Simplified method for calculation of response spectrum at the decoupling point of two vibrating systems

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1. INTRODUCTION

The main problem of the separate calculation of secondary vibrating systems decoupled from main systems is the determination of the response spectrum in the decoupling point.

In [1] an equation was developed for the direct calculation of the response spectra in a point of the vibrating system on the basis of the input response spectra for the fixed points. In the developed equation the same excitation in all fixed points was assumed.

During earthquakes,aircraft crashes or explosions for a lot of vibrating systems such as pipes, cranes, columns and big machines fixed at different floors of the buildings different excitations at various fixed points occur. For this reason a new extended equation was developed in this paper that can also be applied to multiple support excitations

2. CALCULATION OF RESPONSE SPECTRUM AT THE DECOUPLING POINT OF TWO VIBRATING SYSTEMS WITH THE SAME EXCITATION IN THE FIXED POINTS

The general equation according to [1] for the calculation of response spectrum $S_s(\omega)$ at the decoupling point s of two vibrating systems with the excitation at the fest points $S_h(\omega)$ is:

$$S_s(\omega) = S_h(\omega)[1 + \Psi_j \alpha_{sh}(F_m - 1)] \tag{1}$$

where:

Ψ_j is the normalised participation factor $\overline{\Psi}_j$ for mode j. $\Psi_j = \dfrac{\overline{\Psi}_j}{\sum_{i=0}^{n} \overline{\Psi}_i}$

α_{sh} is the influence coefficient calculated from the displacement δ_s of the point s and the maximum displacement δ_m of the vibration system in the action of the response spectrum $S_h(\omega)$ $\alpha_{sh} = \dfrac{\delta_s}{\delta_m}$

F_m is the amplification factor at resonance calculated from the damping ratio of the vibrating system (ξ_1) and the damping ratio (ξ_2) of the response spectrum $S_s(\omega)$ to be calculated.

$$F_m = \frac{1}{\xi_1 + \xi_2}$$

The equations of the movement of the vibrating system with n masses(see Fig. 1) excited with $\ddot{U}_o(t) = \ddot{U}_{n+1}(t) = \ddot{U}(t)$ are:

$$M_1\ddot{X}_1 + C_{11}\dot{X}_1 + \cdots + C_{1n}\dot{X}_n + K_{11}X_1 + + \cdots + K_{1n}X_n = -M_1\ddot{U}(t)$$

$$M_2\ddot{X}_2 + C_{21}\dot{X}_1 + \cdots + C_{2n}\dot{X}_n + K_{21}X_1 + \cdots + K_{2n}X_n = -M_2\ddot{U}(t)$$

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$$M_n\ddot{X}_n + C_{n1}\dot{X}_1 + \cdots + C_{nn}\dot{X}_n + K_{n1}X_1 + \cdots + K_{nn}X_n = -M_n\ddot{U}(t)$$

After the transformation into the coordinates of the normal modes

$$X(t) = [\Phi]\, z(t) \tag{3}$$

we can write equation (1) in the matrix form as:

$$[M][\Phi]\ddot{z} + [C][\Phi]\dot{z} + [K][\Phi]z = [M]\{1\}\ddot{U}(t) \tag{4}$$

Equation (4) will be multiplied by $\underline{\Phi}_n^T$ to obtain the uncoupled equation of the movement

$$M_n\ddot{z} + C_n\dot{z} + K_n z = \underline{\Phi}_n^T[M]\{1\}\ddot{U}(t) \tag{5}$$

where:

$$M_n = \underline{\Phi}_n^T[M]\underline{\Phi}_n$$

$$C_n = \underline{\Phi}_n^T[C]\underline{\Phi}_n$$

$$K_n = \underline{\Phi}_n^T[K]\underline{\Phi}_n$$

Further the response spectrum calculation method will be used to determine the modal displacement (see [1] page 95):

$$\underline{v}_{n,max} = \frac{1}{\omega_n^2}\underline{\Phi}_n\frac{E_n}{M_n}S(\omega_n) \tag{6}$$

where:

$$E_n = \underline{\Phi}_n^T[M]\{1\}$$

The expression $\frac{E_n}{M_n}$, the mass participation factor, is constant for the eigenvalue n:

$$\overline{\Psi}_n = \frac{E_n}{M_n} = \frac{\Phi_n^T[M]\{1\}}{M_n} \tag{7}$$

3. CALCULATION OF RESPONSE SPECTRUM AT THE DECOUPLING POINT OF TWO VIBRATING SYSTEMS WITH DIFFERENT EXCITATIONS IN THE FIXED POINTS

