Flow Induced Vibrations of Pipes with Attached Valves

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The solution to problems of vibrations of pipes induced by internal flow has undeniable academic as well as engineering interest. The theoretical stimulus for their study derives from the nonconservative character of the system and the physical concretization of follower type excitation. The results can be applied, e.g., in the design of pipelines, cylindrical fuel elements in nuclear reactors and on the analysis of unwanted vibrations of nuclear piping sometimes detected at the testing stage of power plants.

The conjugation of academic and design interest resulted in a vast number of publications that have significantly contributed for the understanding of the phenomenon. A review of such extensive available literature reveals, however, that systems with attached eccentric lumped masses appear not to have been considered so far. Those mechanical configurations are, nevertheless, common in industrial piping, e.g., where large, heavy valves are directly attached to pipes of comparatively small diameter.

The influence of the corresponding lumped translational and rotatory inertia terms on the stability of the motion induced by axial, internal flow is examined for typical boundary conditions and constant speed flow.

The governing equations are found by application of the extended Hamilton principle and a reduced Galerkin's method is utilized to approximate the complex eigen-frequencies, for each value of flow speed. The associated eingenvalue problem is formulated in a manner that permits straight application of current numerical algorithms.

Appropriate dimensionless parameters are defined and the numerical results are displayed for representative values of those parameters.

The study reveals that the regions of divergence (where existing) and coupled flutter are highly dependent both on the magnitude of the lumped mass and on its eccentricity.

The magnitude of the frequencies on the regions of periodic motion is also strongly dependent on the same parameters. It is further shown that appropriate location of the valves may reduce the probability of dynamic unstability and the resultant mal-function of the piping system.

1. Introduction and Scope

The rapid development of dynamic stability analysis as a branch of applied mechanics was closely associated with the failure of classical Euler-type methods to study non-conservative systems [1] .

A typical example of the shortcomings of the bifurcation of equilibrium approach is the cantilevered column subjected to a follower force. The interest of engineers on early theoretical results, obtained on idealized mathematical models, was limited until the physical realization of follower forces was perceived. This occured when vibrations of pipes due to internal flow were observed and their study became a design requirement for some pipelines, cooling units and like systems.

The concurrent theoretical and practical importance of these problems provided stimulus for additional research in the area, led to a large number of studies and permitted the experimental corroboration of attained analytical results [e.g.2] . Many available papers shed light on the characteristics of the phenomena and a review of the state of knowledge in the field may be construed from the survey by S.S.Chen [3] , supplemented by a number of publications of the past four years, e.g. [4] . Investigations have been performed for cases of constant and pulsatile flow, for different support and boundary conditions, for curved and straight pipe configurations. The importance of nonlinearities has also been examined and adequate techniques to deal with them have been proposed [5] .

In spite of the mentioned thorough coverage of those topics, the seemingly important possibility of having large, misaligned masses lumped on the pipe had apparently not been reported prior to some effort initiated by the author [6] . Concentrated masses may represent valves often directly attached to piping. Valves have frequently very high rigidity, compared with that of piping, and their centers of mass can be located at non-negligible distance from the axis of the pipe. The scope of the present study is the assessment of the effects that lumped rotatory and/or translational inertia have on the stability of motion for constant speed of flow. Simple geometric configurations were selected and the masses placed so that the analysis of the effects of translational and rotatory inertia can be made when they act either individually or simultaneously.

The investigation does not include out-of-plane vibrations and is limited to straignt pipe segments. The coupling of bending and torsion modes, inherent to out-of-plane vibrations, is not considered herein. Similarly, the effects of changes of direction of the axis of the piping are also ignored. Both conditions are, nonetheless, believed important [7] and their study is regarded as a natural and required extension to be undertaken as soon as feasible.

2. Equations of Motion and Technique of Solution

The equations of motion are usually derived either from the equilibrium of infinitesimal elements or from the stationarity of the functional associated with Hamilton's generalized principle. The latter method is more suitable to the ensuing numerical analysis, leads to the natural boundary conditions and appears altogether less likely to allow for ommissions or errors. The procedure followed here is an extension of the analysis presented in [8] . The stationarity conditions may be written as:

$$\int_{\tau_1}^{\tau_2}\int_{0}^{1} \left(\ddot{\bar{y}} + \bar{y}^{IV} + 2\,\bar{u}\sqrt{\beta}\,\dot{\bar{y}}' + \bar{u}^2\,\bar{y}'' \right) \delta\bar{y}\, d\bar{x}\, d\tau + \bar{M}\int_{\tau_1}^{\tau_2} \ddot{\bar{y}}\,(\bar{x}_o)\;\delta\bar{y}\,(\bar{x}_o)\, d\tau + \bar{M}\,\bar{h}^2\int_{\tau_1}^{\tau_2} \ddot{\bar{y}}\,\times$$

$$(\bar{x}_o)\;.\;\delta\bar{y}'\,(\bar{x}_o)\,d\tau + \delta\left[\text{Boundary Terms}\right] = o \qquad (1)$$

