Spectrum analysis with quadratic participation for seismic design of light secondary system

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1 INTRODUCTION

Response spectrum method based on the square root sum of the squares (SRSS) modal superposition technique is widely used in seismic design. Also, this method has recently been expanded by Wu et al. (1978), Leimbach et al. (1979) and Lin et al. (1980) to be capable of treating the multiple support response spectra input. The SRSS technique is justifiable by the statistical theory for earthquake motions which have smooth response spectra over a wide range of frequencies. However, it can cause significant errors in the response prediction of the light secondary or appended system such as the piping system in a reactor building. This drawback is brought about mostly by the improper application of SRSS rule to the modal superposition without regard to the predominant characteristic of the floor response spectra input.

In this paper, a new approach, "quadratic perticipation method", is presented. This simple approach is demonstrated to be capable of the acculate response prediction for the light secondary system which subject to either uniform or multiple support excitation.

2 BASIC EQUATIONS

2.1 Equation of motion and solution

The equation of motion of a multiple degrees-of-freedom (DOF) secondary system subject to multiple support excitation is written in the matrix form as

$$(1)\quad [M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M][\eta](\{\ddot{x}_b\} + \{1\}\ddot{x}_g)$$

$$(2)\quad \{v\} = [\eta]\{x_b\}$$

where [M], [C], [K] and [η] are the mass matrix, damping matrix, stiffness matrix and the pseudostatic influence matrix, respectively of the secondary system. Also, {u} and {xb} are the relative displacement vector from the pseudostatic displacement vector, {v}, and the relative displacement of the supporting points from the ground motion, xg.

Using normal mode approach, the solution of eq.(1) results in the following response equation.

$$(3)\quad \{\ddot{X}_a\} = \{\ddot{u}\} + \{\ddot{v}\} + \{1\}\ddot{x}_g = \sum_{j=1}^{n} \{\phi_a\}_j \sum_{p=1}^{L} \Gamma_{jp}\, \ddot{Z}_{pj}$$

where $\{\phi a\}$ is the vector of the orthonormalized mode shape, Γjp is the modal-support participation coefficient for the j-th mode and the p-th support, and $\ddot{Z}pj$ is the j-th response acceleration for p-th support motion. The response equations could be given for other physical values such as displacement, stress, etc. This paper, however, treats only the absolute acceleration to spare the duplication.

Instead of eq.(3), the following formulation is possible.

$$(4)\quad \{\ddot{X}_a\} = \sum_{j=1}^{n} \sum_{J=1}^{m} \{\xi\}_{jJ}\, \ddot{Z}_{jJ}$$

where $\ddot{Z}jJ$ is the response acceleration of the single DOF oscillator for the j-th mode of the secondary system which is appended on the single DOF oscillator for the J-th mode of the supporting system. Also, $\{\xi\}jJ$ is the vector of the quadratic perticipation factor for the j/J-th secondary-on-primary system and is defined as

$$(5)\quad \{\xi\}_{jJ} = \sum_{p=1}^{L} \{\phi_a\}_j\, \Gamma_{jp}\, \phi_{bpJ}\, \psi_{bJ}$$

where ϕbpJ and ψbJ are the orthonormalized mode shape of the p-th support and the participation coefficient, respectively for the J-th mode of the supporting system.

2.2 Response Spectrum Method

Eq.(3) results in the following response equations as shown by Lin et al. (1980) for the multiple support response spectra input.

$$(6)\quad \{\ddot{X}_{amax}\}_j = \{\phi_a\}_j \left[\sum_{p=1}^{L} (\Gamma_{jp}\, S_{pj})^2 + 2 \sum_{q=p+1}^{L} \sum_{p=1}^{L-1} \beta_{pq}\, \Gamma_{jp}\, \Gamma_{jq}\, S_{pj}\, S_{qj}\right]^{\frac{1}{2}}$$

$$(7)\quad \{|\ddot{X}_a|_{max}\} = \left[\sum_{j=1}^{m} \{\ddot{X}_{amax}\}_j^2\right]^{\frac{1}{2}}$$

where $Spj(\omega aj)$, βpq and $\{\ddot{X}amax\}j$ are peak acceleration of the j-th mode which is read from the FRS of the p-th support, the coefficient of inter support relation and the peak responses of the system for the j-th mode in gloval coordinates which retain signs, respectively. Also, $\{|\ddot{X}a|max\}$ is the peak responses of the system which is obtained by SRSS modal superposition technique. For the coefficient of inter support relation, $\beta pq = 0$ (i.e. SRSS) or $\beta pq = 1$ (i.e. algebraic sum) is selected. This selection is dependent on the judgement on the difference between the response spectrum of support p and that of support q. As a conservative measure, the absolute sum superposition may be selectable.