The excitation movement $U_{n+1}(t)$ of the point N+1 (see Fig. 1) can be decomposed at any time t_j into the excitation $U_o(t_j)$ and the excitation $U_{n+1}(t_j) - U_o(t_j)$. Then the movement of the vibrating system in Fig.1 can be analysed in two steps:

a. The analysis of the displacements of the vibrating system in Fig.1 with the same excitation $U_o(t)$ acting in all fixed points.

b. The analysis of the displacements of the vibrating system in Fig.1 with the excitation $U_{n+1}(t) - U_o(t)$ acting only in point N+1. Point 0 in this step will be considered the fixed point of the system.

a. THE ANALYSIS FOR THE SAME EXCITATION $U_o(t)$ AT ALL FIXED POINTS.

The response spectrum at the point s of the system can be determined for the first step a according to eq.(1) as:

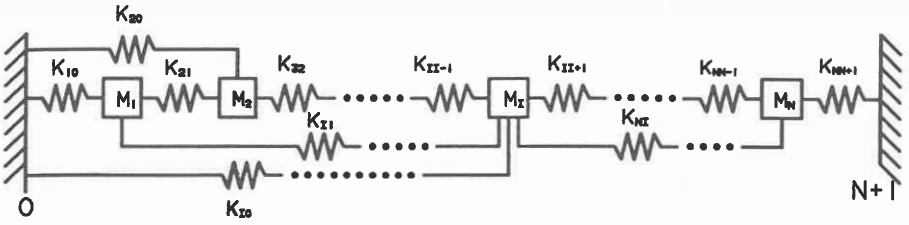

FIG. 1

$$S_s^a(\omega) = S_o(\omega)\left[1 + \Psi_f \alpha_{sh}(F_m - 1)\right] \tag{8}$$

b. THE ANALYSIS OF THE DISPLACEMENT FOR THE EXCITATION $U_{n+1}(t) - U_o(t)$ AT THE POINT N+1

For the relative displacement $U_{n+1}(t_j) - U_o(t_j)$ at the point N+1 results a relative displacement $U_{ri}(t_j)$ at the point I, different from $U_{n+1}(t_j) - U_o(t_j)$. The displacement $U_{ri}(t_j)$ will be determined on the basis of the static equilibrium conditions. The relative displacement $U_{ri}(t_j)$ depends on the stiffnesses of the vibrating system. It is also proportional to the relative displacement $U_{n+1}(t_j) - U_o(t_j)$

$$U_{ri}(t_j) = \beta_i\left[U_{n+1}(t_j) - U_o(t_j)\right] \tag{9}$$

where β_i , the coefficient of influence, is:

$$\beta_i = \frac{\sum_{x=1}^{I} K_{x,x-1} + K_{x,x-2} + \cdots + K_{x,x-i}}{\sum_{x=1}^{n+1} K_{x,x-1} + K_{x,x-2} + \cdots + K_{x,x-i}} \tag{10}$$

$K_{x,n}$ is the stiffness of the elastic element between the points x and n.

For the use of equation (10) we assume that all local stiffnesses $K_{x,n}$ change their values for the rapid movements with the same proportionality constant. With this assumption the influence coefficient β_i keeps its value constant with different kinds of excitations. With the correct analysis of the vibrating system in Fig.1 we normally calculate the stresses and strains for the dynamic excitation and for the static load owing to the relative displacement $\left[U_{n+1}(t) - U_o(t)\right]_{max}$ in the point N+1. The so calculated static displacement in the point I for the maximum relative displacement in the point N+1, divided by the maximum relative displacement in the point N+1 gives the numerical value of β_i .

