (n)	GDP Ratio	Foreign Cons. Home bias (ν)
0.2	0.25	0.99
0.27	0.37	0.97
0.34	0.515	0.94
0.41	0.694	0.91
0.48	0.923	0.86
0.5	1	0.85
0.55	1.22	0.80
0.62	1.63	0.72
0.69	2.22	0.60
0.76	3.166	0.42
0.83	4.88	0.074

case $n < 0.5$, the home country is smaller than the foreign country. When the country size is 0.2, the GDP of the home country is 25 percent of the foreign country's while the home country's GDP is almost 5 times as much as the foreign country's when the country size is 0.83.

1.4 Solution Method

In the open macroeconomic model, the standard linear approximation is applied around the non-stochastic steady state. However, the linear approximation has unsolvable difficulties for the portfolio choices for two reasons. First, the portfolio choices have the indeterminacy problem which is not uniquely pinned down in a non-stochastic steady state. This problem can be overcome by treating the steady state value of the portfolios as endogenous. Second, the portfolio choices are indeterminate in the first order approximation because all the assets are perfect substitutes. This problem can be also solved by considering the second order approximation. I follow the methods described in Devereux and Sutherland (2008) that develop technical solutions for the steady state and dynamic portfolios in a general DSGE model with multiple assets. Their work can be applied for both complete and incomplete markets.

1.4.1 Solving the Model

I follow Devereux and Sutherland (2008) that developed numerical approximation methods suitable for solving for portfolio allocations in this type of model. The solution procedures stated in the paper emphasize several important properties to calculate the steady state and first order portfolios. First, the first-order behavior of the non-portfolio parts of the model is not influenced by the deviation of portfolio holdings around the solution of the steady state portfolio. It is important that the first order macroeconomic solution is sufficient to solve the value of the steady state portfolios. Second, the second-order approximation of the portfolio problem that appears in the Euler equation has the terms of the solution of the first-order approximation for the non-portfolio parts. This means that the steady state value of portfolio holdings is solved by the first-order approximation of the macroeconomic parts. The portfolio holdings only directly enter the non-portfolio part through the budget constraint taking $b_H r_{x,t}$ as a mean-zero i.i.d. random variable, ξ_t. This is derived from the third property that the expected excess return, $E_t[r_{x,t+1}]$, is zero in the first-order approximation of the model. The solution for the first-order portfolio problem requires the higher-order approximation. As in the case of the steady state portfolio solution, the first-order portfolio holdings only directly enter the non-portfolio part through the second-order approximation of the budget constraint defining $b_{H,t-1} r_{x,t}$ as the random variable, ξ_t. The fourth important property is that the covariance matrix of the innovations, Σ is constant. Therefore, the solutions for the second moments are non-time varying. The system of the second-order approximations of the model can be written in the state-space representation. This paper uses the solution model developed by Lombard and Sutherland (2007).

1.4.2 Steady state portfolios

In order to calculate the steady state portfolios, the first order approximation of non-portfolio parts and the second-order approximation of Euler equations of asset holdings are required. The steady state portfolios only appear in the non-portfolio parts of the model through the random variable of the budget constraint.
The net foreign asset is defined in real terms.

$$NFA_{t-1} = \tilde{\alpha}^{E}_{F,t-1} + \tilde{\alpha}^{B}_{F,t-1} - \tilde{\alpha}^{*E}_{H,t-1} - \tilde{\alpha}^{*B}_{H,t-1} \tag{1.42}$$

The budget constraint can be written by the net foreign asset NFA_t assuming a net zero steady state value and by the excess return on portfolios ξ_t.

$$NFA_t = NFA_{t-1} r^{B}_{H,t} + w_t h_t - c_t + d_t - \frac{p^I_t}{p_t} i_t + \xi_t \tag{1.43}$$

where the random variable ξ_t is defined by

$$\xi_t = -\tilde{\alpha}^{*E}_{H,t-1} \left(r^{E}_{H,t} - r^{B}_{H,t} \right) + \tilde{\alpha}^{E}_{F,t-1} \left(r^{E}_{F,t} - r^{B}_{H,t} \right) + \tilde{\alpha}^{B}_{F,t-1} \left(r^{B}_{F,t} - r^{B}_{H,t} \right) \tag{1.44}$$

Up to the first order approximation, the excess returns $r_{x,t}$ are mean-zero random variables.

$$r_{x,t} = \left[\left(r^E_{H,t} - r^B_{H,t} \right) \left(r^E_{F,t} - r^B_{H,t} \right) \left(r^B_{F,t} - r^B_{H,t} \right) \right] \tag{1.45}$$

Therefore, ξ_t is also an i.i.d. exogenous variable which I refer to as "the excess return shock". Dynare produces the following solution form in the first order approximation.

$$y_t = y^s + A y^h_{t-1} + B u_t \tag{1.46}$$

where y_t is a vector of all endogenous variables which array the order of static variables, purely backward, mixed and finally purely forward variables. y^s is the vector of the steady state values, y^h_{t-1} is the vector of state variables and u_t is the vector of exogenous variables.
In order to calculate the steady state value of the portfolios, the necessary variables c_t, c^*_t and $r_{x,t}$ can be extracted from the rows of y_t of the above Dynare solution. u_t includes the excess return shock, ξ_t, and other exogenous shocks, ε_t. Therefore, the variables c_t, c^*_t and $r_{x,t}$ depend on the shocks ξ_t and other exogenous shocks, ε_t.

$$\begin{bmatrix} c_t \\ c^*_t \\ r_{x,t} \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} \xi_t \\ \varepsilon_t \end{bmatrix}$$

By combining the optimality condition $E[(c_{t+1} - c^*_{t+1}) r_{x,t+1}] = 0$ and the excess return shock $\xi_t = \tilde{\alpha} r_{x,t}$, the steady state value of the portfolios can be obtained.

1.5 Results

I study the sensitivity of the equilibrium home bond and home equity holdings to the choices of parameters; the degree of relative risk aversion, the elasticity of substitution, the degree of price stickiness, the persistence of the technology shock, the share of capital, the degree of pegged exchange rate and country size.

The relative risk aversion
Figure 1.1 shows the relationship between the relative risk aversion and the equilibrium bond and equity. The degree of risk aversion affects households' consumption through changes in the inter-temporal elasticity of substitution. The more risk averse, the lower the inter-temporal elasticity of substitution. Suppose that home relative price is high. Households can choose either the purchase of goods at a cheaper price or the generation of more income. The dominant effect

depends on the desired consumption smoothing of households. If households are sufficiently risk averse, they want to smooth consumption by increasing their income with more equity holdings. If households are less risk averse, they substitute into purchasing of the cheaper price goods while hedging the real exchange rate risk with nominal bonds. As households are more risk averse, they tend to have less bond holdings and more domestic equity. Both bond and equity dramatically change for the risk aversion smaller than 2.

Price Adjustment Cost

I compare not only different effects of flexible price setting and sticky price setting on the equilibrium portfolios but also consider the sensitivity of the degree of price stickiness. I find that equity is more home biased and nominal bond holdings get higher as the average duration of price adjustment gets longer. Under the sticky price model, the fluctuation of the real exchange rate is closely related to the fluctuation of the nominal exchange rate which is associated with the nominal bond rate. Nominal bonds can be a good hedge for the real exchange rate risk. Under a flexible price system, the nominal exchange rate is less correlated with the real exchange rate and the nominal bond is no longer effective to hedge the real exchange rate risk. Higher degrees of price stickiness imply higher equilibrium nominal bond holdings. On the other hand, the higher nominal rigidity generates higher home bias in equity. Under sticky prices, a positive technology shock increases output and decreases labor and the wage rate. The equity return rises due to the positive productivity shock and the lower wage rate. The equity holdings is a good hedge for the labor income risk. Under flexible prices, the increase in output by the technology shock reduces the price and raises labor and the wage rate which implies a lower dividend. The equity holdings with the lower real returns is no longer efficient to hedge the income risk.

Substitution Elasticity of Home and Foreign goods

Figure 1.3 shows that easier substitution between home and foreign goods makes households have more diversified asset holdings, lower equity holdings and higher bond holdings. Output declines by the negative productivity shock and the real exchange rate appreciates with the fall in home equity returns. Foreign equities are more valuable and home bias in equity declines. The easier changes in the consumption of the home and foreign goods by changes in the relative prices raise the bond holdings which is effective as a hedge for the real exchange rate risk. For the relatively low elasticity, $\eta < 1$, both bond and equity holdings have large changes.

Persistence of Productivity shock

When there are sufficient nominal price rigidities and endogenous capital accumulation, productivity shocks have real effects on output, the wage rate and investment causing change in equity positions. In this model, the increase in investment is larger than the increase in output and decrease in the wage rate. Because of the smaller impacts on equity returns by the output and wage rate than that on investment, equity return decreases by lower payouts. If the productivity shock is more persistent, households prefer lower equity holdings and higher bond holdings.

Response to Exchange rate

Figure 1.5 shows how the degree of flexibility of the nominal exchange rate has an influence on households' asset holdings. Both home bond and home equity holdings dramatically change from the free floating exchange rate regime to the managed exchange rate regime of the parameter ϕ_s, 0.1. As the nominal exchange rate gets less volatile, households are exposed less currency risk. Households move toward holding more foreign equities. The consumption and real exchange rate correlation is pretty high (close to 0.9) like the complete market model with perfect risk sharing. The consumption-real exchange rate anomaly (Backus-Smith puzzle) is generated in my model. Riskless bond assets are preferred to the equity holdings.

Capital Share

As the capital share increases, the increase in output from the productivity shock is significant. Due to the higher productivity and lower wage rate, higher equity returns cause the home bias in equity. In the opposite way, if both countries have large labor shares, households are strongly exposed to the home labor risk. If the capital share declines in this state, households sell home equities. The larger the labor share, the smaller home bias in equity.

Country Size

Figure 1.7 compares the small open economy and large open economy portfolio positions, as the country size ranges from 0.2 to 0.83. When the home country size paramter is 0.2, the GDP of the home country is 1/4 as much as that of large country. The foreign country's openness of domestic consumption goods is 0.01 which means domestic exports consist almost entirely of investment goods. When the home country size parameter is 0.76, the GDP of the home country is three times that of foreign country. The domestic bond holding is 2.9 percent and the domestic equity holding is 92 percent. According to US data, the domestic bond holding is 3 percent of total financial assets and the domestic equity holding is 89 percent of total financial assets.

1.6 Conclusions

This paper investigates the equilibrium international portfolio choices and the home bias puzzle in a New Open Macroeconomic model with incomplete markets, nominal rigidity and capital accumulation. For the benchmark calibration, I find that domestic households hold around 93 percent of the financial wealth in the home equities and 7 percent in the foreign equities and also hold around 14 percent of the total financial wealth for the home bond. The key parameters have effects on equilibrium asset choices changing the degree of the home bias in equity. The degree of nominal rigidity, the elasticity of substitution and capital shares have crucial impacts on the home bias in equity. Equity and bond holdings are changed by changes in parameter values. Especially, the exchange rate regime shows a high degree of sensitivity of home bias in equity by affecting the volatility of the nominal exchange rate. Households pay the cost to reduce the fluctuation of the nominal exchange rate and obtain the benefit generated by the diversification of the portfolio.
Although results have no perfect match to that of the numerical iteration method, the interpretation of the equilibrium of the endogenous portfolio choices is still meaningful and can provide the component for the future work on dynamic portfolio choices.

1.7 Figures

Figures show the sensitivity of the equilibrium home bond and equity by the changes in the seven parameters. The percent of the bond and equity is plotted along the Y axis and the degree of the parameter value is plotted along the X axis. The negative value of the Y axis of the bond/GDP means borrowing. 1 of Y axis of the equity means 100 percent of the home equity holding.

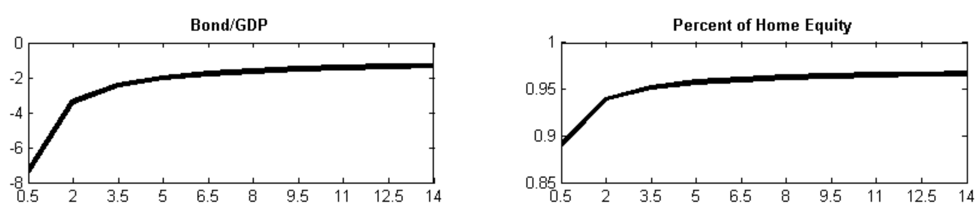


Figure 1.2: Relative Risk Aversion

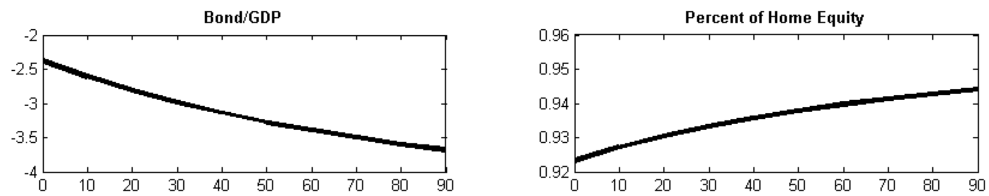


Figure 1.3: Price Adjustment cost

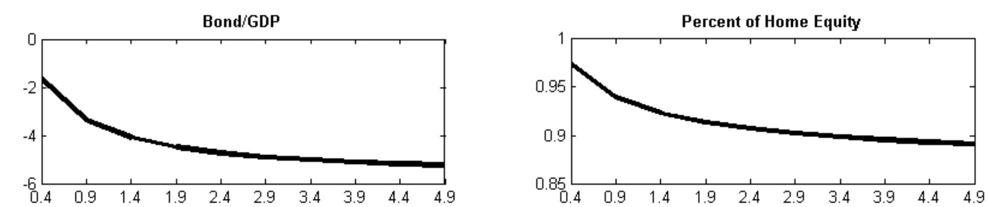


Figure 1.4: Substitution Elasticity of Home and Foreign goods

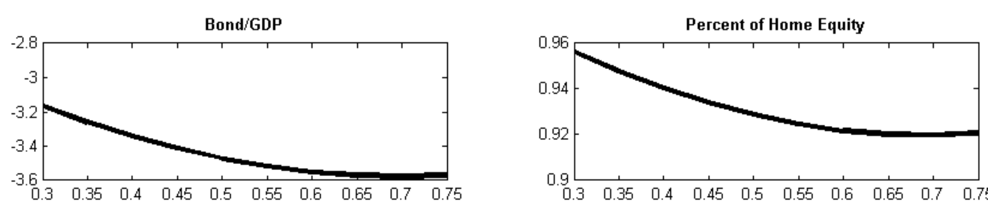


Figure 1.5: Persistence of Productivity shock

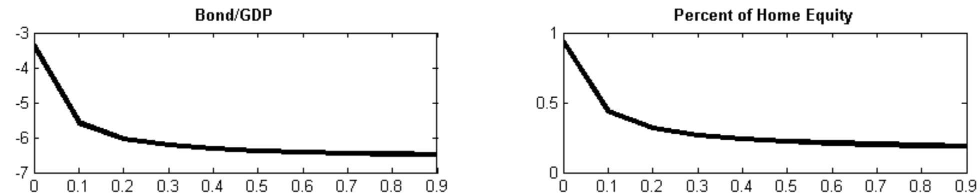


Figure 1.6: Response to Exchange rate

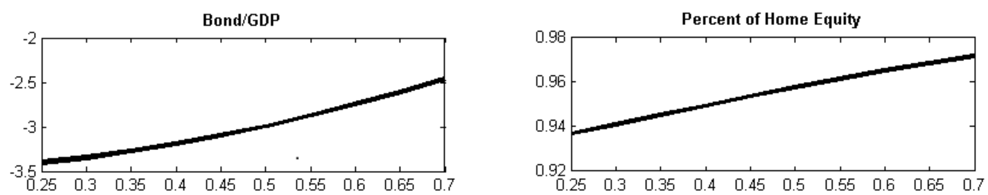


Figure 1.7: Capital Share

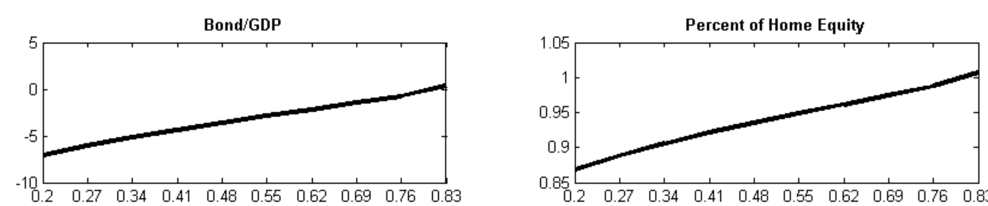


Figure 1.8: Country Size

Chapter 2

Dynamic portfolio choices

2.1 Introduction

The improvements in international financial markets have stimulated interest in the structure of capital outflows and inflows and resulted in changes in country portfolio positions.
Lane and Milesi-Ferretti (2006) document the development of the external positions of emerging markets and provide data on the net and gross external positions and the composition of international portfolios. The remarkable feature of emerging countries is not only the large growth of the net international capital flows and net external position but also that of the gross external positions in the financial global markets in the recent decades. The graph below shows the gross assets and liabilities of the small Asian countries from 1970. Since the mid 1990s, they have seen huge development of their external positions.

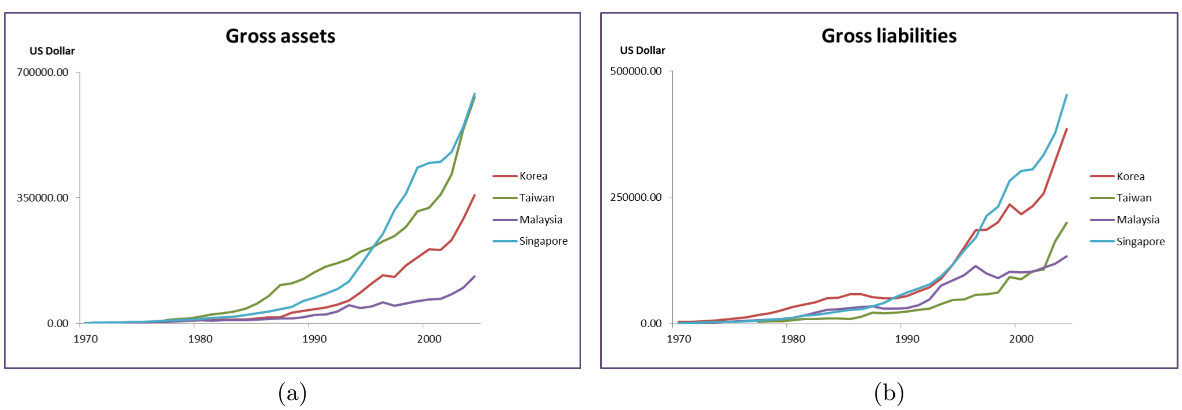


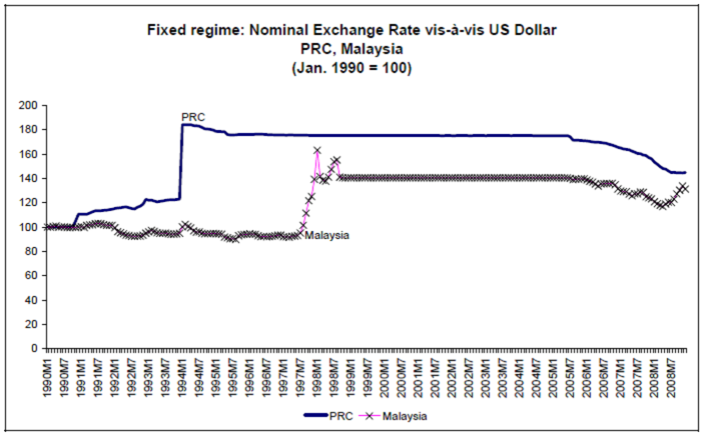
Figure 2.1: Assets and Liabiilities for Asian Coutries

These aspects of international financial markets lead to an interest in the structure of capital outflows and inflows, paying attention to country portfolio positions. Emerging markets are experiencing a surge in capital inflows that expose them to higher risks of financial instability and higher interdependence on the financial states of other countries. There is no doubt that emerging markets respond very sensitively to changes in the financial circumstances from large economies. The recent, global financial crisis brought more attention to the role of international capital flows and the monetary policies affecting them. In particular, large growth of the gross capital inflows from developed economies to emerging economies has caused financial instability in the developed economies to be spread to many emerging economies. The strongly interconnected global financial system means that monetary policies in the developed economies affect emerging economies directly through their effects on capital flows.

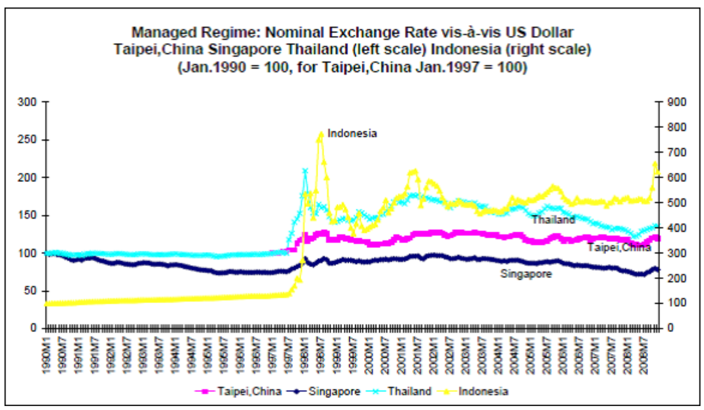
Kim and Yang(2009) empirically analyze the transmission of US monetary policy shocks to the East Asian countries. These countries use different exchange rate regimes, fixed, managed and free-floating. They show that Indonesia, the Philippines, Thailand, Korea, and Japan all follow the inflation-targeting rules based on the classification of Stone and Bhundia (2004) but they have a variety of exchange rate regimes. The impacts of US monetary policy shocks on the interest rates of the Asian countries depend on the exchange rate regime.

This paper follows the formulation of Monacelli(2004) to model a managed/fixed exchange rate regime as an extension of a Taylor-style interest rate rule. It is consistent with the empirical insensitivity of output volatility to the type of exchange rate regime. The relationship between exchange rate regime and real exchange rate volatility holds independently of the source of shocks. Introduction of incomplete exchange rate pass through generates no correlation between the nominal exchange rate and inflation differentials. Therefore, the representation of a managed/fixed exchange rate regime by means of an interest rate rule under sticky prices and incomplete exchange rate pass-through is suitable for an open economy policy rule. The paper shows the effect of monetary policy rules on optimal portfolios in an asymmetric two-country, sticky-price open economy macro model with two different price setting systems - producer-currency pricing (PCP) for the advanced economy and local-currency pricing (LCP) for the emerging economy - under fixed and flexible exchange rate regimes.

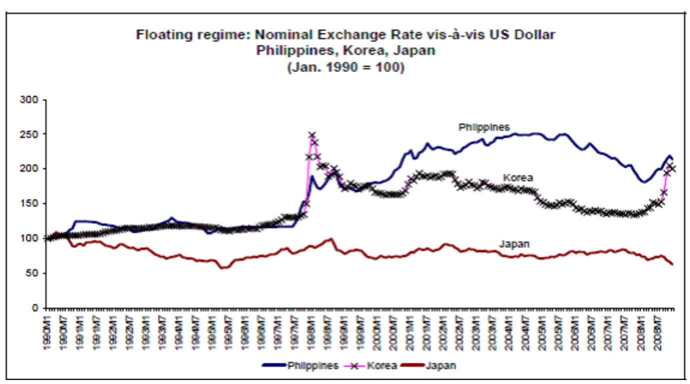
The standard DSGE model with only one risk-free bond has no portfolio diversification and no valuation effect. These classes of models therefore have no implications for capital outflows and inflows caused by movements in expected excess returns to external assets or liabilities. Despite the model having various financial assets, a technical deficiency left an indeterminacy problem unsolvable for a steady state portfolio. A number of authors have developed different methods for equilibrium portfolios, such as Kollmann (2006) and Coeurdacier, Kollmann and Martin (2010). Furthermore, Devereux and Sutherland (2010) and Wincoop and Tille (2010) calculate the time-varying dynamics of international portfolios. Utilizing the DSGE model, I



(a) Fixed Peg (1990 = 100)



(b) Managed Regime



(c) Free Floating Regime

Figure 2.2: **Exchange Rate Regime.** Source: Kim and Yang(2009) "International Monetary Transmission and Exchange Rate Regimes"

solve for the steady state value of the portfolio, which is indeterminate in the basic DSGE model, and show the impulse responses of the home country household's dynamic portfolio choices to the foreign country's monetary policy shocks. I employ a two-country, open economy model with incomplete asset markets that features sticky prices with Rottemberg price adjustment costs, capital accumulation, monetary policy shocks under fixed or flexible exchange rate regimes and different types of exporting price setting systems for each country. I calibrate the benchmark model using empirical estimates based on Korean and US quarterly data. The model is solved using the numerical methods based on Taylor series higher order approximations developed in Devereux and Sutherland (2010). First, a solution of the first order linear approximation of the non-portfolio part of the model is derived and used to solve for the steady state portfolio holdings. Like the steady state portfolio decisions, the dynamic portfolio choice problem can also be solved from the solution of a second- order linear approximation of the non-portfolio side of the model at the approximation point obtained from the first step. This chapter is organized as follows. Section 2 describes the model in detail. The benchmark calibration is described in section 3. Section 4 states the solution methods. The impulse responses are shown in section 5. Section 6 discusses the dynamic portfolio holdings generated by the asymmetric benchmark parameterization. Section 7 concludes the paper.

2.2 Literature review

As the international financial market has grown, the importance of research on the external asset positions, international capital flows and valuation effects has increased. However, macroeconomic models generally only deal with net capital flows with one non-contingent bond or complete markets with Arrow-Debreu securities in which capital flows are all zero after an initial period. Devereux - Sutherland (2010) and Tille - Wincoop (2010) independently achieved methodological advances for the optimal portfolio decision with both gross and net asset positions and for changes in expected returns or risks. Tille and Wincoop (2010) develop a numerical iterative solution method to solve portfolio choices in any higher order approximation. As in the analytical method of Devereux and Sutherland (2011), the steady state portfolio decision can be solved by the first order approximation of the non-portfolio equations and the second-order approximation of the portfolio equations. The computation of the dynamic portfolio decisions requires the higher order approximations. Tille and Wincoop (2010) not only focus on the computational method for the portfolios but also valuation effects and capital flows as a component of the external position changes. And the capital flows are decomposed into two sources of portfolio growth and portfolio reallocation. But numerical iteration requires more sophisticated efforts. I follow the more general and simple method by Devereux and Sutherland (2010). They provide an analytical solution method for the portfolio choice problem. In standard linearization up to the first order approximation, the portfolio choice problem is unsolvable since assets are perfect substitutes with the same expected excess returns. They solve the indeterministic steady state problem that portfolio choice is not pinned down by adapting second-order approximations of the portfolio equations while the macroeconomic equations are evaluated to a first order approximation. For the dynamic portfolio choices, one order higher approximations for the portfolio and non-portfolio equations need to be considered. The Devereux and Sutherland method is the most popular in recent international macroeconomic papers which focus on the dynamic portfolio problems. Civelli (2008) investigates the valuation effects from productivity shocks and demand shocks. Under flexible prices, the change in the supply of home goods resulting from the productivity shock causes terms of trade adjustments which can offset the consumption risks of shocks by changes in the relative price of exports without the trade of assets. Coeurdacier and Rey (2013) review recent papers about the home bias puzzle in an open economy financial macroeconomic model and explain the phenomenon with three categories of effects. Firstly, the source of risks causing optimal portfolio decisions is broken down by the effects of the real exchange rate risks and the non-tradable income risks in a frictionless financial market in which homogeneous investors can access identical portfolios. Secondly, the main source of friction is asset trade costs in foreign portfolio investments such as transaction costs or different tax treatments in international financial markets. Finally, informational heterogeneity is an important source that causes different portfolio decisions of foreign investors from domestic investors. While there are other papers that investigate the dynamic portfolios in the new Keynesian model, under sticky prices, terms-of-trade movements cannot offset the effects of productivity shocks because goods prices are preset. Therefore, portfolio diversification can have a different influence under price stickiness. Rahbari (2009) reproduces the features of the unconditionally positive correlation between equity returns and human capital returns and the negative correlation conditional on bond returns addressed by Engel and Matsumoto (2005), who show that monetary policy shocks can be hedged by a forward contract assuming nominal price rigidity. He also provides empirical evidence that international portfolio patterns match the data on G7 countries under nominal rigidities and endogenous capital accumulation. These papers concentrate on complete asset market models or international real business cycle models that show the effect of productivity shocks. Bui (2009) examines impulse responses of dynamic portfolio choices from productivity shocks, investment shocks and monetary policy shocks with an incomplete asset market in a new open macroeconomic model. He breaks down the responses by shocks as valuation effects and volume effects and finds that the net changes in foreign asset positions are small due to the offset of the two effects. He mainly emphasizes the effects of the dynamic portfolio choices on the monetary policy shocks under a simple Taylor-type rule. Despite these developments, to the author's knowledge this is the first paper to examine the impacts on the dynamic asset holdings of an emerging economy transmitted from an advanced economy's monetary policy shocks in a Dynamic Stochastic General Equilibrium (DSGE) model. This paper suggests an alternative monetary policy rule similar to Monacelli (2004) which formulates monetary policy in a managed/fixed exchange rate regime as an extension of the Taylor-type interest rate rule. The paper compares alternative monetary policy rules in a model of a small open economy which experiences monetary policy shocks from a large economy when international portfolio choice is endogenous.