2.3 Quadratic participation method

As the response equation for the response spectrum approach has been derived from eq.(3), so the following equation is derived from eq.(4) for the quadratic participation method.

$$(8)\quad \{|\ddot{x}_a|\}_{max} = \sum_{j=1}^{m}\sum_{J=1}^{n}\sum_{k=1}^{m}\sum_{K=1}^{n} \varepsilon_{jJkK}\,(\xi_{jJ}\,S_{jJ})(\xi_{kK}\,S_{kK})$$

where $S_{jJ}(\omega_{aj},\omega_{bJ})$ is the peak response of $\ddot{Z}_{jJ}$ which would be read from the secondary response spectrum (SRS) instead of FRS for the response spectrum method, and ε_{jJkK} is the coefficient of inter modal relation between S_{jJ} and S_{kK}.

Unlike the response spectrum method, the effect of the multiple support excitation is reduced in the quadratic participation function as formulated in eq.(5). Consequently, the response prediction by eq.(8) is free from errors due to misinterpretation of the inter support relation.

3 THE COEFFICIENT OF INTER MODAL RELATION

The assumption that an earthquake motion is a stationary random process is often justifiable. Under this assumption, the coefficient of inter modal relation, ε_{jJkK}, between S_{jJ} and S_{kK} is equivalent to the coefficient of correlation between $\ddot{Z}_{jJ}$ and $\ddot{Z}_{kK}$, and can be written as

$$(9)\quad \varepsilon_{jJkK} = \frac{\kappa_{jJkK}}{\sigma_{jJ}\,\sigma_{kK}}$$

where σ_{jJ} is the standard deviation of $\ddot{Z}_{jJ}$ and κ_{jJkK} is the covariance of $\ddot{Z}_{jJ}$ and $\ddot{Z}_{kK}$.

Fig.1 shows the variation of ε_{jJkK} for the natural frequency ratios, $\nu_1 = \omega_{bJ}/\omega_{aj}$ and $\nu_2 = \omega_{bK}/\omega_{ak}$, as variables, which is obtained by the computation with Kanai-Tajimi (1960) type ground motion input.

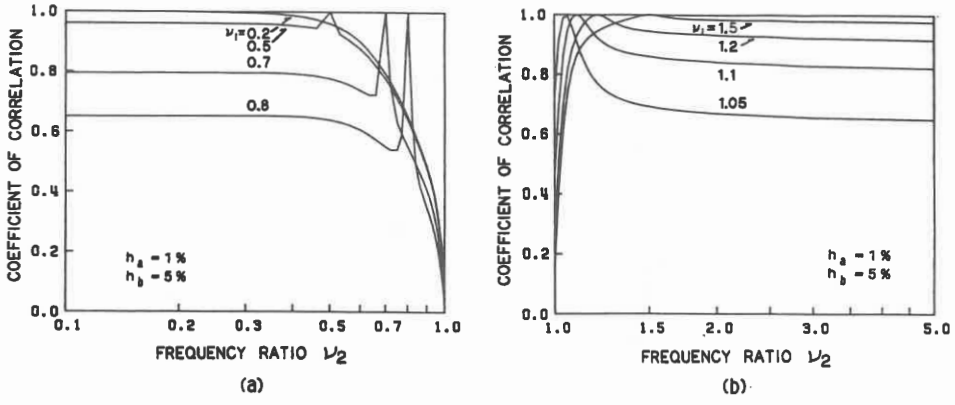

Figure 1. Coefficient of correlation for typical frequency, (a) ω_b = 5 Hz and (b) ω_a = 10 Hz.

Fig.1 (a) and (b) indicate the typical characteristic of ε_{jJkK} in the case where ω_{bJ} is close to ω_{bK} by taking $\omega_{bJ} = \omega_{bK} = 5$ Hz as an example and that in the case where ω_{aj} is close to ω_{ak} by taking $\omega_{aj} = \omega_{ak} = 10$ Hz as an example, respectively. It is shown that the correlation between $\ddot{Z}_{jJ}$ and $\ddot{Z}_{kK}$ gets stronger as ν_1 and ν_2 get smaller in the former case or larger in the latter case.