For different displacements of the masses $M_1, M_2, \ldots, M_n$ equation (2) can be written as:

$$\begin{aligned}
M_1\ddot{X}_1 + C_{11}\dot{X}_1 + \cdots + C_{1n}\dot{X}_n + K_{11}X_1 + \cdots + K_{1n}X_n &= -M_1\beta_1\left[\ddot{U}_{n+1}(t) - \ddot{U}_o(t)\right] \\
M_2\ddot{X}_2 + C_{21}\dot{X}_1 + \cdots + C_{2n}\dot{X}_n + K_{21}X_1 + \cdots + K_{2n}X_n &= -M_2\beta_2\left[\ddot{U}_{n+1}(t) - \ddot{U}_o(t)\right] \\
&\qquad\qquad (11) \\
M_n\ddot{X}_n + C_{n1}\dot{X}_1 + \cdots + C_{nn}\dot{X}_n + K_{n1}X_1 + \cdots + K_{nn}X_n &= -M_n\beta_n\left[\ddot{U}_{n+1}(t) - \ddot{U}_o(t)\right]
\end{aligned}$$

After the transformation into the modal coordinates and the multiplication by $\underline{\Phi}_n^T$ the result is (see also eq.5):

$$M_n\ddot{z} + C_n\dot{z} + K_n z = \underline{\Phi}_n^T[M]\{1\}\beta_n\left[\ddot{U}_{n+1}(t) - U_o(t)\right] \tag{12}$$

Equation (6) of the modal displacement can be expressed here as:

$$\underline{v}_{n,max} = \frac{1}{\omega_n^2}\underline{\Phi}_n\frac{E_n}{M_n}\beta_n\left[S_{n+1}(\omega) - S_o(\omega)\right] \tag{13}$$

The comparison of the equations (6) und (13) shows that the modal displacements at different points of a multi-degree-of-freedom system excitated with $S_{n+1}(\omega) - S_o(\omega)$ only at the point N+1 can be determined like the modal displacement of the same system excited with $S_{n+1}(\omega) - S_o(\omega)$ at all fixed points, with the only difference that the modal participation factors must be reduced by multiplying them by β_i

$$\overline{\Psi}_i^x = \overline{\Psi}_i\beta_i \tag{14}$$

The considerations made for the simplified formula for the calculation of the response spectrum at the decoupling point of a vibrating system with the same excitation value at all fixed points can be applied here to different exitation values at the fixed points. These considerations are explained in detail in [1] page 35 to 45.

The response spectrum at the point s will be calculated according to eq.(1) as:

$$S_s^b(\omega) = (S_{n+1}(\omega) - S_o(\omega))\left[1 + \Psi_j^x\alpha_{ih}(F_m - 1)\right] \tag{14}$$

where Ψ_j^x is the normalised value of the reduced mass participation factor $\overline{\Psi}_j^x$.

$$\Psi_i^x = \frac{\overline{\Psi}_i\beta_i}{\sum_{i=1}^{n}\overline{\Psi}_i\beta_i} \tag{15}$$

$$\Psi_j^x = \max\left[\Psi_1^x, \Psi_2^x, \ldots, \Psi_i^x, \ldots, \Psi_n^x\right] \tag{16}$$

The response spectrum at the point s is the sum of the response spectra calculated at steps a and b :

$$S_s(\omega) = S_s^a(\omega) + S_s^b(\omega) \tag{17}$$

Considering equations (8) and (14) we can write the equation (17) as:

$$S_s(\omega) = S_o(\omega)\left[1 + \Psi_j\alpha_{sh}(F_m - 1)\right] + \left[S_{n+1}(\omega) - S_o(\omega)\right]\left[1 + \Psi_j^x\alpha_{sh}(F_m - 1)\right] \tag{18}$$

Equation (18) is the general equation for the direct calculation of response spectrum $S_s(\omega)$ at any point s of the vibrating system on the basis of input response spectra for the fixed points in the case of multiple support excitations.
The significance of the different terms in eq.(18) is already given with equations (1), (10), (14), (15) and (16).

REFERENCES

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[2] CLOUGH, R. W., PENZIEN, J Dynamics of Structures, Mc Graw Hill, New York, 1975.