

COMPARISON BETWEEN TIME-STEP-INTEGRATION AND PROBABILISTIC METHODS IN SEISMIC ANALYSIS OF A LINEAR STRUCTURE

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SUMMARY

Assuming that earthquake ground motion is a stationary time function, the seismic analysis of a linear structure can be done by probabilistic methods using the "power spectral density function" (PSD), instead of applying the more traditional time-step-integration using earthquake time histories (TH).

A given structure was analyzed both by PSD and TH methods, computing and comparing "floor response spectra".

For the purpose of having "equivalent" TH and PSD to use in the comparison, both were derived artificially from the USNRC- response spectra. The TH were readily available; the PSD was derived directly from the USNRC-response spectra by iteration.

The analysis using TH was performed for two different TH and different frequency intervals for the "floor-response-spectra".

The analysis using PSD first produced PSD functions of the responses of the floors and these were then converted into "floor-response-spectra".

Plots of the resulting "floor-response-spectra" show:

- (1) The agreement of TH and PSD results is quite close.
- (2) The curves produced by PSD are much smoother than those produced by TH and mostly form an envelope of the latter.
- (3) The curves produced by TH are quite jagged with the location and magnitude of the peaks depending on the choice of frequencies at which the "floor-response-spectra" were evaluated and on the choice of TH.

1. Introduction

There are several methods available for the seismic analysis of structures. The time-history-integration (TH-method) is the best known of them. The power-spectral-density-method (PSD-method) is getting more importance lately. This method is well-known for a long time in the analysis of aircrafts and also of sea-structures exposed to waves.

These two methods base on very different theories as explained in Section 2 with more details. However, the most important difference is that: whereas the TH-method analyses a specific earthquake as a deterministic time function, the PSD-method starts from probabilistic assumptions and considers an earthquake as a stationary time function defined by variance and correlation function.

In order to get familiar with these two so different methods, it is of great interest to compare results calculated on the same basis. This comparative analysis was performed by the authors on a linear structure. It is presented in Section 3.

2. Methods

2.1 Time-History-Integration (TH-Method)

Here, the structure is strained by the acceleration function of an earthquake. Assuming a linear behaviour, the dynamic equilibrium is fulfilled at every moment by solving the equation system (1):

$$m\ddot{v} + c\dot{v} + kv = -m \cdot \ddot{u}_g \quad \text{eq. (1)}$$

where m , c and k are the mass-, damping- and stiffness-matrix respectively, v the relative displacement and \ddot{u}_g the ground motion.

We utilize modal analysis; i.e. to solve system (1), it has to be uncoupled, solved as individual modal equations, and then superponed. The damping is introduced as modal damping for every mode considered.

2.2 Analysis using probabilistic assumptions (PSD-Method)

The bases of this theory are exposed by Robson in Reference [2]. An earthquake is considered as a stationary time function, i.e. for several long enough periods of time, variance and correlative function are identical.

For such a stationary function, the so-called PSD-function $S(f)$ over the frequency axis can be derived by using a fourier transformation and forming a marginal value:

$$S(f) = \lim_{T \rightarrow \infty} \left[\frac{2}{T} \cdot |A_T(if)|^2 \right] \quad \text{eq. (2)}$$

where T is the period of the time function, $A_T(if)$ the fourier transform of the stationary time function, i the imaginary unit and f the frequency as function parameter.

The variance σ^2 of the time function is then determined through integration:

$$\sigma^2 = \int_0^{\infty} S(f) df \quad \text{eq. (3)}$$

In Figure 1, the acceleration-PSD-function derived from the NRC-spectrum with 5% damping is shown. The variance is represented by the area under that curve. The square root gives the standard deviation.

The value S_k of the PSD at a certain frequency f_k is a measure for the share of that frequency on the time function. If the time function represents an acceleration, the PSD-function is then proportional to the power of this vibration for a certain mass.

The response of a structure to a periodical loading of the form

$$P(t) = P_0 \cdot e^{i\omega t} \quad \text{eq. (4)}$$

can be expressed by the equation (5)

$$x(t) = \alpha(if) \cdot P_0 \cdot e^{i\omega t} \quad \text{eq. (5)}$$

$\alpha(if)$ is defined as the complex response function or simply the receptance of the structure. A structure has as many α -functions as modes. The modes of the structure are to be extracted in the same way as for the TH-method. The α -functions are only dependent upon the structure and describe exactly the response to a certain frequency of the periodical loading.