The dimensionless parameters in (1) are defined by:

$$\bar{x} = x\,/\,L,\ \bar{y} = y\,/\,L,\ \bar{h} = h\,/\,L, \tau = t\,/\,T,\ T = \left[(\rho_f + \rho_p)\,L^4\,/\,(E\,I)\right]^{1/2},$$

$$\bar{u} = \{\rho_p\,L\,/\,(EI)\}^{1/2}\,u,\ \beta = \rho_f\,/\,(\rho_f + \rho_p),\ \bar{M} = M\,/\,\{(\rho_f + \rho_p)\,L\} \qquad (2)$$

were M = lumped mass of eccentricity h, located at the section of abscissa x_o, (EI) = bending stiffness of pipe of length L and mass density ρ_p, and ρ_f = mass density of fluid that travels at speed u along the pipe. A geometric sketch of the system is shown in Fig. 1.

The term on $(\partial^2\,\bar{y}\,/\,\partial\bar{x}\,\partial\tau)$ in (1) reflects the presence of Coriolis forces due to the axial flow and the rotation of the fluid. It is noted that $\partial^2\bar{y}\,/\,\partial\bar{x}\partial\tau = \partial/\partial\tau\,(\partial\bar{y}/\partial\bar{x})$ measures the speed of rotation and changes sign along the axis of the pipe at any given instant. This term causes the well known "distortion" of the classical modes of vibration. The curvature $\bar{y}''$ of the deformed configuration introduces a term similar to that due to compressive axial loads and reduces the natural frequencies of the system.

The results to be reported in the sequel pertain either to a simply supported(s.s.) or to a cantilevered pipe. The discretization of the equations of motion is based on a Galerkin approximation, utilizing for admissible functions (specewise) the modes of vibration $g_i\,(\bar{x})$ of the corresponding pipe (for $\bar{M} = \bar{u} = o$):

$$\bar{y}\,(\bar{x},\,\tau) \simeq \sum_{1}^{N} g_i\,(\bar{x})\,f_i\,(\tau) \qquad (3)$$

Introducing (3) into (1) and performing the space integrations, the stationarity conditions are reduced to a system of ordinary differential equations:

$$A\,\ddot{\underset{\sim}{f}} + V\,\dot{\underset{\sim}{f}} + D\,\underset{\sim}{f} = o \qquad (4)$$

where the matrices A,V,D depend on the geometric constraints. For a simply supported pipe the elements of the matrices (obtained with $g_i = \sqrt{2}\,\sin i\,\pi\,\bar{x}$) are:

$$A_{ij} = \delta_{ij} + \bar{M}\,\phi_i(\bar{x}_o)\,\phi_j(\bar{x}_o) + \bar{M}\,\bar{h}^2\,\phi'_i\,(\bar{x}_o)\,\phi'_j\,(\bar{x}_o)$$

$$V_{ij} = 2\,\bar{u}\,\beta\,(1 - \delta_{ij})\,i\left[\frac{1-(-1)^{j-i}}{j - i} + \frac{1-(-1)^{i+j}}{i + j}\right] \qquad (5)$$

$$D_{ij} = (\pi^4\,i^4 - \pi^2\,\bar{u}^2\,j^2)\,\delta_{ij}$$

where δ_{ij} is Kronecker's delta. For a cantilevered pipe the admissible functions are $g_j = \cosh\alpha_j\,\bar{x} - \cos\alpha_j\,\bar{x} - \sigma_j\,(\sinh\alpha_j\,\bar{x} - \sin\alpha_j\,\bar{x})$, with

$\sigma_j = (\sinh\alpha_j - \sin\alpha_j)\,/\,(\cosh\alpha_j + \cos\alpha_j)$. Parameters α_j can be found in standard textbooks in dynamics of structures. The elements of the corresponding A,V,D matrices are readily obtained by quadrature and ommitted here for shortness.

