

## TORSIONAL STRUCTURAL RESPONSE FROM FREE-FIELD GROUND MOTION

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### ABSTRACT

Torsional response of structures subjected to the action of both the free-field torsional inputs and external torque is investigated. By expanding the work of Scanlan, both lateral and torsional foundation inputs due to a travelling shear wave are derived from the free-field point motion. These free-field torsional motions are used as the basis of numerical studies. Response for different soil stiffness and structural characteristics are studied, as well as different dynamic models. In one dynamic model the structure is coupled to the soil using a compliance spring matrix and in the second model the structure coupled to an elastic half-space. Results of these two basic models are compared and found to be in good agreement. Finally, torsional structural response caused by torsional inputs is compared with lateral response caused by modified lateral inputs to determine the significance of torsional excitation on the seismic response of building structures. Numerical results show that these torsional seismic loads are as large or larger than those from modified lateral inputs.

## INTRODUCTION

In most seismic analyses of building structures, the earthquake free-field ground motion is assumed to be identical at all coordinate points at a given level beneath the structure. This basic assumption implies that the wave length of the seismic ground motion is long compared to the structure foundation. However, for some soil conditions, the wave lengths of the dominant frequencies in earthquake motions are comparable to the foundations of large structures. Thus, the particle displacements of S-waves under these structures tend to oppose each other in a linear direction causing a "self-cancellation" effect. In addition, these displacements create a free-field torsional input motion. It is the purpose of the paper to report results of a study which treats these two effects in an approximate manner.

The average effective translational and torsional ground input motions are derived by assuming that the base mass of the structure to be rigid and rectangular, resting upon distributed soil springs, with seismic shear travelling waves passing under the springs. This approach was proposed by Scanlan [1] and was further expanded by Wolf [2]. Since the base mass of the structure is rigid it will tend to average the ground motion forcing the soil particles to displace as a rigid body. As a result both lateral and torsional foundation inputs are obtained from free-field point motion. By abandoning the uniformity hypothesis of the free-field input motion, the lateral displacement components will generally be reduced. The significance of the torsional soil-structure interaction effects are evaluated by comparing changes in output acceleration spectrum with the free-field acceleration spectrum.

Numerical results of this investigation are obtained by iteration for a simplified structural model subjected to the El Centro, California earthquake of 1940. Responses for different soil stiffness and structural characteristics are studied with two basic dynamic models: 1) a lumped parameter system with compliance spring model of the soil, and 2) a lumped parameter system coupled to an elastic half-space. Results of these two basic dynamic models are compared. In addition, torsional structural response caused by torsional inputs is compared with lateral response induced by modified lateral inputs to determine the significance of torsional excitation on the seismic response of the building structures.

### Theory

#### Part I: Average Effective Seismic Input Motions

In this investigation it is assumed that the base mass of the structure is rigid and rectangular, with seismic waves traversing the plan of the structural site. This approach considers the dynamic characteristics of the structure and wave propagation velocity. Furthermore, the assumption tends to average the ground motion forcing the soil particles to displace as a rigid body. The idealized model of this system is shown in Figure (1).

Free-field ground excitations used in this study are derived by an approximate procedure proposed by Scanlan [1]. Since seismic motions are caused by waves, the free-field lateral acceleration in the x-direction can be expressed in terms of the Fourier series. Specifically,

$$\ddot{U}_F(x,t) = \sum_{n=1}^N A_n \sin(k_n t + \theta_n - \frac{k_n}{c} x) \quad (1)$$

where  $c$  is the shear wave velocity,  $N$  is the number of harmonics to be considered,  $A_n$  are the Fourier amplitudes of the earthquake,  $\theta_n$  is the  $n$ th component phase angle, and  $k_n$  are the circular frequencies.

Due to propagating seismic waves, the average lateral ground acceleration can be determined from

$$(\ddot{U}_F)_{ave} = \frac{1}{A} \int_{-r_0}^{r_0} \ddot{U}_F(x, t) dA \quad (2)$$

where  $A = 4r_0^2$  and  $dA = 2r_0 dx$ . Therefore,

$$(\ddot{U}_F)_{ave} = \frac{1}{2r_0} \int_{-r_0}^{r_0} \sum_{n=1}^N A_n \sin(k_n t + \theta_n - \frac{k_n}{c} x) dx \quad (3)$$

or

$$(\ddot{U}_F)_{ave} = \sum_{n=1}^N \frac{A_n c}{r_0 k_n} \sin(k_n t - \theta_n) \sin \frac{k_n r_0}{c} \quad (4)$$

