Elasto/Visco-Plastic Analysis of Orthotropic Moderately Thick Shells of Revolution

S. Takezono, K. Tao
Kumamoto University, Dept. of Mechnical Engineering, Kumamoto 860, Japan
S. Nagano
Mazda Automobile Co., Ltd., Hiroshima 735, Japan

ABSTRACT

This paper describes an analytical formulation and a numerical analysis on the elasto/visco-plastic problems of orthotropic moderately thick shells of revolution under axi-symmetrical loads with application to a cylindrical shell, and with comparison to experimental results.

The analytical formulation is developed by extension of the Reissner-Naghdi theory in elastic shells where a consideration on the effect of shear deformation is given. As the constitutive equation, Hooke's law for orthotropic materials is used in the elastic range, and equations based on the orthotropic visco-plastic theory derived from the orthotropic plastic theory by Hill are employed in the plastic range. The visco-plastic strain rates are related to the stresses by Perzyna's equation. In order to check up the adequacy of the numerical analysis, experiments on elasto/visco-plastic deformation of a titanium cylindrical shell subject to internal axi-symmetrical loads are performed. Good agreement is obtained between experimental results and analytical solutions.

1. INTRODUCTION

It is well known from experiments that in plastic range the influence of viscosity of the materials appears in the mechanical behaviour of shells made of not only high polymers but also metals at room temperature. The influence of viscosity in a plastic range becomes remarkable under high stress. However, mechanical behaviour of machinery and structures subjected to loads over an elastic limit has been almost all analyzed by the conventional elasto-plastic theory which time does not enter directly into consideration. The authors make much of this fact and have dealt with statical problems and dynamical ones of the elasto/visco-plastic shells until now [1–6]. These studies, however, are almost concerned with the case of thin shells and based on the assumption that the material is isotropic.

In recent years the stress analyses on moderately thick shells have drawn special attention because they are used in many fields. These shells being moderately thick, there may be question in applicability of the classical theory neglecting shear deformation effects.

In the present paper the authors study the elasto/visco-plastic deformation problems of general, moderately thick shells of revolution made of homogeneous and orthotropic material under axi-symmetrical load. The analytical formulation is developed by extension of the Reissner-Naghdi theory for elastic moderately thick shells[7,8]. The constitutive relations are based on the orthotropic visco-plastic theory derived from the orthotropic plastic theory by Hill and the elasto/visco-plastic strain rates are related to the stresses by Perzyna's equation[9]. The basic differential equations derived for increments are numerically solved by the finite difference method, and the solutions are obtained by integration of the increments.

Although it may be often obserbed in experiments that influence of viscosity appears in behavior of structures in a plastic range, there are only a few experiments handling with this problem[10, 11]. The authors perform the experiment on titanium cylindrical shells with free ends subject to internal axi-symmetrical loads. By comparing with calculating results, the adequacy of this numerical method is examined.

2. FUNDAMENTAL EQUATIONS

The parametric expressions of a middle surface of shells of revolution may be written as:

$$r = r(\xi), \quad z = z(\xi) \tag{1}$$

where ξ is the meridional length along the middle surface from a boundary, and r and z are coordinates of the middle surface, as shown in Fig. 1. The relations between the coordinates r, z and the angle φ in Fig. 1 are as follows:

$$r' = \cos(\Phi - \vartheta), \quad z' = \sin(\Phi - \vartheta), \quad (r')^2 + (z')^2 = 1, \quad ' = d/d\xi \tag{2}$$

where $\Phi = \varphi + \vartheta$ and ϑ is a change of the angle φ due to deformation.