2.3 Model

The world economy consists of two countries of different sizes with a continuum of infinitely-lived households. The representative households denoted by the unit interval $j \in [0,1]$ reside in one of two countries, Home (H) and Foreign (F). The interval $j \in [0,n]$ denotes home country households and the interval $j \in [n,1]$ denotes foreign country households. When the home country is defined as the small country, the parameter n that measures the relative size of the country is close to zero and the small country's policy decisions have negligible impacts on the large country. Both countries have monopolistically competitive firms producing differentiated intermediate goods and perfectly competitive firms producing final goods. The firms are represented by the interval $i \in [0,1]$. The home and foreign firms are indexed by the interval $i \in [0,n]$ and $i \in [n,1]$, respectively. The intermediate goods firms set nominal prices in a staggered fashion, as in Rotemberg (1982). The price setting problems for the export market have different assumptions for each country. For the asymmetric case, the small country exporters set prices using the importing market's currency under Local-Currency Pricing (LCP). The large foreign country exporters set their prices in their own currency under Producer-Currency Pricing (PCP).

2.3.1 Consumers

The representative household j maximizes the lifetime utility function

$$U\left(C,H\right) = \frac{{C_t}^{1-\gamma}}{1-\gamma} - \frac{H_t^{1+\omega}}{1+\omega} \tag{2.1}$$

where C_t^j is a composite consumption index and H_t^j denotes hours of labor. The index C_t^j is defined by the CES function

$$C_t^j = \left[(1-\eta)^{\frac{1}{\mu}}\left(C_{H,t}^j\right)^{\frac{\mu-1}{\mu}} + \eta^{\frac{1}{\mu}}\left(C_{F,t}^j\right)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{2.2}$$

where parameter $\eta \in [0,1]$ inversely represents a degree of home bias in consumption and the parameter $\mu > 0$ measures the elasticity of substitution between domestic and foreign goods.

$C^j_{H,t}$ is defined by the CES composite consumption index of domestic goods

$$C^j_{H,t} = \left[\left(\frac{1}{n} \right)^{\frac{1}{\varepsilon}} \left(\int_0^n C^j_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right) \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.3}$$

where $i \in [0, n]$ denotes the variety of the domestic goods. $C^j_{F,t}$ is defined by the CES composite consumption index of goods imported and consumed by domestic households.

$$C^j_{F,t} = \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\varepsilon}} \left(\int_n^1 C^j_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right) \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2.4}$$

where $\varepsilon > 1$ measures the elasticity of substitution between varieties produced in the same country. The optimal $C^j_{H,t}(i)$ and $C^j_{F,t}(i)$ of any given levels of $C^j_{H,t}$ and $C^j_{F,t}$ are respectively

$$C^j_{H,t}(i) = \left(\frac{1}{n} \right) \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C^j_{H,t} \tag{2.5}$$

$$C^j_{F,t}(i) = \left(\frac{1}{1-n} \right) \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C^j_{F,t} \tag{2.6}$$

where $P_{H,t}$ is the domestic currency price index of domestically produced goods and $P_{F,t}$ is the import price index from the foreign country. The corresponding price index is

$$P_t = \left[(1-\eta) P_{H,t}^{1-\mu} + \eta P_{F,t}^{1-\mu} \right]^{\frac{1}{1-\mu}} \tag{2.7}$$

With

$$P_{H,t} = \left[\frac{1}{n} \left(\int_0^n P^j_{H,t}(i)^{1-\varepsilon} di \right) \right]^{\frac{1}{1-\varepsilon}} \tag{2.8}$$

$$P_{F,t} = \left[\frac{1}{1-n} \left(\int_n^1 P^j_{F,t}(i)^{1-\varepsilon} di \right) \right]^{\frac{1}{1-\varepsilon}} \tag{2.9}$$

where P_t is the Consumer Price Index (CPI) in the model. It follows that $\int_0^n P_{H,t}(i)\, C_{H,t}^j(i)\, di = P_{H,t}C_{H,t}^j$ and $\int_n^1 P_{F,t}(i)\, C_{F,t}^j(i)\, di = P_{F,t}C_{F,t}^j$. The household j's optimal allocation between home and foreign goods is represented by

$$C_{H,t}^j = (1-\eta)\left(\frac{P_{H,t}(i)}{P_t}\right)^{-\mu} C_t^j \tag{2.10}$$

$$C_{F,t}^j = \eta\left(\frac{P_{F,t}(i)}{P_t}\right)^{-\mu} C_t^j \tag{2.11}$$

The aggregate demand functions for the home and foreign good i are represented by

$$C_{H,t}(i) = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \tag{2.12}$$

$$C_{F,t}(i) = \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{2.13}$$

Finally, the optimal allocation of expenditures between domestic and foreign goods is given by

$$C_{H,t} = (1-\eta)\left(\frac{P_{H,t}(i)}{P_t}\right)^{-\mu} C_t \tag{2.14}$$

$$C_{F,t} = \eta\left(\frac{P_{F,t}(i)}{P_t}\right)^{-\mu} C_t \tag{2.15}$$

Therefore, the home country's total consumption expenditure is $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$, where $P_{H,t}C_{H,t} = \int_0^n P_{H,t}(i)\, C_{H,t}(i)\, di$ and $P_{F,t}C_{F,t} = \int_n^1 P_{F,t}(i)\, C_{F,t}(i)\, di$. Accordingly, the household maximization of the utility function

$$\mathrm{U}(C,H) = E_t \sum_{t=0}^{\infty} \beta_t \left(\frac{C_t^{\,1-\gamma}}{1-\gamma} - \frac{H_t^{1+\omega}}{1+\omega}\right) \tag{2.16}$$

is subject to a real budget constraint given by

$$b_{H,t}+e_t b_{F,t}+j_t x_{H,t}+e_t j_t^* x_{F,t}=w_t h_t+r_t^k k_t-c_t-\frac{p_{i,t}}{p_t}I_t$$
$$+r_{H,t}b_{H,t-1}+e_t r_{F,t}b_{F,t-1}+(j_t+d_t)x_{H,t-1}+e_t(j_t^*+d_t^*)x_{F,t-1}$$

The discount factor β_t is endogenously determined by the function of the aggregate home consumption as follows

$$\beta_{t+1}=\beta_t\beta\left(C_t\right),\beta_0=1 \tag{2.17}$$

where $0<\beta(C_t)<1$, $\beta'(C_t)<0$, and C_t is the aggregate home consumption. Schmitt-Grohe and Uribe (2003) consider five different specifications for stationarity in incomplete asset market models. In this paper, I employ the endogenous discount factor to eliminate the unit root and solve the non-stationarity problem with the functional form of $\beta(C_t)$

$$\beta\left(C_t\right)=\tilde{\beta}C_t^{-\nu} \tag{2.18}$$

where $0\leq\nu<\gamma$ and $0<\beta C_t^{-\nu}<1$. Home agents hold two bonds, b_H and b_F, respectively denominated in the home and foreign currency, where the real returns on bonds are $r_{H,t}^b$ and $r_{F,t}^b$ and two equities, x_H and x_F, respectively denominated in the home and foreign currency, where the real returns on equities are $r_{H,t}^e$ and $r_{F,t}^e$ The real exchange rate e_t is defined by the price of the foreign good in terms of the home good as following $e_t=S_t\frac{P_t^*}{P_t}$. S_t is the nominal exchange rate. Real exchange rate appreciation corresponds to a decrease in e_t, while depreciation corresponds to an increase in e_t. By defining net wealth W_t as the differences between gross assets and gross liabilities, i.e. $NFA_t=e_t j_t^* x_{F,t}+e_t b_{F,t}-j_t x_{H,t}^*-b_{H,t}^*$, the budget constraints can be rewritten as

$$NFA_t=r_{H,t}^b NFA_{t-1}+w_t h_t+r_t^k k_t-c_t+d_t-\frac{p_{i,t}}{p_t}I_t$$
$$+\alpha_{F,t-1}^b(r_{F,t}^b-r_{H,t}^b)-\alpha_{H,t-1}^e(r_{H,t}^e-r_{H,t}^b)+\alpha_{F,t-1}^e(r_{F,t}^e-r_{H,t}^b)$$

where $r_{x,t}$ is a real excess return on bonds which is defined by

$$r_{x,t}=\left[(r_{F,t}^b-r_{H,t}^b)\quad(r_{H,t}^e-r_{H,t}^b)\quad(r_{F,t}^e-r_{H,t}^b)\right] \tag{2.19}$$

The real returns on bonds and equities are defined by

$$r^b_{H,t} = R_{H,t-1}\frac{P_{t-1}}{P_t} \qquad r^e_{H,t} = \frac{J_t + D_t}{J_{t-1}}\frac{P_t}{P_{t-1}}$$
$$r^b_{F,t} = R_{F,t-1}\frac{P^*_{t-1}}{P^*_t}\frac{e_t}{e_{t-1}} \qquad r^e_{F,t} = \frac{J^*_t + D^*_t}{J^*_{t-1}}\frac{P^*_t}{P^*_{t-1}}\frac{e_t}{e_{t-1}}$$

The real holdings of bonds and equities are defined by

$$\alpha^b_{H,t} = \frac{B_{H,t}}{R_{H,t-1}P_t} \qquad \alpha^e_{H,t} = \frac{J_t x_{H,t}}{P_t}$$
$$\alpha^b_{F,t} = e_t\frac{B_{F,t}}{R_{F,t-1}P^*_t} \qquad \alpha^e_{F,t} = e_t\frac{J^*_t x_{F,t}}{P^*_t}$$

From the first order conditions for bond assets are

$$C_t^{\nu-\gamma} = \beta E_t C_{t+1}^{-\gamma} r_{H,t+1} \tag{2.20}$$

$$\beta E_t \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}}\left(r_{H,t+1} - r_{F,t+1}\right) = 0 \tag{2.21}$$

2.3.2 Firms

In each country, a continuum of monopolistically competitive firms produces the differentiated intermediate goods using capital and labor. A standard Cobb-Douglas production function is

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} \tag{2.22}$$

where A_t is a technology shock that follows the AR(1) process

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t \tag{2.23}$$

The law of capital evolution is

$$k_{t+1} = (1-\delta)\,k_t + I_t - \phi\left(I_t, I_{t-1}\right)_t \tag{2.24}$$

where δ is the depreciation rate of capital, I_t is investment and $\phi(I_t, I_{t-1})$ is a measure of investment adjustment costs. I assume a specific functional form for the investment adjustment costs.

$$\phi\left(I_t, I_{t-1}\right) = \frac{\kappa}{2} I_t \left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{2.25}$$

where κ is the investment adjustment cost parameter. I assume Rotemberg-style sluggish price adjustment so that a firm sets the price with a quadratic adjustment cost in the inflation rate of the goods. Each firm faces two separate pricing problems, one for the domestic market and one for the export market and the export market has two different pricing scenarios. The optimal price-setting strategy for the domestic market can be approximated by the typical New Keynesian Phillips curve for domestic inflation. The home firm maximizes the present value of expected profits by resetting price $\overline{p}_{H,t}$ for the domestic market. I consider both the PCP and the LCP strategy for the export market. When the exporting goods price is determined by PCP, the home firms set their domestic goods prices for the domestic market. The home firm's nominal profit is

$$\begin{aligned} D_t\left(i\right) &= P_{H,t}\left(i\right) Y_t\left(i\right) - w_t h_t\left(i\right) - R_t^k k_t\left(i\right) \\ &- \frac{\varphi_H}{2}\left[\frac{P_{H,t}\left(i\right)}{\bar{\pi}_H P_{H,t-1}\left(i\right)} - 1\right]^2 P_{H,t} Y_t \end{aligned} \tag{2.26}$$

The firms real profit maximization can be written as

$$\begin{aligned} \max_{H_t, K_t, P_{H,t}, P_{H,t}^*} E_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ &\frac{P_{H,t}\left(i\right)}{P_t}\left(\frac{P_{H,t}\left(i\right)}{P_{H,t}}\right)^{-\varepsilon} Y_t - \frac{w_t h_t\left(i\right) - R_t^k k_t\left(i\right)}{P_t} \\ &- \frac{\varphi_H}{2}\left[\frac{P_{H,t}\left(i\right)}{\bar{\pi}_H P_{H,t-1}\left(i\right)} - 1\right]^2 \frac{P_{H,t} Y_t}{P_t} \end{aligned}$$

Subject to $Y_t\left(i\right) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t$, firm is output equals its total demands. The foreign firms also set their foreign goods prices for the foreign market. The expected profits of the foreign firm in the foreign market can be written as

$$\begin{aligned} \max_{H_t^*, K_t^*, P_{F,t}^*} E_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ &\frac{P_{F,t}^*\left(i\right)}{P_t^*}\left(\frac{P_{F,t}^*\left(i\right)}{P_{F,t}^*}\right)^{-\varepsilon} Y_t^* - \frac{w_t^* h_t^*\left(i\right) - R_t^{k^*} k_t^*\left(i\right)}{P_t^*} \\ &- \frac{\varphi_F^*}{2}\left[\frac{P_{F,t}^*\left(i\right)}{\bar{\pi}_F^* P_{F,t-1}^*\left(i\right)} - 1\right]^2 \frac{P_{F,t}^* Y_t^*}{P_t^*} \} \end{aligned}$$

subject to the production function,$y_t^*\left(i\right) = \left(\frac{P_{F,t}^*(i)}{P_{F,t}^*}\right)^{-\varepsilon} Y_t^*$.

Under PCP, the export price is determined by the domestic goods price and the exchange rate

under the law of one price (LOP).
The first order conditions for this problem are given by

$$\lambda_t w_t = (1-\alpha)\,\zeta_t A_t k_t^{\alpha} h_t^{-\alpha} \tag{2.27}$$

$$\lambda_t r_t^k = \alpha \zeta_t A_t k_t^{\alpha-1} h_t^{1-\alpha} \tag{2.28}$$

$$\zeta_t = \frac{1}{A_t} w_t^{1-\alpha} \left(r_t^k\right)^{\alpha} \left[\alpha^{-1}(1-\alpha)^{-(1-\alpha)}\right] \lambda_t \tag{2.29}$$

where $\varepsilon/(\varepsilon-1)$ is the constant gross mark-up over marginal cost. With the assumed technology, the firms real marginal cost is

$$mc_t = \frac{1}{A_t} w_t^{1-\alpha} \left(r_t^k\right)^{\alpha} \left[\alpha^{-1}(1-\alpha)^{-(1-\alpha)}\right] \tag{2.30}$$

The optimal rule for setting prices by home firms can be written as

$$\begin{aligned} \varphi_H \left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right] \frac{\pi_{H,t}}{\bar{\pi}_H} &= (1-\varepsilon) + \varepsilon\left[(1-\eta) + \eta \tau_t^{1-\mu}\right]^{\frac{1}{1-\mu}} mc_t \\ &+ \tilde{\beta} c_t^{-\nu} \varphi_H E_t \left[\frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right] \frac{\pi_{H,t+1}^2}{\bar{\pi}_H} \frac{y_{t+1}}{y_t} \frac{1}{\pi_{t+1}}\right]^2 \end{aligned}$$

The optimal rule for setting prices by foreign firms can be written as

$$\begin{aligned} \varphi_F \left[\frac{\pi_{F,t}^*}{\bar{\pi}_F^*} - 1\right] \frac{\pi_{F,t}^*}{\bar{\pi}_F^*} &= (1-\varepsilon) + \varepsilon\left[(1-\eta) + \eta \tau_t^{1-\mu} *\right]^{\frac{1}{1-\mu}} mc_t^* \\ &+ \tilde{\beta} c_t^* {-\nu} \varphi_F E_t \left[\frac{c_{t+1}^* {-\gamma}}{c_t^* {-\gamma}} \left[\frac{\pi_{F,t}^*}{\bar{\pi}_H} - 1\right] \frac{\pi_{F,t+1}^2 *}{\bar{\pi}_F^*} \frac{y_{t+1}^*}{y_t^*} \frac{1}{\pi_{t+1}^*}\right]^2 \end{aligned}$$

Home firms' exporting goods prices are determined by

$$\pi_{H,t}^* \frac{S_t}{S_{t-1}} = \pi_{H,t} \tag{2.31}$$

Foreign firms' exporting goods prices are determined by

$$\pi_{F,t} = \pi_{F,t}^* \frac{S_t}{S_{t-1}} \tag{2.32}$$

The nominal export prices in a producer currency move one-to-one with the fluctuations of the nominal exchange rate, S_t.
When the home firms use the LCP strategy for the export market, they have to set different prices for the domestic market and for the export market. The home firms nominal profit is

$$\begin{aligned} D_t(i) &= P_{H,t}(i)\,Y_{H,t}(i) + S_t P^*_{H,t}(i)\,Y^*_{H,t}(i) - w_t H_t(i) - R^K_t K_t(i) \\ &- \frac{\varphi_H}{2}\left[\frac{P_{H,t}(i)}{\bar{\pi}_H P_{H,t-1}(i)} - 1\right]^2 P_{H,t}Y_{H,t} - \frac{\varphi^*_H}{2}\left[\frac{P^*_{H,t}(i)}{\bar{\pi}^*_H P^*_{H,t}(i)} - 1\right]^2 S_t P^*_{H,t}Y^*_{H,t} \end{aligned} \tag{2.33}$$

The firms real profit maximization can be written as

$$\begin{aligned} \max_{H_t, K_t, P_{H,t}, P^*_{H,t}} E_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ & \frac{P_{H,t}(i)}{P_t}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_{H,t} + \frac{S_t P^*_{H,t}(i)}{P_t}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon} Y^*_{H,t} \\ & - \frac{w_t H_t(i) - R^K_t K_t(i)}{P_t} - \frac{\varphi_H}{2}\left[\frac{P_{H,t}(i)}{\bar{\pi}_H P_{H,t-1}(i)} - 1\right]^2 \frac{P_{H,t}Y_{H,t}}{P_t} \\ & - \frac{\varphi^*_H}{2}\left[\frac{P^*_{H,t}(i)}{\bar{\pi}^*_H P^*_{H,t-1}(i)} - 1\right]^2 \frac{S_t P^*_{H,t}Y^*_{H,t}}{P_t} \} \end{aligned}$$

Subject to $y_t(i) = y_{H,t}(i) + y^*_{H,t}(i)$, firm is output equals its total demands. The optimal rule for setting export prices of home firm can be written as

$$\begin{aligned} \varphi^*_H \left[\frac{\pi^*_{H,t}}{\bar{\pi}^*_H} - 1\right]\frac{\pi^*_{H,t}}{\bar{\pi}^*_H} &= (1-\varepsilon) + \varepsilon[(1-\eta)\,\tau^*_t\mu - 1 + \eta]^{\frac{1}{1-\mu}}\frac{mc_t}{e_t} \\ &+ \tilde{\beta} c_t^{-\nu}\varphi_H E_t\left[\frac{c^*_{t+1}{-\gamma}}{c^*_t{-\gamma}}\left[\frac{\pi^*_{H,t}}{\bar{\pi}^*_H} - 1\right]\frac{\pi^*_{H,t+1}2}{\bar{\pi}^*_H}\frac{y^*_{H,t+1}}{y_H, t^*}\frac{e_{t+1}}{e_t}\frac{1}{\pi^*_{t+1}}\right]^2 \end{aligned}$$

The optimal rule for setting export prices of foreign firm can be written as

$$\begin{aligned} \varphi_F \left[\frac{\pi_{F,t}}{\bar{\pi}_F} - 1\right]\frac{\pi_{F,t}}{\bar{\pi}_F} &= (1-\varepsilon) + \varepsilon\Big[(1-\eta)\,\tau_t^{\mu-1} + \eta\Big]^{\frac{1}{1-\mu}} mc^*_t \\ &+ \tilde{\beta} c^*_t{-\nu}\varphi_F E_t\left[\frac{c^*_{t+1}{-\gamma}}{c^*_t{-\gamma}}\left[\frac{\pi^*_{F,t}}{\bar{\pi}_H} - 1\right]\frac{\pi^2_{F,t+1}}{\bar{\pi}_F}\frac{y_{F,t+1}}{y_F, t}\frac{e_t}{e_{t+1}}\frac{1}{\pi^*_{t+1}}\right]^2 \end{aligned}$$

2.3.3 Law of one Price

In this paper, the deviation from the law of one price (LOP) is a big difference between LCP and PCP. z_t is the deviation from LOP which is defined by

$$\widehat{z}_t \equiv \widehat{p}^*_{F,t} + \widehat{s}_t - \widehat{p}_{F,t} \tag{2.34}$$

$$\widehat{z}_t^* \equiv \widehat{p}_{H,t} - \widehat{p}_{H,t}^* - \widehat{s}_t \tag{2.35}$$

where s_t is the nominal exchange rate. Under PCP, the law of one price holds when $z_t = 0$ and $z_t^* = 0$. Under LCP, however, the law of one price doesnt hold and both z_t and z_t^* are not zero.

2.3.4 Monetary Policy and the exchange rate regime

Monetary policy is assumed to follow an extended form of a Taylor-type interest rate rule in which deviations of inflation, output and the nominal exchange rate from the long run targets have a feed back on short run movements of the nominal interest rate. The log linear approximated form is

$$\widehat{R}_{H,t} = \rho_H \widehat{R}_{H,t-1} + (1-\rho_H)\,\phi_\pi \widehat{\pi}_t + (1-\rho_H)\,\phi_y \widehat{y}_t + (1-\rho_H)\,\phi_s \widehat{s}_t + \zeta_{m,t} \tag{2.36}$$

where $\zeta_{m,t}$ is monetary policy shock. The rule describes an economy in which monetary policy is constrained by a managed or fixed exchange rate regime. The parameter allows the monetary authority to smooth changes in the interest rate. The parameter $(1-\rho_H)\,\phi_\pi$ and $(1-\rho_H)\,\phi_y$ reflect the responses to the changes in the CPI inflation rate and real GDP, respectively. The parameter $(1-\rho_H)\,\phi_s$ reflects the degree of the managed exchange rate between fixed and floating regimes. A floating exchange rate regime implies $\phi_s = 0$ while a fixed exchange rate implies $\phi_s = \infty$.

2.3.5 Market clearing conditions

Goods market clearing conditions for the producer currency pricing for the producer currency pricing for the home country is

$$Y_t^* = C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^* - \frac{\varphi_H}{2}\left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right]^2 Y_t \tag{2.37}$$

Goods market clearing conditions for the producer currency pricing for the foreign country is

$$Y_t^* = C_{F,t}^* + C_{F,t} + I_{F,t}^* + I_{F,t} - \frac{\varphi_F^*}{2}\left[\frac{\pi_{F,t}^*}{\bar{\pi}_F^*} - 1\right]^2 Y_t^* \tag{2.38}$$

Goods market clearing conditions for the local currency pricing for the domestic country are

$$Y_{H,t} = C_{H,t} + I_{H,t} - \frac{\varphi_H}{2}\left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right]^2 Y_{H,t} \tag{2.39}$$

$$Y_{H,t}^{*} = C_{H,t}^{*} + I_{H,t}^{*} - \frac{\varphi_H^{*}}{2}\left[\frac{\pi_{H,t}^{*}}{\bar{\pi}_H^{*}} - 1\right]^2 Y_{H,t}^{*} \tag{2.40}$$

$$Y_t = Y_{H,t} + Y_{H,t}^{*} \tag{2.41}$$

Domestic output Y_t equals the sum of aggregate domestic demand, $Y_{H,t}$ and aggregate export demand, $Y_{H,t}^{*}$. Goods market clearing conditions for the local currency pricing for the foreign country are

$$Y_{F,t} = C_{F,t} + I_{F,t} - \frac{\varphi_F}{2}\left[\frac{\pi_{F,t}}{\bar{\pi}_F} - 1\right]^2 Y_{F,t} \tag{2.42}$$

$$Y_{F,t}^{*} = C_{F,t}^{*} + I_{F,t}^{*} - \frac{\varphi_F^{*}}{2}\left[\frac{\pi_{F,t}^{*}}{\bar{\pi}_F^{*}} - 1\right]^2 Y_{F,t}^{*} \tag{2.43}$$

$$Y_t = Y_{F,t} + Y_{F,t}^{*} \tag{2.44}$$

Foreign output Y_t equals the sum of aggregate domestic demand, $Y_{F,t}^{*}$ and aggregate export demand, $Y_{F,t}$.

Bond market clearing conditions are

$$b_{H,t} + b_{H,t}^{*} = 0 \tag{2.45}$$

$$b_{F,t}^{*} + b_{F,t} = 0 \tag{2.46}$$

The bond assets $b_{H,t}$ and $b_{F,t}$ are issued in the home and foreign currencies, respectively. The bond holdings by foreign agents are denoted by an asterisk (*).

Equity market clearing conditions are

$$x_{H,t} + x_{H,t}^{*} = 1 \tag{2.47}$$

$$x_{F,t}^{*} + x_{F,t} = 1 \tag{2.48}$$

The equity assets $x_{H,t}$ and $x_{F,t}$ are issued in the home and foreign currencies, respectively. The equity holdings by foreign agents are denoted by an asterisk (*).

2.4 Calibration

In this paper, for the symmetric case, I calibrate the benchmark model using empirical estimates based on US quarterly data. The endogenous discount factor, β is calibrated as 0.99 in the steady sate so that the annual nominal interest rate is 4 percent. The time-varying discount factor depends on the aggregate consumption such as $\tilde{\beta}C_t^{-\nu}$. In the steady state, $\tilde{\beta}$ is calibrated assuming $\nu = 0.01$ and $\tilde{\beta}C^{-\nu} = \beta$. The Frisch labor supply elasticity $1/\omega$ is set equal to 0.33. The consumption home bias $\eta = 0.85$, implying a 15 percent import to GDP ratio and the elasticity of substitution $\mu = 0.9$ following Heathcote and Perri (2002). The capital share in the production function α is set at 0.28. The depreciation rate of capital is 0.025 which is commonly used for US quarterly data, implying a 10 percent annual rate of depreciation. The Rotemberg price adjustment parameter ϕ and the markup ε are 58 and 6, equivalent to setting the Calvo price parameter equal to 0.75 which is about 4 quarters for the average duration of the price adjustment. The capital adjustment parameter is set equal to 4.5. For the monetary policy rule, I adopt the benchmark Taylor rule with partially adjustment calibrated by Canzoneri et. al. (2007) as follows

$$\widehat{R}_{H,t} = \rho_H \widehat{R}_{H,t-1} + (1-\rho_H)\,\phi_\pi \widehat{\pi}_t + (1-\rho_H)\,\phi_y \widehat{y}_t + (1-\rho_H)\,\phi_s \widehat{s}_t + \zeta_{m,t} \tag{2.49}$$

For the foreign country's monetary policy rule, the degree of the response to the nominal exchange rate is negative due to the definition of the exchange rate.

$$\widehat{R}_{F,t} = \rho_F \widehat{R}_{F,t-1} + (1-\rho_F)\,\phi_\pi^* \widehat{\pi}_t^* + (1-\rho_F)\,\phi_y^* \widehat{y}_t^* - (1-\rho_H)\,\phi_s^* \widehat{s}_t + \zeta_{m,t}^* \tag{2.50}$$

For the asymmetric case, the size of home country changes between interval [0.2, 0.83]. In the case $n < 0.5$, the home country is smaller than the foreign country. When the country size is 0.2, the GDP of the home country is 25 percent of the foreign country's while the home country's GDP is almost 5 times as much as the foreign country's when the country size is 0.83.

2.5 Solving the Model

I follow Devereux and Sutherland (2008) that developed numerical approximation methods suitable for this complicated model. The solution procedures stated in the paper emphasize several important properties to calculate the steady state and first order portfolios. First, the first-order behavior of the non-portfolio parts of the model is not influenced by the deviation

Table 2.1: Calibration

Parameter	Interpretation	Value
β	Discount factor	0.99
γ	Risk aversion	2
ω	1/ Elasticity of Labor supply	0.33
η	Consumption home bias	0.85
η^i	Investment home bias	0.75
μ	Subst. Elasticity for Consumption Home and Foreign	0.9
μ^i	Subst. Elasticity for Investment Home and Foreign	1.2
α	capital share in production	0.28
δ	Depreciation Rate	0.025
φ	Price Adjustment cost	58
κ	Investment Adjustment cost	4.5
ε	Subst. Elasticity for Domestic Varieties	6
ρ_r	Persistence of Interest rate	0.824
ρ_a	Persistence of Productivity shock	0.923
ρ_m	Persistence of Monetary shock	0
ρ_i	Persistence of Investment shock	0.85
ϕ_y	Response to Output gap	0.184
ϕ_π	Response to Inflation	2.02
ϕ_s	Response to Exchange rate	0
σ_A	Std of Productivity shock	0.000861
σ_M	Std of Monetary shock	0.000245
σ_I	Std of Investment shock	0.00036
n	Country Size	0.5

Table 2.2: Country Size

Country size (n)	GDP Ratio	Foreign Cons. Home bias (ν)
0.2	0.25	0.99
0.27	0.37	0.97
0.34	0.515	0.94
0.41	0.694	0.91
0.48	0.923	0.86
0.5	1	0.85
0.55	1.22	0.80
0.62	1.63	0.72
0.69	2.22	0.60
0.76	3.166	0.42
0.83	4.88	0.074

of portfolio holdings around the solution of the steady state portfolio. It is important that the first order macroeconomic solution is sufficient to solve the value of the steady state portfolios. Second, the second-order approximation of the portfolio problem that appears in the Euler equation has the terms of the solution of the first-order approximation for the non-portfolio parts. It means that the steady state value of portfolio holdings is solved by the first-order approximation of the macroeconomic parts. The portfolio holdings only directly enter the non-portfolio part through the budget constraint taking $b_H r_{x,t}$ as a mean-zero i.i.d. random variable, ξ_t. This is derived from the third property that the expected excess return, $E_t[r_{x,t+1}]$ is zero in the first-order approximation of the model. The solution for the first-order portfolio problem requires the higher-order approximation. As in the case of the steady state portfolio solution, the first-order portfolio holdings only directly enter the non-portfolio part through the second-order approximation of the budget constraint defining $b_{H,t-1} r_{x,t}$ as the random variable, ξ_t. The fourth important property is that the covariance matrix of the innovations, Σ is constant. Therefore, the solutions for the second moments are non-time varying. The system of the second-order approximations of the model can be written in the state-space representation. This paper uses the solution model developed by Lombard and Sutherland (2007).