The wave number which is a parameter proportional to the ratio of structural x-dimension to the particular wave-length is defined by

$$R_n = \frac{4\pi r_0}{\lambda_n} \quad (5)$$

where  $\lambda_n$  is the wavelength for the  $n^{\text{th}}$  harmonic and is related to the shear wave velocity by

$$c = \frac{k_n}{2\pi} \lambda_n \quad (6)$$

In terms of the wave number, equation (4) can be written as

$$(\ddot{U}_F)_{ave} = \sum_{n=1}^N \frac{2A_n}{R_n} \sin(k_n t + \theta_n) \sin(\frac{R_n}{2}) \quad (7)$$

Equation (7) represents the average free-field lateral ground motion of the earthquake in the x-direction. Utilizing a similar procedure, the lateral free-field ground acceleration of the earthquake in y-direction can be derived as

$$(\ddot{V}_F)_{ave} = \sum_{m=1}^M \frac{2B_m}{R_m} \sin(k_m t + \phi_m) \sin(\frac{R_m}{2}) \quad (8)$$

where  $B_m$  are the Fourier amplitudes spectrum of the y-direction earthquake.

In order to evaluate the free-field rotational ground input about the vertical axis, the lateral acceleration divided by the corresponding lever arm is integrated over the foundation,

$$\ddot{\phi}_{fx}(t) = \frac{1}{2r_0} \int_{-r_0}^{r_0} \frac{U_F(x, t)}{x} dx \quad (9)$$

Substitution of equation (1) into equation (9) and after integrating it, equation (9) can be rewritten as

$$\ddot{\phi}_{fx}(t) = - \sum_{n=1}^N \frac{A_n}{r_0} \cos(k_n t + \theta_n) S_1(\frac{R_n}{2}) \quad (10)$$

where the function

$$S_1(x) = \int_{\sigma}^x \frac{\sin t}{t} dt \quad (11)$$

The rotational ground input due to the y-direction lateral excitation becomes

$$\ddot{\phi}_{FY}(t) = - \sum_{m=1}^M \frac{B_m}{r_o} \cos(k_m t + \phi_m) S_1 \left( \frac{R_m}{2} \right) \quad (12)$$

By superposition, the total torsional ground acceleration due to earthquakes in both lateral directions is then given by

$$\ddot{\phi}_F(t) = \ddot{\phi}_{FX}(t) + \ddot{\phi}_{FY}(t) \quad (13)$$

The torsional acceleration  $\ddot{\phi}_F(t)$  defined by equation (13) is expressed in terms of the wave number,  $R_m$ , and the Fourier amplitudes of the earthquake. In the following section, the torsional half-space closed-form solution as derived in Reference [3] is combined with  $\ddot{\phi}_F(t)$  to form the torsional soil-structure interaction expression.

#### Part II: Torsional Soil-Structure Interaction Expressions

By using a semi-inverse method with the aid of Laplace and Hankel transforms, reference [3] gives the torsional response of the elastic half-space resulting from a linear shear stress distribution varying arbitrarily with time over the surface of the boundary. The torsional soil-structure interaction expression derived in [3] includes the effects of interaction between inertia moments and torsional free-field ground rotation,  $\ddot{\phi}_F(t)$ . For an N-mass torsional mathematical model (Figure 2), the torsional soil-structure interaction equation can be written as

$$\begin{aligned} \ddot{\phi}(r_o, 0, t) = & - \frac{3c}{2\pi^3 r_o^3 G} \left[ \sum_{j=1}^n Q_j \omega_j^3 \right] \int_0^t \int_0^{t-\tau} \ddot{\phi}(\xi) \sin \omega_j (t-\tau-\xi) F(r_o, \tau) d\xi d\tau \\ & - \sum_{j=1}^n Q_j \omega_j^2 \int_0^t \ddot{\phi}(t-\tau) F(r_o, \tau) d\tau + \ddot{\phi}_F(t) \end{aligned} \quad (14)$$

In equation (14) the actual angular acceleration of the base of the structure,  $\ddot{\phi}(t)$ , is expressed in terms of the free-field ground acceleration  $\ddot{\phi}_F(t)$ , the properties of the soil which are represented by the shear wave velocity  $c$  and the shear modulus of elasticity,  $G$ , and the properties of the structure which are described by the radius of the base and the modal parameters. The function  $F(r_o, t)$  is defined by

$$F(r_o, t) = \int_0^1 \int_0^1 \frac{\beta(1-2\alpha^2)}{\sqrt{(1-\beta^2)(1-\alpha^2)}} \left[ \frac{1}{\sqrt{(r_o\alpha + r_o\beta)^2 - (ct)^2}} - \frac{1}{\sqrt{(r_o\alpha - r_o\beta)^2 - (ct)^2}} \right] d\beta d\alpha \quad (15)$$

The importance of the torsional soil-structure interaction effects is evaluated by noting changes in output spectrum responses of  $\ddot{\phi}(t)$  from free-field response spectrum.