The equations of equilibrium are given in the differential form with time or load by:

$$\dot{\psi}_V' + r\dot{p}_V = 0, \quad \dot{\psi}_H' - \dot{N}_\theta + r\dot{p}_H = 0, \quad (r\dot{M}_\xi)' - \dot{M}_\theta \cos\Phi + \dot{\vartheta} M_\theta \sin\Phi - r\dot{Q} = 0 \tag{3}$$

where

$$\dot{\psi}_H = r\dot{H}, \quad \dot{\psi}_V = r\dot{V} \tag{4}$$

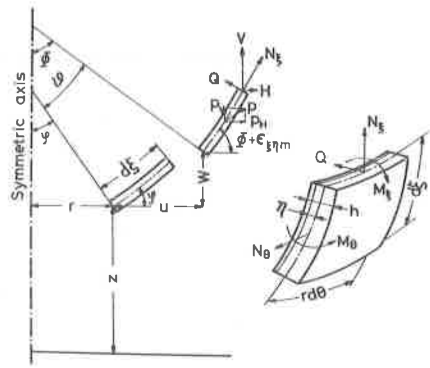

Fig. 1 Notations of shells of revolution.

M_ξ and M_θ are meridional and circumferential bending moments per unit length, respectively, and p_H and p_V are components of distributed loads per unit area of the middle surface, as shown in Fig. 1. p_H and p_V are connected with internal loads $\{p_H^+, p_V^+\}$ and external ones $\{p_H^-, p_V^-\}$ by following relations:

$$p_H = p_H^+ h^+ - p_H^- h^-, \qquad p_V = p_V^+ h^+ - p_V^- h^- \tag{5}$$

where

$$h^\pm = 1 \mp \frac{h}{2}\left(\frac{1}{R_\xi} + \frac{1}{R_\theta}\right) + \frac{h^2}{4}\frac{1}{R_\xi R_\theta} \tag{6}$$

h is thickness of the shell and R_ξ, R_θ are the principal radii of curvature in the meridional and circumferential directions of the middle surface, respectively.

The relations among internal forces per unit length are:

$$r\dot{N}_\xi = \dot{\psi}_H \cos\Phi + \dot{\psi}_V \sin\Phi - (\psi_H \sin\Phi - \psi_V \cos\Phi)\dot{\vartheta}$$
$$r\dot{Q} = -\dot{\psi}_H \sin\Phi + \dot{\psi}_V \cos\Phi - (\psi_H \cos\Phi + \psi_V \sin\Phi)\dot{\vartheta} \tag{7}$$

Denoting radial and axial displacement components by u and w, respectively, the strains and the changes of curvature of the middle surface are expressed by [8]:

$$\dot{\epsilon}_{\xi m} = \dot{u}' \cos\Phi + \dot{w}' \sin\Phi + \dot{\vartheta}(w' \cos\Phi - u' \sin\Phi - \sin\vartheta), \qquad \dot{\epsilon}_{\theta m} = \dot{u}/r$$
$$2\dot{\epsilon}_{\xi\eta m} = \dot{w}' \cos\Phi - \dot{u}' \sin\Phi - \dot{\vartheta}(w' \sin\Phi + u' \cos\Phi + \cos\vartheta) \qquad \dot{\kappa}_\xi = \dot{\vartheta}', \quad \dot{\kappa}_\theta = \dot{\vartheta}\cos\Phi / r \tag{8}$$

where $\epsilon_{\xi\eta m}$ is half the usual engineering shear strain.

The strain components at a distance η from the middle surface, ϵ_ξ, ϵ_θ and $\epsilon_{\xi\eta}$ are given as:

$$\dot{\epsilon}_\xi = (\dot{\epsilon}_{\xi m} - \eta\dot{\kappa}_\xi)/(1 - \eta/R_\xi), \quad \dot{\epsilon}_\theta = (\dot{\epsilon}_{\theta m} - \eta\dot{\kappa}_\theta)/(1 - \eta/R_\theta), \quad \dot{\epsilon}_{\xi\eta} = \dot{\epsilon}_{\xi\eta m}/(1 - \eta/R_\xi)$$
$$R_\xi = 1/\Phi', \quad R_\theta = r/\sin\Phi \tag{9}$$

The terms η/R_ξ and η/R_θ are ordinarily neglected in the thin shell theory.