2.6 Solution Method

In the open macroeconomic model, the standard linear approximation is applied around the non-stochastic steady state. However, the linear approximation has unsolvable difficulties for the portfolio choices for two reasons. First, the portfolio choices have the indeterminacy problem which is not uniquely pinned down in a non-stochastic steady state. This problem can be overcome by treating the steady state value of the portfolios as endogenous. Second, the portfolio choices are indeterminate in the first order approximation because all the assets are perfect substitutes. This problem can be also solved by considering second order approximation. I follow the methods described in Devereux and Sutherland (2008) that develop technical solutions for the steady state and dynamic portfolios in a general DSGE model with multiple assets. Their work can be applied for both complete and incomplete markets.

2.6.1 Steady state portfolios

In order to calculate the steady state portfolios, the first order approximation of non-portfolio parts and the second-order approximation of Euler equations of asset holdings are required. The steady state portfolios only appear in the non-portfolio parts of the model through the random variable of the budget constraint.

The net foreign asset is defined in real terms.

$$NFA_{t-1} = \tilde{\alpha}^{E}_{F,t-1} + \tilde{\alpha}^{B}_{F,t-1} - \tilde{\alpha}^{*E}_{H,t-1} - \tilde{\alpha}^{*B}_{H,t-1} \tag{2.51}$$

The budget constraint can be written by the net foreign asset NFA_t assuming a net zero steady state value and by the excess return on portfolios ξ_t.

$$NFA_t = NFA_{t-1} r^B_{H,t} + w_t h_t - c_t + d_t - \frac{p^I_t}{p_t} i_t + \xi_t \tag{2.52}$$

where the random variable xi_t is defined by

$$\xi_t = -\tilde{\alpha}^{*E}_{H,t-1}\left(r^E_{H,t} - r^B_{H,t}\right) + \tilde{\alpha}^{E}_{F,t-1}\left(r^E_{F,t} - r^B_{H,t}\right) + \tilde{\alpha}^{B}_{F,t-1}\left(r^B_{F,t} - r^B_{H,t}\right) \tag{2.53}$$

Up to the first order approximation, the excess returns $r_{x,t}$ are mean-zero random variables.

$$r_{x,t} = \left[\left(r^E_{H,t} - r^B_{H,t}\right)\left(r^E_{F,t} - r^B_{H,t}\right)\left(r^B_{F,t} - r^B_{H,t}\right)\right] \tag{2.54}$$

Therefore, ξ_t is also an i.i.d. exogenous variable which calls "excess return shock".

Dynare produces the following solution form in the first order approximation.

$$y_t = y^s + A y^h_{t-1} + B u_t \tag{2.55}$$

where y_t is a vector of all endogenous variables which array the order of static variables, purely backward, mixed and finally purely forward variables. y^s is the vector of the steady state values, y^h_{t-1} is the vector of state variables and u_t is the vector of exogenous variables.
In order to calculate the steady state value of the portfolios, the necessary variables c_t, c^*_t and $r_{x,t}$ can be extracted from the rows of y_t of the above Dynare solution. u_t includes the excess return shock, ξ_t and other exogenous shocks, ε_t. Therefore, the variables c_t, c^*_t and $r_{x,t}$ depend on the shocks ξ_t and other exogenous shocks, ε_t.

$$\begin{bmatrix} c_t \\ c^*_t \\ r_{x,t} \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \end{bmatrix} \begin{bmatrix} \xi_t \\ \varepsilon_t \end{bmatrix}$$

By combining the optimality condition $E[(c_{t+1} - c^*_{t+1})r_{x,t+1}] = 0$ and the excess return shock $\xi_t = \tilde{\alpha} r_{x,t}$, the steady state value of the portfolios can be obtained.

2.6.2 Dynamic portfolios

In order to compute the dynamic portfolios, the higher order approximation is necessary. Only the second order approximation of the budget constraint includes the time-varying portfolios. The second order approximation of non-portfolio parts and the third order approximation of Euler equations of asset holdings are required.
Dynare produces the following solution form in the second order approximation.

$$y_t = y^s + 0.5\Delta^2 + Ay^h_{t-1} + Bu_t + 0.5C\left(y^h_{t-1} \otimes y^h_{t-1}\right) + 0.5D\left(u_t \otimes u_t\right) + E\left(y^h_{t-1} \otimes u_t\right) \quad (2.56)$$

where $\left(y^h_{t-1} \otimes y^h_{t-1}\right)$ is the Kronecker product of the state variables, $(u_t \otimes u_t)$ is the Kronecker product of the vector of the exogenous variables, and $\left(y^h_{t-1} \otimes u_t\right)$ is the Kronecker product of the vector of state variables by the vector of exogenous variables.
The third order approximation of the optimality condition can be evaluated using the first and second order solutions for the behavior of the consumption and excess returns. The excess return shock, ξ_t is defined by $\hat{\alpha_{t-1}} \hat{r_{x,t}}$ in the second order approximation where the hat denotes the first order deviation.
Unlike the case for the steady state value of the portfolios, time-varying optimal portfolios are affected by the changes in the covariance between $(c_t - c^*_t)$ and $r_{x,t}$. The second order term, $\left(y^h_{t-1} \otimes u_t\right)$ captures how the consumption difference responds to the movement of the state variables to the stochastic shocks.

2.7 Results

2.7.1 Productivity shock

Flexible exchange rate regime

PCP Figure 1(a) shows dynamic asset changes for the home country from the home country's positive productivity shock assuming PCP. The productivity shock implies that less labor is needed to produce the same amount as before the shock. Home output, consumption and investment increase and the wage rate decreases. Foreign output and consumption increases because of higher demand for home export goods. The real exchange rate depreciates and net exports increase. Due to higher productivity and a lower wage rate, the real return on home equity relative to the real return on the home bond increases the most compared to the real returns on the foreign bond and equity as soon as the shock is recognized. Foreign country asset investors try to hedge the real exchange rate risk which motivates the holding of the foreign bond with higher real returns relative to real returns on home bond. Home country households that experience higher wealth by positive productivity shocks to home firms invest their extra wealth in home and foreign equities. Home households hold the foreign equity because they sell their home equity by a fixed supply of equity claims for the foreign households' risk hedging. The foreign country's holdings of home equity is higher than the home country's holdings of foreign equity at the beginning. Increases in home investment implies increases in the demand for labor wage rate and decreases in payouts to shareholders. The home country's holdings of foreign equity increase and the foreign country's holdings of home equity decrease. Net foreign assets for the home country are positive.

LCP Figure 1(b) shows dynamic asset changes for the home country from the home country's positive productivity shock under LCP. Unlike PCP, local currency pricing system leads to the appreciation of the real exchange rate and a trade balance deficit. There's no expenditure switching effects because preset import prices in the local currency don't cause changes in home good prices by the home currency's strongness. Instead, home exporters' revenues in the home currency falls and foreign exporters' revenues in foreign currency rises by changes in markups at given marginal costs. Foreign output rises and consumption falls in the LCP while they are opposite in the PCP. Home households raise the holdings of home bonds with relatively higher real returns than foreign bonds and the holdings of foreign equity with higher real equity returns. Increases in foreign country exporters' income induce foreign households to hold more home and foreign equity but to take short positions in foreign equity by long position of home households. Equity holdings by both countries gradually diminish from increases in investment of both countries. Home country's net foreign assets decline.

Managed exchange rate regime

Figure 2(a) and 2(b) show responses of dynamic portfolios from the home productivity shock under PCP and LCP, respectively, under a managed exchange rate regime. Unlike the flexible exchange rate regime, real effects under both PCP and LCP are mitigated by the managed nominal exchange rate. In both cases, home output and investment increase and home consumption decreases by the home productivity shock. Foreign country output initially increases and decreases gradually. Foreign consumption and investment increase. The effect also leads to a depreciation in the real exchange rate and trade balance surplus. Home imports decrease and home exports increase under a managed exchange rate regime while home imports are very small and exports increase a lot under a flexible exchange rate regime. Decreases in home consumption result from decreases in consumption goods produced by home country and import consumption goods produced by foreign country.

PCP Both real returns on home equity and foreign equity increase similarly. The real return on home bonds increases and the real return on foreign bond decreases. The foreign country has the motivation to hedge real exchange rate risks with higher relative foreign prices. The foreign country holds more home bonds, home equity and foreign equity. The net foreign asset position is negative as home equity holdings by the foreign country rise and foreign equity holdings by the home country fall.

LCP Both real returns on home and foreign bonds and equities increase. The home real return on bonds is higher than the foreign one while real returns on home and foreign equities increase similarly. Foreign country exporters' incomes decline from the depreciation in real exchange rate. Foreign households hold more foreign bonds with higher returns. Home country exporters' incomes increase due to the real exchange rate depreciation. Home households hold more home and foreign equities, which reflects the wealth effect. The net foreign position is positive as foreign equity holdings by the home country rise and home equity holdings by the foreign country fall.

2.7.2 Monetary shock

Flexible exchange rate regime

Figures 3(a) and 3(b) show the response of home country net foreign asset positions to the home contractionary monetary policy shock under the flexible exchange rate regime. Monetary policy affects dynamic portfolios through two channels. Monetary policy has direct impacts on asset returns from nominal interest rate shocks. Monetary policy also has real effects under nominal price rigidity. A domestic contractionary monetary policy that causes 1 percent increase in the home interest rate reduces home output, consumption and investment.

PCP Home contractionary monetary shocks lead to falls in home output, consumption and investment. Foreign output, consumption and investment decrease. The real exchange rate appreciates and home net exports decrease. Real returns on home bonds and equity increase and real returns on foreign bonds and equity decrease. There's no real exchange rate risk since the higher relative price decreases both home exports and imports. Trade deficits depend on the rate of decrease in exports and imports due to both decreases in exports and imports. Home households try to hedge their income risks by holding more home equity. Foreign equity holdings by the home country immediately fall much more than home equity holdings by the foreign country.

LCP Home country's responses to the real macro variables in the LCP are similar to the PCP while foreign country's responses are different since foreign output decreases and consumption and investment increase. The real exchange rate initially appreciates but immediately depreciates and home net exports increase. Home real returns on bonds and equity increase more than foreign real return on bond and equity, respectively. Due to the LCP properties, foreign country exporters experience lower income rather than higher relative export prices. Foreign households hold more home bond with higher real returns to hedge the risk. Home households hold more home equity to hedge their lower labor income risks.

Managed exchange rate regime

Figures 4(a) and 4(b) show the impacts of net asset positions from home contractionary monetary shocks under a managed exchange rate regime. Especially, in the framework of the extension of Taylor type rule, the degree of managed nominal exchange rate is strongly related to asset returns. Nominal bond returns depend on the fluctuations of the nominal exchange rate in the sticky price model. Home contractionary monetary shocks increase not only home real returns on bond and equity but also foreign real returns on bonds and equity under a managed exchange rate regime. As mitigated differences between PCP and LCP under managed exchange rate regime, real effects in both cases are similar, decreases in home and foreign output and investment, increase in home consumption, decrease in foreign consumption, appreciation in the real exchange rate and home trade deficits.

PCP Home country imports increase by lower importing prices. Home households hedge real exchange rate risks initially by holding more home bonds with the higher real returns than foreign bonds and also hedge income risks of lower labor income by holding more home and foreign equities with the same rate of increase. Foreign households sell their home and foreign equities which implies negative net foreign asset positions.

LCP Both home exports and imports decrease. Home households have no motivation to hedge the real exchange rate risk by the relative price differences. Home households try to hedge their income risks by holding more home and foreign equities. Foreign households sell their home and foreign equities for the fixed supply of equities.

2.7.3 Investment shock

Flexible exchange rate regime

Figures 5(a) and 5(b) show the responses to the positive investment shock under the flexible exchange rate regime. The response of the net foreign asset position to the positive investment shock is similar under PCP and LCP. In both cases, improvement in the home investment leads to initial drops in home and foreign real returns on equities.

PCP Home output increases and home consumption initially drops and rebounds gradually. Foreign output, consumption and investment decline. Home net exports initially decrease and increase after as the real exchange rate initially appreciates and depreciates. Initial drops in home net exports result from the increase in importing foreign investments and the decrease in exporting home investments by the improvement of home investments not from the consumption swtiching effect. Therefore, home households have no motivation to hedge real exchange rate risks. As the real exchange rate depreciates, home net exports increase with higher exports of home consumption goods and lower imports of foreign consumption goods. Foreign households hold more home bonds with higher real returns for real exchange rate risk hedging and hold more foreign equity for income risk hedging.

LCP Foreign output increases and foreign consumption and investment decrease while the real effect on the home country is similar that under PCP. Appreciation in the real exchange rate leads to a fall in home exporters' income and a rise in foreign exporters' income under LCP. Home households hold more home equity and foreign households hold more foreign equity.

Managed exchange rate regime

Figures 6(a) and 6(b) show the impacts to shocks in home investment under a managed exchange rate regime. Real effects are similar under PCP and LCP. A positive investment shock leads to increases in home output, consumption and investment, decreases in foreign output, consumption and investment, appreciation in the real exchange rate and trade balance deficits.

PCP The real return on home bonds initially drops more than the real return on foreign bonds and the real return on foreign bonds then rebounds higher than the real return on home bonds. The real return on home equity initially drops and rebounds immediately while the real return on foreign equity increases. Home households hold more foreign bonds with higher real returns to hedge the real exchange rate risk. Households also have more home and foreign equities to invest their extra wealth.

LCP Real returns on home and foreign bonds move as under PCP and real returns on home and foreign equities initially drop and rebound immediately. Decreases in home exporters' incomes induce to have more foreign bond and the wealth effect lead to have more home and foreign equities.

2.7.4 Policy transmission

To investigate the impact on net foreign asset positions for the small country from foreign shocks, country size n is calibrated as 0.76 for the large country and 0.24 for the small country. The large country's export price is determined by producer currency pricing while the small country's export price is determined by local currency pricing. Therefore, consumption and investment goods are traded in the large country's currency in the world market except for domestic goods of the small country. The small country is the foreign country and the large country is the home country. Figures from 7(a) to 9(b) show how the home country's policy changes affect net foreign asset positions for the foreign country under different exchange rate regimes.

Flexible exchange rate regime

Productivity shock

Figure 7(a) shows effects on foreign net foreign asset positions from the home country's positive productivity shock under a flexible exchange rate regime. The home country's positive productivity shock leads to increases in output, consumption and investment. The foreign country's output and investment increase but consumption decreases. Due to the local currency pricing system for the foreign country's exports, the real exchange rate appreciates while the real exchange rate depreciates in the case of producer currency pricing for both countries. The home country's export price increases and the foreign country exporter's revenues increase from appreciation in the real exchange rate because both the home country and foreign country export in the home currency. Foreign households also have the motivation for the lower labor income risk hedge. Foreign households have more home and foreign equities. Therefore, foreign country's net exports increase. The foreign country's net foreign asset position is improved. In the symmetric case, home equity holdings by the foreign country and foreign equity holdings by the home country increase. Net foreign asset positions depend on the rate of increase. In the asymmetric case, home large country takes a short position in the foreign equity and the foreign small country takes a long position in the home equity.

Monetary shock

Figure 8(a) shows the effects on foreign net foreign asset positions of the home country's contractionary monetary policy shock under a flexible exchange rate regime. The home country's contractionary monetary policy shock raises home country real returns on bonds and equity while foreign country real returns on bonds initially drop and rebound and real returns on equity fall. The real exchange rate initially appreciates and then depreciates and home net exports increase. Home consumption and investment exporting goods prices fall from the depreciation in the real exchange rate. Therefore, foreign consumption and investment rise while foreign output falls. Home imports decrease and home exports increase. The home country's trade surplus goes down as the rate of decrease in home imports goes down. Initial drops of the home bond holdings by the foreign country depend on the initial jump in the foreign exporters' income. As the foreign exporters' income decreases, home bond holdings by foreign households rebound to hedge real exchange rate risk. This is comparable to the initial appreciation in the real exchange rate raising home relative prices and inducing home households to hold more home bonds. Changes in home and foreign equities result from both home and foreign households' hedging income risks.

Investment shock

Figure 9(a) shows the effects on foreign net foreign asset positions from the home country's positive investmen shock under a flexible exchange rate regime. Improvements in home investment lead to increases in home output, consumption and investment, trade deficits and appreciation in the real exchange rate. Decreases in home consumption and investment exporting goods and increases in foreign consumption and investment exporting goods from the appreciation in the real exchange rate lead to decreases in foreign consumption and investment. Foreign output increases due to a increase in foreign exporter's revenues under local currency pricing. Home households hold more foreign bonds and foreign households hold more home bonds to hedge the real exchange rate risk. Foreign households hold more home and foreign equities for the hedge of the lower labor income and the foreign equity holding is higher than home equity holding. Home equity holdings by foreign country change a little but foreign equity holdings by home country decrease a lot.

Managed exchange rate regime

Productivity shock

Figure 7(b) shows the effects on foreign net foreign asset positions from the home country's positive productivity shock under the managed exchange rate regime. Home positive productivity shock leads to increases in home output, consumption and investment. Depreciation in the real exchange rate induces home consumption and investment exporting goods to increase. Despite the increase in foreign investment exports, foreign exporters' revenues decrease due to local currency pricing. Therefore, foreign output and investment decrease and foreign consumption increases. This is different from the symmetric case of PCP and LCP in which home output and investment increase and home consumption decrease but foreign output, consumption and investment increase. Foreign households hedge the real exchange rate risk by holding more foreign bonds with higher returns. Home households hold more home and foreign equities through the wealth effect. the home country takes a long position in foreign equity and the foreign country takes a short position in home equity which implies negative net foreign asset position for the foreign country.

Monetary shock

Figure 8(b) shows the effects on foreign net foreign asset positions from a home country's contractionary monetary shock under the managed exchange rate regime. The home contractionary monetary policy shock induces both countries' output, consumption and investment to decrease. Under the managed exchange rate regime, both countries' economies goes into recession. The home country's trade balance worsens because the decrease in exports is greater than the decrease in imports. The home country's real returns on bonds and equity increase and the foreign country's real returns on bonds and equity also increase but by less than home country real returns. Both home and foreign households change their equity positions to hedge the risk of lower labor incomes. Increases in foreign exporters' income lead to change in the holdings of both home and foreign bonds.

Investment shock

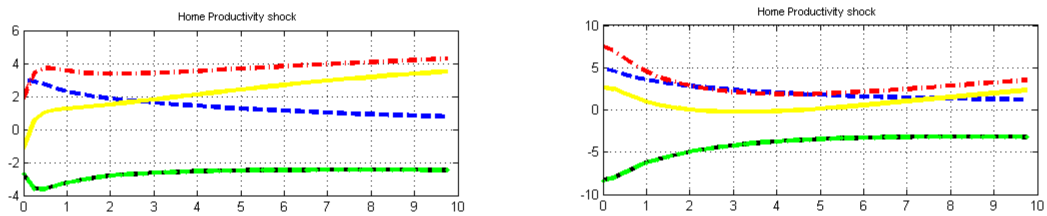
Figure 9(b) shows the effects on foreign net foreign asset positions from a home country's positive investment shock under the managed exchange rate regime. Responses to the investment shock are similar to responses under the flexible exchange rate regime. Improvements in home investment efficiency increase home consumption and investment goods produced by the home country and imported from the foreign country. Foreign consumption and investment decreases by falls in both foreign consumption and investment goods produced by the foreign country and imported from the home country. Foreign output increases because increases in exporting goods to the home country are greater than decreases in goods produced by foreign country. As the real exchange rate turns from appreciation to depreciation, foreign exporters' income rises then falls. Foreign households hedge the real exchange rate risk by switching home and foreign bond holdings. The home country also hedges the real exchange rate risk by switching home and foreign bond holdings. Foreign households hold more home and foreign equities to hedge the lower labor income risks. The home positive investment shock improves the foreign trade balance and net foreign asset positions.

2.8 Conclusions

In this chapter, I compare the effects on dynamic portfolio choices from productivity shocks, monetary policy shocks and investment shocks with three types of export price setting systems under the flexible and the managed exchange rate regimes. Each country's households choose financial assets to hedge the source of risks affecting their income stream. Their exposures to risks depend on the different types of shocks, the different price setting systems and the different exchange rate regimes. It is important to understand how bonds and equities are related to the hedging motivation. I find that bonds are a good hedge for real exchange rate risks affecting their purchasing power for higher relative prices and equities are a good hedge for labor income risks and a good asset investing extra wealth. For the symmetric case, I examine a two-country open macroeconomic model with nominal rigidity, producer currency pricing and local currency pricing system for exporting goods, capital accumulation, an extension of Taylor-type monetary rules and incomplete asset markets. For the asymmetric case, I add dollar dominant pricing system for exporting goods and country size for distinction between large and small open economies in the symmetric version. In the symmetric case, capital inflows and outflows move together under the flexible exchange rate regime in which the equilibrium equity is home biased for each country and real returns on bonds and equities are adjusted by nominal exchange rate fluctuations. Under the managed exchange rate regime, capital inflows and outflows move in the opposite direction caused by the diversification of the equilibrium home and foreign equities for each country and the same rate of increases of real returns on home and foreign equities by low fluctuations in the nominal exchange rate. In the asymmetric case, the large country exports goods under PCP and small country exports goods under LCP. Responses of real variables to shocks under the flexible exchange rate regime are similar to that in the symmetric case but the effect on dynamic asset and liability holdings is different due to the dollarization in the world market that affects home equity holdings by the foreign country. Further study can be extended to welfare issues with endogenous portfolio choices and optimal monetary policy. Besides the three types of shocks are used in this paper; technology shock, monetary policy shock and investment shock, additional shocks such as those due to changes in fiscal policy, also could be interesting to invest. Empirical studies of dynamic portfolios can apply the structural VAR framework. Bayesian framework can be applied for future work.

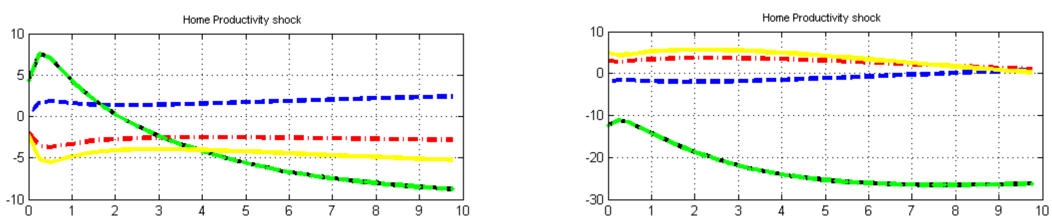
2.9 Figures

Figures show responses of dynamic portfolios to the productivity shock, monetary policy shock and investment shock under flexible and managed exchange rate regimes. Net foreign asset position, NFA_t is marked by yellow solid line. Home bond holdings by foreign country, $\alpha^{*B}_{H,t}$ is marked by black solid line. Home equity holdings by foreign country, $\alpha^{*E}_{H,t}$ is indicated by blue dashed line. Foreign bond by home country, $\alpha^{B}_{F,t}$ is indicated by green dashed line. Foreign equity by home country, $\alpha^{E}_{F,t}$ is marked by red dashed line.



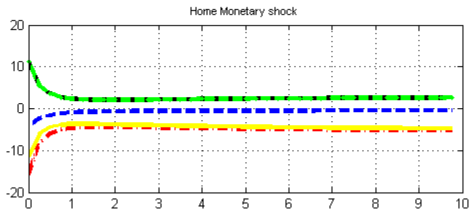
(a) Producer currency pricing (b) Local currency pricing

Figure 2.3: **Home Productivity shock** under flexible exchange rate regime $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed,$\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid

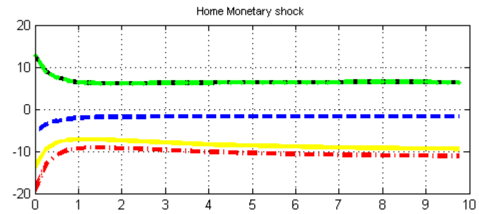


(a) Producer currency pricing (b) Local currency pricing

Figure 2.4: **Home Productivity shock** under managed exchange rate regime ($\phi_s = 0.9$) $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid

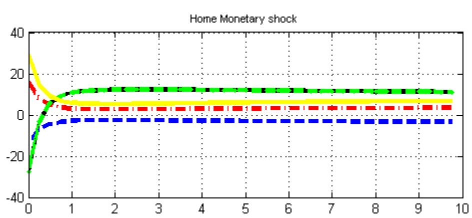


(a) Producer currency pricing

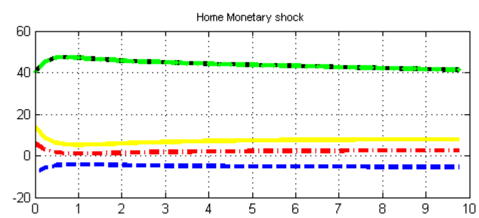


(b) Local currency pricing

Figure 2.5: **Home Monetary policy shock** under flexible exchange rate regime $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid

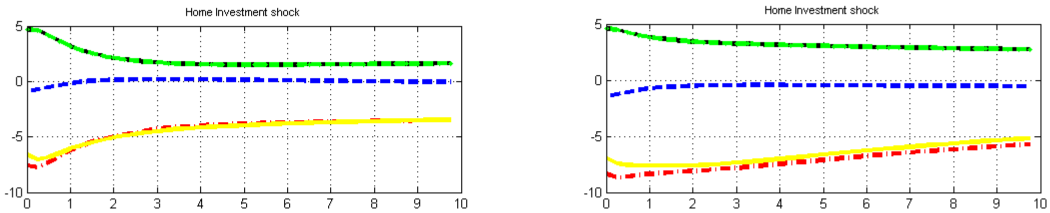


(a) Producer currency pricing



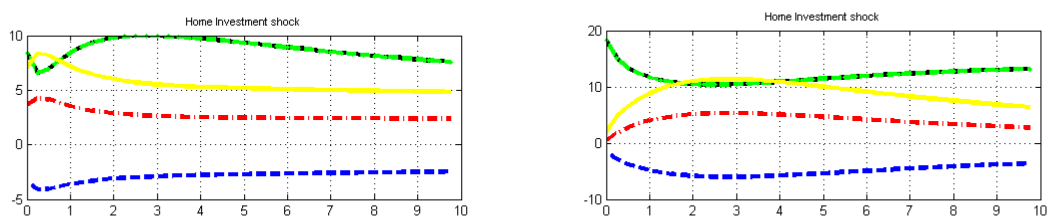
(b) Local currency pricing

Figure 2.6: **Home Monetary policy shock** under managed exchange rate regime ($\phi_s = 0.9$) $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid



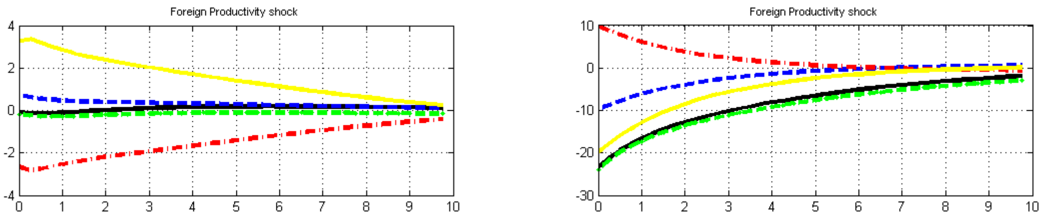
(a) Producer currency pricing (b) Local currency pricing

Figure 2.7: **Home Investment shock** under flexible exchange rate regime $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid



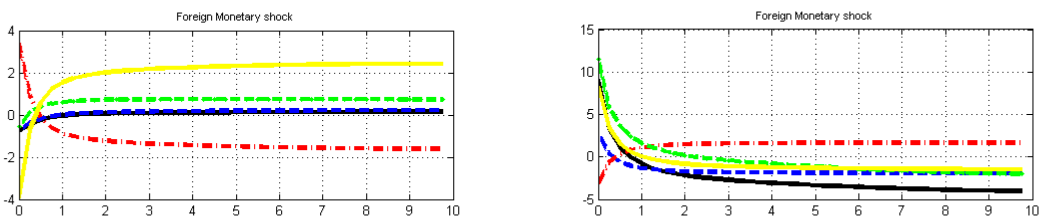
(a) Producer currency pricing (b) Local currency pricing

Figure 2.8: **Home Investment shock** under managed exchange rate regime ($\phi_s = 0.9$) $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid



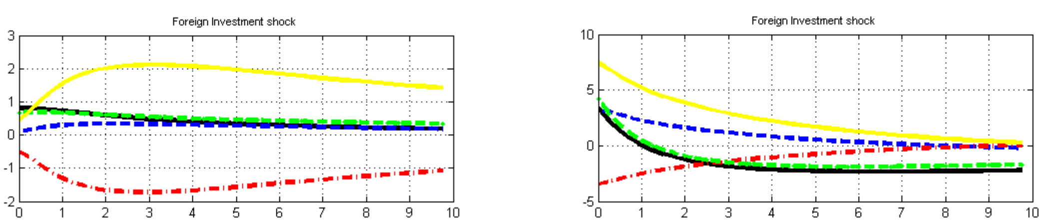
(a) Flexible exchange rate regime (b) Managed exchange rate regime

Figure 2.9: **Productivity shock** $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid



(a) Flexible exchange rate regime (b) Managed exchange rate regime

Figure 2.10: **Monetary shock** $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid

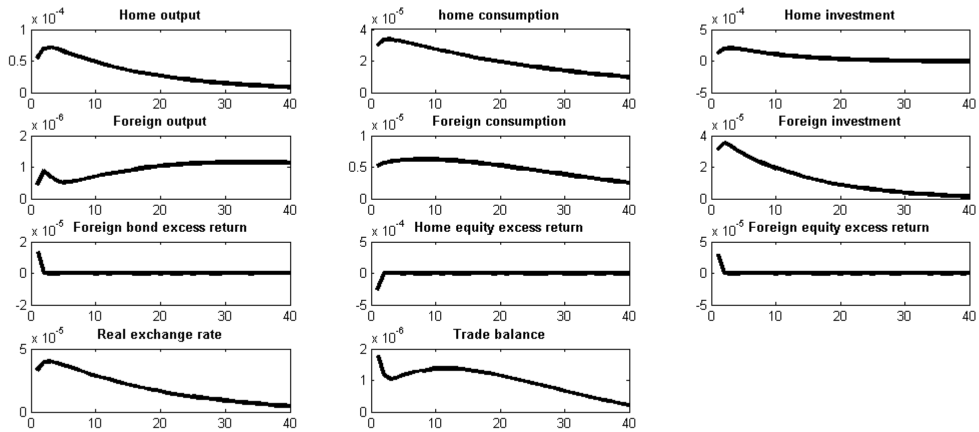


(a) Flexible exchange rate regime (b) Managed exchange rate regime

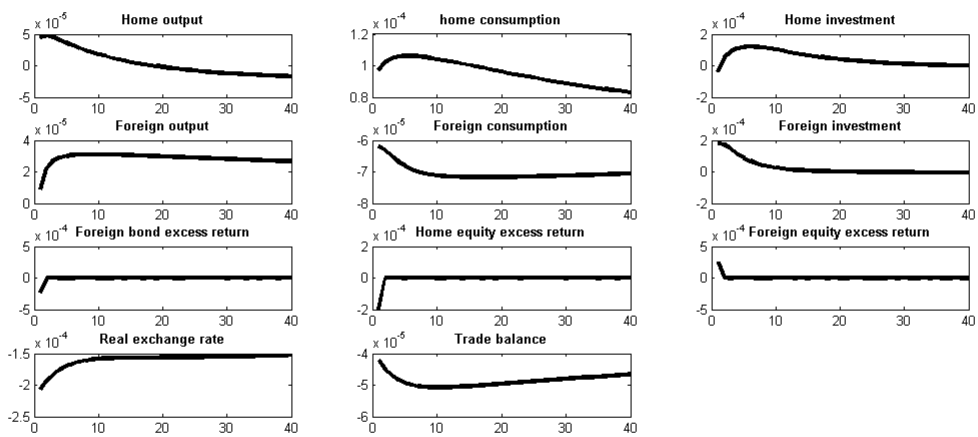
Figure 2.11: **Investment shock** $\alpha^{*B}_{H,t}$ -black solid, $\alpha^{*E}_{H,t}$ -blue dashed, $\alpha^{B}_{F,t}$ -green dashed, $\alpha^{E}_{F,t}$ -red dashed, NFA_t -yellow solid

2.10 Figures

Figures from 2.12 to 2.20 show the impulse responses of real macro variables to the productivity shock, monetary policy shock and investment shock under flexible and managed exchange rate regimes.