In addition to using the half-space formulation of the soil presented above, a compliance spring-damper model of the soil was also developed. The mathematical formulation and iteration procedure presented in Reference [4] was applied. The compliance springs and equivalent viscous dampers were based on the work of Bycroft [5].

#### DISCUSSIONS

##### Part I: Input Motions

Free-field ground inputs used in this analysis are based on data from the El Centro, California earthquake. For the torsional soil-structure interaction studies, the soil-

structure combination is subjected to the first ten seconds of the combined North-South and East-West lateral components of the free-field ground accelerations. The North-South components of the El Centro earthquake is shown in Figure(3). By including the averaging effects of the structure foundation, the modified lateral ground acceleration of the El Centro earthquake calculated from equation (7) would have smaller amplitudes throughout the time history. This reduction is compared to the input motion on Figure (3).

To further examine the influence of the foundation averaging effects, acceleration spectrum curves of the North-South lateral component of the El Centro earthquake for soils correspond to shear wave velocities of 152.4, 304.8, 609.6 and 3048 meters/sec (500, 1000, 2000, 10,000 ft/sec) were plotted versus frequency in Figure (4). For the stiffest soil (corresponding to the 3048 m/s shear wave velocity), results indicate that the acceleration spectrum values are very close to that of the free-field motion's values. In this case, the free-field ground acceleration is a very good representation of the input excitation applied at the base of the structure. On the other hand, for fairly soft soil (152.4 m/s shear wave velocity), "self-cancellating" and torsional effects would alter the input acceleration drastically. Therefore, if the free-field ground motion is used to perform a seismic analysis of the structural response to the earthquake, this model indicates that significant errors may occur.

The torsional free-field angular acceleration time curves as derived by equation (13) for different shear wave velocities are shown in Figures (3) to (7). These torsional free-field input acceleration,  $\ddot{\phi}_F(t)$ , are substituted into the soil-structure interaction expression, equation (14), which is solved numerically for the computed angular output acceleration of the base of the structure,  $\ddot{\phi}(t)$ , for a specified soil-structure combination.

#### Part 2: The Response to the Combined El Centro Input

Torsional soil-structure interaction effects are investigated based on the idealized dynamic model shown in Figure (8). This relatively simple model consists of a rigid massless base and two structural masses of 178,260 kg (393,000 lbm). The fixed-base frequency of the structural masses corresponds to the frequency at which the peak input amplitude of the free-field ground motion occurred. For the seismic design of structure, it must be emphasized that only the spectrum values at the fixed-base frequency are used to calculate the maximum shock loads. Thus, the significance of torsional interaction effects can be evaluated by comparing the response spectra of the free-field motion to that of the foundation motion at the fixed-base frequency.

In Figures (9) to (11), the spectrum response of the free-field angular acceleration is shown as a solid line. In addition the spectrum values determined from the base accelerations calculated from equation (14) are plotted at various frequencies for the three soils considered. The input and output acceleration spectrum values calculated from both the half-space solution and lumped parameter method are summarized in Table 1. Also included in the table are the percentage reduction in the input spectrum values at the structural fixed-base frequencies.

As observed on Figures (9) to (11), the largest torsional interaction effects occur at the structural fixed-base frequency and the change in the free-field input acceleration spectrum value is most pronounced for the case of the softer soil. It can be noted from Table (1) that the percentage reduction in the input spectrum values increases as the soil

shear wave velocity decreases. At the fixed-base frequencies, reductions in the free-field acceleration spectra of between 23 to 66% are calculated. Furthermore, Table (1) indicated that agreement between the two methods at the fixed-base frequencies was found to be very good.