Now, we shall develope the Perzyna's elasto/visco-plastic constitutive equations [9] into the case of orthotropic materials by using Hill's orthotropic plastic theory. If ξ, θ and η denote principal axes of anisotropy and the normal stress σ_η directed to the shell thickness is neglected, the equivalent stress $\bar{\sigma}$ in orthotropic materials is expressed as follows:

$$\bar{\sigma} = \sqrt{\frac{3}{2(F+G+H)}}[(G+H)\sigma_\xi^2 - 2H\sigma_\xi\sigma_\theta + (F+H)\sigma_\theta^2 + 2M\sigma_{\xi\eta}{}^2]^{1/2} \tag{10}$$

where F , G , H and M are parameters of anisotropy. The gradient $[\,\partial\bar{\sigma}/\partial\sigma\,]$ is given as follows by employing $\bar{\sigma}$ for the plastic potential:

$$[\frac{\partial\bar{\sigma}}{\partial\sigma}] = \frac{3}{2\bar{\sigma}}S_{ij} = \frac{3}{2\bar{\sigma}}[\,S_\xi,\, S_\theta,\, 2S_{\xi\eta}\,] \tag{11}$$

where,

$$S_\xi = \frac{(G+H)\sigma_\xi - H\sigma_\theta}{F+G+H}, \quad S_\theta = \frac{(F+H)\sigma_\theta - H\sigma_\xi}{F+G+H} \quad S_{\xi\eta} = \frac{M\sigma_{\xi\eta}}{F+G+H} \tag{12}$$

In the present theory when we assume that the total strain rate can be resolved into an elastic and a visco-plastic part, the constitutive equations for orthotropic material are written as follows:

$$\begin{aligned} &\dot{\sigma}_\xi = \frac{E_\xi}{1-\nu_{\xi\theta}\nu_{\theta\xi}}[\,\dot{\epsilon}_\xi + \nu_{\theta\xi}\dot{\epsilon}_\theta - (\dot{\epsilon}_\xi{}^{vp} + \nu_{\theta\xi}\dot{\epsilon}_\theta{}^{vp})\,], \qquad \dot{\sigma}_\theta = \frac{E_\theta}{1-\nu_{\xi\theta}\nu_{\theta\xi}}[\,\nu_{\xi\theta}\dot{\epsilon}_\xi + \dot{\epsilon}_\theta - (\nu_{\xi\theta}\dot{\epsilon}_\xi{}^{vp} + \dot{\epsilon}_\theta{}^{vp})\,], \\ &\dot{\sigma}_{\xi\eta} = 2G_{\xi\eta}\,[\,\dot{\epsilon}_{\xi\eta} - \dot{\epsilon}_{\xi\eta}{}^{vp}\,] \end{aligned} \tag{13}$$

where the dot denotes the partial differentiation with respect to time, E_ξ , E_θ , $G_{\xi\eta}$ are Young's modulus, and $\nu_{\xi\theta}$, $\nu_{\theta\xi}$ are Poisson's ratio. The visco-plastic strain rates $\dot{\epsilon}_{ij}{}^{vp}$ in eqs. (13) are from Perzyna's equation [9]:

$$\dot{\epsilon}_{ij}{}^{vp} = \gamma < \Psi(f) > [\frac{\partial\bar{\sigma}}{\partial\sigma}] = \frac{3}{2\bar{\sigma}}\gamma < \Psi(f) > S_{ij} \tag{14}$$

where γ is a viscosity parameter of the material. The notation $< \Psi(f) >$ is defined as:

$$< \Psi(f) > = 0 \; : \; f \leqq 0, \qquad < \Psi(f) > = \Psi(f) \; : \; f > 0 \tag{15}$$

where function f is

$$f = (\,\bar{\sigma} - \bar{\sigma}^*\,)\,/\,\bar{\sigma}^* \tag{16}$$

In eq. (16) $\bar{\sigma}^*$ is a function of equivalent visco-plastic strain $\bar{\epsilon}^{vp}$:

$$\bar{\epsilon}^{vp} = \sqrt{\frac{2}{3}(F+G+H)} \int \left\{ \frac{(F+H)\ d\epsilon_\xi{}^{vp\,2} + 2H\ d\epsilon_\xi{}^{vp} d\epsilon_\theta{}^{vp} + (G+H)\ d\epsilon_\theta{}^{vp\,2}}{FG+GH+HF} + \frac{2\ d\epsilon_{\xi\eta}{}^{vp\,2}}{M} \right\}^{1/2} \tag{17}$$

and the function is obtained from the usual tension test when a strain rate is enough small. Also $f = 0$ becomes Hill's yield surface.