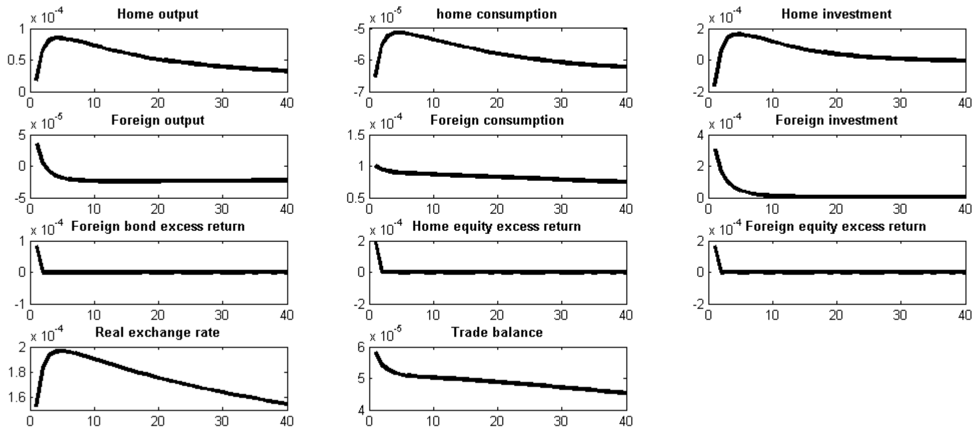


(a) Producer currency pricing

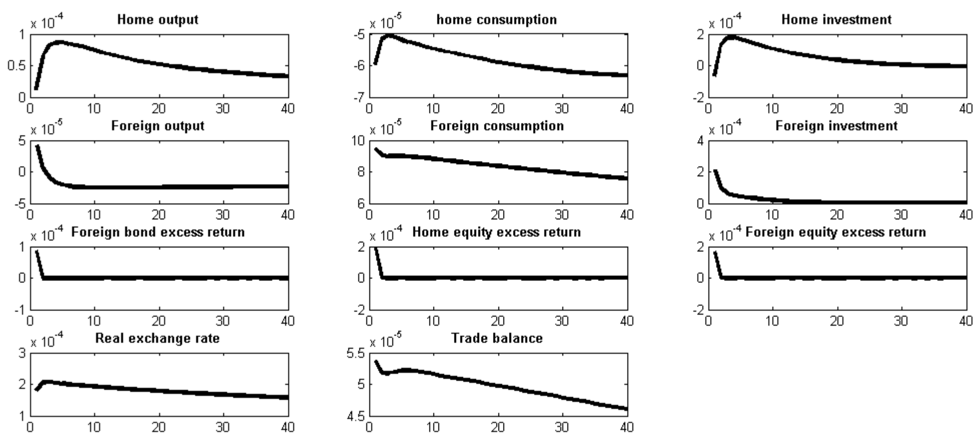


(b) Local currency pricing

Figure 2.12: **Home Productivity shock** under flexible exchange rate regime

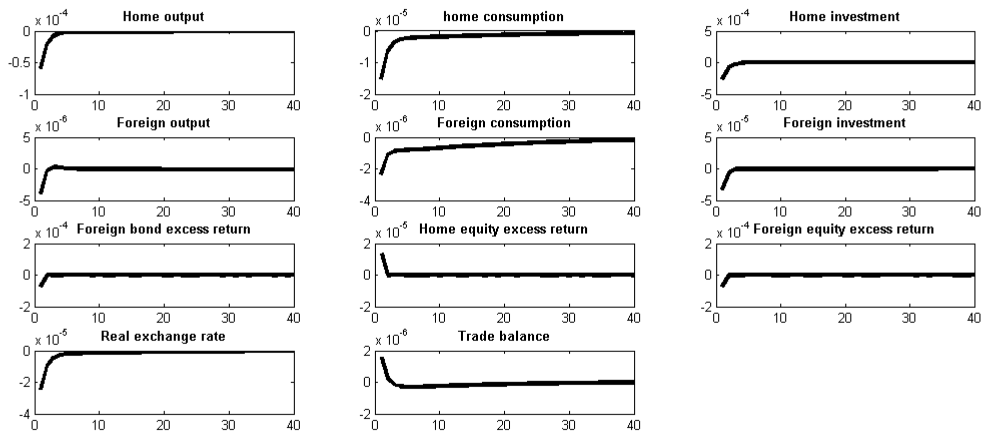


(a) Producer currency pricing

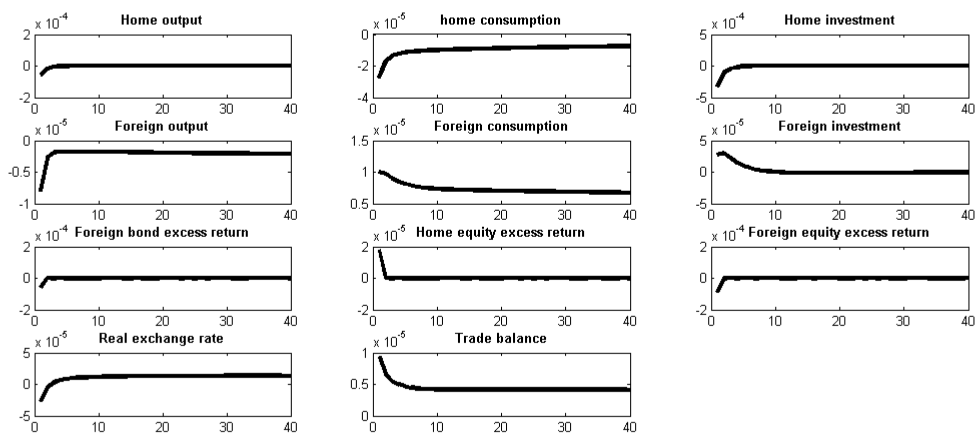


(b) Local currency pricing

Figure 2.13: **Home Productivity shock** under managed exchange rate regime

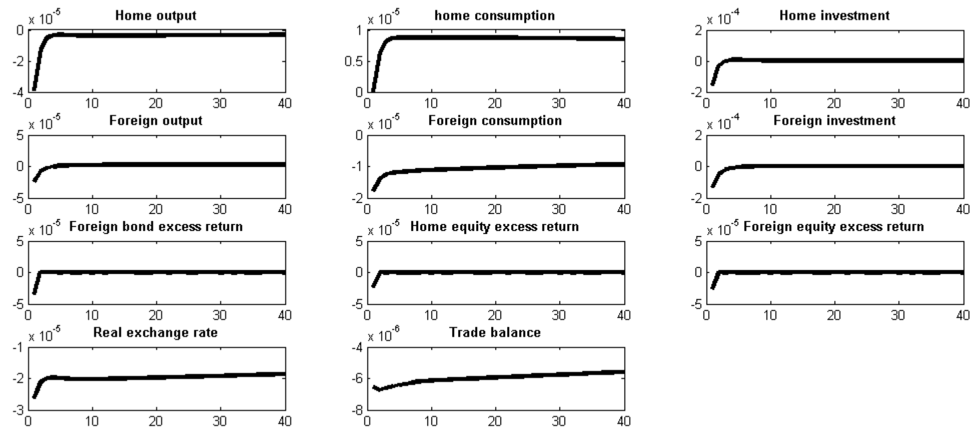


(a) Producer currency pricing

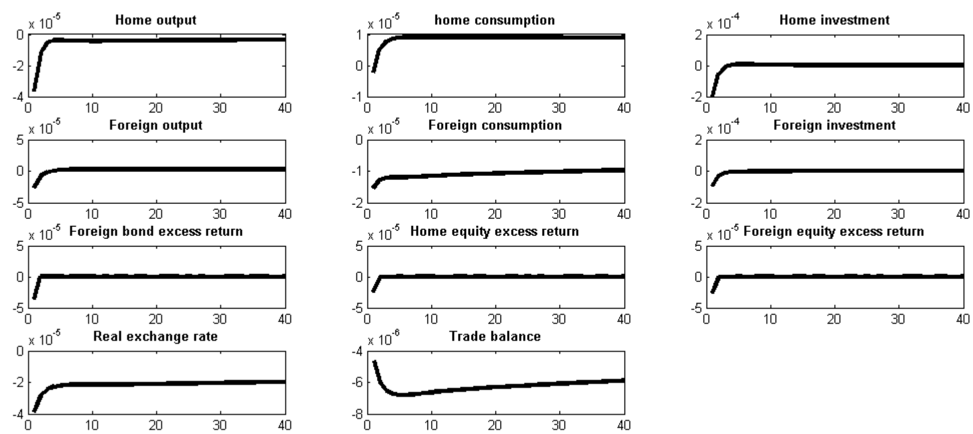


(b) Local currency pricing

Figure 2.14: **Home Monetary policy shock** under flexible exchange rate regime

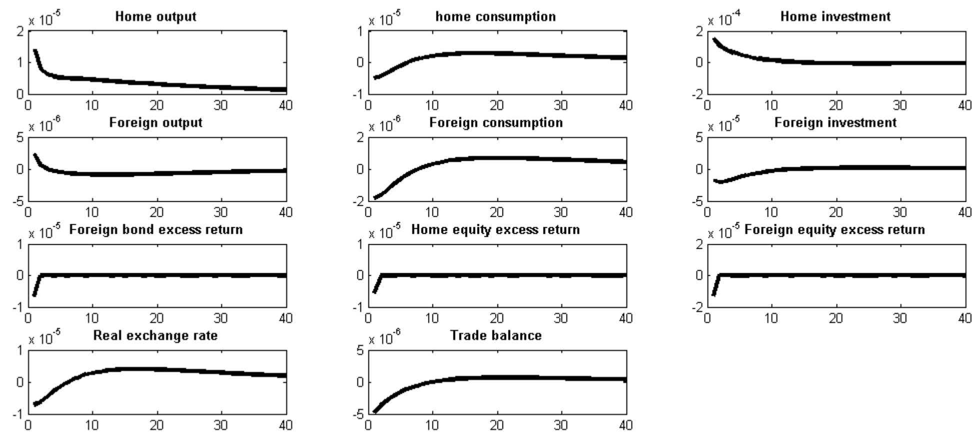


(a) Producer currency pricing

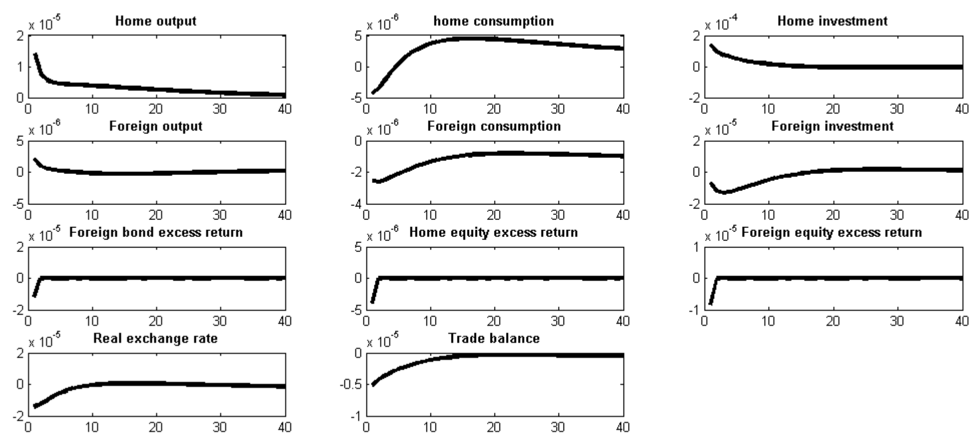


(b) Local currency pricing

Figure 2.15: **Home Monetary policy shock** under managed exchange rate regime

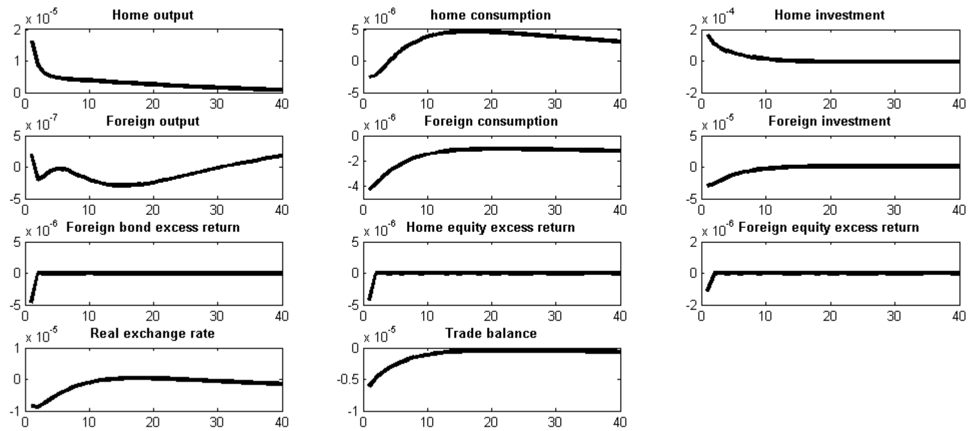


(a) Producer currency pricing

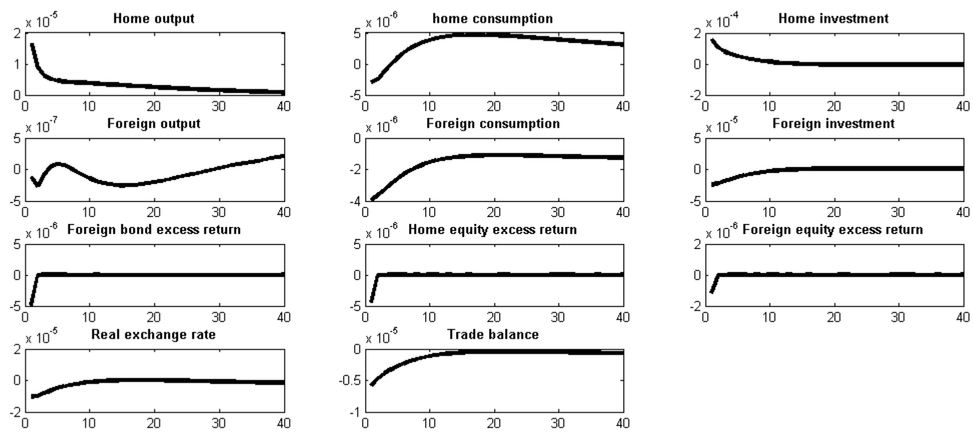


(b) Local currency pricing

Figure 2.16: **Home Investment shock** under flexible exchange rate regime

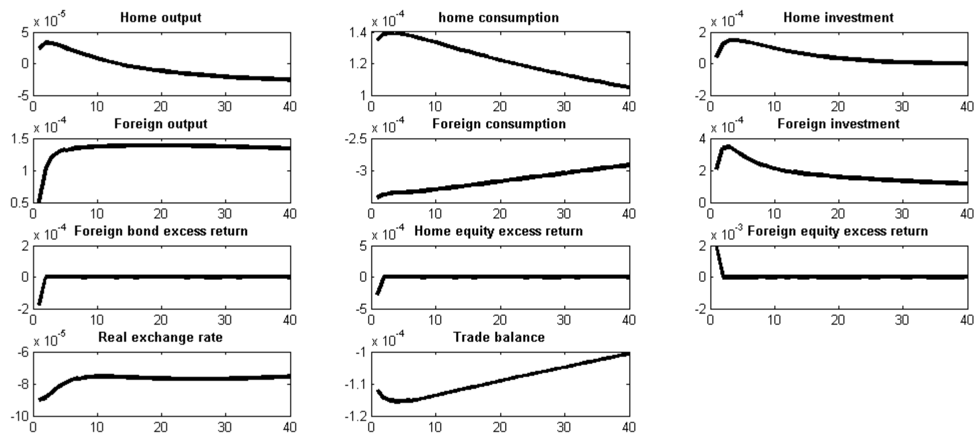


(a) Producer currency pricing

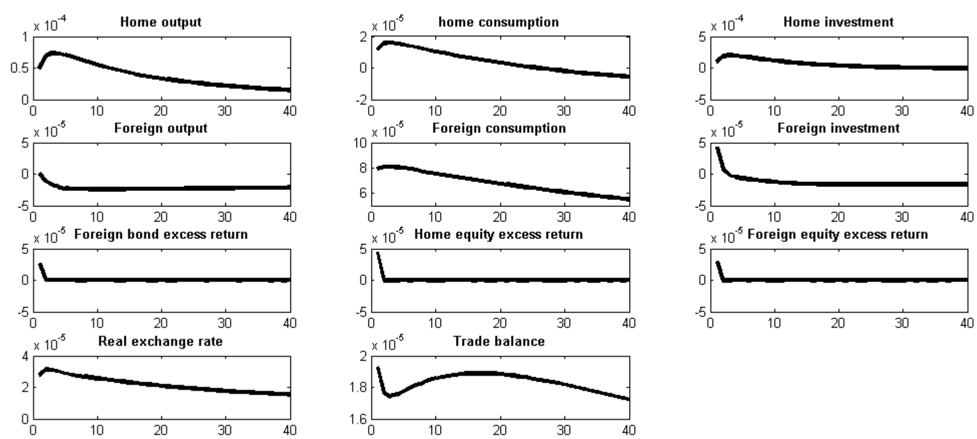


(b) Local currency pricing

Figure 2.17: **Home Investment shock** under managed exchange rate regime

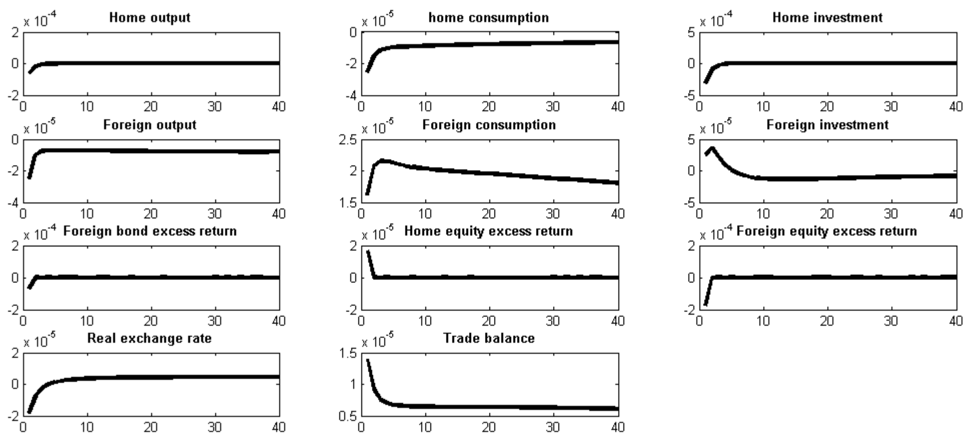


(a) Flexible exchange rate regime

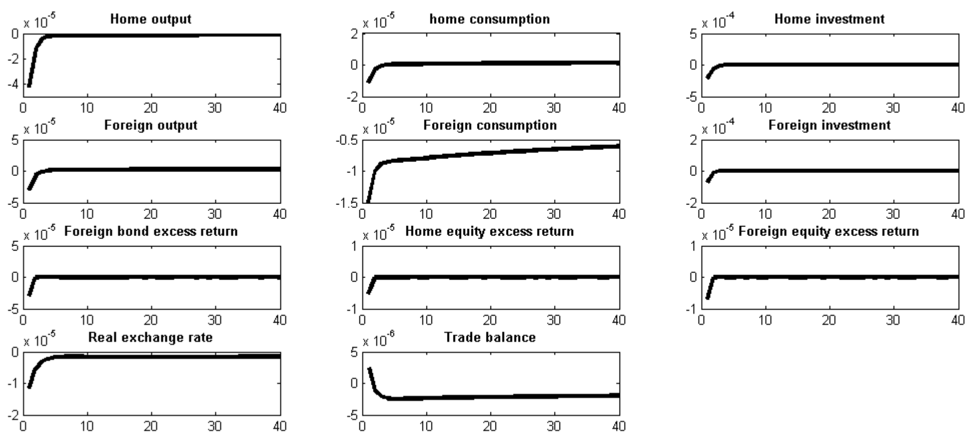


(b) Managed exchange rate regime

Figure 2.18: **Productivity shock** home country size = 0.24, foreign country size = 0.76

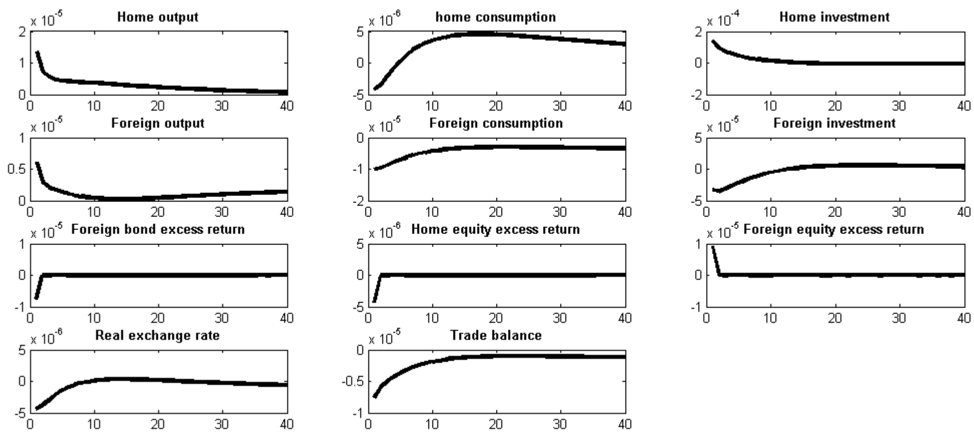


(a) Flexible exchange rate regime

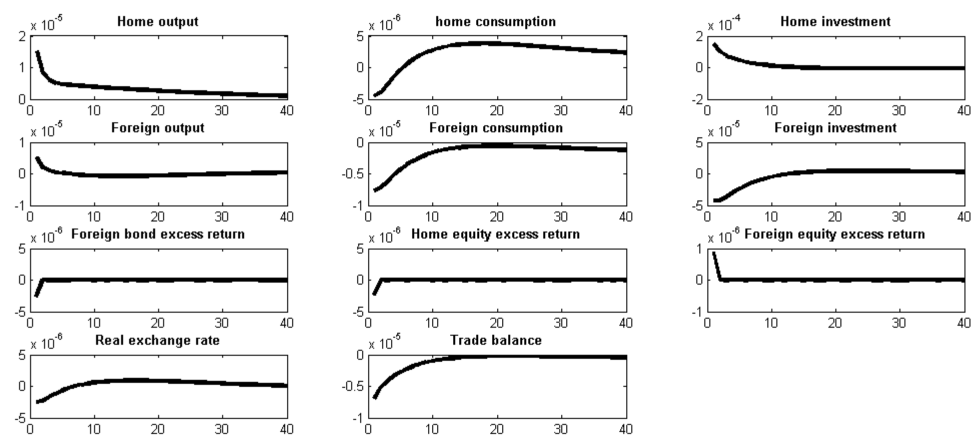


(b) Managed exchange rate regime

Figure 2.19: **Monetary shock** home country size = 0.24, foreign country size = 0.76



(a) Flexible exchange rate regime



(b) Managed exchange rate regime

Figure 2.20: **Investment shock** home country size = 0.24, foreign country size = 0.76

Chapter 3

Optimal Monetary Policy

3.1 Introduction

This chapter provides a welfare based evaluation of the nominal exchange rate stabilization in an open economy with an endogenous portfolio decision. Friedman (1953) argued that a freely floating exchange rate regime is required for optimal monetary policy. If differences in relative prices between two countries are immediately adjusted under flexible price setting, the exchange rate regime is irrelevant for optimality. On the other hand, the freely floating exchange rate regime is required to adjust relative prices under sluggish price setting. The instantaneous movement of the exchange rate has real impacts called the expenditure switching effect. Devereux and Engel (2003) throw doubt on this conventional wisdom that the flexible exchange rate is necessary for optimal monetary policy. They incorporate local currency pricing that characterize the imperfect exchange rate pass-through based on the work of Obstfeld and Rogoff (1995, 1998). They find that a fixed exchange rate regime is the optimal monetary policy in the local currency pricing model. Unlike the producer currency pricing, the flexible exchange rate regime plays no role in the optimal monetary policy under LCP, which produces no expenditure switching effect. However, they assume a money supply rule and rule out the redistribution of the dynamic wealth effects without an endogenous portfolio problem. Similarly, Monacelli (2005) allows for local currency pricing that alters the optimal monetary policy in a small open economy. The incomplete exchange rate pass-through generates a trade-off in the stabilization of inflation and the output gap. Justiniano and Preston (2009) analyze the optimal policy in the small open economy with generalized Taylor rules which respond to nominal exchange rate fluctuations. The result is somewhat different from the papers above in that the stabilization of the nominal exchange rate amplifies the variability in inflation, output and nominal interest rates. Chun et al.(2007) also compare the welfare from alternative monetary policy rules such as a simple Taylor-type interest rule which uses inflation and output gap variables, a general inflation targeting rule which uses inflation, output and exchange rate gap variables, a consumer price index inflation targeting rule, a domestic price index inflation targeting rule and a pegged exchange rate regime. They show that an inflation targeting rule is still desirable in a new Keynesian model with imperfect exchange rate pass through effects. However, the financial constraints such as financial frictions or liability dollarization may yield different results. When liabilities are denominated in dollars but assets are in the domestic currency, sudden nominal depreciations might reduce net worth. Therefore, while in the original framework the effect of monetary policy is amplified indirectly, under liability dollarization there is also a direct channel which further emphasizes the importance of financial frictions for policy design. Palacios-Salguero (2009) develops the choice problem between fixed and flexible exchange rate regimes in a small open economy where the degree of dollarization is endogenous. He calibrates the parameters of the model for Argentina, Canada and Mexico. The specific monetary policy rule is not implemented. Instead, the monetary authority is assumed to supply the amount of money that maintains the pegged value of the nominal exchange rate under the fixed exchange rate regime and constantly maintains the domestic export price under the flexible exchange rate regime. The flexible exchange rate policy is optimal based on the conditional expected value of welfare. He follows the recommendation of Kim et al.(2003) that consider the welfare effects of a transition from a particular initial state to the stochastic steady state induced by the policy.

This chapter explores the optimal exchange rate regime between managed and flexible exchange rate policies by changes in the parameter value of the exchange rate gap variable in the modified Taylor type interest rate rule for the asymmetric two country model. The asymmetric model with the LCP and the PCP for each country's price setting system produces the effect of the dollarization. Furthermore, the parameter value that provides the greatest value of the conditional expected value of welfare can be compared to the calibrated value from the other papers studying a similar framework. Kim and Kim (2008) argue that the second-order terms are necessary for analyzing welfare effects of changes in the environment of the financial market and the conditional welfare measure is required for an accurate measure. The unconditional welfare has no reflection of the welfare effects from one steady state to a new steady state. In this chapter, I compare the flexible exchange rate regime and the managed exchange rate regime by the conditional welfare measure.

3.2 Model

The world economy consists of two countries of different sizes with a continuum of infinitely-lived households. The representative households denoted by the unit interval $j \in [0,1]$ reside in one of two countries, Home (H) and Foreign (F). The interval $j \in [0,n]$ denotes home country households and the interval $j \in [n, 1]$ denotes foreign country households. When the home country is defined as the small country, the parameter n that measures the relative size of the country is close to zero and the small country's policy decisions have negligible impacts on the large country. Both countries have monopolistically competitive firms producing differentiated intermediate goods and perfectly competitive firms producing final goods. The firms are represented by the interval $i \in [0, 1]$. The home and foreign firms are indexed by the interval $i \in [0, n]$ and $i \in [n, 1]$, respectively. The intermediate goods firms set nominal prices in a staggered fashion, as in Rotemberg (1982). The price setting problems for the export market have different assumptions for each country. For the asymmetric case, the small country exporters set prices using the importing market's currency under Local-Currency Pricing (LCP). The large foreign country exporters set their prices in their own currency under Producer-Currency Pricing (PCP).