By using the PSD-function of a load with stationary properties, it is now possible to describe the response of the structure in the form of a PSD-function also:

$$S_x(f) = |\alpha(if)|^2 \cdot S_p(f) \quad \text{eq. (6)}$$

where $S_x(f)$ is the PSD-function of the response, $S_p(f)$ the PSD-function of the excitation and $\alpha(if)$ the receptance of the structure.

The variance σ_x^2 of the response of the structure can be determined as

$$\sigma_x^2 = \int_0^{\infty} S_x(f) df \quad \text{eq. (7)}$$

and the maximum value of the response as

$$x_{\max} = n \cdot \sigma_x \quad \text{eq. (8)}$$

where the factor n is determined in such a way that $n \cdot \sigma_{\max}$ is reached only once during the course of the earthquake.

A floor-response-spectrum can be determined by using formula (6) correspondingly, applying $S_x(f)$ as PSD of the loading to a SDOF-system in the frequency range considered.

The basic requirement for the applicability of the PSD-method is the presentation of the design-earthquake as a PSD-function. The co-author, E.H. Ermutlu, has developed an iteration procedure allowing the presentation of the NRC-design spectrum as a PSD-function: A response spectrum is calculated using an assumed PSD as a first approximation. This is compared to the design spectrum. The PSD-function has to be corrected until the resultant response spectrum corresponds to the design spectrum with sufficient accuracy for the total frequency domain considered. The PSD-function shown in Figure 1 is produced by this iteration procedure.

3. Example for Comparison

3.1 The Structure

The numerical model of the structure shown in Fig. 2 represents one of the layouts considered for a reactor utility building. The superstructure was separated into two buildings of different height, standing on basemats with two narrow connections.

The buildings are replaced by the commonly used stick models, taking into consideration the major eccentricities and the corresponding massdistribution.

The soil is represented by horizontal (Z-direction), vertical and rotational (around X-axis) springs with corresponding additional masses.

This system was further reduced to 6 modes in the Z-direction with frequencies between 4.23 and 22.93 Hz.

Damping was assigned to each mode in proportion to the strainenergies in the different materials.

3.2 The Loading

Two artificial horizontal earthquake acceleration timehistories, labeled H1 and H2, were scaled to .15 G and used in time-step-integration.

The power spectral density function (PSD) discussed earlier and shown in Fig. 1. was derived from the USNRC design response spectrum for 5 % damping scaled to .15 G.

Figures 3, 4 and 5 compare response spectra derived from H1, H2 and the PSD for damping values of 5, 10 and 15 %.

Figures 6 and 7 further compare response spectra from the PSD with the original USNRC response spectra for damping values 5 and 10 %.

Figures 3 and 6 as well as Figures 4 and 7 form pairs to be regarded together. As can be seen, the correspondences on the whole are quite good. The PSD results adhere much more uniformly to the original USNRC response spectra but give slightly higher values in the high frequency range.

The peaks and dips of response spectra produced from timehistories are shown to depend on the choice of frequencies at which the evaluations are made (Fig. 16) and on the timehistory used, whereas the response spectra issuing from the PSD are much smoother and suggest less necessity to draw envelopes. The same can be said of results produced at points within the structure.

3.3 Results

Acceleration timehistories of horizontal motions in direction Z were produced for points 1, 6, 12 and 30 (Fig. 2) by applying H2 in direction Z in the ground.

Floor response spectra were then computed from these four timehistories for damping values 2 and 5 %.

The same eight spectra were created by use of the PSD.

Figures 8 thru 15 show the comparison between the corresponding response spectra.

The PSD-based spectra mostly envelope those derived by time-step-integration of timehistory H2 and are much smoother, but the agreement over all is quite close.