The resultant forces and the resultant moments per unit length are as follows:

$$\left\{ \dot{N}_\xi\, .\, \dot{Q}\, .\, \dot{M}_\xi \right\} = \int_{-h/2}^{h/2} \left\{ \dot{\sigma}_\xi\, ,\, \dot{\sigma}_{\xi\eta},\, \dot{\sigma}_\xi\,\eta \right\} (\,1-\eta\,/\,R_\theta\,)\,d\eta\ ,\ \left\{ \dot{N}_\theta\, .\, \dot{M}_\theta \right\} = \int_{-h/2}^{h/2} \left\{ \dot{\sigma}_\theta\, ,\, \dot{\sigma}_\theta\,\eta \right\} (\,1-\eta\,/\,R_\xi\,)\,d\eta \tag{18}$$

Substitution of eqs. (9) , (13) into eq. (18) is:

$$\begin{Bmatrix} \dot{N}_\xi \\ \dot{N}_\theta \\ \dot{Q} \\ \dot{M}_\xi \\ \dot{M}_\theta \end{Bmatrix} = \begin{bmatrix} TC_\xi & T\nu_{\theta\xi}C_\xi & 0 & -D_\xi\lambda & 0 \\ T\nu_{\xi\theta}C_\theta & TC_\theta & 0 & 0 & D_\theta\lambda \\ 0 & 0 & {}^5/{}_6G_{\xi\eta}h & 0 & 0 \\ D_\xi\lambda & 0 & 0 & -D_\xi & -\nu_{\theta\xi}D_\xi \\ 0 & -D_\theta\lambda & 0 & -\nu_{\xi\theta}D_\theta & -D_\theta \end{bmatrix} \begin{Bmatrix} \dot{\epsilon}_{\xi m} \\ \dot{\epsilon}_{\theta m} \\ 2\dot{\epsilon}_{\xi\eta m} \\ \dot{\kappa}_\xi \\ \dot{\kappa}_\theta \end{Bmatrix} - \begin{Bmatrix} \dot{N}_\xi{}^{vp} \\ \dot{N}_\theta{}^{vp} \\ \dot{Q}^{vp} \\ \dot{M}_\xi{}^{vp} \\ \dot{M}_\theta{}^{vp} \end{Bmatrix} \tag{19}$$

where

$$C_\xi = E_\xi\,h\,,\ C_\theta = E_\theta\,h\,,\ D_\xi = \frac{E_\xi h^3}{12(1-\nu_{\xi\theta}\nu_{\theta\xi})},\ D_\theta = \frac{E_\theta h^3}{12(1-\nu_{\xi\theta}\nu_{\theta\xi})},\ \lambda = \frac{1}{R_\xi} - \frac{1}{R_\theta},\ T = \frac{1}{1-\nu_{\xi\theta}\nu_{\theta\xi}}$$

In eq. (19) $(\ )^{vp}$ denotes apparent internal forces due to viscoplastic strains as follows:

$$\begin{aligned} &\left\{ \dot{N}_\xi{}^{vp}\, ,\, \dot{Q}^{vp}\, ,\, \dot{M}_\xi{}^{vp} \right\} = \int_{-h/2}^{h/2} \left\{ \dot{\sigma}_\xi{}^{vp}\, ,\, \dot{\sigma}_{\xi\eta}{}^{vp}\, ,\, \dot{\sigma}_\xi{}^{vp}\eta \right\} (\,1-\eta\,/\,R_\theta\,)\,d\eta \\ &\left\{ \dot{N}_\theta{}^{vp}\, ,\, \dot{M}_\theta{}^{vp} \right\} = \int_{-h/2}^{h/2} \left\{ \dot{\sigma}_\theta{}^{vp},\ \dot{\sigma}_\theta{}^{vp}\eta \right\} (\,1-\eta\,/\,R_\xi\,)\ d\eta \end{aligned} \tag{20}$$