3.2.1 Consumers

The representative household j maximizes the lifetime utility function

$$U\left(C, H\right) = \frac{{C_t}^{1-\gamma}}{1-\gamma} - \frac{H_t^{1+\omega}}{1+\omega} \tag{3.1}$$

where C_t^j is a composite consumption index and H_t^j denotes hours of labor. The index C_t^j is defined by the CES function

$$C_t^j = \left[(1-\eta)^{\frac{1}{\mu}}\left(C_{H,t}^j\right)^{\frac{\mu-1}{\mu}} + \eta^{\frac{1}{\mu}}\left(C_{F,t}^j\right)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{3.2}$$

where parameter $\eta \in [0, 1]$ inversely represents a degree of home bias in consumption and the parameter $\mu > 0$ measures the elasticity of substitution between domestic and foreign goods. $C_{H,t}^j$ is defined by the CES composite consumption index of domestic goods

$$C_{H,t}^j = \left[\left(\frac{1}{n}\right)^{\frac{1}{\varepsilon}}\left(\int_0^n C_{H,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.3}$$

where $i \in [0, n]$ denotes the variety of the domestic goods. $C_{F,t}^j$ is defined by the CES composite consumption index of goods imported and consumed by domestic households.

$$C_{F,t}^j = \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\varepsilon}}\left(\int_n^1 C_{F,t}^j(i)^{\frac{\varepsilon-1}{\varepsilon}} di\right)\right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{3.4}$$

where $\varepsilon > 1$ measures the elasticity of substitution between varieties produced in the same country. The optimal $C^j_{H,t}(i)$ and $C^j_{F,t}(i)$ of any given levels of $C^j_{H,t}$ and $C^j_{F,t}$ are respectively

$$C^j_{H,t}(i) = \left(\frac{1}{n}\right)\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C^j_{H,t} \tag{3.5}$$

$$C^j_{F,t}(i) = \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C^j_{F,t} \tag{3.6}$$

where $P_{H,t}$ is the domestic currency price index of domestically produced goods and $P_{F,t}$ is the import price index from the foreign country. The corresponding price index is

$$P_t = \left[(1-\eta)\, P_{H,t}^{1-\mu} + \eta P_{F,t}^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{3.7}$$

With

$$P_{H,t} = \left[\frac{1}{n}\left(\int_0^n P^j_{H,t}(i)^{1-\varepsilon} di\right)\right]^{\frac{1}{1-\varepsilon}} \tag{3.8}$$

$$P_{F,t} = \left[\frac{1}{1-n}\left(\int_n^1 P^j_{F,t}(i)^{1-\varepsilon} di\right)\right]^{\frac{1}{1-\varepsilon}} \tag{3.9}$$

where P_t is the Consumer Price Index (CPI) in the model. It follows that $\int_0^n P_{H,t}(i)\, C^j_{H,t}(i)\, di = P_{H,t} C^j_{H,t}$ and $\int_n^1 P_{F,t}(i)\, C^j_{F,t}(i)\, di = P_{F,t} C^j_{F,t}$. The household j's optimal allocation between home and foreign goods is represented by

$$C^j_{H,t} = (1-\eta)\left(\frac{P_{H,t}(i)}{P_t}\right)^{-\mu} C^j_t \tag{3.10}$$

$$C^j_{F,t} = \eta\left(\frac{P_{F,t}(i)}{P_t}\right)^{-\mu} C^j_t \tag{3.11}$$

The aggregate demand functions for the home and foreign good i are represented by

$$C_{H,t}(i) = \frac{1}{n}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \tag{3.12}$$

$$C_{F,t}(i) = \left(\frac{1}{1-n}\right)\left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\varepsilon} C_{F,t} \tag{3.13}$$

Finally, the optimal allocation of expenditures between domestic and foreign goods is given by

$$C_{H,t} = (1-\eta)\left(\frac{P_{H,t}(i)}{P_t}\right)^{-\mu} C_t \tag{3.14}$$

$$C_{F,t} = \eta\left(\frac{P_{F,t}(i)}{P_t}\right)^{-\mu} C_t \tag{3.15}$$

Therefore, the home country's total consumption expenditure is $P_tC_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$, where $P_{H,t}C_{H,t} = \int_0^n P_{H,t}(i)\, C_{H,t}(i)\, di$ and $P_{F,t}C_{F,t} = \int_n^1 P_{F,t}(i)\, C_{F,t}(i)\, di$. Accordingly, the household maximization of the utility function

$$\mathrm{U}(C,H) = E_t \sum_{t=0}^{\infty} \beta_t \left(\frac{C_t^{1-\gamma}}{1-\gamma} - \frac{H_t^{1+\omega}}{1+\omega}\right) \tag{3.16}$$

is subject to a real budget constraint given by

$$b_{H,t} + e_t b_{F,t} + j_t x_{H,t} + e_t j_t^* x_{F,t} = w_t h_t + r_t^k k_t - c_t - \frac{p_{i,t}}{p_t} I_t$$
$$+ r_{H,t} b_{H,t-1} + e_t r_{F,t} b_{F,t-1} + (j_t + d_t) x_{H,t-1} + e_t (j_t^* + d_t^*) x_{F,t-1}$$

The discount factor β_t is endogenously determined by the function of the aggregate home consumption as follows

$$\beta_{t+1} = \beta_t \beta(C_t), \beta_0 = 1 \tag{3.17}$$

where $0 < \beta(C_t) < 1$, $\beta'(C_t) < 0$, and C_t is the aggregate home consumption. Schmitt-Grohe and Uribe (2003) consider five different specifications for stationarity in incomplete asset market models. In this paper, I employ the endogenous discount factor to eliminate the unit root and solve the non-stationarity problem with the functional form of $\beta(C_t)$

$$\beta(C_t) = \tilde{\beta} C_t^{-\nu} \tag{3.18}$$

where $0 \leq \nu < \gamma$ and $0 < \beta C_t^{-\nu} < 1$. Home agents hold two bonds, b_H and b_F, respectively denominated in the home and foreign currency, where the real returns on bonds are $r^b_{H,t}$ and $r^b_{F,t}$ and two equities, x_H and x_F, respectively denominated in the home and foreign currency, where the real returns on equities are $r^e_{H,t}$ and $r^e_{F,t}$ The real exchange rate e_t is defined by the price of the foreign good in terms of the home good as following $e_t = S_t \frac{P_t^*}{P_t}$. S_t is the nominal exchange rate. Real exchange rate appreciation corresponds to a decrease in e_t, while depreciation corresponds to an increase in e_t. By defining net wealth W_t as the differences between gross assets and gross liabilities, i.e. $NFA_t = e_t j_t^* x_{F,t} + e_t b_{F,t} - j_t x^*_{H,t} - b^*_{H,t}$, the budget constraints can be rewritten as

$$\begin{aligned} NFA_t = {} & r^b_{H,t} NFA_{t-1} + w_t h_t + r^k_t k_t - c_t + d_t - \frac{p_{i,t}}{p_t} I_t \\ & + \alpha^b_{F,t-1}(r^b_{F,t} - r^b_{H,t}) - \alpha^e_{H,t-1}(r^e_{H,t} - r^b_{H,t}) + \alpha^e_{F,t-1}(r^e_{F,t} - r^b_{H,t}) \end{aligned}$$

where $r_{x,t}$ is a real excess return on bonds which is defined by

$$r_{x,t} = \begin{bmatrix} (r^b_{F,t} - r^b_{H,t}) & (r^e_{H,t} - r^b_{H,t}) & (r^e_{F,t} - r^b_{H,t}) \end{bmatrix} \tag{3.19}$$

The real returns on bonds and equities are defined by

$$r^b_{H,t} = R_{H,t-1} \frac{P_{t-1}}{P_t} \qquad r^e_{H,t} = \frac{J_t + D_t}{J_{t-1}} \frac{P_t}{P_{t-1}}$$

$$r^b_{F,t} = R_{F,t-1} \frac{P^*_{t-1}}{P^*_t} \frac{e_t}{e_{t-1}} \qquad r^e_{F,t} = \frac{J^*_t + D^*_t}{J^*_{t-1}} \frac{P^*_t}{P^*_{t-1}} \frac{e_t}{e_{t-1}}$$

The real holdings of bonds and equities are defined by

$$\alpha^b_{H,t} = \frac{B_{H,t}}{R_{H,t-1} P_t} \qquad \alpha^e_{H,t} = \frac{J_t x_{H,t}}{P_t}$$

$$\alpha^b_{F,t} = e_t \frac{B_{F,t}}{R_{F,t-1} P^*_t} \qquad \alpha^e_{F,t} = e_t \frac{J^*_t x_{F,t}}{P^*_t}$$

From the first order conditions for bond assets are

$$C_t^{\nu-\gamma} = \beta E_t C_{t+1}^{-\gamma} r_{H,t+1} \tag{3.20}$$

$$\beta E_t \frac{C_{t+1}^{-\gamma}}{C_t^{-\gamma}} \left(r_{H,t+1} - r_{F,t+1}\right) = 0 \tag{3.21}$$

3.2.2 Firms

In each country, a continuum of monopolistically competitive firms produces the differentiated intermediate goods using capital and labor. A standard Cobb-Douglas production function is

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} \tag{3.22}$$

where A_t is a technology shock that follows the AR(1) process

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t \tag{3.23}$$

The law of capital evolution is

$$k_{t+1} = (1-\delta)\, k_t + I_t - \phi\left(I_t, I_{t-1}\right)_t \tag{3.24}$$

where δ is the depreciation rate of capital, I_t is investment and $\phi(I_t, I_{t-1})$ is a measure of investment adjustment costs. I assume a specific functional form for the investment adjustment costs.

$$\phi\left(I_t, I_{t-1}\right) = \frac{\kappa}{2} I_t \left(\frac{I_t}{I_{t-1}} - 1\right)^2 \tag{3.25}$$

where κ is the investment adjustment cost parameter. I assume Rotemberg-style sluggish price adjustment so that a firm sets the price with a quadratic adjustment cost in the inflation rate of the goods. Each firm faces two separate pricing problems, one for the domestic market and one for the export market and the export market has two different pricing scenarios. The optimal price-setting strategy for the domestic market can be approximated by the typical New Keynesian Phillips curve for domestic inflation. The home firm maximizes the present value of expected profits by resetting price $\overline{p}_{H,t}$ for the domestic market. I consider both the PCP and the LCP strategy for the export market. When the exporting goods price is determined by PCP, the home firms set their domestic goods prices for the domestic market. The home firm's nominal profit is

$$\begin{aligned} D_t(i) &= P_{H,t}(i)\, Y_t(i) - w_t h_t(i) - R_t^k k_t(i) \\ &- \frac{\varphi_H}{2} \left[\frac{P_{H,t}(i)}{\bar{\pi}_H P_{H,t-1}(i)} - 1\right]^2 P_{H,t} Y_t \end{aligned} \tag{3.26}$$

The firms real profit maximization can be written as

$$\max_{H_t, K_t, P_{H,t}, P^*_{H,t}} E_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ \frac{P_{H,t}(i)}{P_t} \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t - \frac{w_t h_t(i) - R^k_t k_t(i)}{P_t} - \frac{\varphi_H}{2} \left[\frac{P_{H,t}(i)}{\bar{\pi}_H P_{H,t-1}(i)} - 1 \right]^2 \frac{P_{H,t} Y_t}{P_t}$$

Subject to $Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} Y_t$, firm is output equals its total demands. The foreign firms also set their foreign goods prices for the foreign market. The expected profits of the foreign firm in the foreign market can be written as

$$\max_{H^*_t, K^*_t, P^*_{F,t}} E_t \sum_{t=0}^{\infty} \beta_t \lambda_t \{ \frac{P^*_{F,t}(i)}{P^*_t} \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}} \right)^{-\varepsilon} Y^*_t - \frac{w^*_t h^*_t(i) - R^{k^*}_t k^*_t(i)}{P^*_t} - \frac{\varphi^*_F}{2} \left[\frac{P^*_{F,t}(i)}{\bar{\pi}^*_F P^*_{F,t-1}(i)} - 1 \right]^2 \frac{P^*_{F,t} Y^*_t}{P^*_t} \}$$

subject to the production function, $y^*_t(i) = \left(\frac{P^*_{F,t}(i)}{P^*_{F,t}} \right)^{-\varepsilon} Y^*_t$.

Under PCP, the export price is determined by the domestic goods price and the exchange rate under the law of one price (LOP).

The first order conditions for this problem are given by

$$\lambda_t w_t = (1-\alpha) \zeta_t A_t k^{\alpha}_t h^{-\alpha}_t \tag{3.27}$$

$$\lambda_t r^k_t = \alpha \zeta_t A_t k^{\alpha-1}_t h^{1-\alpha}_t \tag{3.28}$$

$$\zeta_t = \frac{1}{A_t} w^{1-\alpha}_t \left(r^k_t \right)^{\alpha} \left[\alpha^{-1} (1-\alpha)^{-(1-\alpha)} \right] \lambda_t \tag{3.29}$$

where $\varepsilon/(\varepsilon - 1)$ is the constant gross mark-up over marginal cost. With the assumed technology, the firms real marginal cost is

$$mc_t = \frac{1}{A_t} w^{1-\alpha}_t \left(r^k_t \right)^{\alpha} \left[\alpha^{-1} (1-\alpha)^{-(1-\alpha)} \right] \tag{3.30}$$

The optimal rule for setting prices by home firms can be written as

$$\varphi_H\left[\frac{\pi_{H,t}}{\bar{\pi}_H}-1\right]\frac{\pi_{H,t}}{\bar{\pi}_H}=(1-\varepsilon)+\varepsilon\Big[(1-\eta)+\eta\tau_t^{1-\mu}\Big]^{\frac{1}{1-\mu}}mc_t$$
$$+\tilde{\beta}c_t^{-\nu}\varphi_H E_t\left[\frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}}\left[\frac{\pi_{H,t}}{\bar{\pi}_H}-1\right]\frac{\pi_{H,t+1}^2}{\bar{\pi}_H}\frac{y_{t+1}}{y_t}\frac{1}{\pi_{t+1}}\right]^2$$

The optimal rule for setting prices by foreign firms can be written as

$$\varphi_F\left[\frac{\pi_{F,t}^*}{\bar{\pi}_F^*}-1\right]\frac{\pi_{F,t}^*}{\bar{\pi}_F^*}=(1-\varepsilon)+\varepsilon\Big[(1-\eta)+\eta\tau_t^{1-\mu}*\Big]^{\frac{1}{1-\mu}}mc_t^*$$
$$+\tilde{\beta}c_t^*-\nu\varphi_F E_t\left[\frac{c_{t+1}^*-\gamma}{c_t^*-\gamma}\left[\frac{\pi_{F,t}^*}{\bar{\pi}_H}-1\right]\frac{\pi_{F,t+1}^2*}{\bar{\pi}_F^*}\frac{y_{t+1}^*}{y_t^*}\frac{1}{\pi_{t+1}^*}\right]^2$$

Home firms' exporting goods prices are determined by

$$\pi_{H,t}^*\frac{S_t}{S_{t-1}}=\pi_{H,t} \tag{3.31}$$

Foreign firms' exporting goods prices are determined by

$$\pi_{F,t}=\pi_{F,t}^*\frac{S_t}{S_{t-1}} \tag{3.32}$$

The nominal export prices in a producer currency move one-to-one with the fluctuations of the nominal exchange rate, S_t.

When the home firms use the LCP strategy for the export market, they have to set different prices for the domestic market and for the export market. The home firms nominal profit is

$$\begin{aligned}D_t(i) &= P_{H,t}(i)Y_{H,t}(i)+S_tP_{H,t}^*(i)Y_{H,t}^*(i)-w_tH_t(i)-R_t^KK_t(i)\\ &- \frac{\varphi_H}{2}\left[\frac{P_{H,t}(i)}{\bar{\pi}_HP_{H,t-1}(i)}-1\right]^2P_{H,t}Y_{H,t}-\frac{\varphi_H^*}{2}\left[\frac{P_{H,t}^*(i)}{\bar{\pi}_H^*P_{H,t}^*(i)}-1\right]^2S_tP_{H,t}^*Y_{H,t}^*\end{aligned} \tag{3.33}$$

The firms real profit maximization can be written as

$$\max_{H_t,K_t,P_{H,t},P^*_{H,t}} E_t\sum_{t=0}^{\infty}\beta_t\lambda_t\{\frac{P_{H,t}(i)}{P_t}\left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon}Y_{H,t}+\frac{S_tP^*_{H,t}(i)}{P_t}\left(\frac{P^*_{H,t}(i)}{P^*_{H,t}}\right)^{-\varepsilon}Y^*_{H,t}$$
$$-\frac{w_tH_t(i)-R^K_tK_t(i)}{P_t}-\frac{\varphi_H}{2}\left[\frac{P_{H,t}(i)}{\bar{\pi}_HP_{H,t-1}(i)}-1\right]^2\frac{P_{H,t}Y_{H,t}}{P_t}$$
$$-\frac{\varphi^*_H}{2}\left[\frac{P^*_{H,t}(i)}{\bar{\pi}^*_HP^*_{H,t-1}(i)}-1\right]^2\frac{S_tP^*_{H,t}Y^*_{H,t}}{P_t}\}$$

Subject to $y_t(i)=y_{H,t}(i)+y^*_{H,t}(i)$, firm is output equals its total demands. The optimal rule for setting export prices of home firm can be written as

$$\varphi^*_H\left[\frac{\pi^*_{H,t}}{\bar{\pi}^*_H}-1\right]\frac{\pi^*_{H,t}}{\bar{\pi}^*_H}=(1-\varepsilon)+\varepsilon[(1-\eta)\tau^*_t\mu-1+\eta]^{\frac{1}{1-\mu}}\frac{mc_t}{e_t}$$
$$+\tilde{\beta}c_t^{-\nu}\varphi_HE_t\left[\frac{c^*_{t+1}{}^{-\gamma}}{c^*_t{}^{-\gamma}}\left[\frac{\pi^*_{H,t}}{\bar{\pi}^*_H}-1\right]\frac{\pi^*_{H,t+1}2}{\bar{\pi}^*_H}\frac{y^*_{H,t+1}}{y_H,t^*}\frac{e_{t+1}}{e_t}\frac{1}{\pi^*_{t+1}}\right]^2$$

The optimal rule for setting export prices of foreign firm can be written as

$$\varphi_F\left[\frac{\pi_{F,t}}{\bar{\pi}_F}-1\right]\frac{\pi_{F,t}}{\bar{\pi}_F}=(1-\varepsilon)+\varepsilon\Big[(1-\eta)\tau_t^{\mu-1}+\eta\Big]^{\frac{1}{1-\mu}}mc^*_t$$
$$+\tilde{\beta}c^*_t-\nu\varphi_FE_t\left[\frac{c^*_{t+1}{}^{-\gamma}}{c^*_t{}^{-\gamma}}\left[\frac{\pi^*_{F,t}}{\bar{\pi}_H}-1\right]\frac{\pi^2_{F,t+1}}{\bar{\pi}_F}\frac{y_{F,t+1}}{y_F,t}\frac{e_t}{e_{t+1}}\frac{1}{\pi^*_{t+1}}\right]^2$$

3.2.3 Law of one Price

In this paper, the deviation from the law of one price (LOP) is a big difference between LCP and PCP. z_t is the deviation from LOP which is defined by

$$\widehat{z}_t\equiv\widehat{p}^*_{F,t}+\widehat{s}_t-\widehat{p}_{F,t} \tag{3.34}$$

$$\widehat{z}^*_t\equiv\widehat{p}_{H,t}-\widehat{p}^*_{H,t}-\widehat{s}_t \tag{3.35}$$

where s_t is the nominal exchange rate. Under PCP, the law of one price holds when $z_t=0$ and $z^*_t=0$. Under LCP, however, the law of one price doesnt hold and both z_t and z^*_t are not zero.

3.2.4 Monetary Policy and the exchange rate regime

Monetary policy is assumed to follow an extended form of a Taylor-type interest rate rule in which deviations of inflation, output and the nominal exchange rate from the long run targets have a feed back on short run movements of the nominal interest rate. The log linear approximated form is

$$\widehat{R}_{H,t} = \rho_H \widehat{R}_{H,t-1} + (1-\rho_H)\,\phi_\pi \widehat{\pi}_t + (1-\rho_H)\,\phi_y \widehat{y}_t + (1-\rho_H)\,\phi_s \widehat{s}_t + \zeta_{m,t} \tag{3.36}$$

where $\zeta_{m,t}$ is monetary policy shock. The rule describes an economy in which monetary policy is constrained by a managed or fixed exchange rate regime. The parameter allows the monetary authority to smooth changes in the interest rate. The parameter $(1-\rho_H)\,\phi_\pi$ and $(1-\rho_H)\,\phi_y$ reflect the responses to the changes in the CPI inflation rate and real GDP, respectively. The parameter $(1-\rho_H)\,\phi_s$ reflects the degree of the managed exchange rate between fixed and floating regimes. A floating exchange rate regime implies $\phi_s = 0$ while a fixed exchange rate implies $\phi_s = \infty$.

3.2.5 Market clearing conditions

Goods market clearing conditions for the producer currency pricing for the producer currency pricing for the home country is

$$Y_t^* = C_{H,t} + C_{H,t}^* + I_{H,t} + I_{H,t}^* - \frac{\varphi_H}{2}\left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right]^2 Y_t \tag{3.37}$$

Goods market clearing conditions for the producer currency pricing for the foreign country is

$$Y_t^* = C_{F,t}^* + C_{F,t} + I_{F,t}^* + I_{F,t} - \frac{\varphi_F^*}{2}\left[\frac{\pi_{F,t}^*}{\bar{\pi}_F^*} - 1\right]^2 Y_t^* \tag{3.38}$$

Goods market clearing conditions for the local currency pricing for the domestic country are

$$Y_{H,t} = C_{H,t} + I_{H,t} - \frac{\varphi_H}{2}\left[\frac{\pi_{H,t}}{\bar{\pi}_H} - 1\right]^2 Y_{H,t} \tag{3.39}$$

$$Y_{H,t}^* = C_{H,t}^* + I_{H,t}^* - \frac{\varphi_H^*}{2}\left[\frac{\pi_{H,t}^*}{\bar{\pi}_H^*} - 1\right]^2 Y_{H,t}^* \tag{3.40}$$

$$Y_t = Y_{H,t} + Y_{H,t}^* \tag{3.41}$$

Domestic output Y_t equals the sum of aggregate domestic demand, $Y_{H,t}$ and aggregate export demand, $Y^*_{H,t}$. Goods market clearing conditions for the local currency pricing for the foreign country are

$$Y_{F,t} = C_{F,t} + I_{F,t} - \frac{\varphi_F}{2}\left[\frac{\pi_{F,t}}{\bar{\pi}_F} - 1\right]^2 Y_{F,t} \tag{3.42}$$

$$Y^*_{F,t} = C^*_{F,t} + I^*_{F,t} - \frac{\varphi^*_F}{2}\left[\frac{\pi^*_{F,t}}{\bar{\pi}^*_F} - 1\right]^2 Y^*_{F,t} \tag{3.43}$$

$$Y_t = Y_{F,t} + Y^*_{F,t} \tag{3.44}$$

Foreign output Y_t equals the sum of aggregate domestic demand, $Y^*_{F,t}$ and aggregate export demand, $Y_{F,t}$.
Bond market clearing conditions are

$$b_{H,t} + b^*_{H,t} = 0 \tag{3.45}$$

$$b^*_{F,t} + b_{F,t} = 0 \tag{3.46}$$

The bond assets $b_{H,t}$ and $b_{F,t}$ are issued in the home and foreign currencies, respectively. The bond holdings by foreign agents are denoted by an asterisk (*).
Equity market clearing conditions are

$$x_{H,t} + x^*_{H,t} = 1 \tag{3.47}$$

$$x^*_{F,t} + x_{F,t} = 1 \tag{3.48}$$

The equity assets $x_{H,t}$ and $x_{F,t}$ are issued in the home and foreign currencies, respectively. The equity holdings by foreign agents are denoted by an asterisk (*).

3.3 Welfare

The exchange rate policy is evaluated as the measure of conditional welfare which maximizes the conditional expected utility of the representative household given a particular initial state of the economy. The solution method produces a second order approximation of the value function

V_t.

$$V_t = E_t \left\{ \sum_{s-t}^{\infty} \beta^{s-t} U\left(C_s, H_s\right) \right\} \tag{3.49}$$

The law of motion for the conditional expected utility is following

$$V_t - \beta E_t \left\{ V_{t+1} \right\} = U\left(C_s, H_s\right) \tag{3.50}$$

The fixed versus floating exchange rate regimes can be characterized by different values of the parameter φ_s. The optimal policy is obtained by maximizing conditional welfare with respect to the choice of φ_s.

3.4 Calibration

In this paper, for the symmetric case, I calibrate the benchmark model using empirical estimates based on US quarterly data. The endogenous discount factor, β is calibrated as 0.99 in the steady sate so that the annual nominal interest rate is 4 percent. The time-varying discount factor depends on the aggregate consumption such as $\tilde{\beta} C_t^{-\nu}$. In the steady state, $\tilde{\beta}$ is calibrated assuming $\nu = 0.01$ and $\tilde{\beta} C^{-\nu} = \beta$. The Frisch labor supply elasticity $1/\omega$ is set equal to 0.33. The consumption home bias $\eta = 0.85$, implying a 15 percent import to GDP ratio and the elasticity of substitution $\mu = 0.9$ following Heathcote and Perri (2002). The capital share in the production function α is set at 0.28. The depreciation rate of capital is 0.025 which is commonly used for US quarterly data, implying a 10 percent annual rate of depreciation. The Rotemberg price adjustment parameter ϕ and the markup ε are 58 and 6, equivalent to setting the Calvo price parameter equal to 0.75 which is about 4 quarters for the average duration of the price adjustment. The capital adjustment parameter is set equal to 4.5. For the monetary policy rule, I adopt the benchmark Taylor rule with partially adjustment calibrated by Canzoneri et. al. (2007) as follows

$$\widehat{R}_{H,t} = \rho_H \widehat{R}_{H,t-1} + \left(1 - \rho_H\right) \phi_\pi \widehat{\pi}_t + \left(1 - \rho_H\right) \phi_y \widehat{y}_t + \left(1 - \rho_H\right) \phi_s \widehat{s}_t + \zeta_{m,t} \tag{3.51}$$

For the foreign country's monetary policy rule, the degree of the response to the nominal exchange rate is negative due to the definition of the exchange rate.

$$\widehat{R}_{F,t} = \rho_F \widehat{R}_{F,t-1} + \left(1 - \rho_F\right) \phi_\pi^* \widehat{\pi}_t^* + \left(1 - \rho_F\right) \phi_y^* \widehat{y}_t^* - \left(1 - \rho_H\right) \phi_s^* \widehat{s}_t + \zeta_{m,t}^* \tag{3.52}$$

Table 3.1: Calibration

Parameter	Interpretation	Value
β	Discount factor	0.99
γ	Risk aversion	2
ω	1/ Elasticity of Labor supply	0.33
η	Consumption home bias	0.85
η^i	Investment home bias	0.75
μ	Subst. Elasticity for Consumption Home and Foreign	0.9
μ^i	Subst. Elasticity for Investment Home and Foreign	1.2
α	capital share in production	0.28
δ	Depreciation Rate	0.025
φ	Price Adjustment cost	58
κ	Investment Adjustment cost	4.5
ε	Subst. Elasticity for Domestic Varieties	6
ρ_r	Persistence of Interest rate	0.824
ρ_a	Persistence of Productivity shock	0.923
ρ_m	Persistence of Monetary shock	0
ρ_i	Persistence of Investment shock	0.85
ϕ_y	Response to Output gap	0.184
ϕ_π	Response to Inflation	2.02
ϕ_s	Response to Exchange rate	0
σ_A	Std of Productivity shock	0.000861
σ_M	Std of Monetary shock	0.000245
σ_I	Std of Investment shock	0.00036
n	Country Size	0.5

For the asymmetric case, the size of home country changes between interval [0.2, 0.83]. In the case $n < 0.5$, the home country is smaller than the foreign country. When the country size is 0.2, the GDP of the home country is 25 percent of the foreign country's while the home country's GDP is almost 5 times as much as the foreign country's when the country size is 0.83.

3.5 Results

In order to study the optimal monetary policy, I compare the welfare between the free floating exchange rate regime and the managed exchange rate regime through the sensitivity of the degree of the pegged exchange rate. Figure 3.1 shows changes in welfare, ranging from 0 to 0.9.

Table 3.2: Country Size

Country size (n)	GDP Ratio	Foreign Cons. Home bias (ν)
0.2	0.25	0.99
0.27	0.37	0.97
0.34	0.515	0.94
0.41	0.694	0.91
0.48	0.923	0.86
0.5	1	0.85
0.55	1.22	0.80
0.62	1.63	0.72
0.69	2.22	0.60
0.76	3.166	0.42
0.83	4.88	0.074

Productivity shock

Under the flexible exchange regime, a positive home productivity shock increases the home and foreign interest rates. The real exchange rate depreciates leading to the trade balance surplus. Under the managed exchange rate regime, a positive home productivity shock increases home real interest rate and decreases the foreign real interest rate. The foreign country has the motives to hedge the real exchange rate risk and the labor income risk. The cost of the diversification of financial assets dominates the benefit so there is no incentive for hedging risks.