References

- [1] Ermutlu, H.E. - "Dynamic Analysis of Arch Dams subjected to Seismic Disturbances" - Thesis submitted for the Degree of Doctor of Philosophy, Institute of Sound and Vibration Research, The University of Southampton, U.K. Dec. 1968
- [2] Robson, J.D. - "An Introduction to Random Vibration" - Edinburgh Univ. Publications, Science and Mathematics Texts, 5, Edinburgh Univ. Press, 1963
- [3] U.S. Atomic Energy Commission, Regulatory Guide 1.60 - "Design Response Spectra" for Seismic Design of Nuclear Power Plants - Revision 1, Dec. 1973
- [4] Rice, S.O. - "Mathematical Analysis of Random Noise" - Bell System Tech. J., Vol 23, 1944, pp 282, and Vol. 24, 1945, pp 44
- [5] USNRC - "238 NI GESSAR - General Electric Standard Safety Analysis Report" - Section 3.7 - Seismic Design
- [6] NASTRAN Theoretical Manual, NASA, SP - 221 (01), Ed. R.H. Mac Neal, The Mac Neal - Schwendler Corp., Ap. 1972
- [7] NASTRAN User's Manual, NASA SP, Ed. C.W. McCormick, The Mac Neal - Schwendler Corp., 1972 and Update Documentation