From above equations (3) ∿ (20) , the following simultaneous differential equations of second order with respect to $\dot{\vartheta}$ and $\dot{\psi}_H$ can be derived as follows:

$$\begin{bmatrix} a_1 , a_2 \\ b_1 , b_2 \end{bmatrix} \dot{x}'' + \begin{bmatrix} a_3 , a_4 \\ b_3 , b_4 \end{bmatrix} \dot{x}' + \begin{bmatrix} a_5 , a_6 \\ b_5 , b_6 \end{bmatrix} \dot{x} = \begin{Bmatrix} d_1 \\ d_2 \end{Bmatrix} , \quad \dot{x} = \begin{Bmatrix} \dot{\vartheta} \\ \dot{\psi}_H \end{Bmatrix} \tag{21}$$

where $a_1 \sim a_6$, $b_1 \sim b_6$ and d_1 , d_2 are the constants determined from the shell geometrics, elastic constants, loads, apparent internal forces and the solutions at the before calculating stage.

Equation (21) is the fundamental equation for elasto/visco-plastic problems of orthotropic moderately thick shells of revolution. In the equation if C_ξ , D_ξ and $\nu_{\xi\theta}$ are equal to C_θ , D_θ and $\nu_{\theta\xi}$, respectively, this reduces to the equation for isotropic shells derived previously by Takezono and Tanoue [5].

3. NUMERICAL METHOD

A finite difference method is employed for solutions of eq. (21). The usual central difference formulas are used for every mesh point except boundary points of the shell. For the boundary points forward and backward differnce equations are employed. Integrations of eqs.(20) are carried out by Simpson's 1/3 rule in the use of the values at K points where are taken for equal distance through shell thickness.

The above numerical method is concerned with a handling of a spatial aspect of the problem. In this elasto/visco-plastic problem a temporal aspect also must be considered. The solutions at any stage of the problem are obtained by integration of internal forces and displacements due to the load increment and the time increment.

4. EXPERIMENTS

4. 1 Test pieces and material constants

The cylindrical test pieces used in experiment were cut out from a titanium drawn tube with outer diameter 60 mm and inner diameter 46 mm. The dimensions of the test piece are 25.5 mm in mean radius, 3 mm in thickness and 86 mm in length (Fig. 4).

To know this material constants three directional uniaxial tension test specimens, axial, circumferential and 45° directions, were cut out from the drawn tube (Fig. 2). Tension tests of these three kinds of test pieces were carried out under various constant strain rates ($\dot{\epsilon}$ = 0.05∼300 μ / s). Figure 3 illustrates the stress-strain curves obtained from the tests for axial specimens. Further consideration of the test results for circumferential and 45° directional specimens gave us next material constant values.

$$\begin{aligned} &\text{Young's modulus : } E_\xi = E_\theta = 107 \text{ GPa}, \quad \text{Poisson's ratio : } \nu_{\xi\theta} = \nu_{\theta\xi} = 0.35 \\ &\text{Statical hardening stress - strain relation : } \bar{\sigma}^* = 482\,(\,9.18 \times 10^{-5} + \bar{\epsilon}^{vp})^{0.13} \text{ MPa} \\ &\text{Initial yield stress : } \sigma_Y = 144 \text{ MPa}, \quad \text{Viscosity parameter : } \gamma = 0.0015 \ 1/s \\ &\text{Parameter of anisotropy : } G/F = 1.59,\ H/F = 2.20,\ N/F = 6.30 \\ &\Psi(f) = [\,(\bar{\sigma} - \bar{\sigma}^*)/\bar{\sigma}^*\,]^{3.8} \end{aligned} \tag{22}$$

The broken lines in Fig. 3 indicate the results calculated from the above material constants. It is seen from the figure that the calculated lines approximately agree with the experimental results in the whole strain and strain rate ranges.

4. 2 Experimental method

Schematic view of the experiments is shown in Fig. 4. Using an electro-hydraulic servo testing machine, axial compressive forces W are applied to the upper and the lower cylinders ①, ②, and the brass ring divided into twelve parts ③ , is spreaded to the radial direction by the tapered part of the cylinder ① lubricated by means of grease. Then the titanium cylindrical test piece ④ is subjected to the radial axisymmet-

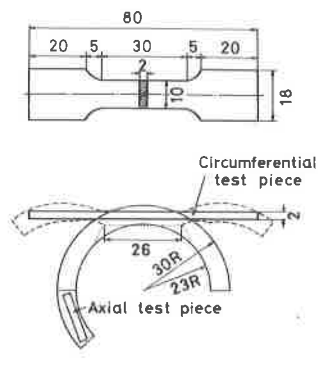

Fig. 2 Uniaxial tensile test piece.

rical load (mean contact pressure $\bar{P}$). Strain gages were attached on the outer surface and axial distributions of axial and circumferential strains ϵ_ξ , ϵ_θ were measured.