Monetary Policy shock

Under the flexible exchange regime, a home contractionary monetary policy shock increases the home interest rate and decreases the foreign interest rate. The value of the domestic currency deposit is higher relative to the value of the foreign currency deposit and so the real exchange rate appreciates leading to the trade balance deficit. The wealth from the foreign equity decreases due to the lower real return on equity. Under the managed exchange rate regime, a home contractionary monetary policy shock increases both home and foreign interest rates. The hedge for the real exchange rate risk is paid as the cost for the lower fluctuation of the exchange rate. Home households hold more home and foreign equities. The diversification of financial assets increases home wealth.

Investment shock

Under the flexible exchange regime, a positive home investment shock increases the home and foreign real interest rates after initial drops. The real exchange rate initially appreciates and then depreciates leading to an initial trade balance deficit and then a trade balance surplus. Under the managed exchange rate regime, a positive home investment shock increases both home and foreign interest rates. In both cases, the wealth effect is dominant in the home country. The diversification of financial assets increases home wealth. The managed exchange rate regime is better than the flexible exchange rate regime under the investment shock. As the degree of managed exchange rate increases, the welfare decreases by the higher cost of the lower exchange rate fluctuation.

3.6 Conclusion

The welfare effects depend on the type of shocks. When the country experiences the positive productivity shock, the flexible exchange rate regime generates higher welfare than the managed exchange rate regime. Due to the small exposure of risks to home households, it is unnecessary to pay the cost of the diversification of financial assets caused by managing fluctuations of the nominal exchange rate. On the other hand, the monetary policy shock and the investment shock provide different welfare effects. The home contractionary monetary policy shock raises not only the home real interest rate but also the foreign real interest rate under the managed exchange rate regime while the foreign interest rate falls under the flexible exchange rate regime. The increased wealth from foreign assets hedges the real exchange rate risk and dominates the cost of the diversification of financial assets. The home country has the wealth effect by the positive investment shock rather than motives to hedge risks under both the flexible and the managed exchange rate regimes. Diversified financial assets create more wealth and offer the incentive to pay the cost of the lower fluctuation of the exchange rate.

3.7 Figure

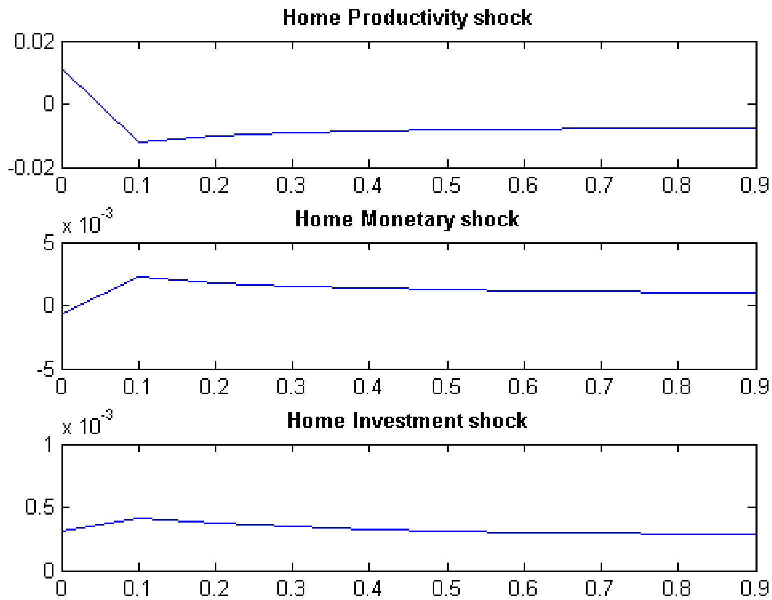


Figure 3.1: **Sensitivity of Welfare by degree of pegged exchange rate**

REFERENCES

[1] Baxter, M. and Jermann, U.J. (1997) *The International Diversification Puzzle Is Worse than You think.* American Economic Review 87, 170-180, March

[2] Bui, T.(2009) *International Asset Portfolios in the New Open Economy Macroeconomics model.* Job Market paper.

[3] Christoffel, K. and Coenen, G. and Warne, A. (2008) *The New Area-Wide Model of the euro area: a micro-founded open-economy model for forecasting and policy analysis.* European Central Bank working paper series.

[4] Coenen, G. and McAdam, P. and Straub, R. (2008) *Tax reform and labour-market performance in the euro area: A simulation-based analysis using the New Area-Wide Model.* Journal of Economic Dynamics and Control, Elsevier, vol. 32(8), pages 2543-2583, August.

[5] Civelli, A.(2008) *International Real Business Cycles, Portfolio Diversification and Valuation Effects.* Job Market paper.

[6] Coeurdacier, N., Kollmann, R. and Martin, P. (2010) *International portfolios, capital accumulation and foreign assets dynamics.* Journal of International Economics, Elsevier, vol. 80(1), pages 100-112, January.

[7] Coeurdacier, N. and Rey,H. (2013) *Home Bias in Open Economy Financial Macroeconomics.* Journal of Economic Literature, American Economic Association, vol. 51(1), pages 63-115, March.

[8] Corsetti, G. (2007) *New Open Economy Macroeconomics.* CEPR Discussion Papers 6578.

[9] Devereux, M. and Engel, C. (2003) *Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility.* Review of Economic Studies, Oxford University Press, vol. 70(4), pages 765-783.

[10] Devereux, M. and Sutherland, A.(2007) *Monetary policy and Portfolio Choice in an Open Economy Macro Model.* Journal of European Economic Association, MIT Press, vol. 5(2-3), pages 491-499,April-May.

[11] Devereux, M. and Sutherland, A.(2011) *Country Portfolios in Open Economy Macro Models.* Journal of European Economic Association, vol. 9(2), pages 337-369, April.

[12] Devereux, M. and Sutherland, A.(2010) *Country Portfolio Dynamics.* Journal of Economic Dynamics & Control, Elsevier, vol. 34(7), pages 1325-1342, July.

[13] Engel,C and A. Matsumoto,(2005) *Portfolio Choice in a Monetary Open Economy DSGE Model.* NBER Working Papers 12214.

[14] Friedman, M. (1953) *The Case for Flexible Exchange Rates.* In Essays in Positive Economics, University of Chicago Press, pages 157-203.

[15] Heathcote, J. and F.Perri (2002) *Financial Autarky and International Business Cycles.* Journal of Monetary Economics 49, 601-627, April.

[16] Heathcote, J. and F.Perri (2013) *The International Diversification Puzzle is Not As Bad As You think.* Journal of Political Economy, University of Chicago Press, vol. 121(6), pages 1108 - 1159, December.

[17] Justiniano, A. and Preston, B. (2009) *Monetary policy and uncertainty in an empirical small open economy model.* Working Paper Series WP-09-21, Federal Reserve Bank of Chicago.

[18] Keen, B. and Wang, Y. (2007) *What is a realistic value for price adjustment costs in New Keynesian models?* Applied Economics Letters, 14, 789-793, Month.

[19] Kim, J. and Kim, S.H. (2008) *Calculating and using second-order accurate solutions of discrete time dynamic equilibrium models.* Journal of Economic Dynamics & Control, vol, pp, month.

[20] Kim, S.H. and Kim, J. (2003) *Welfare Effects of Tax Policy in Open Economies: Stabilization and Cooperation.* Computing in Economics and Finance 2003 259, Society for Computational Economics.

[21] Kim, S. and Yang, D. (2009) *International Monetary Transmission and Exchange Rate Regimes: Floaters vs. Non-Floaters.* Asian Development Bank Institute working paper series.

[22] Kollmann, R. (2006) *International Portfolio Equilibrium and the Current Account.* CEPR Discussion Paper 3819.

[23] Lane, P. and Milesi-Ferretti, G.M. (2007) *The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities 1970-2004.* Journal of International Economics, Elsevier, vol. 73(2), pages 223-250, November.

[24] Lombardo, G. and Sutherland, A. (2007) *Computing second-order-accurate solutions for rational expectation models using linear solution methods.* Journal of Economic Dynamics and Control, Elsevier, vol. 31(2), pages 515-530, February.

[25] Monacelli, T. (2004) *Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy.* Journal of International Economics, vol 62(1), Pages 191217, January

[26] Monacelli, T. (2005) *Monetary Policy in a Low Pass-through Environment.* Journal of Money, Credit and Banking, 37, Pages 10471066.

[27] Mussa, M. (1986) *Nominal exchange rate regimes and the behavior of real exchange rates: Evidence and implications.* Carnegie-Rochester Conference Series on Public Policy, 36, pages 163-209.

[28] Obstfeld, M. and Rogoff, K. (1995) *Exchange Rate Dynamic Redux.* Journal of Political Economy,103, pages 624-660.

[29] Obstfeld, M. and Rogoff, K. (1998) *Risk and Exchange Rates.* Working paper No.6694, National Bureau of Economic Research.

[30] Rahbari, E. (2009) *Portfolios and Business Cycles in an Open Economy.* Job market paper.

[31] Rotemberg, J.J. (1982) *Sticky Prices in the United States.* Journal of Political Economy 90, 1187-1211, month.

[32] Schmitt-Grohe, S. and Uribe, M. (2003) *Closing small open economy models.* Journal of International Economics, Elsevier, vol. 61(1), pages 163-185, October.

[33] Sims, Christopher A. (2002) *Solving Linear Rational Expectations Models.* Computational Economics, Society for Computational Economics, vol. 20(1-2), pages 1-20, October.

[34] Stone, M. and Bhundia, R. (2004) *A New Taxonomy of Monetary Regimes.* International Monetary Fund working paper.

[35] Sutherland, A. (2002) *A Simple Second-Order Solution Method For Dynamic General Equilibrium Models.* CEPR Discussion Papers 3554, C.E.P.R. Discussion Papers.

[36] Tille, C. and Wincoop, E. (2010). *International capital flows.* Journal of International Economics, Elsevier, vol. 80(2), pages 157-175, March.

APPENDICES

Appendix A

Equations

Steady state relations

$$\bar{r}_H = \bar{r}_F = \frac{1}{\beta}$$

$$\overline{mc} = \frac{\varepsilon - 1}{\varepsilon}$$

$$\frac{\bar{w}}{\bar{P}} = \left[\frac{\varepsilon - 1}{\varepsilon} \left(\frac{\bar{R}^k}{\bar{P}} \right)^{-\alpha} \left(\alpha^{\alpha} (1-\alpha)^{1-\alpha} \right) \right]^{\frac{1}{1-\alpha}}$$

$$\frac{\bar{R}^k}{\bar{P}} = \frac{1}{\beta} - (1 - \delta)$$

$$\frac{\bar{h}}{\bar{y}} = (1-\alpha) \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\bar{w}}{\bar{P}} \right)^{-1}$$

$$\frac{\bar{k}}{\bar{y}} = (\alpha) \left(\frac{\varepsilon - 1}{\varepsilon} \right) \left(\frac{\bar{R}^k}{\bar{P}} \right)^{-1}$$

$$\bar{d} = \bar{y} - \overline{wh} - \overline{r^k k}$$

$$\bar{y} = \bar{k}^{\alpha} \bar{h}^{1-\alpha}$$

$$\bar{y}^* = \bar{k}^{\alpha *} \bar{h}^{1-\alpha *}$$

$$\bar{W} = \bar{b}_H + \bar{b}_F = 0$$

$$\phi = log \frac{R^{(1-\rho)}}{y^{(1-\rho)\varphi_y}}$$

A.1 System

Budget constraint

$$F_t = F_{t-1} + e_t P^*_{H,t} NX_t - P_{F,t} IM_t + \xi_t \tag{A.1}$$

Consumption F.O.Cs

$$\lambda_t = c_t^{-\sigma} \tag{A.2}$$

$$\lambda_t^* = c_t^{*-\sigma} \tag{A.3}$$

Euler equations

$$\lambda_t = \beta_t E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \right] R_{H,t} \tag{A.4}$$

$$\lambda_t^* = \beta_t^* E_t \left[\frac{\lambda_{t+1}^*}{\pi_{t+1}^*} \right] R_{F,t} \tag{A.5}$$

$$\lambda_t = \beta_t E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}} \frac{(j_{t+1} + d_{t+1})}{j_t} \right] \tag{A.6}$$

$$\lambda_t^* = \beta_t^* E_t \left[\frac{\lambda_{t+1}^*}{\pi_{t+1}^*} \frac{(j_{t+1}^* + d_{t+1}^*)}{j_t^*} \right] \tag{A.7}$$

Investment F.O.Cs

$$p_t^I = q_t \cdot u_t^I \tag{A.8}$$

$$p_t^{I*} = q_t^* \cdot u_t^{I*} \tag{A.9}$$

Capital Accumulation

$$k_t = (1 - \delta) k_{t-1} + i_t \cdot u_t^I \tag{A.10}$$

$$k_t^* = (1 - \delta) k_{t-1}^* + i_t^* \cdot u_t^{I*} \tag{A.11}$$

Tobin's Q

$$q_t = \beta_t E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \alpha \frac{mc_{t+1} y_{t+1}}{k_{t+1}} + (1-\delta) q_{t+1} \right] \tag{A.12}$$

$$q_t^* = \beta_t^* E_t \left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \alpha \frac{mc_{t+1}^* y_{t+1}^*}{k_{t+1}^*} + (1-\delta) q_{t+1}^* \right] \tag{A.13}$$

Phillips curves (PCP)

$$\begin{aligned} p_{H,t} = & \left(\frac{\omega}{\omega - 1} \right) mc_t - \left(\frac{\chi_p}{\omega - 1} \right) (\pi_{H,t+1} - 1) \pi_{H,t} p_{H,t} \\ & + \left(\frac{\chi_p}{\omega - 1} \right) \beta_t E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \pi_{H,t+1} \frac{mc_{t+1} y_{t+1}}{y_t} p_{H,t+1} \right] \end{aligned} \tag{A.14}$$

$$\begin{aligned} p_{F,t}^* = & \left(\frac{\omega}{\omega - 1} \right) mc_t^* - \left(\frac{\chi_p}{\omega - 1} \right) (\pi_{F,t+1}^* - 1) \pi_{F,t}^* p_{F,t}^* \\ & + \left(\frac{\chi_p}{\omega - 1} \right) \beta_t^* E_t \left[\frac{\lambda_{t+1}^*}{\lambda_t^*} \pi_{F,t+1}^* \frac{mc_{t+1}^* y_{t+1}^*}{y_t^*} p_{F,t+1}^* \right] \end{aligned} \tag{A.15}$$

$$\pi_{F,t} = \pi_{F,t}^* \frac{e_t}{e_{t-1}} \tag{A.16}$$

$$\pi_{H,t}^* \frac{e_t}{e_{t-1}} = \pi_{H,t} \tag{A.17}$$

Union F.O.Cs for wage

$$\lambda_t w_t = h_t^{\psi} \tag{A.18}$$

$$\lambda_t^* w_t^* = h_t^{\psi *} \tag{A.19}$$

Marginal cost

$$w_t = (1-\alpha) mc_t \frac{y_t}{h_t} \tag{A.20}$$

$$w_t^* = (1-\alpha) mc_t^* \frac{y_t^*}{h_t^*} \tag{A.21}$$

Production Function

$$y_t = a_t k_t^{\alpha} h_t^{(1-\alpha)} \tag{A.22}$$

$$y_t^* = a_t^* k_t^{*\alpha} h_t^{*(1-\alpha)} \tag{A.23}$$

Taylor Rules

$$R_{H,t} = \phi R_{H,t-1}^{\rho} \pi_t^{(1-\rho)\varphi_\pi} y_t^{(1-\rho)\varphi_y} \varepsilon_{m,t} \tag{A.24}$$

$$R_{F,t} = \phi R_{F,t-1}^{\rho} \pi_t^{*(1-\rho)\varphi_\pi} y_t^{*(1-\rho)\varphi_y} \varepsilon_{m,t}^* \tag{A.25}$$

Aggregate Resource constraint

$$y_t = c_{H,t} + i_{H,t} + c_{H,t}^* + i_{H,t}^* + 0.5\chi_p y_t (\pi_{H,t} - 1)^2 \tag{A.26}$$

$$y_t^* = c_{F,t} + i_{F,t} + c_{F,t}^* + i_{F,t}^* + 0.5\chi_p y_t^* (\pi_{F,t}^* - 1)^2 \tag{A.27}$$

Consumption final goods

$$c_t = \left[\mu^{c(1/\eta^c)} c_{H,t}^{(\eta^c-1)/\eta^c} + (1-\mu^c)^{1/\eta^c} c_{F,t}^{(\eta^c-1)/\eta^c}\right]^{\eta^c/(\eta^c-1)} \tag{A.28}$$

$$c_t^* = \left[\mu^{c(1/\eta^c}) c_{F,t}^{*(\eta^c-1)/\eta^c} + (1-\mu^c)^{1/\eta^c} c_{H,t}^{*(\eta^c-1)/\eta^c}\right]^{\eta^c/(\eta^c-1)} \tag{A.29}$$

Consumption price index

$$1 = \mu^c p_{H,t}^{(1-\eta^c)} + (1-\mu^c) p_{F,t}^{(1-\eta^c)} \tag{A.30}$$

$$1 = \mu^c p_{F,t}^{*(1-\eta^c)} + (1-\mu^c) p_{H,t}^{*(1-\eta^c)} \tag{A.31}$$

Foreign consumption demand

$$c_{F,t} = (1-\mu^c) p_{F,t}^{-\eta^c} \cdot c_t \tag{A.32}$$

$$c_{H,t}^* = (1-\mu^c) p_{H,t}^{*-\eta^c} \cdot c_t^* \tag{A.33}$$

Investment final goods

$$i_t = \left[\mu^{i(1/\eta^i)} i_{H,t}^{(\eta^i-1)/\eta^i} + (1-\mu^i)^{1/\eta^i} i_{F,t}^{(\eta^i-1)/\eta^i} \right]^{\eta^i/(\eta^i-1)} \tag{A.34}$$

$$i_t^* = \left[\mu^{i(1/\eta^i)} i_{F,t}^{*(\eta^i-1)/\eta^i} + (1-\mu^i)^{1/\eta^i} i_{H,t}^{*(\eta^i-1)/\eta^i} \right]^{\eta^i/(\eta^i-1)} \tag{A.35}$$

Investment price index

$$p_t^{I(1-\eta^i)} = \mu^i p_{H,t}^{(1-\eta^i)} + (1-\mu^i) p_{F,t}^{(1-\eta^i)} \tag{A.36}$$

$$p_t^{I*(1-\eta^i)} = \mu^i p_{F,t}^{*(1-\eta^i)} + (1-\mu^i) p_{H,t}^{*(1-\eta^i)} \tag{A.37}$$

Foreign investment demand

$$i_{F,t} = (1-\mu^i) \left(\frac{p_{F,t}}{p_t^I} \right)^{-\eta^i} \cdot i_t \tag{A.38}$$

$$i^*_{H,t} = (1-\mu^i)\left(\frac{p^*_{H,t}}{p^{I*}_t}\right)^{-\eta^i} \cdot i^*_t \tag{A.39}$$

Imports

$$IM_t = c_{F,t} + i_{F,t} \tag{A.40}$$

$$IM^*_t = c^*_{H,t} + i^*_{H,t} \tag{A.41}$$

Domestic Terms of Trade

$$T_t = \frac{p_{F,t}}{e_t p^*_{H,t}} \tag{A.42}$$

$$T^*_t = \frac{p^*_{H,t} e_t}{p_{F,t}} \tag{A.43}$$

Home inflation relative price link

$$\pi_{H,t} = \pi_t \frac{p_{H,t}}{p_{H,t-1}} \tag{A.44}$$

$$\pi^*_{F,t} = \pi^*_t \frac{p^*_{F,t}}{p^*_{F,t-1}} \tag{A.45}$$

Export Inflation Relative price link

$$\pi^*_{H,t} = \pi_t{}^* \frac{p^*_{H,t}}{p^*_{H,t-1}} \tag{A.46}$$

$$\pi_{F,t} = \pi_t \frac{p_{F,t}}{p_{F,t-1}} \tag{A.47}$$

Real Nominal Exchange rate

$$\frac{e_t}{e_{t-1}} = \frac{s_t}{s_{t-1}} \frac{\pi_t^*}{\pi_t} \tag{A.48}$$

Fisher equation

$$r_{H,t}^B = \frac{R_{H,t}}{\pi_t} \tag{A.49}$$

$$r_{F,t}^B = \frac{R_{F,t} e_t}{\pi_t^* e_{t-1}} \tag{A.50}$$

UIP condition

$$\frac{R_{H,t}}{R_{F,t}} = \frac{e_{t+1}\pi_{t+1}}{e_t \pi_{t+1}^*} \tag{A.51}$$

Endogenous discount factor

$$\beta_t = \widetilde{\beta} c_t^{-\vartheta} \tag{A.52}$$

$$\beta_t^* = \widetilde{\beta} c_t^{*-\vartheta} \tag{A.53}$$

Excess return

$$r_{x,t}^1 = r_{F,t}^B - r_{H,t}^B \tag{A.54}$$

$$r_{x,t}^2 = r_{H,t}^E - r_{H,t}^B \tag{A.55}$$

$$r_{x,t}^3 = r_{F,t}^E - r_{H,t}^B \tag{A.56}$$

Equity price

$$r_{H,t}^{E} = \frac{(j_t + d_t)}{j_{t-1}} \pi_t \tag{A.57}$$

$$r_{F,t}^{E} = \frac{(j_t^* + d_t^*)}{j_{t-1}^*} \pi_t^* \tag{A.58}$$

Profits

$$e_t p_{H,t}^* IM_t^* - p_{F,t} IM_t = w_t h_t - c_t + d_t \tag{A.59}$$

$$p_{F,t} IM_t / e_t - p_{H,t}^* IM_t^* = w_t^* h_t^* - c_t^* + d_t^* \tag{A.60}$$

Trade balance

$$T_t = e_t p_{H,t}^* IM_t^* - p_{F,t} IM_t \tag{A.61}$$

$$T_t^* = p_{F,t} IM_t / e_t - p_{H,t}^* IM_t^* \tag{A.62}$$

Shocks

$$\xi_t = \rho_\xi \xi_{t-1} + \epsilon_{\xi,t} \tag{A.63}$$

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \tag{A.64}$$

$$a_t^* = \rho_a a_{t-1}^* + \epsilon_{a,t}^* \tag{A.65}$$

$$m_t = \rho_m m_{t-1} + \epsilon_{m,t} \tag{A.66}$$

$$m_t^* = \rho_m m_{t-1}^* + \epsilon_{m,t}^* \tag{A.67}$$

$$u_t^I = \rho_i u_{t-1}^I + \epsilon_{i,t} \tag{A.68}$$

$$u_t^{I*} = \rho_i u_{t-1}^{I*} + \epsilon_{i,t}^* \tag{A.69}$$

A.2 Solution method

Net Foreign Asset The net foreign asset is defined by the total assets minus the total liabilities (foreign assets owned by domestic households minus the domestic assets owned by foreign households).

$$NFA_t = e_t \frac{J_t^*}{P_t^*} X_{F,t} + e_t \frac{B_{F,t}}{R_{F,t} P_t^*} - \frac{J_t}{P_t} X_{H,t}^* - \frac{B_{H,t}^*}{R_{H,t} P_t} \tag{A.70}$$

Substituting $B_{H,t}^*$ into $B_{H,t}$ by a bond market clearing condition, $B_{H,t} + B_{H,t}^* = 0$, the net foreign asset position is following.

$$NFA_t = e_t \frac{J_t^*}{P_t^*} X_{F,t} + e_t \frac{B_{F,t}}{R_{F,t} P_t^*} - \frac{J_t}{P_t} X_{H,t}^* + \frac{B_{H,t}}{R_{H,t} P_t} \tag{A.71}$$

The left-hand side of the budget constraint can be re-written by the net foreign asset position and market clearing condition for equities, $X_{H,t} + X_{H,t}^* = 1$.

$$\begin{aligned} \frac{J_t}{P_t} X_{H,t} + e_t \frac{J_t^*}{P_t^*} X_{F,t} + \frac{B_{H,t}}{R_{H,t} P_t} + e_t \frac{B_{F,t}}{R_{F,t} P_t^*} &= \frac{J_t}{P_t}(1 - X_{H,t}^*) + e_t \frac{J_t^*}{P_t^*} X_{F,t} + \frac{B_{H,t}}{R_{H,t} P_t} + e_t \frac{B_{F,t}}{R_{F,t} P_t^*} \\ &= NFA_t + \frac{J_t}{P_t} \end{aligned} \tag{A.72}$$

The financial part of the right-hand side of the budget constraint can also be re-written by

$$\begin{aligned} & \frac{(J_t + D_t)}{P_t} X_{H,t-1} + e_t \frac{(J_t^* + D_t^*)}{P_t^*} X_{F,t-1}^* + \frac{B_{H,t-1}}{P_t} + e_t \frac{B_{F,t-1}}{P_t^*} \\ = & \frac{(J_t + D_t)}{J_{t-1}} \frac{P_{t-1}}{P_t} \frac{J_{t-1}}{P_{t-1}} (1 - X_{H,t-1}^*) + e_{t-1} \frac{(J_t^* + D_t^*)}{J_{t-1}^*} \frac{P_{t-1}^*}{P_t^*} \frac{J_{t-1}^*}{P_{t-1}^*} X_{F,t-1}^* \frac{e_t}{e_{t-1}} \\ & + \frac{B_{H,t-1}}{R_{H,t-1} P_{t-1}} \frac{P_{t-1}}{P_t} R_{H,t-1} + e_{t-1} \frac{B_{F,t-1}}{R_{F,t-1} P_{t-1}^*} \frac{P_{t-1}^*}{P_t^*} \frac{e_t}{e_{t-1}} R_{F,t-1} \end{aligned} \tag{A.73}$$

Multiplying the real return on home bond to the net foreign asset position, $R_{H,t-1}P_{t-1}/P_t = r^B_{H,t}$

$$NFA_{t-1}\frac{P_{t-1}}{P_t}R_{H,t-1} = -\frac{J_{t-1}}{P_{t-1}}X^*_{H,t-1}r^B_{H,t} + e_{t-1}\frac{J^*_{t-1}}{P^*_{t-1}}X_{F,t-1}r^B_{H,t} + \frac{B_{H,t-1}}{R_{H,t-1}P_{t-1}}r^B_{H,t} + e_{t-1}\frac{B_{F,t-1}}{R_{F,t-1}P^*_{t-1}}r^B_{H,t} \tag{A.74}$$

Subtracting this equation from the financial part of the budget constraint,

$$NFA_{t-1}r^B_{H,t} - \frac{J_{t-1}}{P_{t-1}}X^*_{H,t-1}\left(r^E_{H,t} - r^B_{H,t}\right) + e_{t-1}\frac{J^*_{t-1}}{P^*_{t-1}}X_{F,t-1}\left(r^E_{F,t} - r^B_{H,t}\right) + e_{t-1}\frac{B_{F,t-1}}{R_{F,t-1}P^*_{t-1}}\left(r^B_{F,t} - r^B_{H,t}\right) + \frac{(J_t + D_t)}{P_t} \tag{A.75}$$

The real return on bond and equities are defined by following,

$$r^B_{H,t} = \frac{R_{H,t}}{\pi_t}$$
$$r^B_{F,t} = \frac{R_{F,t}e_t}{\pi^*_t e_{t-1}}$$
$$r^E_{H,t} = \frac{(j_t + d_t)}{j_{t-1}}\pi_t$$
$$r^E_{F,t} = \frac{(j^*_t + d^*_t)}{j^*_{t-1}}\pi^*_t$$

The real bond holdings and equities are defined by

$$\tilde{\alpha}^E_{H,t} = \frac{J_t}{P_t}X_{H,t}$$
$$\tilde{\alpha}^{E*}_{H,t} = \frac{J_t}{P_t}X^*_{H,t}$$
$$\tilde{\alpha}^E_{F,t} = e_t\frac{J^*_t}{P^*_t}X_{F,t}$$
$$\tilde{\alpha}^B_{H,t} = \frac{B_{H,t}}{R_{H,t}P_t}$$
$$\tilde{\alpha}^B_{F,t} = e_t\frac{B_{F,t}}{R_{F,t}P^*_t}$$

Using the real asset holdings, the financial part of the right-hand side of the budget constraint is

$$NFA_{t-1}r^B_{H,t} - \tilde{\alpha}^{*E}_{H,t-1}\left(r^E_{H,t} - r^B_{H,t}\right) + \tilde{\alpha}^E_{F,t-1}\left(r^E_{F,t} - r^B_{H,t}\right) + \tilde{\alpha}^B_{F,t-1}\left(r^B_{F,t} - r^B_{H,t}\right) + \frac{(J_t + D_t)}{P_t} \tag{A.76}$$

Therefore, the budget constraint is

$$\begin{aligned} & NFA_{t-1} + \frac{J_t}{P_t} \\ &= NFA_{t-1}r^B_{H,t} - \tilde{\alpha}^{*E}_{H,t-1}\left(r^E_{H,t} - r^B_{H,t}\right) + \tilde{\alpha}^E_{F,t-1}\left(r^E_{F,t} - r^B_{H,t}\right) + \tilde{\alpha}^B_{F,t-1}\left(r^B_{F,t} - r^B_{H,t}\right) \\ &\qquad + \frac{(J_t + D_t)}{P_t} + w_t h_t - c_t - \frac{p^I_t}{p_t} i_t \end{aligned}$$

Using an excess return shock ξ_t,

$$\xi_t = -\tilde{\alpha}^{*E}_{H,t-1}\left(r^E_{H,t} - r^B_{H,t}\right) + \tilde{\alpha}^E_{F,t-1}\left(r^E_{F,t} - r^B_{H,t}\right) + \tilde{\alpha}^B_{F,t-1}\left(r^B_{F,t} - r^B_{H,t}\right) \tag{A.77}$$

The budget constraint can be re-written by

$$NFA_{t-1} = NFA_{t-1}r^B_{H,t} + w_t h_t - c_t + d_t - \frac{p^I_t}{p_t} i_t + \xi_t \tag{A.78}$$

Solving the steady state portfolios The optimality conditions for the assets can be written by the excess returns and the difference between the home and foreign consumptions.

$$E_t\left[\left(C_{t+1} - C^*_{t+1}\right) r_{x,t+1}\right] = 0 \tag{A.79}$$

$$r_{x,t} = \left(r^E_{H,t} - r^B_{H,t}\right) + \left(r^E_{F,t} - r^B_{H,t}\right) + \left(r^B_{F,t} - r^B_{H,t}\right) \tag{A.80}$$

The second order approximation of the optimality condition yields

$$E_t\left[\left(\hat{C}_{t+1} - \hat{C}^*_{t+1}\right) \hat{r}_{x,t+1}\right] = 0 \tag{A.81}$$

Extract the rows from the state space solution.

$$\hat{r}_{x,t+1} = R_1 \xi_{t+1} + R_2 \varepsilon_{t+1} \tag{A.82}$$

From the excess return shock,

$$\xi_{t+1} = \tilde{\alpha}\hat{r}_{x,t+1} \tag{A.83}$$

Combine the above two equations

$$\hat{r}_{x,t+1} = R_1\tilde{\alpha}\hat{r}_{x,t+1} + R_2\varepsilon_{t+1}$$
$$(I - R_1\tilde{\alpha}\hat{r}_{x,t+1}) = R_2\varepsilon_{t+1} \Rightarrow \hat{r}_{x,t+1} = [I - R_1\tilde{\alpha}]^{-1} R_2\varepsilon_{t+1}$$

The difference between home and foreign consumptions is

$$\left(\hat{C}_{t+1} - \hat{C}^*_{t+1}\right) = D_1\xi_{t+1} + D_2\varepsilon_{t+1} \tag{A.84}$$

Using the excess return shock

$$\left(\hat{C}_{t+1} - \hat{C}^*_{t+1}\right) = D_1\tilde{\alpha}\hat{r}_{x,t+1} + D_2\varepsilon_{t+1}$$
$$\left(\hat{C}_{t+1} - \hat{C}^*_{t+1}\right) = D_1\tilde{\alpha}[I - R_1\tilde{\alpha}]^{-1}R_2\varepsilon_{t+1}$$
$$= \left(D_1\tilde{\alpha}[I - R_1\tilde{\alpha}]^{-1}R_2 + D_2\right)\varepsilon_{t+1}$$

Substitute $\left(\hat{C}_{t+1} - \hat{C}^*_{t+1}\right)$ and $\hat{r}_{x,t+1}$ into the optimality condition

$$\left(D_1\tilde{\alpha}\left[I - R_1\tilde{\alpha}\right]^{-1} R_2 + D_2\right)\Sigma R_2^{'}\left(\left[I - R_1\tilde{\alpha}\right]^{-1}\right)^{'} = 0$$

$$D_1\tilde{\alpha}\left[I - R_1\tilde{\alpha}\right]^{-1} R_2\Sigma R_2{}^{'}\left(\left[I - R_1\tilde{\alpha}\right]^{-1}\right)^{'} = -D_2\Sigma R_2{}^{'}\left(\left[I - R_1{}^{\tilde{\alpha}}\right]^{-1}\right)^{'}$$

$$D_1\tilde{\alpha}\left[I - R_1\tilde{\alpha}\right]^{-1} R_2\Sigma R_2^{'} = -D_2\Sigma R_2^{'}D_1\tilde{\alpha}\left[I - R_1\tilde{\alpha}\right]^{-1} = -D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1}$$

$$D_1\tilde{\alpha} = -D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1}\left[I - R_1\tilde{\alpha}\right]$$
$$= -D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1} I + D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1} R_1\tilde{\alpha}$$
$$D_1\tilde{\alpha} - D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1} R_1\tilde{\alpha} = -D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1} I$$
$$\left(D_1 - D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1} R_1\right)\tilde{\alpha} = -D_2\Sigma R_2^{'}\left(R_2\Sigma R_2^{'}\right)^{-1}$$

Steady state portfolios are

$$\tilde{\alpha} = -\frac{D_2 \Sigma R_2' \left(R_2 \Sigma R_2' \right)^{-1}}{\left(D_1 - D_2 \Sigma R_2' \left(R_2 \Sigma R_2' \right)^{-1} R_1 \right)} \tag{A.85}$$

$\tilde{\alpha}$ is 1x3 vector which means each $-\tilde{\alpha}_{H,t}^{E*}$, $\tilde{\alpha}_{F,t}^{E}$ and $\tilde{\alpha}_{F,t}^{B}$.