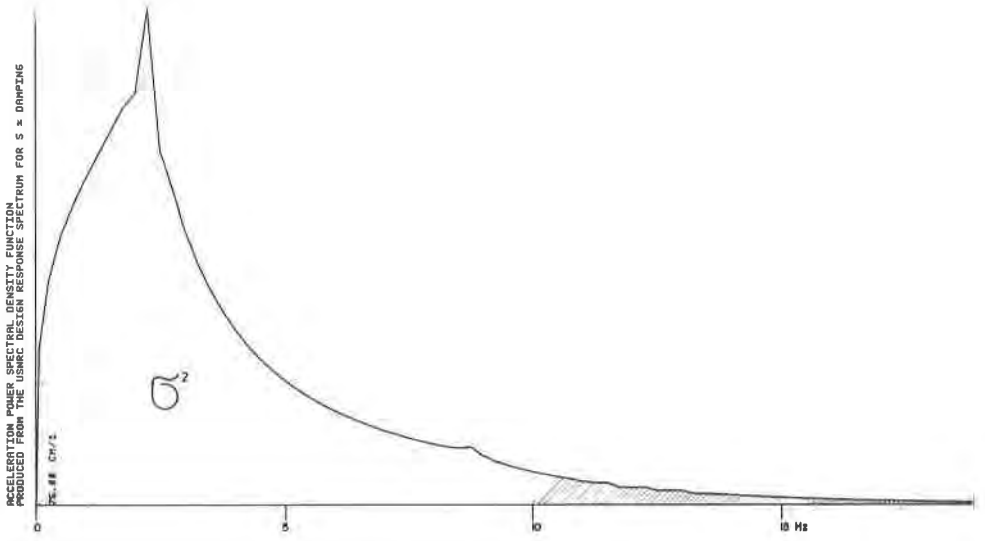


FIG 1

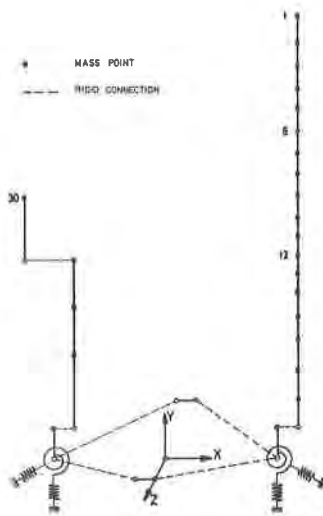


FIG 2

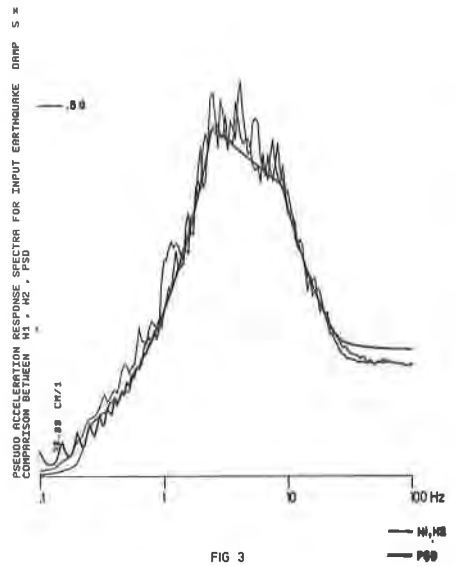


FIG 3

— N₁, H₈
 - - - P₀₀

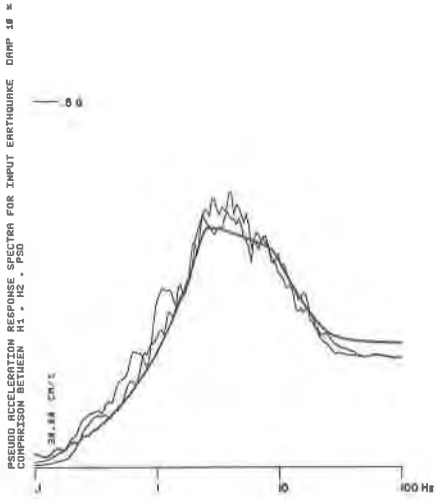


FIG 4

— HI, H2
— PSD

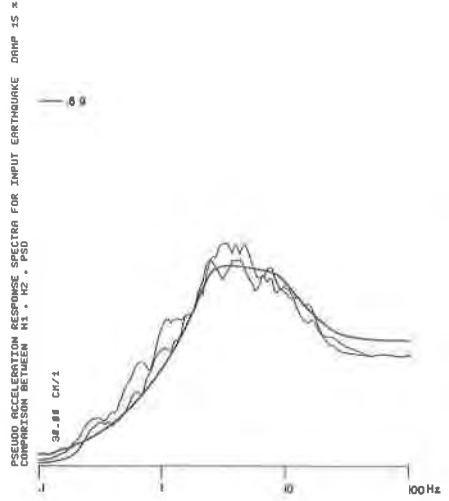


FIG 5

— HI, H2
— PSD

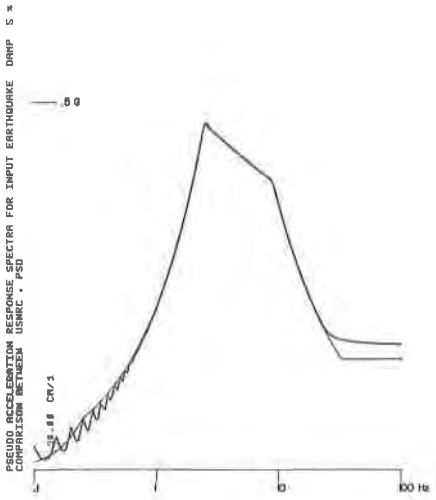


FIG 6

— USNRC
— PSD

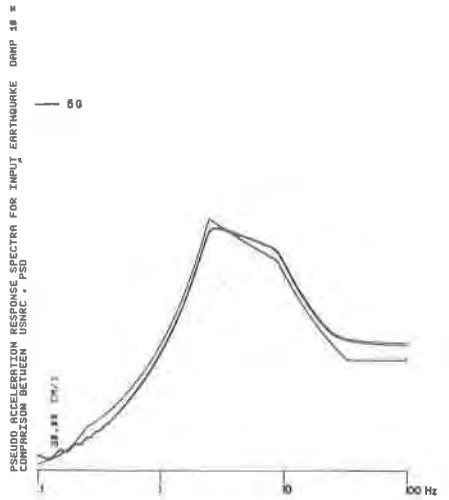
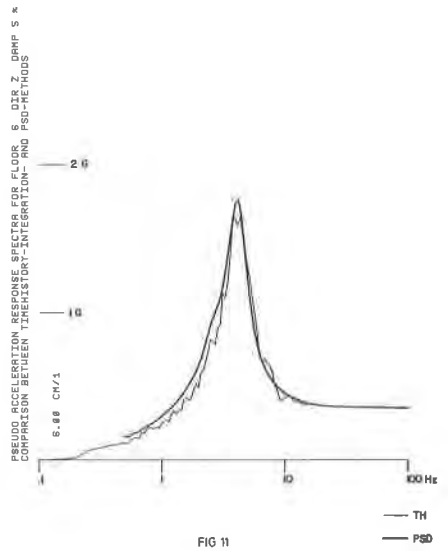
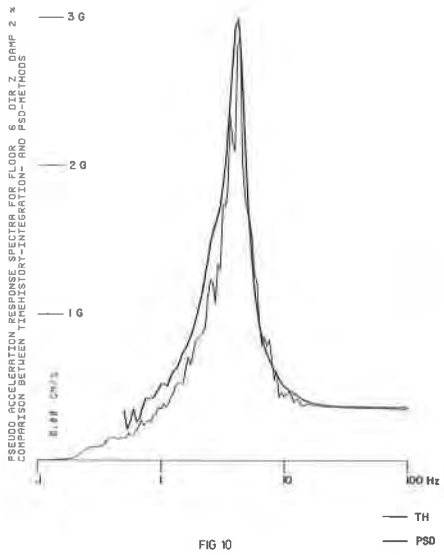
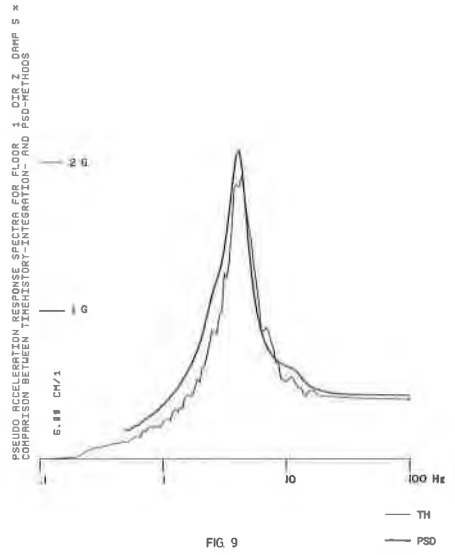
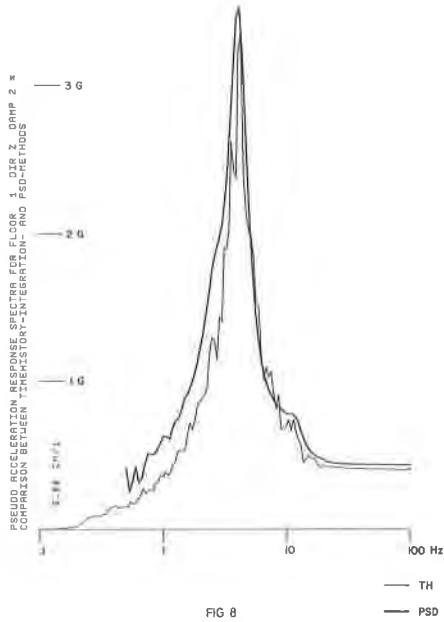