4 NUMERICAL EXAMPLE

As an illustration of the advantage of the quadratic participation method over the existing methods, the example computations are carried out. Fig.2 and 3 show the spring-mass models and the characteristics of modal vibration, which simulate the reactor building and an appended equipment, respectively. Two cases, case A where the equipment is supported at 3rd, 6th and 9th floor of the building and case B where at 4th, 5th and 6th floor, are involved.

El Centro (1940) NS is used for the input ground motion. The damping ratios are 5% for the building and 1% for the equipment.

For design purpose, the broadened FRS is commonly used to account for the uncertainty of the system frequency. However, the use of such a FRS or SRS is excluded here to avoid clouding the issue with additional variables. Instead, the numerically exact values are used for S_{pj} of eq.(6) and S_{jJ} of eq.(8).

For the coefficient of inter modal relation, the following realistic rules are applied without exception: (1) $\varepsilon_{jJkJ} = 1$ where ν_{jJ} and $\nu_{kJ} < 0.8$, (2) $\varepsilon_{jJjK} = 1$ where ν_{jJ} and $\nu_{jK} > 1.2$, (3) otherwise $\varepsilon_{jJkK} = 0$.

As it seems difficult to judge the inter support relation without any dispute, the SRSS (MRSM1), the algebraic sum (MRSM2) and the absolute sum (MRSM3) are all employed for the support superposition in the multiple response spectrum method (MRSM). Also, the single response spectrum method (SRSM) using the enveloped FRS is added for comparison.

Fig.4 compares the solutions by the various methods with the exact solution by the time-history computation. In case A, the results by MRSM or SRSM indicate that the response distributions along the system are far from the exact distribution. As to MRSM, this drawback is attributable to the SRSS modal superposition in eq.(7) which allows the missing of the signs of $\{\ddot{X}_{max}\}_j$. Only the quadratic participation method agrees with the exact solution. In case B, most of the methods agree with the exact solution. Only exception is MRSM1 which yields unconservative solutions.