Appendix B

Log-linearized System

B.1 First order approximation

The consumer's budget constraint

$$\begin{aligned}\hat{W}_t &= \overline{wh} \cdot \hat{w}_t + \overline{wh} \cdot \hat{h}_t + \overline{r^k k} \cdot \hat{r}_t^k + \overline{r^k k} \cdot \hat{k}_t - \bar{c} \cdot \hat{c}_t - \bar{i} \cdot \hat{i}_t - \bar{i} \cdot \hat{p}_t^i + \bar{i} \cdot \hat{p}_t + \bar{d} \cdot \hat{d}_t + \bar{r}_F \hat{W}_{t-1} \\ &+ \hat{\xi}_t \end{aligned} \tag{B.1}$$

The Euler equations

$$(\nu - \gamma)\,\widehat{c}_t - \widehat{R}_{H,t} = -\gamma E_t \widehat{c}_{t+1} - E_t \widehat{\pi}_{t+1} \tag{B.2}$$

$$(\nu - \gamma)\,\widehat{c}_t - \widehat{R}_{F,t} + \widehat{e}_t = -\gamma E_t \widehat{c}_{t+1} - E_t \widehat{\pi}^*_{t+1} + E_t \widehat{e}_{t+1} \tag{B.3}$$

International Risk Sharing conditon

$$-\gamma E_t \widehat{c}_{t+1} + \gamma^* E_t \widehat{c}^*_{t+1} + E_t \widehat{e}_{t+1} = (\nu - \gamma)\,\widehat{c} - (\nu - \gamma^*)\,\widehat{c}^*_t + \widehat{e}_t \tag{B.4}$$

The excess return on bonds

$$\widehat{r}_{x,t} = \overline{r}_H \cdot \widehat{r}_{H,t} - \overline{r}_F \cdot \widehat{r}_{F,t} \tag{B.5}$$

Optimal capital decision

$$(\nu - \gamma)\,\widehat{c}_t + \widehat{q}_t = -\gamma E_t \widehat{c}_{t+1} + \beta \overline{r}^k E_t \widehat{r}^k_{t+1} + \beta\,(1-\delta)\,E_t \widehat{q}_{t+1} \tag{B.6}$$

$$(\nu - \gamma)\,\widehat{c}^*_t + \widehat{q}^*_t = -\gamma E_t \widehat{c}^*_{t+1} + \beta \overline{r}^k E_t \widehat{r}^{k*}_{t+1} + \beta\,(1-\delta)\,E_t \widehat{q}^*_{t+1} \tag{B.7}$$

Optimal Investment

$$\beta\varphi E_t \widehat{i}_{t+1} = \varphi\,(1+\beta)\,\widehat{i}_t - \varphi \widehat{i}_{t-1} - \widehat{q}_t + \hat{p}^i_t - \hat{p}_t \tag{B.8}$$

$$\beta\varphi E_t \widehat{i}^*_{t+1} = \varphi\,(1+\beta)\,\widehat{i}^*_t - \varphi \widehat{i}^*_{t-1} - \widehat{q}^*_t + \hat{p}^{i*}_t - \hat{p}^*_t \tag{B.9}$$

CPI inflation

$$\widehat{\pi}_t = (1-\eta)\,\widehat{\pi}_{H,t} + \eta \widehat{\pi}_{F,t} \tag{B.10}$$

$$\widehat{\pi}^*_t = (1-\eta)\,\widehat{\pi}^*_{F,t} + \eta \widehat{\pi}^*_{H,t} \tag{B.11}$$

Nominal profit

$$\bar{d} \cdot \hat{d}_t = \bar{y} \cdot \hat{y}_t - \overline{wh} \cdot \hat{w}_t - \overline{wh} \cdot \hat{h}_t - \overline{r^k k} \cdot \hat{r}^k_t - \overline{r^k k} \cdot \hat{k}_t \tag{B.12}$$

Real exchange rate

$$\widehat{e}_t = \widehat{p}^*_t - \widehat{p}_t + \widehat{s}_t \tag{B.13}$$

Production function

$$\widehat{y}_t = \widehat{a}_t + \alpha \widehat{k}_t + (1-\alpha)\,\widehat{h}_t \tag{B.14}$$

$$\widehat{y}_t^* = \widehat{a}_t^* + \alpha \widehat{k}_t^* + (1-\alpha)\,\widehat{h}_t^* \tag{B.15}$$

Fisher equation

$$\overline{r}_H \cdot \widehat{r}_{H,t} + \overline{r}_H \cdot \widehat{\pi}_t = \overline{r}_H \cdot \widehat{R}_{H,t-1} \tag{B.16}$$

$$\overline{r}_F \cdot \widehat{r}_{F,t} + \overline{r}_F \cdot \widehat{\pi}_t^* = \overline{r}_F \cdot \widehat{R}_{F,t-1} \tag{B.17}$$

Capital evolution

$$\overline{k} \cdot \widehat{k}_{t+1} = \overline{i} \cdot \widehat{i}_t + (1-\delta)\,\overline{k} \cdot \widehat{k}_t \tag{B.18}$$

$$\overline{k}^* \cdot \widehat{k}_{t+1}^* = \overline{i}^* \cdot \widehat{i}_t^* + (1-\delta)\,\overline{k}^* \cdot \widehat{k}_t^* \tag{B.19}$$

Labor supply

$$-\gamma \widehat{c}_t + \widehat{w}_t = \omega \widehat{h}_t \tag{B.20}$$

$$-\gamma \widehat{c}_t^* + \widehat{w}_t^* = \omega \widehat{h}_t^* \tag{B.21}$$

Real Marginal cost

$$\widehat{mc}_t = (1-\alpha)\,\widehat{w}_t + \alpha \widehat{r}_t^k - \widehat{a}_t \tag{B.22}$$

$$\widehat{mc}_t^* = (1-\alpha)\,\widehat{w}_t^* + \alpha \widehat{r}_t^{k*} - \widehat{a}_t^* \tag{B.23}$$

capital labor ratio

$$\widehat{w}_t + \widehat{h}_t = \widehat{r}_t^k + \widehat{k}_t \tag{B.24}$$

$$\widehat{w}_t^* + \widehat{h}_t^* = \widehat{r}_t^{k*} + \widehat{k}_t^* \tag{B.25}$$

Monetary Policy rules

$$\widehat{R}_{H,t} = \rho_H \widehat{R}_{H,t-1} + (1-\rho_H)\varphi_\pi \widehat{\pi}_t + (1-\rho_H)\varphi_y \widehat{y}_t + (1-\rho_H)\varphi_s \widehat{s}_t + \zeta_{m,t} \tag{B.26}$$

$$\widehat{R}_{F,t} = \rho_F \widehat{R}_{F,t-1} + (1-\rho_F)\varphi_\pi^* \widehat{\pi}_t^* + (1-\rho_F)\varphi_y^* \widehat{y}_t^* + (1-\rho_H)\varphi_s^* \widehat{s}_t + \zeta_{m,t}^* \tag{B.27}$$

Terms of trade

$$\widehat{\tau}_t = \hat{p}_{F,t} - \hat{p}_{H,t} \tag{B.28}$$

$$\widehat{\tau}_t^* = \hat{p}_{H,t}^* - \hat{p}_{F,t}^* \tag{B.29}$$

Utility function

$$\bar{u} \cdot \widehat{u}_t = \bar{c}^{1-\gamma} \cdot \widehat{c}_t - \bar{h}^{1+\omega} \cdot \widehat{h}_t \tag{B.30}$$

Value function

$$\hat{V}_t + \nu\beta\hat{c}_t - \beta E_t \hat{V}_{t+1} = (1-\beta)\hat{u}_t \tag{B.31}$$

In PCP,
Market Clearing Conditions

$$\overline{y}\widehat{y}_t = \mu\eta(1-\eta)\overline{c}\widehat{\tau}_t - \mu\eta(1-\eta)\overline{c}^*\widehat{\tau}_t^* + (1-\eta)\overline{c}\widehat{c}_t + \eta\overline{c}^*\widehat{c}_t^* + \overline{i}\,\widehat{i}_t \tag{B.32}$$

$$\overline{y}^*\widehat{y}_t^* = -\mu\eta(1-\eta)\overline{c}\widehat{\tau}_t + \mu\eta(1-\eta)\overline{c}^*\widehat{\tau}_t^* + \eta\overline{c}\widehat{c}_t + (1-\eta)\overline{c}^*\widehat{c}_t^* + \overline{i}^*\widehat{i}_t^* \tag{B.33}$$

Phillips curves

$$\phi_H \widehat{\pi}_{H,t} = \beta \phi_H E_t \widehat{\pi}_{H,t+1} + \eta (\theta - 1) \widehat{\tau}_t + (\theta - 1) \widehat{mc}_t \tag{B.34}$$

$$\phi_F^* \widehat{\pi}_{F,t}^* = \beta \phi_F^* E_t \widehat{\pi}_{F,t+1}^* + (\theta - 1) \eta \widehat{\tau}_t^* + (\theta - 1) \widehat{mc}_t^* \tag{B.35}$$

$$\widehat{\pi}_{H,t}^* = \widehat{\pi}_{H,t} + \hat{s}_{t-1} - \hat{s}_t \tag{B.36}$$

$$\widehat{\pi}_{F,t} = \widehat{\pi}_{F,t}^* + \hat{s}_t - \hat{s}_{t-1} \tag{B.37}$$

In LCP,
Market Clearing Conditions

$$\overline{y}_H \widehat{y}_{H,t} = \mu \eta (1 - \eta) \overline{c} \widehat{\tau}_t + (1 - \eta) \overline{c} \widehat{c}_t + \overline{i} \, \widehat{i}_t \tag{B.38}$$

$$\overline{y}_H^* \widehat{y}_{H,t}^* = -\mu \eta (1 - \eta) \overline{c}^* \widehat{\tau}_t^* + \eta \overline{c}^* \widehat{c}_t^* \tag{B.39}$$

$$\overline{y}_F^* \widehat{y}_{F,t}^* = \mu \eta (1 - \eta) \overline{c}^* \widehat{\tau}_t^* + (1 - \eta) \overline{c}^* \widehat{c}_t^* + \overline{i}^* \, \widehat{i}_t^* \tag{B.40}$$

$$\overline{y}_F \widehat{y}_{F,t} = -\mu \eta (1 - \eta) \overline{c} \widehat{\tau}_t + \eta \overline{c} \widehat{c}_t \tag{B.41}$$

$$\overline{y} \widehat{y}_t = \overline{y}_H \widehat{y}_{H,t} + \overline{y}_H^* \widehat{y}_{H,t}^* \tag{B.42}$$

$$\overline{y}^* \widehat{y}_t^* = \overline{y}_F \widehat{y}_{F,t} + \overline{y}_F^* \widehat{y}_{F,t}^* \tag{B.43}$$

Phillips curves

$$\phi_H \widehat{\pi}_{H,t} = \beta \phi_H E_t \widehat{\pi}_{H,t+1} + \eta (\theta - 1) \widehat{\tau}_t + (\theta - 1) \widehat{mc}_t \tag{B.44}$$

$$\phi_F^* \widehat{\pi}_{F,t}^* = \beta \phi_F^* E_t \widehat{\pi}_{F,t+1}^* + (\theta - 1)\, \eta \widehat{\tau}_t^* + (\theta - 1)\, \widehat{mc}_t^* \tag{B.45}$$

$$\phi_H^* \widehat{\pi}_{H,t}^* = \beta \phi_H^* E_t \widehat{\pi}_{H,t+1}^* + (\eta - 1)(\theta - 1)\, \widehat{\tau}_t^* + (\theta - 1)\, \widehat{mc}_t - (\theta - 1)\, \widehat{e}_t \tag{B.46}$$

$$\phi_F \widehat{\pi}_{F,t} = \beta \phi_F E_t \widehat{\pi}_{F,t+1} + (\eta - 1)(\theta - 1)\, \widehat{\tau}_t + (\theta - 1)\, \widehat{mc}_t^* + (\theta - 1)\, \widehat{e}_t \tag{B.47}$$

Deviations of law of price

$$\widehat{z}_t = \widehat{p}_{F,t}^* - \widehat{p}_{F,t} + \widehat{s}_t \tag{B.48}$$

$$\widehat{z}_t^* = \widehat{p}_{H,t} - \widehat{p}_{H,t}^* - \widehat{s}_t \tag{B.49}$$

B.2 Second order approximation

The consumer's budget constraint

$$\begin{aligned}
\hat{W}_t &= \overline{wh} \cdot \hat{w}_t + \overline{wh} \cdot \hat{h}_t + \overline{r^k k} \cdot \hat{r}_t^k + \overline{r^k k} \cdot \hat{k}_t - \bar{c} \cdot \hat{c}_t - \bar{i} \cdot \hat{i}_t - \bar{i} \cdot \hat{p}_t^i + \bar{i} \cdot \hat{p}_t + \bar{d} \cdot \hat{d}_t \\
&+ \bar{r}_F \hat{W}_{t-1} + \overline{b_H r}_H \cdot \hat{r}_{H,t} + \frac{1}{2}\overline{wh} \cdot \hat{w}_t^2 + \frac{1}{2}\overline{wh} \cdot \hat{h}_t^2 + \frac{1}{2}\overline{r^k k} \cdot \hat{r}_t^{k^2} + \frac{1}{2}\overline{r^k k} \cdot \hat{k}_t^2 - \frac{1}{2}\bar{c} \cdot \hat{c}_t^2 \\
&- \frac{1}{2}\bar{i} \cdot \hat{i}_t^2 - \frac{1}{2}\bar{i} \cdot \hat{p}_t^{i^2} - \frac{1}{2}\bar{i} \cdot \hat{p}_t^2 + \frac{1}{2}\bar{d} \cdot \hat{d}_t^2 + \frac{1}{2}\overline{b_H r}_H \cdot \hat{r}_{H,t}^2 - \frac{1}{2}\overline{b_H r}_H \cdot \hat{r}_{F,t}^2 + \overline{wh} \cdot \hat{w}_t \hat{h}_t \\
&+ \overline{r^k k} \cdot \hat{r}_t^k \hat{k}_t - \bar{i} \cdot \hat{p}_t^i \hat{i}_t + \bar{i} \cdot \hat{p}_t \hat{i}_t + \bar{i} \cdot \hat{p}_t^i \hat{p}_t + \bar{r}_F \hat{W}_{t-1} \hat{r}_{F,t} + \hat{\xi}_t
\end{aligned} \tag{B.50}$$

The Euler equations

$$\begin{aligned}
(\nu - \gamma)\, \hat{c}_t - \hat{R}_{H,t} &= -\gamma E_t \hat{c}_{t+1} - E_t \hat{\pi}_{t+1} \\
&- \frac{1}{2}(\nu - \gamma)^2 \hat{c}_t^2 - \frac{1}{2}\hat{R}_{H,t}^2 + (\nu - \gamma)\, \hat{c}_t \hat{R}_{H,t} \\
&+ \frac{1}{2}\gamma^2 E_t \hat{c}_{t+1}^2 + \frac{1}{2} E_t \hat{\pi}_{t+1}^2 + \gamma E_t \hat{c}_{t+1} \hat{\pi}_{t+1}
\end{aligned} \tag{B.51}$$

$$
\begin{aligned}
(\nu-\gamma)\,\hat{c}_t - \hat{R}_{F,t} + \hat{e}_t &= -\gamma E_t\hat{c}_{t+1} - E_t\hat{\pi}^*_{t+1} + E_t\hat{e}_{t+1} \\
&- \frac{1}{2}(\nu-\gamma)^2\hat{c}_t^2 - \frac{1}{2}\hat{R}^2_{F,t} - \frac{1}{2}\hat{e}_t^2 + \frac{1}{2}\gamma^2 E_t\hat{c}^2_{t+1} + \frac{1}{2}E_t\hat{\pi}^{*2}_{t+1} + \frac{1}{2}E_t\hat{e}^2_{t+1} \\
&+ (\nu-\gamma)\,\hat{c}_t\hat{R}_{F,t} - (\nu-\gamma)\,\hat{c}_t\hat{e}_t + \hat{R}_{F,t}\hat{e}_t + \gamma E_t\hat{c}_{t+1}\hat{\pi}^*_{t+1} \\
&- \gamma E_t\hat{c}_{t+1}\hat{e}_{t+1} - E_t\hat{\pi}^*_{t+1}\hat{e}_{t+1} \qquad \text{(B.52)}
\end{aligned}
$$

International Risk Sharing conditon

$$
\begin{aligned}
(\nu-\gamma)\,\hat{c} - (\nu-\gamma^*)\,\hat{c}^*_t + \hat{e}_t &= -\gamma E_t\hat{c}_{t+1} + \gamma^* E_t\hat{c}^*_{t+1} + E_t\hat{e}_{t+1} \\
&- \frac{1}{2}(\nu-\gamma)^2\hat{c}_t^2 + \frac{1}{2}(\nu-\gamma)^2\hat{c}_t^{*2} + \frac{1}{2}\hat{e}_t^2 + \frac{1}{2}\gamma^2 E_t\hat{c}^2_{t+1} \\
&- \frac{1}{2}\gamma^2 E_t\hat{c}^{*2}_{t+1} - \frac{1}{2}E_t\hat{e}^2_{t+1} - (\nu-\gamma)\,\hat{c}^*_t\hat{e}_t - \gamma E_t\hat{c}_{t+1}\hat{r}_{H,t+1} \\
&+ \gamma E_t\hat{c}^*_{t+1}\hat{r}_{H,t+1} - \gamma E_t\hat{c}^*_{t+1}\hat{e}_{t+1} + E_t\hat{r}_{H,t+1}\hat{e}_{t+1} \qquad \text{(B.53)}
\end{aligned}
$$

The excess return on bonds

$$
\widehat{r}_{x,t} = \overline{r}_H \cdot \widehat{r}_{H,t} - \overline{r}_F \cdot \widehat{r}_{F,t} \qquad \text{(B.54)}
$$

Optimal capital decision

$$
\begin{aligned}
(\nu-\gamma)\,\hat{c}_t + \hat{q}_t &= -\gamma E_t\hat{c}_{t+1} + \beta\bar{r}^k E_t\hat{r}^k_{t+1} + \beta\,(1-\delta)\,E_t\hat{q}_{t+1} \\
&- \frac{1}{2}(\nu-\gamma)^2\hat{c}_t^2 - \frac{1}{2}\hat{q}_t^2 + \frac{1}{2}\gamma^2 E_t\hat{c}^2_{t+1} + \frac{1}{2}\beta\bar{r}^k E_t\hat{r}^{k^2}_{t+1} + \frac{1}{2}\beta\,(1-\delta)\,E_t\hat{q}^2_{t+1} \\
&- (\nu-\gamma)\,\hat{c}_t\hat{q}_t - \gamma\beta\bar{r}^k E_t\hat{c}_{t+1}\hat{r}^k_{t+1} - \gamma\beta\,(1-\delta)\,E_t\hat{c}_{t+1}\hat{q}_{t+1} \qquad \text{(B.55)}
\end{aligned}
$$

$$
\begin{aligned}
(\nu-\gamma)\,\hat{c}^*_t + \hat{q}^*_t &= -\gamma E_t\hat{c}^*_{t+1} + \beta\bar{r}^k E_t\hat{r}^{*k}_{t+1} + \beta\,(1-\delta)\,E_t\hat{q}^*_{t+1} \\
&- \frac{1}{2}(\nu-\gamma)^2\hat{c}_t^{*2} - \frac{1}{2}\hat{q}_t^{*2} + \frac{1}{2}\gamma^2 E_t\hat{c}^{*2}_{t+1} + \frac{1}{2}\beta\bar{r}^k E_t\hat{r}^{*k^2}_{t+1} + \frac{1}{2}\beta\,(1-\delta)\,E_t\hat{q}^{*2}_{t+1} \\
&- (\nu-\gamma)\,\hat{c}^*_t\hat{q}^*_t - \gamma\beta\bar{r}^k E_t\hat{c}^*_{t+1}\hat{r}^{*k}_{t+1} - \gamma\beta\,(1-\delta)\,E_t\hat{c}^*_{t+1}\hat{q}^*_{t+1} \qquad \text{(B.56)}
\end{aligned}
$$

Optimal Investment

$$\begin{aligned}
\beta\varphi E_t\hat{i}_{t+1} &= \varphi(1+\beta)\hat{i}_t - \varphi\hat{i}_{t-1} - \hat{q}_t + \hat{p}^i_t - \hat{p}_t \\
&+ \frac{1}{2}\hat{p}^{i^2}_t + \frac{1}{2}\hat{p}^2_t + \frac{1}{2}(4-5\beta)\varphi\hat{i}^2_t + 2\varphi\hat{i}^2_{t-1} - \frac{5}{2}\beta\varphi E_t\hat{i}^2_{t+1} - \frac{1}{2}\hat{q}^2_t \\
&+ (\nu-\gamma)\hat{p}^i_t\hat{c}_t - (\nu-\gamma)\hat{p}_t\hat{c}^{-\gamma}_t - \hat{p}^i_t\hat{p}_t - (\nu-\gamma)\hat{c}_t\hat{q}_t + (\nu-\gamma)\varphi\hat{c}_t\hat{i}_t + \varphi\hat{i}_t\hat{q}_t \\
&- 4\varphi_t\hat{i}_t\hat{i}_{t-1} + \beta\varphi(\nu-\gamma)E_t\hat{c}_{t+1}\hat{i}_t + \beta\varphi E_t\hat{q}_{t+1}\hat{i}_t + 5\beta\varphi E_t\hat{i}_{t+1}\hat{i}_t - \varphi\hat{i}_{t-1}\hat{q}_t \\
&- \beta\varphi(\nu-\gamma)E_t\hat{c}_{t+1}\hat{i}_{t+1} + \beta\varphi E_t\hat{q}_{t+1}\hat{i}_t + \beta\varphi E_t\hat{i}_{t+1}\hat{q}_{t+1} \qquad \text{(B.57)}
\end{aligned}$$

$$\begin{aligned}
\beta\varphi E_t\hat{i}^*_{t+1} &= \varphi(1+\beta)\hat{i}^*_t - \varphi\hat{i}^*_{t-1} - \hat{q}^*_t + \hat{p}^{*i}_t - \hat{p}^*_t \\
&+ \frac{1}{2}\hat{p}^{*i^2}_t + \frac{1}{2}\hat{p}^{*2}_t + \frac{1}{2}(4-5\beta)\varphi\hat{i}^{*2}_t + 2\varphi\hat{i}^{*2}_{t-1} - \frac{5}{2}\beta\varphi E_t\hat{i}^{*2}_{t+1} - \frac{1}{2}\hat{q}^{*2}_t \\
&+ (\nu-\gamma^*)\hat{p}^{*i}_t\hat{c}^*_t - (\nu-\gamma^*)\hat{p}^*_t\hat{c}^{*-\gamma}_t - \hat{p}^{*i}_t\hat{p}^*_t - (\nu-\gamma^*)\hat{c}^*_t\hat{q}^*_t + (\nu-\gamma^*)\varphi\hat{c}^*_t\hat{i}^*_t + \varphi\hat{i}^*_t\hat{q}^*_t \\
&- 4\varphi_t\hat{i}^*_t\hat{i}^*_{t-1} + \beta\varphi(\nu-\gamma^*)E_t\hat{c}^*_{t+1}\hat{i}^*_t + \beta\varphi E_t\hat{q}^*_{t+1}\hat{i}^*_t + 5\beta\varphi E_t\hat{i}^*_{t+1}\hat{i}^*_t - \varphi\hat{i}^*_{t-1}\hat{q}^*_t \\
&- \beta\varphi(\nu-\gamma^*)E_t\hat{c}^*_{t+1}\hat{i}^*_{t+1} + \beta\varphi E_t\hat{q}^*_{t+1}\hat{i}^*_t + \beta\varphi E_t\hat{i}^*_{t+1}\hat{q}^*_{t+1} \qquad \text{(B.58)}
\end{aligned}$$

CPI inflation

$$\begin{aligned}
\hat{\pi}_t &= (1-\eta)\hat{\pi}_{H,t} + \eta\hat{\pi}_{F,t} \\
&- \frac{1}{2}(1-\mu)\hat{\pi}^2_t + \frac{1}{2}(1-\mu)(1-\eta)\hat{\pi}^2_{H,t} + \frac{1}{2}(1-\mu)\eta\hat{\pi}^2_{F,t} \\
&- \eta(1-\mu)(1-\eta)\hat{\pi}_{H,t}\hat{\tau}_{t-1} + \eta(1-\mu)(1-\eta)\hat{\pi}_{F,t}\hat{\tau}_{t-1} \qquad \text{(B.59)}
\end{aligned}$$

$$\begin{aligned}
\hat{\pi}^*_t &= (1-\eta)\hat{\pi}^*_{F,t} + \eta\hat{\pi}^*_{H,t} \\
&- \frac{1}{2}(1-\mu)\hat{\pi}^{*2}_t + \frac{1}{2}(1-\mu)(1-\eta)\hat{\pi}^{*2}_{F,t} + \frac{1}{2}(1-\mu)\eta\hat{\pi}^{*2}_{H,t} \\
&- \eta(1-\mu)(1-\eta)\hat{\pi}^*_{F,t}\hat{\tau}^*_{t-1} + \eta(1-\mu)(1-\eta)\hat{\pi}^*_{H,t}\hat{\tau}^*_{t-1} \qquad \text{(B.60)}
\end{aligned}$$