FIG 7

— USNRC
— PSD



PSEUDO ACCELERATION RESPONSE SPECTRA FOR FLOOR 12, DIR Z, DRMP 2 %
COMPARISON BETWEEN TIME-HISTORY-INTEGRATION- AND PSD-METHODS

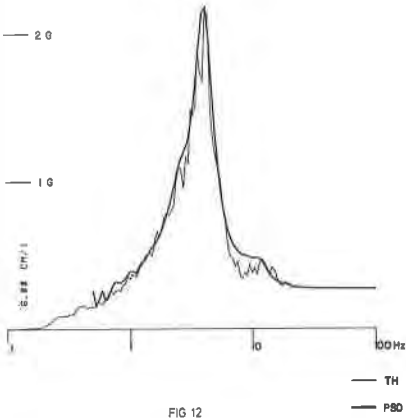


FIG 12

PSEUDO ACCELERATION RESPONSE SPECTRA FOR FLOOR 12, DIR Z, DRMP 5 %
COMPARISON BETWEEN TIME-HISTORY-INTEGRATION- AND PSD-METHODS

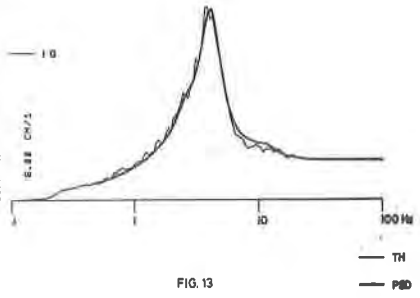


FIG 13

PSEUDO ACCELERATION RESPONSE SPECTRA FOR FLOOR 34, DIR Z, DRMP 2 %
COMPARISON BETWEEN TIME-HISTORY-INTEGRATION- AND PSD-METHODS

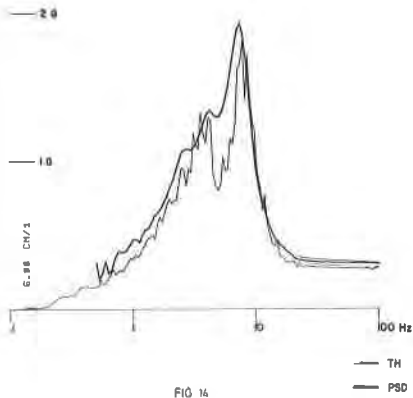


FIG 14

PSEUDO ACCELERATION RESPONSE SPECTRA FOR FLOOR 34, DIR Z, DRMP 5 %
COMPARISON BETWEEN TIME-HISTORY-INTEGRATION- AND PSD-METHODS