The characterization of the stability of motion is accomplished by examining the real parts of the complex eigenvalues of (4). This approach, if applied directly, might be somewhat cumbersome and it is preferred to replace (4) by another system of equations more easily amenable to numerical treatment 9 . Equations (4) are premultiplied by A^{-1}, vectors $\underset{\sim}{f}$ and $\dot{\underset{\sim}{f}}$ grouped into a single vector and simple linear algebraic operations lead to:

$$\begin{bmatrix} \dot{\underset{\sim}{f}} \\ \\ \dot{\underset{\sim}{f}} \end{bmatrix} = \begin{bmatrix} -A^{-1}V & -A^{-1}D \\ \\ I & O \end{bmatrix} \begin{bmatrix} \dot{\underset{\sim}{f}} \\ \\ \underset{\sim}{f} \end{bmatrix} = S \begin{bmatrix} \dot{\underset{\sim}{f}} \\ \\ \underset{\sim}{f} \end{bmatrix} \tag{6}$$

The eigenvalues λ of matrix S are the same as those of the problem posed by (4). Numerical techniques to obtain the complex $\lambda\lambda$ for a real, non-symmetric matrix have long been established and efficient computer routines are available for their implementation. The kinetic approach to investigate the existence of instabilities from the behavior of $e^{\lambda\tau}$ states basically that unbounded motion arises for Re (λ) > o. Oscillatory instability may be present for λ = o. The knowledge of the eigenvalues, over a range of speed flow $\bar{u}$, permits the establishment of stability curves λ vs. $\bar{u}$. The study of these curves will show divergence and flutter instabilities both for pinned-pinned and clamped-free pipes. Earlier studies of pipes with fixed ends were confined to the determination of bifurcation type of instabilities, since the systems were considered "conservative", the Coriolis forces being "gyroscopic conservative". The fact is that the systems are non-conservative, although they behave as if they were conservative for some values of $\bar{u}$ [10] .

3. Results

The results were generated through a computer program that found the eigenvalues for dimensionless speeds $\bar{u}$ varied from zero up to 12.5. The number N of admissible functions required to attain acceptable accuracy depends on the value of $\bar{u}$ (with more functions needed in the high speed region) and on the number of eigenvalues under study. Sufficiently accurate approximations of the three lowest eigenvalues are obtained with N = 6 for all $\bar{u}\bar{u}$ considered.

The results are illustrated in Figs. 2 thru 5, selected as representative of patterns of behavior. Numerous other cases were solved and some results reported elsewhere [11]. Fig. 2 displays both Im (λ) and Re (λ) versus $\bar{u}$, designated shortly as stability curves, for $\bar{M}$ = 0, 1, 2.5 at the mid-span of a s.s. pipe, with h = o and β = 0.25. The symmetry of Re (λ) with respect to the $\bar{u}$ -axis is always verified for these boundary conditions. Fig. 3 compares results for β = 0.25 and β = 0.01, in a simply supported system with a centrally located lumped mass $\bar{M}$ = 1. Due to the above mentioned symmetry, values of Re $\lambda < 0$ are not shown. Fig. 4 displays the stability curves for a cantilever pipe with β = 0.5 and an aligned mass $\bar{M}$ = 1.25 at the tip. Finally, Fig. 5 illustrates the behavior of the eigenvalues for a cantilever with β = 0.5 and $\bar{M}$ = 2.5, varying the rotary inertia of the valve through a changing $\bar{h}$ that assumes the values 0,0.25 and 0.50.

The effects of rotatory inertia alone may be studied e.g. by attaching a mass over one of the supports in a s.s. pipe. The variation of results with $\bar{M}$ in such a system is illustrated in the table below, where $\bar{h} = 0.50$ and $\bar{M} = 0.5, 1.0, 2. , 5.0$. The parametric speed values were selected at a region where modifications on λ are readily observed. More detailed analysis can be found in [12].

INFLUENCE OF $\bar{M}$ ON EIGENVALUES ($\beta = 0.25$, $\bar{x}_o = 1$, $\bar{h} = 0.50$)

		$\bar{M} = 0.5$			$\bar{M} = 1.0$			$\bar{M} = 2.5$			$\bar{M} = 5.0$		
$\bar{u}$		λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
5.5	(Reλ)	10.38	0.	.0.	10.15	0.	0.	9.97	0..	0..	9.90	0.	0.
	(Imλ)	0.	6.08	40.93	0.	4.45	40.54	0.	2.89	40.31	0.	2.07	40.23
6.5	(Reλ)	13.88	3.34	0.	14.10	2.37	0.	14.24	1.50	0.	14.29	1.06	0.
	(Imλ)	0.	0.	34.38	0.	0.	33.84	0.	0.	33.51	0.	0.	33.40
7.5	(Reλ)	12.55	12.55	0.	14.99	7.87	0.	15.61	4.96	0.	15.78	3.52	0.
	(Imλ)	1.16	1.16	24.76	0.	0.	23.63	0.	0.	22.85	0.	0.	22.57
		Coupling			No coupling								