Another important observation that can be made from the numerical results is that the peak loads caused by torsional excitation are similar in magnitude to the seismic loads caused by the lateral mode of vibration. This fact is determined by comparing the maximum acceleration of the two different modes of excitation. At the fixed-base frequencies of 6 and 6.75 cps, maximum spectrum values of 3.25g and 2.4g (Figure 4) are observed for lateral input motion. These inputs are the basis of the free-field torsional inputs. For the torsional motion, Table (1) predicts the peak acceleration at 6.75 cps to be 1.69g which is obtained by multiplying the output spectrum value by the radius of the foundation ( $A_o(\omega) \times r_o = .111 \times 15.24$ ). The peak acceleration at 6 cps (Figure(12)) is calculated to be 3.1g. Thus, in the seismic design of structures the effects of torsion on the dynamic response of the system may be very significant and must be evaluated.

#### Conclusions

A study has been performed to establish the significance of the torsional mode of excitation in the evaluation of the total response of a structure. Based on numerical results the principal conclusions are:

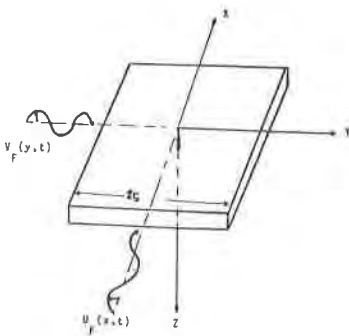
1. Torsional effects are generally excited from lateral transverse waves, and because of the particle motion under the foundation, lateral seismic response (Figure (6)) of the structure can be considerably reduced from the single point uniform motion hypothesis.
2. By using typical free-field inputs, the numerical results of this investigation indicated that the peak torsional accelerations are similar in magnitude to maximum lateral accelerations. Thus, the torsional mode of excitation can increase seismic loads significantly as lateral motions are reduced and, therefore, significantly alter the dynamic response of the structure to the earthquake.
3. By making use of normal mode theory [4] and the solution of a half-space problem torsional interaction expression between a structure and the half-space can be expressed by an integral equation. For the torsional structural model driven by the El Centro earthquake input, Table (1) indicates the largest percentage reduction of output-acceleration spectrum from input-acceleration spectrum at the fixed-base frequency for all three soils occurs when the mode frequency is tuned to the frequency at which a peak occurs in the free acceleration spectrum curve.
4. If proper foundation compliance coefficients are chosen at the fixed-base frequency, comparisons of the output-acceleration spectrum computed by the lumped-mass discrete mathematical formulation have been shown to be in excellent agreement with those determined by the closed-form continuum solutions at the structural fixed-base frequencies. The agreement was not as close at other frequencies. For practical purposes, solutions at the structural fixed-base frequency using springs and dashpots are accurate enough for engineering design calculations and is the most economical and fastest solution when compared to the highly sophisticated analyses, such as closed-form analytical formulation or finite element methods.

## References

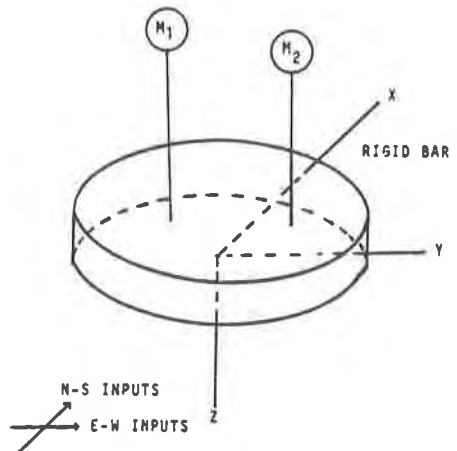
1. SCANLAN, R.H., "Seismic Wave Effects of Soil-Structure Interaction," Paper K 2/1, 3rd SMIRT, London, U.K., 1975.
2. WOLF, J.P., "Seismic Response Due to Traveling Shear Wave Including Soil-Structure Interaction with Base-Mat Uplift," Jr. Earthquake Engineering and Structural Dynamics, Vol. 5, p 337-363, 1977.
3. LAM, P.C., "Torsional Soil-Structure Interaction Effects," Doctoral Dissertation, The University of Akron, June 1978.
4. SCAVUZZO, R.J. RAFTOPOULOS, D.D. and BAILEY, J.L., "Application of Three-Dimensional Normal Mode Theory to Seismic Analysis of Nuclear Power Plants," Paper K 3/4, 2nd SMIRT, Berlin, Germany, 1973.
5. BYCROFT, G.N., "Forced Vibrations of a Rigid Circular Plate on a Semi-Infinite Elastic Space and on an Elastic Stratum," Phil. Trans. Roy. Soc., London, Vol. 248, Ser A 948, 1956.