The loads ($\bar{P}$ = 27.0∿135 MPa) were applied stepwise as shown in Fig. 5 and the strains in the test piece were measured every five minutes from just after loading to the stationary state. When the maximum strain rate becomes less than 5 μ in five minutes, we regarded it as a stationary state and proceeded to the next loading stage. In the figure, P_0 denotes the initial yield load from calculations. Relations between W and $\bar{P}$ were calibrated by means of measuring strains of a circular ring under inner pressure in which strains can be obtained theoretically.

4. 3 Comparison between the experimental and the calculated results

To find the contact pressure distribution pressure distinction sheets were inserted between the split ring ③ and specimen ④ . From the change of color of the sheet we found that the pressure was distributed uniformly in circumferential direction and distributed on a width of 4 mm in axial direction. Therefore, in this analysis, the pressure distribution was approximated with next equations:

$$\begin{aligned} 39 \leqq \xi \leqq 41 \text{ mm} &: \; p_H{}^{+} = P = [\,(\bar{P}/2)/2^4\,]\,(\xi - 39)^4 \\ 41 \leqq \xi \leqq 43 \text{ mm} &: \; p_H{}^{+} = P = \bar{P} - [\,(\bar{P}/2)/2^4\,]\,(\xi - 43)^4 \end{aligned} \qquad (23)$$

The increment of time $(\Delta t)_i$ was decided as $(\Delta t)_i = 0.9\,\Delta t_0$, where

$$\Delta t_0 = \frac{4}{3}\frac{(1+\nu)}{\gamma E}\left(\frac{\bar{\sigma}^*}{\Psi'}\right)_i \qquad (\Psi' = d\Psi/df,\; i:\ \text{calculation stage}) \qquad (24)$$

is the stability limit given by Zienkiewicz and Cormeau [12].

The boundary conditions at the point A (j = 1) and B (j = N) are:

$$\begin{aligned} &\text{Point A,} \quad \dot{M}_\xi = 0,\; \dot{\psi}_H = 0 \\ &\text{Point B,} \quad \dot{\vartheta} = 0,\; \dot{\psi}_H = 0 \end{aligned} \qquad (25)$$

The meridional mesh number N and the division number K through thickness of the shell were chosen to be N = 216 and K = 19, respectively. These numerical values were determined from evaluation of convergency of the solutions according to variation of the values and in consideration of capacity of the computer and computing time.

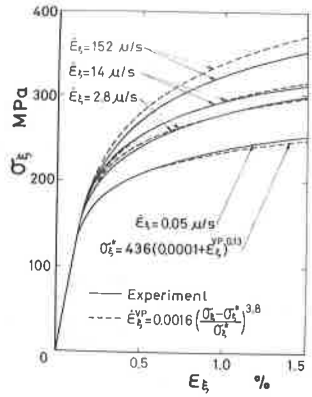

Fig. 3 Stress--strain relations.

Decision of the parameter of anisotropy M is difficult because thickness directional specimen can not be obtained. However, since the influence of the value is small, $S_{\xi\eta}$ and $\sigma_{\xi\eta}$ in eqs. (12) were considered to be equal and M/F = 4.79 was used in this calculation.

Some of the essential features of the solutions are shown in Figs. 6∿11. The results by thin shell theory are indicated by dotted lines and chain lines. In this thin shell theory the effect of the shear strain $\epsilon_{\xi\eta}$ was ignored, while the shear stress $\sigma_{\xi\eta}$ was considered in computing $\bar{\sigma}$.

The adequacy of this numerical analysis seems to be supported because close agreement between experimental and calculated values is seen in Figs. 7 and 8. It is found from these figures that as the load increases, the variation of values with time becomes significant and the diffence between solutions from both theories appears remarkably .