4. Interpretation of Results

Solutions obtained for $\bar{M} = o$ essentially reproduce values already published, though presented in a different manner. Some discrepancies have been found by the author, especially in the neighbourhodd of Im $\lambda_2 = o$ and Im $\lambda_3 = o$. In those regions it may be numerically difficult to ascertain e.g. which values correspond to λ_1 or to λ_2 and some faulty results have been reported in the literature [e.g. 13] .

In s.s. pipes, for $\bar{M} = o$, Fig. 2 shows divergence at $\bar{u} = \pi$ (Im $\lambda_1 = o$, Re $\lambda_1 > o$). Increasing $\bar{u}$, λ_1 and λ_2 coalesce in the upper vicinity of $\bar{u} = 2\pi$ at onset of coupled flutter. The source of energy that sustains this kinetic instability, i.e. the transfer of energy from non-conservative forces into elastic pipes, is explained by Weaver [10] . Making $\bar{M} = 1$ not only decreases the magnitude of the frequencies but also modifies the width and location of the intervals of $\bar{u}$ for which coupled flutter occurs. For $\bar{M} = 2.5$ that oscillatory instability disappears. Notice that for $\bar{u} < \pi$ it is Re $\lambda_i = o$, $i = 1, 2, 3$, and nothing can be said about stability. The curves in Fig. 3 show little qualitative dependence on β. This result is valid for s.s. pipes and it is not to be extrapolated without special caution, because it contradicts conclusions attained in cantilevered pipes [12] .

Fig. 4 compared with Figs. 2 and 3 illustrates the marked difference between systems with fixed ends and those with a free end. The presence of $\bar{M}$ is responsible for the knee behavior of the damping terms Re λ_3. A positive and, therefore, stabilizing damping coefficient (Re $\lambda > o$) appears for moderate speeds of flow. Remark that Im λ_2 never vanishes (contrary to the case $\bar{M} = o$). Im λ_1 is zero in an interval between two points of bifurcation and, hence, no oscillations are associated with λ_1 and the mode is said to be critically damped. Re λ_2 and Re λ_1 are positive for higher $\bar{uu}$ proving the existence of oscillatory instabilities.

The last figure emphasizes the relative importance of the rotatory inertia of the valves, varied through the increase of $\bar{M}\,\bar{h}^2$. The frequencies become smaller, as expected, due to the increase on kinetic energy. The closed locii of double Re λ_1 (for same $\bar{u}$)

shrinks for increasing $\bar{h}$. The effects are less pronounced on λ_2 but the second admissible shape selected and the location of the valve preclude definitive conclusions.

The values tabulated for the s.s. pipe corroborate the importance of rotatory inertia by itself. Notice for instance that coalescent λ_1, λ_2 that appear for $\bar{M} = 0.5$ (at $\bar{u} = 7.5$) are no longer present for larger values of $\bar{M}$. The decrease of Im λ_3 with $\bar{M}$, in spite of zero deflection at the point of application of the mass, is also apparent ($\bar{u}_3 = 3\pi$) and the divergence of the second mode between $\bar{u} = 5.5$ and $\bar{u} = 6.5$ can be predicted.

5. Concluding Remarks

The early motivation for the study resulted from unexpected vibration detected while testing a power plant. Typically, the lines involved were of Ø 2", supported valves weighing 450 lbs, with an eccentricity of 540 mm and conveyed water. The application of the results derived herein revealed that the lowest critical velocity was far higher than the speed of flow. More interesting were the findings for a steam dump line (Ø 8" sch 80) in which the speed of flow reaches near sonic values. The line had an eccentric valve of 1400 lbs attached to it. The existence of spring supports and the complex geometry of the line precluded direct application of the graphs, but a first estimate confirmed the potential influence of the valve and advised a modification of the supports.

A short account of the conclusions drawn from the study reads as follows:

Large, rigid valves influence the dynamic behavior of pipes conveying fluid in a non-negligible manner. The magnitude of the eigenvalues and the character of instabilities are modified due to the rotatory and translational inertia introduced by the concentrated mass of the valves. Modification of β appears to be of importance for cantilevered pipes but not so for pipes with fixed ends. Appropriate location of lumped masses may effectively control the dynamic stability of the system.