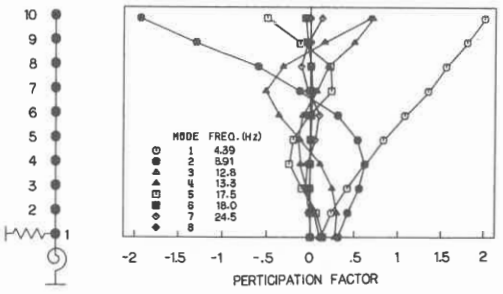

Figure 2. Spring-mass model of a building

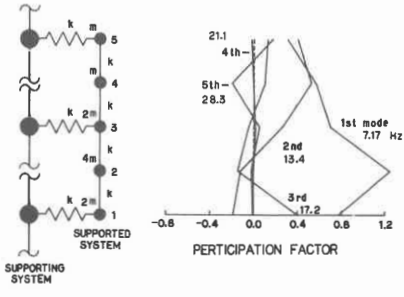

Figure 3. Spring-mass model of an equipment

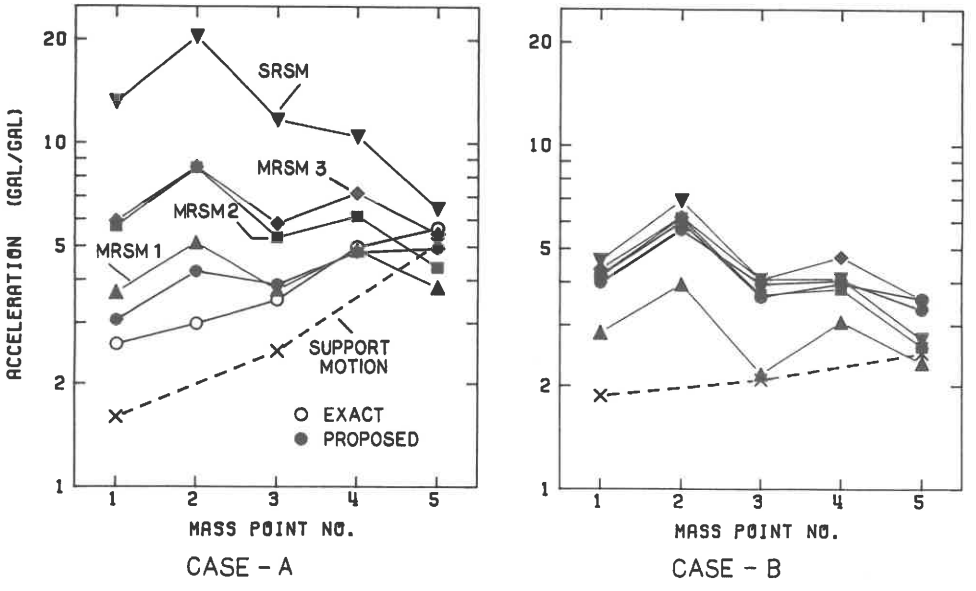

Figure 4. Acceleration amplification of the equipment model

5 SECONDARY RESPONSE SPECTRUM FOR DESIGN PURPOSE

The remaining issue for the application of the quadratic participation method to the seismic design would be to demonstrate how SRS can be specified for design purpose.

Fig.5 shows a contour lines map of an example original SRS which is the envelope of four earthquakes, El Centro (1940), Taft (1952), San Fernand (1971,JPL) and Miyagiken-oki (1978, Tohoku Univ.). Since enveloping like this is not sufficient for the uncertainties of the system frequency, the SRS shall be broadened. In view that the natural frequency would essentially be a probabilistic variable, the realistic definition for the design SRS is as follows.

$$(10) \quad S_D = E[S_o] + 2\sigma_s$$

where S_D is the design SRS, $E[S_o]$ and σ_s are the mathematical expectation and the standard deviation, respectively of S_o.

Fig.6 demonstrates the example design SRS which is computed from the original SRS in Fig.5 under the assumption that the frequency for the secondary system and that for the supporting system are mutually independent variables obedient to the normal distribution with the coefficient of variation of 5% for each of the variables.

By the nature of derivation, thus obtained design SRS would be more reliable than the design FRS obtained by the conventional broadening technique for the uncertainty involved in the frequency characteristics of both the supporting and the secondary system.

SYMBOL	AMPLIFICATION	SYMBOL	AMPLIFICATION	SYMBOL	AMPLIFICATION
0	- 1	1	1 - 2	2	2 - 3
3	3 - 4	4	4 - 5	5	5 - 6
6	6 - 8	7	8 - 10	8	10 - 15
9	15 - 20	A	20 - 30	B	30 - 40
C	40 - 50	D	50 - 60	E	60 - 66.12

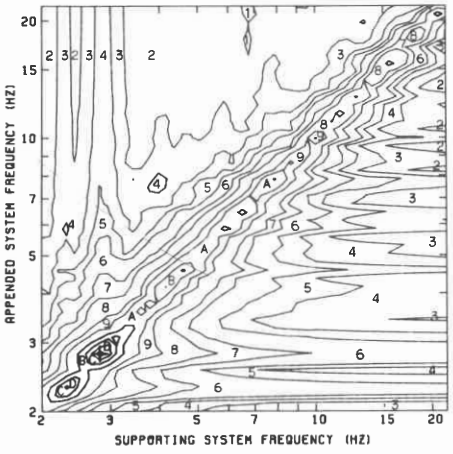

Figure 5. Envelope SRS

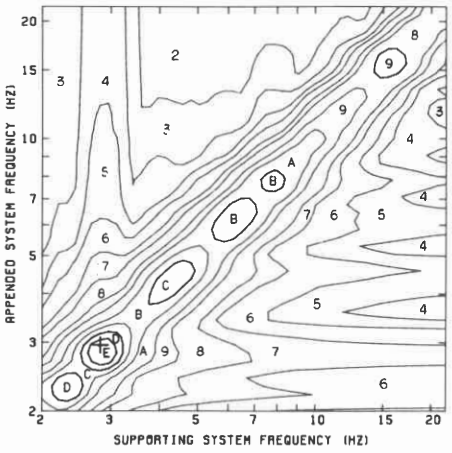

Figure 6. Design SRS

6 CONCLUDING REMARKS

A new approach called "quadratic participation method" has been presented for the realistic prediction of the seismic response of a light secondary system subject to either uniform or multiple support excitation from a supporting system. Unlike the current response spectrum method, the presented approach employs the secondary response spectrum instead of the floor response spectrum and the quadratic participation factor instead of the participation factor in the formulation of the response equation.

Besides being practical, this method is capable of avoiding the serious errors which might be caused by the current method in the prediction of the system response.

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