Table 1    Tabulation of acceleration response spectra values for the different mathematical models subjects to the El Centro earthquake input acceleration. Fixed-base frequency is 6.75 cps.

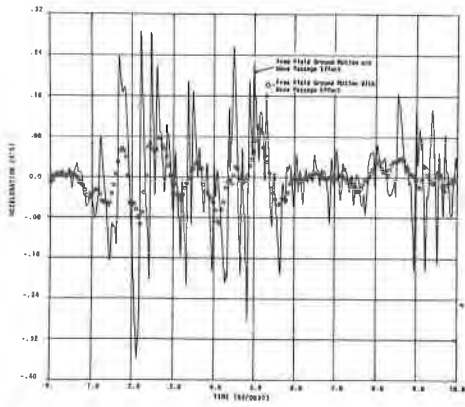
Mathematical Model Description	Shear Wave Velocities		Spectrum Values		
	ft/sec	m/sec	g/ft	g/m	% Reduction
Free-Field Motion	500	152.4	.100	.327	---
	1000	304.8	.089	.292	---
	2000	609.6	.103	.339	---
Continuum Soln. (Linear shear stress assumption) Eq. 14	500	152.4	.034	.111	66.21
	1000	304.8	.051	.166	43.06
	2000	609.6	.077	.251	23.16
Lumped-Mass Discrete Model [ 4 ]	500	152.4	.034	.110	66.34
	1000	304.8	.047	.154	47.13
	2000	609.5	.074	.244	25.53



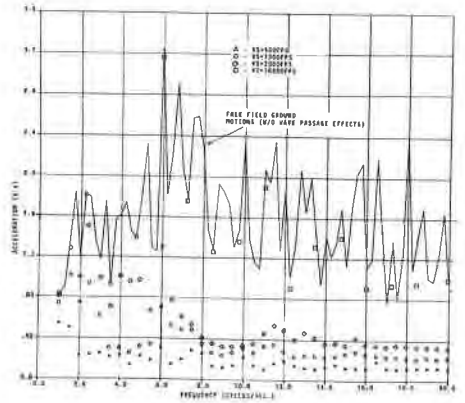
1. Wave Model for foundation input averaging calculations.



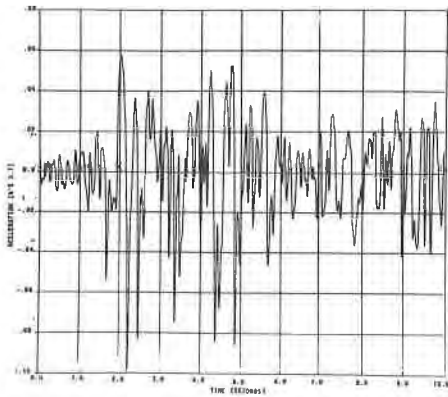
2. Torsional N-Mass mathematical model.



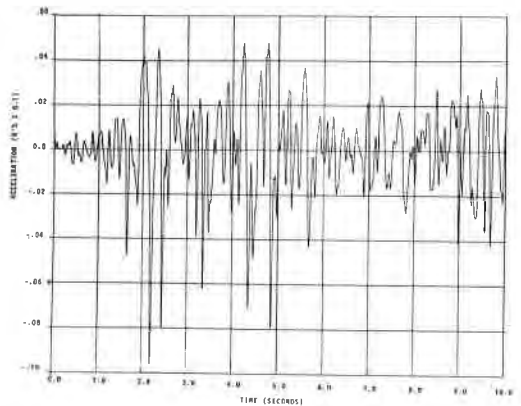
3. Comparison of the N-S lateral ground acceleration of the El Centro earthquake with and without foundation averaging effects.



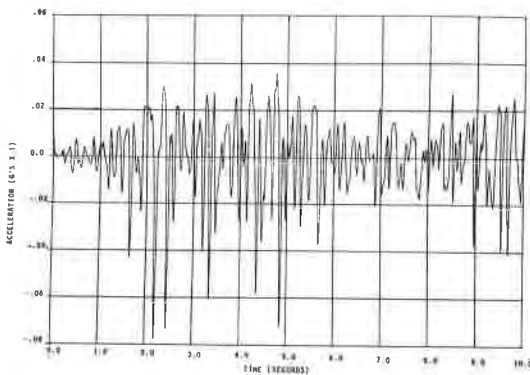
4. The acceleration spectra of the N-S lateral component of the El Centro earthquake are plotted vs. frequency for soils corresponding to shear wave velocities 152.4, 304.8, 609.6 and 3048. m/s.