Nominal profit

$$\begin{aligned}
\bar{d}\cdot\hat{d}_t &= \bar{y}\cdot\hat{y}_t - \overline{wh}\cdot\hat{w}_t - \overline{wh}\cdot\hat{h}_t - \overline{r^k k}\cdot\hat{r}^k_t - \overline{r^k k}\cdot\hat{k}_t \\
&- \frac{1}{2}\bar{d}\cdot\hat{d}^2_t - \frac{1}{2}\bar{y}\cdot\hat{y}^2_t - \frac{1}{2}\overline{wh}\cdot\hat{w}^2_t - \frac{1}{2}\overline{wh}\cdot\hat{h}^2_t - \frac{1}{2}\overline{r^k k}\cdot\hat{r}^{k^2}_t - \frac{1}{2}\overline{r^k k}\cdot\hat{k}^2_t \\
&- \overline{wh}\cdot\hat{w}_t\hat{h}_t - \overline{r^k k}\cdot\hat{r}^k_t\hat{k}_t \qquad \text{(B.61)}
\end{aligned}$$

Real exchange rate

$$
\begin{aligned}
\hat{e}_t &= \hat{p}_t^* - \hat{p}_t + \hat{s}_t \\
&- \frac{1}{2}\hat{e}_t^2 + \frac{1}{2}\hat{p}_t^{*2} + \frac{1}{2}\hat{p}_t^2 + \frac{1}{2}\hat{s}_t^2 \\
&- \hat{p}_t^*\hat{p}_t + \hat{p}_t^*\hat{s}_t - \hat{p}_t\hat{s}_t
\end{aligned} \tag{B.62}
$$

Production function

$$
\begin{aligned}
\hat{y}_t &= \hat{a}_t + \alpha\hat{k}_t + (1-\alpha)\,\hat{h}_t \\
&- \frac{1}{2}\hat{y}_t^2 + \frac{1}{2}\hat{a}_t^2 + \frac{1}{2}\alpha^2\hat{k}_t^2 + \frac{1}{2}(1-\alpha)^2\hat{h}_t^2 \\
&+ \alpha\hat{a}_t\hat{k}_t + (1-\alpha)\,\hat{a}_t\hat{h}_t + \alpha\,(1-\alpha)\,\hat{k}_t\hat{h}_t
\end{aligned} \tag{B.63}
$$

$$
\begin{aligned}
\hat{y}_t^* &= \hat{a}_t^* + \alpha\hat{k}_t^* + (1-\alpha^*)\,\hat{h}_t^* \\
&- \frac{1}{2}\hat{y}_t^{*2} + \frac{1}{2}\hat{a}_t^{*2} + \frac{1}{2}\alpha^{*2}\hat{k}_t^{*2} + \frac{1}{2}(1-\alpha^*)^2\hat{h}_t^{*2} \\
&+ \alpha^*\hat{a}_t^*\hat{k}_t^* + (1-\alpha^*)\,\hat{a}_t^*\hat{h}_t^* + \alpha^*\,(1-\alpha^*)\,\hat{k}_t^*\hat{h}_t^*
\end{aligned} \tag{B.64}
$$

Fisher equation

$$
\begin{aligned}
\hat{r}_{H,t} + \hat{\pi}_t &= \hat{R}_{H,t-1} \\
&- \frac{1}{2}\hat{r}_{H,t}^2 - \frac{1}{2}\hat{\pi}_t^2 + \frac{1}{2}\hat{R}_{H,t-1}^2 - \hat{r}_{H,t}\hat{\pi}_t
\end{aligned} \tag{B.65}
$$

$$
\begin{aligned}
\hat{r}_{F,t} + \hat{\pi}_t^* &= \hat{R}_{F,t-1} \\
&- \frac{1}{2}\hat{r}_{F,t}^2 - \frac{1}{2}\hat{\pi}_t^{*2} + \frac{1}{2}\hat{R}_{F,t-1}^2 - \hat{r}_{F,t}\hat{\pi}_t^*
\end{aligned} \tag{B.66}
$$

Capital evolution

$$
\begin{aligned}
\bar{k}\cdot\hat{k}_{t+1} &= \bar{i}\cdot\hat{i}_t + (1-\delta)\,\bar{k}\cdot\hat{k}_t \\
&- \frac{1}{2}\bar{k}\cdot\hat{k}_{t+1}^2 + \frac{1}{2}\,(1-\varphi)\,\bar{i}\cdot\hat{i}_t^2 - \frac{1}{2}\varphi\bar{i}\cdot\hat{i}_{t-1}^2 + \frac{1}{2}\,(1-\delta)\,\bar{k}\cdot\hat{k}_t^2 + \varphi\bar{i}\cdot\hat{i}_t\hat{i}_{t-1}
\end{aligned} \tag{B.67}
$$

$$
\begin{aligned}
\bar{k}^* \cdot \hat{k}^*_{t+1} &= \bar{i}^* \cdot \hat{i}^*_t + (1-\delta)\,\bar{k}^* \cdot \hat{k}^*_t \\
&- \frac{1}{2}\bar{k}^* \cdot \hat{k}^{*2}_{t+1} + \frac{1}{2}(1-\varphi^*)\,\bar{i}^* \cdot \hat{i}^{*2}_t - \frac{1}{2}\varphi^*\bar{i}^* \cdot \hat{i}^{*2}_{t-1} + \frac{1}{2}(1-\delta)\,\bar{k}^* \cdot \hat{k}^{*2}_t \\
&+ \varphi^*\bar{i}^* \cdot \hat{i}^*_t\hat{i}^*_{t-1} \qquad \text{(B.68)}
\end{aligned}
$$

Labor supply

$$
-\gamma\hat{c}_t + \hat{w}_t = \omega\hat{h}_t - \frac{1}{2}\gamma^2\hat{c}^2_t - \frac{1}{2}\hat{w}^2_t + \frac{1}{2}\omega^2\hat{h}^2_t + \gamma\hat{c}_t\hat{w}_t \qquad \text{(B.69)}
$$

$$
-\gamma\hat{c}^*_t + \hat{w}^*_t = \omega^*\hat{h}^*_t - \frac{1}{2}\gamma^{*2}\hat{c}^{*2}_t - \frac{1}{2}\hat{w}^{*2}_t + \frac{1}{2}\omega^{*2}\hat{h}^{*2}_t + \gamma\hat{c}^*_t\hat{w}^*_t \qquad \text{(B.70)}
$$

Real Marginal cost

$$
\begin{aligned}
\widehat{mc}_t &= (1-\alpha)\,\hat{w}_t + \alpha\hat{r}^k_t - \hat{a}_t \\
&- \frac{1}{2}\widehat{mc}_t + \frac{1}{2}(1-\alpha)^2\hat{w}^2_t + \frac{1}{2}\alpha^2\hat{r}^{k^2}_t + \frac{1}{2}\hat{a}^2_t \\
&+ \alpha(1-\alpha)\,\hat{w}_t\hat{r}^k_t - (1-\alpha)\,\hat{w}_t\hat{a}_t - \alpha\hat{r}^k_t\hat{a}_t \qquad \text{(B.71)}
\end{aligned}
$$

$$
\begin{aligned}
\widehat{mc}^*_t &= (1-\alpha^*)\,\hat{w}^*_t + \alpha^*\hat{r}^{*k}_t - \hat{a}^*_t \\
&- \frac{1}{2}\widehat{mc}^*_t + \frac{1}{2}(1-\alpha^*)^2\hat{w}^{*2}_t + \frac{1}{2}\alpha^{*2}\hat{r}^{*k^2}_t + \frac{1}{2}\hat{a}^{*2}_t \\
&+ \alpha^*(1-\alpha^*)\,\hat{w}^*_t\hat{r}^{*k}_t - (1-\alpha^*)\,\hat{w}^*_t\hat{a}^*_t - \alpha^*\hat{r}^{*k}_t\hat{a}^*_t \qquad \text{(B.72)}
\end{aligned}
$$

capital labor ratio

$$
\begin{aligned}
\hat{w}_t + \hat{h}_t &= \hat{r}^k_t + \hat{k}_t \\
&- \frac{1}{2}\hat{w}^2_t - \frac{1}{2}\hat{h}^2_t + \frac{1}{2}\hat{r}^{k^2}_t + \frac{1}{2}\hat{k}^2_t - \hat{w}_t\hat{h}_t + \hat{r}^k_t\hat{k}_t \qquad \text{(B.73)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{w}^*_t + \hat{h}^*_t &= \hat{r}^{*k}_t + \hat{k}^*_t \\
&- \frac{1}{2}\hat{w}^{*2}_t - \frac{1}{2}\hat{h}^{*2}_t + \frac{1}{2}\hat{r}^{*k^2}_t + \frac{1}{2}\hat{k}^{*2}_t - \hat{w}^*_t\hat{h}^*_t + \hat{r}^{*k}_t\hat{k}^*_t \qquad \text{(B.74)}
\end{aligned}
$$

Terms of trade

$$\begin{aligned}\hat{\tau}_t &= \hat{p}_{F,t} - \hat{p}_{H,t} \\ &- \frac{1}{2}\hat{\tau}_t^2 + \frac{1}{2}\hat{p}_{F,t}^2 + \frac{1}{2}\hat{p}_{H,t}^2 - \hat{p}_{F,t}\hat{p}_{H,t} \end{aligned} \tag{B.75}$$

$$\begin{aligned}\hat{\tau}_t^* &= \hat{p}_{H,t}^* - \hat{p}_{F,t}^* \\ &- \frac{1}{2}\hat{\tau}_t^{*2} + \frac{1}{2}\hat{p}_{H,t}^{*2} + \frac{1}{2}\hat{p}_{F,t}^{*2} - \hat{p}_{H,t}^*\hat{p}_{F,t}^* \end{aligned} \tag{B.76}$$

Utility function

$$\begin{aligned}\bar{u}\cdot\hat{u}_t &= \bar{c}^{1-\gamma}\cdot\hat{c}_t - \bar{h}^{1+\omega}\cdot\hat{h}_t \\ &- \frac{1}{2}\bar{u}\cdot\hat{u}_t^2 + \frac{1}{2}\left(1-\gamma\right)\bar{c}^{1-\gamma}\cdot\hat{c}_t^2 + \frac{1}{2}\left(1+\omega\right)\bar{h}^{1+\omega}\cdot\hat{h}_t^2 \end{aligned} \tag{B.77}$$

Value function

$$\begin{aligned}\hat{V}_t + \nu\beta\hat{c}_t - \beta E_t\hat{V}_{t+1} &= \left(1-\beta\right)\hat{u}_t \\ &- \frac{1}{2}\hat{V}_t^2 + \frac{1}{2}\nu^2\beta\hat{c}_t^2 + \frac{1}{2}\beta E_t V_{t+1}^2 + \frac{1}{2}\left(1-\beta\right)\hat{u}_t^2 - \nu\beta\hat{c}_t E_t\hat{V}_{t+1} \end{aligned} \tag{B.78}$$

In PCP,
Market Clearing Conditions

$$\begin{aligned}\bar{y}\hat{y}_t &= \mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_t - \mu\eta\left(1-\eta\right)\bar{c}^*\hat{\tau}_t^* + \left(1-\eta\right)\bar{c}\hat{c}_t + \eta\bar{c}^*\hat{c}_t^* + \bar{i}\cdot\hat{i}_t \\ &- \frac{1}{2}\bar{y}\hat{y}_t^2 + \frac{1}{2}\left[\mu\eta^2\left(1-\eta\right)\left(2\mu-1\right) + \mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}\hat{\tau}_t^2 + \frac{1}{2}\left(1-\eta\right)\bar{c}\cdot\hat{c}_t^2 \\ &+ \frac{1}{2}\left[\mu\eta(1-\eta)^2\left(2\mu-1\right) + \mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}^*\hat{\tau}_t^{*2} + \frac{1}{2}\eta c^*\cdot\hat{c}_t^{*2} + \frac{1}{2}\bar{i}\cdot\hat{i}_t^2 \\ &+ \frac{1}{2}\phi_H\bar{y}\hat{\pi}_{H,t}^2 + \mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_t\hat{c}_t - \mu\eta\left(1-\eta\right)\bar{c}^*\hat{\tau}_t^*\hat{c}_t^* \end{aligned} \tag{B.79}$$

$$
\begin{aligned}
\bar{y}^{*}\hat{y}_{t}^{*} &= -\mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_{t}+\mu\eta\left(1-\eta\right)\bar{c}^{*}\hat{\tau}_{t}^{*}+\eta\bar{c}\hat{c}_{t}+\left(1-\eta\right)\bar{c}^{*}\hat{c}_{t}^{*}+\bar{i}^{*}\hat{i}_{t}^{*} \\
&- \frac{1}{2}\bar{y}^{*}\hat{y}_{t}^{*2}+\frac{1}{2}\left[\mu\eta(1-\eta)^{2}\left(2\mu-1\right)+\mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}\hat{\tau}_{t}^{*2}+\frac{1}{2}\eta\bar{c}\cdot\hat{c}_{t}^{2} \\
&+ \frac{1}{2}\left[\mu\eta^{2}\left(1-\eta\right)\left(2\mu-1\right)+\mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}^{*}\hat{\tau}_{t}^{*2}+\frac{1}{2}\left(1-\eta\right)c^{*}\cdot\hat{c}_{t}^{*2}+\frac{1}{2}\bar{i}^{*}\cdot\hat{i}_{t}^{*2} \\
&+ \frac{1}{2}\phi_{F}^{*}\bar{y}^{*}\hat{\pi}_{H,t}^{*2}-\mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_{t}\hat{c}_{t}+\mu\eta\left(1-\eta\right)\bar{c}^{*}\hat{\tau}_{t}^{*}\hat{c}_{t}^{*} \qquad \text{(B.80)}
\end{aligned}
$$

Phillips curves

$$
\begin{aligned}
\phi_{H}\hat{\pi}_{H,t} &= \beta\phi_{H}E_{t}\hat{\pi}_{H,t+1}+\eta\left(\theta-1\right)\hat{\tau}_{t}+\left(\theta-1\right)\widehat{mc}_{t} \\
&- \frac{3}{2}\phi_{H}\hat{\pi}_{H,t}^{2}+\frac{5}{2}\beta\phi_{H}E_{t}\hat{\pi}_{H,t+1}^{2}+\frac{1}{2}\left(\theta-1\right)\widehat{mc}_{t}^{2}+\frac{1}{2}\left(\theta-1\right)\left[\mu\eta^{2}+\eta\left(1-\mu\right)\right]\hat{\tau}_{t}^{2} \\
&+ \eta\left(\theta-1\right)\hat{\tau}_{t}\widehat{mc}_{t}+\beta\phi_{H}E_{t}\hat{\pi}_{H,t+1}\hat{y}_{t+1}-\beta\phi_{H}E_{t}\hat{\pi}_{H,t+1}\hat{y}_{t}-\beta\gamma\phi_{H}E_{t}\hat{\pi}_{H,t+1}\hat{c}_{t+1} \\
&+ \beta\gamma\phi_{H}E_{t}\hat{\pi}_{H,t+1}\hat{c}_{t}-\beta\phi_{H}E_{t}\hat{\pi}_{H,t+1}\hat{\pi}_{t+1} \qquad \text{(B.81)}
\end{aligned}
$$

$$
\begin{aligned}
\phi_{F}^{*}\hat{\pi}_{F,t}^{*} &= \beta\phi_{F}^{*}E_{t}\hat{\pi}_{F,t+1}^{*}+\left(\theta-1\right)\eta\hat{\tau}_{t}^{*}+\left(\theta-1\right)\widehat{mc}_{t}^{*} \\
&- \frac{3}{2}\phi_{F}^{*}\hat{\pi}_{F,t}^{*2}+\frac{5}{2}\beta\phi_{F}^{*}E_{t}\hat{\pi}_{F,t+1}^{*2}+\frac{1}{2}\left(\theta-1\right)\widehat{mc}_{t}^{*2}+\frac{1}{2}\left(\theta-1\right)\left[\mu\eta^{2}+\eta\left(1-\mu\right)\right]\hat{\tau}_{t}^{*2} \\
&+ \eta\left(\theta-1\right)\hat{\tau}_{t}^{*}\widehat{mc}_{t}^{*}+\beta\phi_{F}^{*}E_{t}\hat{\pi}_{F,t+1}^{*}\hat{y}_{t+1}^{*}-\beta\phi_{F}^{*}E_{t}\hat{\pi}_{F,t+1}^{*}\hat{y}_{t}^{*}-\beta\gamma^{*}\phi_{F}^{*}E_{t}\hat{\pi}_{F,t+1}^{*}\hat{c}_{t+1}^{*} \\
&+ \beta\gamma^{*}\phi_{F}^{*}E_{t}\hat{\pi}_{F,t+1}^{*}\hat{c}_{t}^{*}-\beta\phi_{F}^{*}E_{t}\hat{\pi}_{F,t+1}^{*}\hat{\pi}_{t+1}^{*} \qquad \text{(B.82)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}_{H,t}^{*} &= \hat{\pi}_{H,t}+\hat{s}_{t-1}-\hat{s}_{t} \\
&- \frac{1}{2}\hat{\pi}_{H,t}^{*2}+\frac{1}{2}\hat{\pi}_{H,t}^{2}+\frac{1}{2}\hat{s}_{t-1}^{2}-\frac{1}{2}\hat{s}_{t}^{2} \\
&+ \hat{\pi}_{H,t}\hat{s}_{t-1}-\hat{\pi}_{H,t}\hat{s}_{t}-\hat{s}_{t-1}\hat{s}_{t} \qquad \text{(B.83)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\pi}_{F,t} &= \hat{\pi}_{F,t}^{*}+\hat{s}_{t}-\hat{s}_{t-1} \\
&- \frac{1}{2}\hat{\pi}_{F,t}^{2}+\frac{1}{2}\hat{\pi}_{F,t}^{*2}+\frac{1}{2}\hat{s}_{t}^{2}+\frac{1}{2}\hat{s}_{t-1}^{2} \\
&+ \hat{\pi}_{F,t}^{*}\hat{s}_{t}-\hat{\pi}_{F,t}^{*}\hat{s}_{t-1}-\hat{s}_{t}\hat{s}_{t-1} \qquad \text{(B.84)}
\end{aligned}
$$

In LCP,

Market Clearing Conditions

$$\begin{aligned}\bar{y}_H\hat{y}_{H,t} &= \mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_t+\left(1-\eta\right)\bar{c}\hat{c}_t+\bar{i}\cdot\hat{i}_t \\ &- \frac{1}{2}\bar{y}_H\hat{y}_{H,t}^2+\frac{1}{2}\left[\mu\eta^2\left(1-\eta\right)\left(2\mu-1\right)+\mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}\hat{\tau}_t^2 \\ &+ \frac{1}{2}\left(1-\eta\right)\bar{c}\cdot\hat{c}_t^2+\frac{1}{2}\bar{i}\cdot\hat{i}_t^2+\frac{1}{2}\phi_H\bar{y}_H\hat{\pi}_{H,t}^2+\mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_t\hat{c}_t \end{aligned} \quad \text{(B.85)}$$

$$\begin{aligned}\bar{y}_H^*\hat{y}_{H,t}^* &= -\mu\eta\left(1-\eta\right)\bar{c}^*\hat{\tau}_t^*+\eta\bar{c}^*\hat{c}_t^* \\ &- \frac{1}{2}\bar{y}_H^*\hat{y}_{H,t}^{*2}+\frac{1}{2}\left[\mu\eta(1-\eta)^2\left(2\mu-1\right)+\mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}^*\hat{\tau}_t^{*2} \\ &+ \frac{1}{2}\eta\bar{c}^*\hat{c}_t^{*2}+\frac{1}{2}\phi_H^*\bar{y}_H^*\hat{\pi}_{H,t}^{*2}-\mu\eta\left(1-\eta\right)\bar{c}^*\hat{\tau}_t^*\hat{c}_t^* \end{aligned} \quad \text{(B.86)}$$

$$\begin{aligned}\bar{y}_F^*\hat{y}_{F,t}^* &= \mu\eta\left(1-\eta\right)\bar{c}^*\hat{\tau}_t^*+\left(1-\eta\right)\bar{c}^*\hat{c}_t^*+\bar{i}^*\hat{i}_t^* \\ &- \frac{1}{2}\bar{y}_F^*\hat{y}_{F,t}^{*2}+\frac{1}{2}\left[\mu\eta^2\left(1-\eta\right)\left(2\mu-1\right)+\mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}^*\hat{\tau}_t^{*2} \\ &+ \frac{1}{2}\left(1-\eta\right)\bar{c}^*\hat{c}_t^{*2}+\frac{1}{2}\bar{i}^*\cdot\hat{i}_t^{*2}+\frac{1}{2}\phi_F^*\bar{y}_F^*\hat{\pi}_{F,t}^{*2}+\mu\eta\left(1-\eta\right)\bar{c}^*\hat{\tau}_t^*\hat{c}_t^* \end{aligned} \quad \text{(B.87)}$$

$$\begin{aligned}\bar{y}_F\hat{y}_{F,t} &= -\mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_t+\eta\bar{c}\hat{c}_t \\ &- \frac{1}{2}\bar{y}_F\hat{y}_{F,t}^2+\frac{1}{2}\left[\mu\eta(1-\eta)^2\left(2\mu-1\right)+\mu\eta\left(1-\eta\right)\left(1-\mu\right)\right]\bar{c}\hat{\tau}_t^2 \\ &+ \frac{1}{2}\eta\bar{c}\cdot\hat{c}_t^2+\frac{1}{2}\phi_F\bar{y}_F\hat{\pi}_{F,t}^2-\mu\eta\left(1-\eta\right)\bar{c}\hat{\tau}_t\hat{c}_t \end{aligned} \quad \text{(B.88)}$$

$$\begin{aligned}\bar{y}\hat{y}_t &= \bar{y}_H\hat{y}_{H,t}+\bar{y}_H^*\hat{y}_{H,t}^* \\ &- \frac{1}{2}\bar{y}\hat{y}_t^2+\frac{1}{2}\bar{y}_H\hat{y}_{H,t}^2+\frac{1}{2}\bar{y}_H^*\hat{y}_{H,t}^{*2} \end{aligned} \quad \text{(B.89)}$$

$$\begin{aligned}\bar{y}^*\hat{y}_t^* &= \bar{y}_F\hat{y}_{F,t}+\bar{y}_F^*\hat{y}_{F,t}^* \\ &- \frac{1}{2}\bar{y}^*\hat{y}_t^{*2}+\frac{1}{2}\bar{y}_F\hat{y}_{F,t}^2+\frac{1}{2}\bar{y}_F^*\hat{y}_{F,t}^{*2} \end{aligned} \quad \text{(B.90)}$$

Phillips curves

$$
\begin{aligned}
\phi_H \hat{\pi}_{H,t} &= \beta\phi_H E_t\hat{\pi}_{H,t+1} + \eta(\theta-1)\hat{\tau}_t + (\theta-1)\widehat{mc}_t \\
&- \frac{3}{2}\phi_H\hat{\pi}^2_{H,t} + \frac{5}{2}\beta\phi_H E_t\hat{\pi}^2_{H,t+1} + \frac{1}{2}(\theta-1)\widehat{mc}^2_t + \frac{1}{2}(\theta-1)\left[\mu\eta^2 + \eta(1-\mu)\right]\hat{\tau}^2_t \\
&+ \eta(\theta-1)\hat{\tau}_t\widehat{mc}_t + \beta\phi_H E_t\hat{\pi}_{H,t+1}\hat{y}_{H,t+1} - \beta\phi_H E_t\hat{\pi}_{H,t+1}\hat{y}_{H,t} - \beta\gamma\phi_H E_t\hat{\pi}_{H,t+1}\hat{c}_{t+1} \\
&+ \beta\gamma\phi_H E_t\hat{\pi}_{H,t+1}\hat{c}_t - \beta\phi_H E_t\hat{\pi}_{H,t+1}\hat{\pi}_{t+1} \qquad \text{(B.91)}
\end{aligned}
$$

$$
\begin{aligned}
\phi^*_F \hat{\pi}^*_{F,t} &= \beta\phi^*_F E_t\hat{\pi}^*_{F,t+1} + (\theta-1)\eta\hat{\tau}^*_t + (\theta-1)\widehat{mc}^*_t \\
&- \frac{3}{2}\phi^*_F\hat{\pi}^{*2}_{F,t} + \frac{5}{2}\beta\phi^*_F E_t\hat{\pi}^{*2}_{F,t+1} + \frac{1}{2}(\theta-1)\widehat{mc}^{*2}_t + \frac{1}{2}(\theta-1)\left[\mu\eta^2 + \eta(1-\mu)\right]\hat{\tau}^{*2}_t \\
&+ \eta(\theta-1)\hat{\tau}^*_t\widehat{mc}^*_t + \beta\phi^*_F E_t\hat{\pi}^*_{F,t+1}\hat{y}^*_{F,t+1} - \beta\phi^*_F E_t\hat{\pi}^*_{F,t+1}\hat{y}^*_{F,t} - \beta\gamma^*\phi^*_F E_t\hat{\pi}^*_{F,t+1}\hat{c}^*_{t+1} \\
&+ \beta\gamma^*\phi^*_F E_t\hat{\pi}^*_{F,t+1}\hat{c}^*_t - \beta\phi^*_F E_t\hat{\pi}^*_{F,t+1}\hat{\pi}^*_{t+1} \qquad \text{(B.92)}
\end{aligned}
$$

$$
\begin{aligned}
\phi^*_H \hat{\pi}^*_{H,t} &= \beta\phi^*_H E_t\hat{\pi}^*_{H,t+1} + (\eta-1)(\theta-1)\hat{\tau}^*_t + (\theta-1)\widehat{mc}_t - (\theta-1)\hat{e}_t \\
&- \frac{3}{2}\phi^*_H\hat{\pi}^{*2}_{H,t} + \frac{5}{2}\beta\phi^*_H E_t\hat{\pi}^{*2}_{H,t+1} + \frac{1}{2}(\theta-1)\left[\mu(1-\eta)^2 + (1-\mu)(1-\eta)\right]\hat{\tau}^{*2}_t \\
&+ \frac{1}{2}(\theta-1)\widehat{mc}^2_t + \frac{1}{2}(\theta-1)\hat{e}^2_t + \beta\phi_H E_t\hat{\pi}^*_{H,t+1}\hat{y}^*_{H,t+1} - \beta\phi^*_H E_t\hat{\pi}^*_{H,t+1}\hat{y}^*_{H,t} \qquad \text{(B.93)} \\
&- \beta\gamma\phi^*_H E_t\hat{\pi}^*_{H,t+1}\hat{c}_{t+1} + \beta\gamma\phi^*_H E_t\hat{\pi}^*_{H,t+1}\hat{c}_t - \beta\phi^*_H E_t\hat{\pi}^*_{H,t+1}\hat{\pi}^*_{t+1} - \beta\phi^*_H E_t\hat{\pi}^*_{H,t+1}\hat{e}_t \\
&+ \beta\phi^*_H E_t\hat{\pi}^*_{H,t+1}\hat{e}_{t+1} - (1-\eta)(\theta-1)\hat{\tau}^*_t\widehat{mc}_t + (1-\eta)(\theta-1)\hat{\tau}^*_t\hat{e}_t - (\theta-1)\widehat{mc}_t\hat{e}_t
\end{aligned}
$$

$$
\begin{aligned}
\phi_F \hat{\pi}_{F,t} &= \beta\phi_F E_t\hat{\pi}_{F,t+1} + (\eta-1)(\theta-1)\hat{\tau}_t + (\theta-1)\widehat{mc}^*_t + (\theta-1)\hat{e}_t \\
&- \frac{3}{2}\phi_F\hat{\pi}^2_{F,t} + \frac{5}{2}\beta\phi_F E_t\hat{\pi}^2_{F,t+1} + \frac{1}{2}(\theta-1)\left[\mu(1-\eta)^2 + (1-\mu)(1-\eta)\right]\hat{\tau}^2_t \\
&+ \frac{1}{2}(\theta-1)\widehat{mc}^{*2}_t + \frac{1}{2}(\theta-1)\hat{e}^2_t + \beta\phi_F E_t\hat{\pi}_{F,t+1}\hat{y}_{F,t+1} - \beta\phi_F E_t\hat{\pi}_{F,t+1}\hat{y}_{F,t} \\
&- \beta\gamma^*\phi_F E_t\hat{\pi}_{F,t+1}\hat{c}^*_{t+1} + \beta\gamma^*\phi_F E_t\hat{\pi}_{F,t+1}\hat{c}^*_t - \beta\phi_F E_t\hat{\pi}_{F,t+1}\hat{e}_{t+1} + \beta\phi_F E_t\hat{\pi}_{F,t+1}\hat{e}_t \\
&- \beta\phi_F E_t\hat{\pi}_{F,t+1}\hat{\pi}_{t+1} - \beta\phi_F E_t\hat{\pi}_{F,t+1}\hat{\pi}_{t+1} - (1-\eta)(\theta-1)\hat{\tau}_t\widehat{mc}^*_t - (1-\eta)(\theta-1)\hat{\tau}_t\hat{e}_t \\
&+ (\theta-1)\widehat{mc}^*_t\hat{e}_t \qquad \text{(B.94)}
\end{aligned}
$$

Deviations of law of price

$$
\begin{aligned}
\hat{z}_t &= \hat{p}^*_{F,t} - \hat{p}_{F,t} + \hat{s}_t \\
&- \frac{1}{2}\hat{z}^2_t + \frac{1}{2}\hat{p}^{*2}_{F,t} + \frac{1}{2}\hat{p}^2_{F,t} + \frac{1}{2}\hat{s}^2_t - \hat{p}^*_{F,t}\hat{p}_{F,t} + \hat{p}^*_{F,t}\hat{s}_t - \hat{p}_{F,t}\hat{s}_t \qquad \text{(B.95)}
\end{aligned}
$$

$$
\begin{aligned}
\hat{z}_t^* &= \hat{p}_{H,t} - \hat{p}_{H,t}^* - \hat{s}_t \\
&- \frac{1}{2}\hat{z}_t^{*2} + \frac{1}{2}\hat{p}_{H,t}^2 + \frac{1}{2}\hat{p}_{H,t}^{*2} + \frac{1}{2}\hat{s}_t^2 - \hat{p}_{H,t}\hat{p}_{H,t}^* + \hat{p}_{H,t}^*\hat{s}_t - \hat{p}_{H,t}\hat{s}_t
\end{aligned} \tag{B.96}
$$