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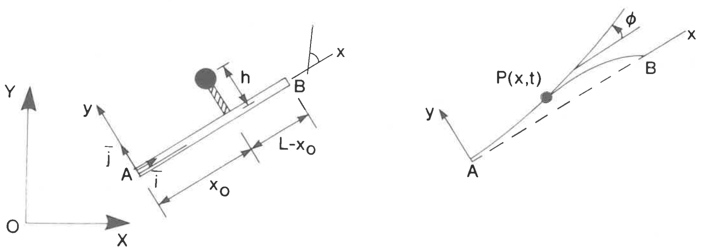

Fig. 1 - General Geometry of System with Attached Mass.

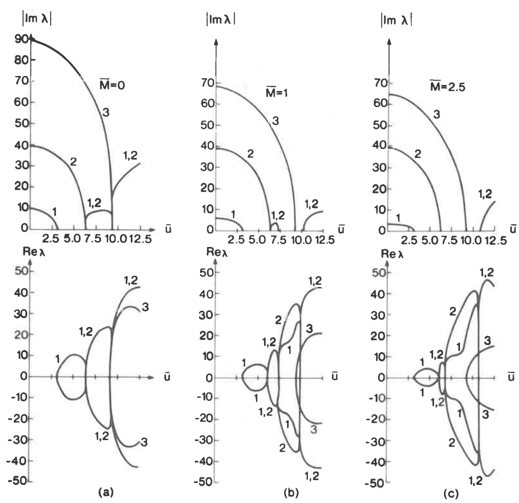

Fig. 2 - Complex Eigenfrequencies for Simply Supported Pipe with Mid-Span Mass ($\bar{M}$ = 0, 1, 2.5).

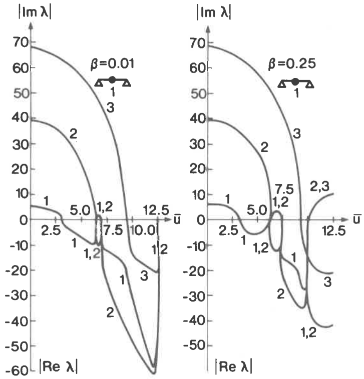

Fig. 3 - Influence of β on Eigenfrequencies in a Simply Suported Pipe.

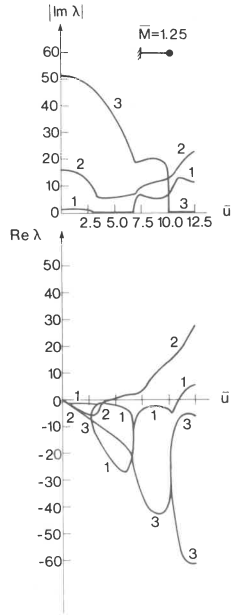

Fig. 4 - Behaviour of Cantilever Pipe with Mass at Tip.

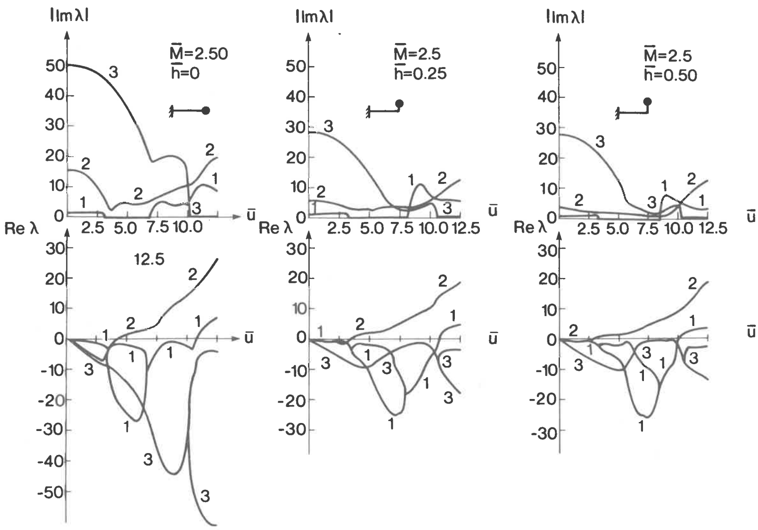

Fig. 5 - Influence of Translational and Rotatory Lumped Inertiae in Stability Curves for Cantilevered Systems.