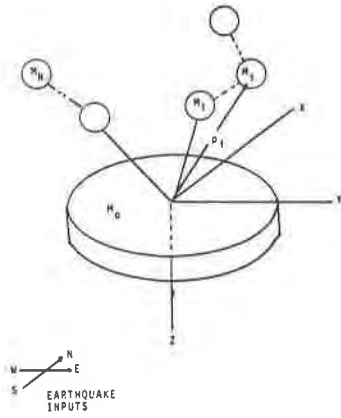
5. The free-field angular acceleration,  $\phi_P$ , derived from equation 13 is plotted vs. time,  $C = 152.4$  m/s.



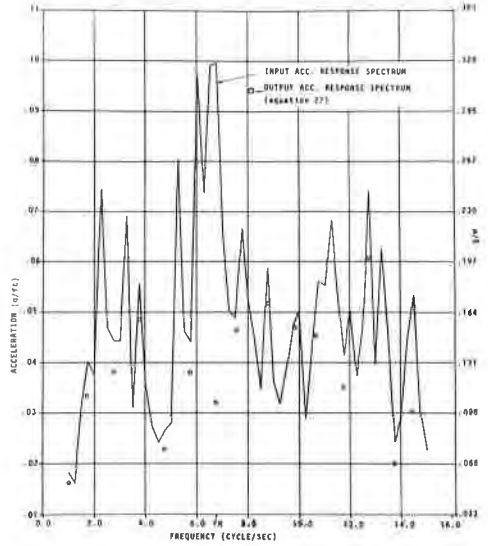
6. The free-field angular acceleration,  $\phi_P$ , derived from equation 13 is plotted vs. time,  $C = 304.8$  m/s.



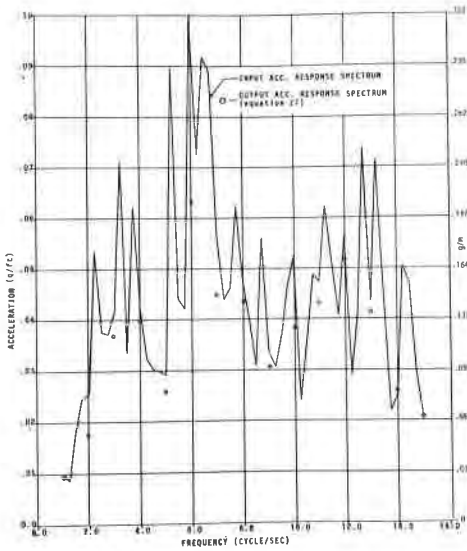
7. The free-field angular acceleration,  $\phi_P$ , derived from equation 13 is plotted vs. time  $C = 609.6$  m/s.



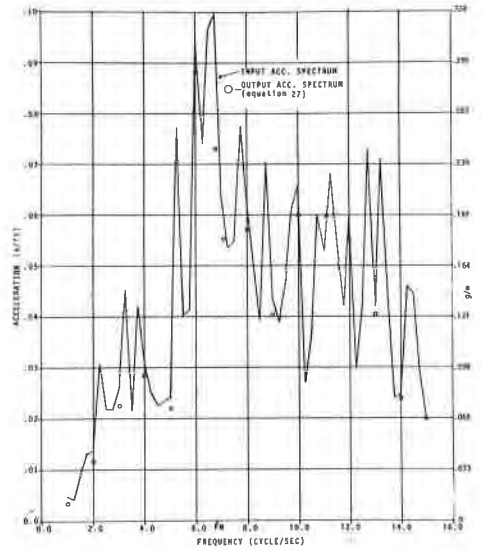
8. Idealized torsional three-mass structural model.



9. The acceleration spectra of the input motion are shown as a solid line. Output spectra (equation 14) are also plotted.  $C = 152.4$  m/s. Fixed-base frequency is 6.75 cps.



10. The acceleration spectra of the input motion are shown as a solid line. Output spectra (equation 14) are also plotted.  $C = 304.8$  m/s. Fixed-base frequency is 6 cps.



11. The acceleration spectra of the input motion are shown as a solid line. Output spectra (equation 14) are also plotted.  $C = 609.6$  m/s. Fixed-base frequency is 6.75 cps.