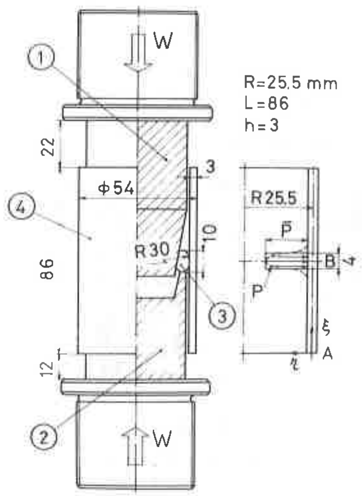

Fig. 4 Schematic view of experiment.

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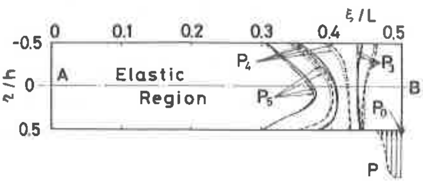

Fig. 6 Progression of yield in meridional section.

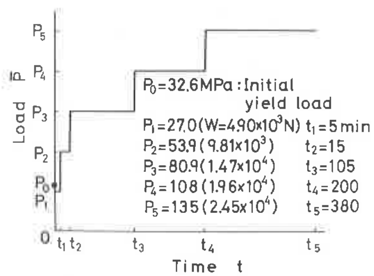

Fig. 5 Load–time relations.

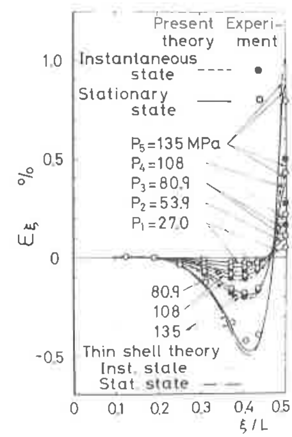

Fig. 7 Distributions of ϵ_ξ .

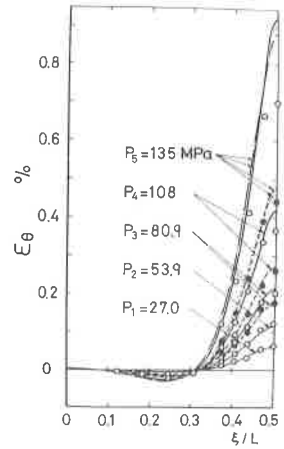

Fig. 8 Distributions of ϵ_θ .

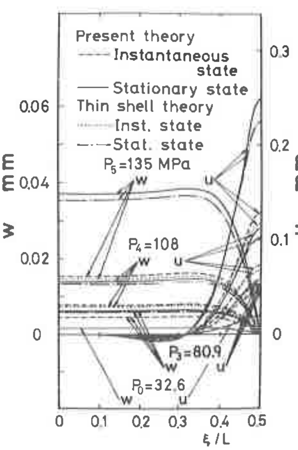

Fig. 9 Distributions of u , w .

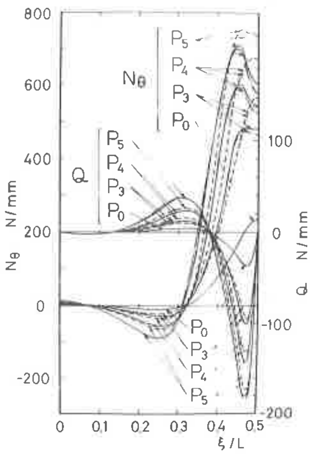

Fig. 10 Distributions of N_θ , Q .

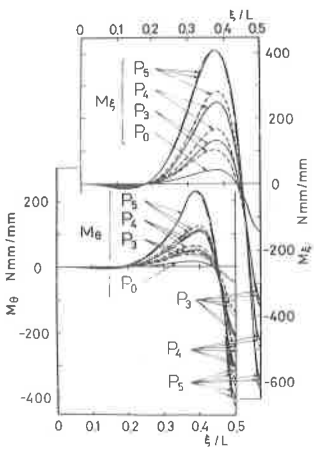

Fig. 11 Distributions of M_ξ , M_θ .