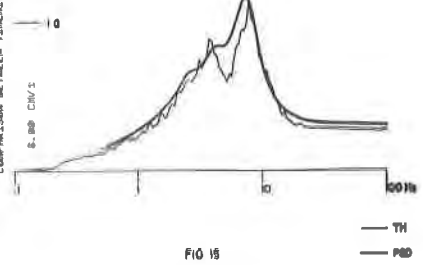


FIG 15

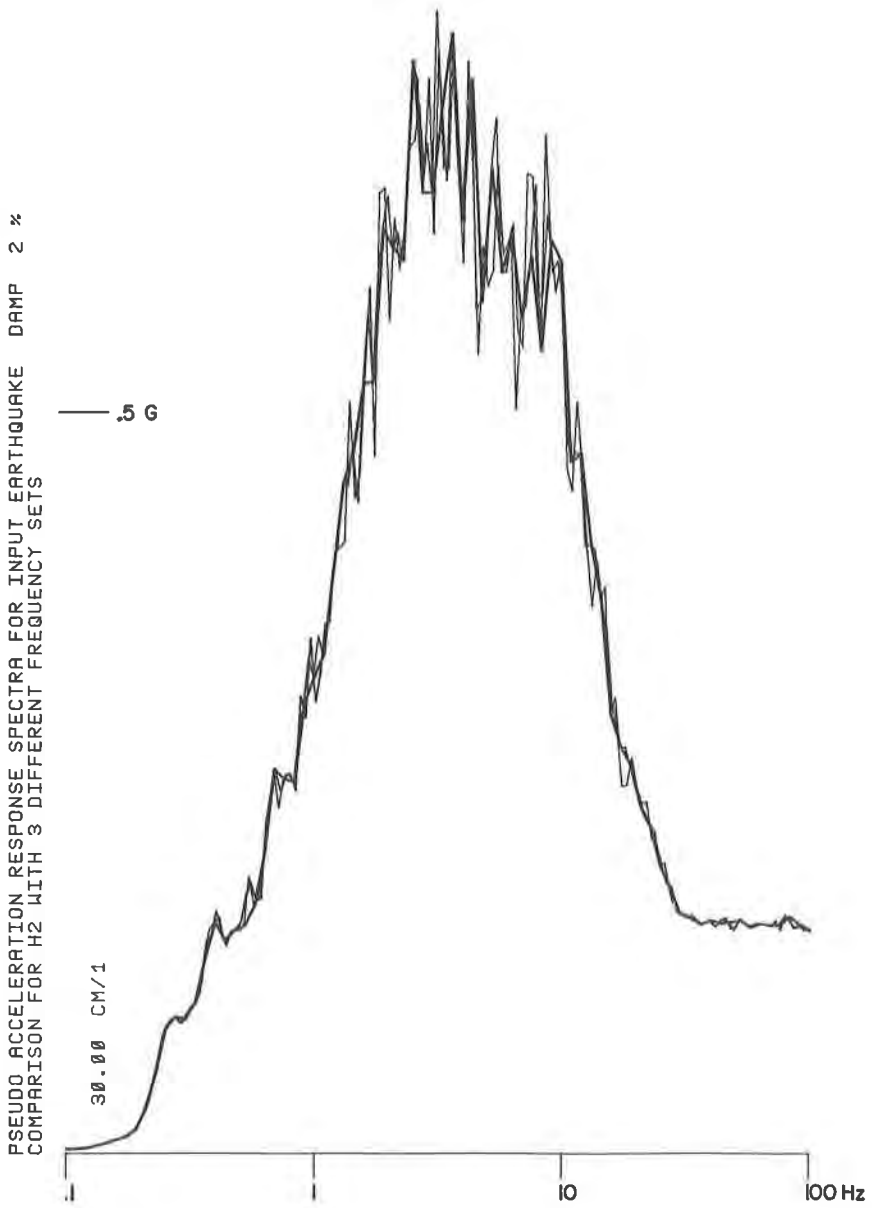


FIG. 16