

Notes on Cycle Dependent Ratcheting under Multiaxial Loads Including Bauschinger Effect and Non-Linear Strain Hardening

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1 INTRODUCTION & BACKGROUND

Many components and structures undergoing mechanical or thermal cyclic (*secondary type*) loads while subjected to sustained (*primary type*) loadings are liable to undergo strain accumulations which can affect their mode of failure. If the mode of failure is ratcheting or fatigue with ratcheting, e. g., as appears to be the case for typical dynamic/seismic type loads (Hara and Shibata, 1981; English et al., 1986–1989; Asada, 1989; and Beaney, 1989), then the allowable stresses based on plastic collapse mode (ASME, 1980) can be overly conservative. This points to the need for better evaluation procedures and/or changes to the Code (Tagart et al., 1990) both of which would be benefited by some theoretical basis. Much of the early work is known to be restricted to idealized *non-workhardening* (or simple isotropic hardening) material behavior and simplified loadings (e. g., uniaxial simplification or two-bar simulations for thermal cycling with steady multiaxial stress conditions). Theoretical evaluation of ratcheting under multiaxial loads (Garud, 1986–1990) suggests that the (material) behavior can be significantly different, and not necessarily obvious, from that under uniaxial loading.

Thus, for certain applications and better utilization of the material/structural strength characteristics, there exists the need to develop and improve existing plasticity based methods (of analysis/design) to deal with the complex problems of multiaxial cyclic loading—including shake-down and strain accumulation phenomena. Presented in this paper are the results and discussion of the author's recent evaluation of *cycle dependent* ratcheting which explicitly accounts for the strain-hardening and multiaxial *Bauschinger* effect through the use of a complete set of material constitutive relations in the form of an incremental plasticity with a generalized kinematic hardening.

It is shown that, under combined state of stress, the incremental plasticity predicts *continued* strain accumulation in each (half) cycle even for a kinematic type strain hardening material. The case of triaxial *steady* stress with imposed *cyclic* strain is analyzed in detail evaluating the *material* ratcheting response; this is similar to a pipe section or a piping component subjected to cyclic axial strain (e. g., *secondary* loading due to thermal or seismic action) while internally pressurized (for *primary* loading). For such a case (in a pipe) the theory predicts (local) wall thinning and circumferential growth as a result of *cycle dependent* ratcheting (limited only by *structural* considerations). Results under other combined (multiaxial) primary and secondary (cyclic) loadings are also presented and discussed in relation to experimental observations.

From results of the analysis (for Type 304 stainless steel and A333-gr6 carbon steel) a graphical representation—called *Ratchet Assessment Diagram or RAD*—is developed showing load-combinations (of primary and secondary stresses) for various rates of strain accumulation (on *per cycle* basis). The results are compared with prior theoretical works and experimental observations. It is suggested that the inclusion of more realistic strain hardening material behavior reduces the over-conservatism in the ratchet evaluation, yet retains some margin due to structural considerations. In addition, a simple relation is proposed (between stress allowables) for a specified total ratchet strain as a criterion which is discussed in relation to the ASME Code considerations.

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2 PLASTICITY THEORY AND ITS IMPLEMENTATION

The type of structural and material deformation phenomena to be addressed under this investigation are such that only *time independent* response will be of dominant concern (notwithstanding the fact that it takes a finite time to apply any loading and to observe the response). That is, *time dependent* phenomena, such as creep and stress relaxation, are excluded from consideration either because their contribution (to the total deformation) is relatively small or that their impact can be assessed independently. Therefore, a suitable formulation of time independent plasticity is selected for the purpose of analysis.

The main focus of this work was to be able to investigate *any type of multiaxiality and any combination of static plus cyclic multiaxial components* of loading; as such, a full implementation (involving all tensorial components of stress-strain quantities) of the plasticity theory was needed—however, details of all the *transient* (cycle-to-cycle changing) material properties need not be *explicitly* accounted for. Also, it was considered important that the *strain hardening* characteristics and the *Bauschinger effect*, commonly exhibited by the metallic materials, be accounted for (under the complex loadings of interest); the latter refers to the lowering of elastic limit in the reverse direction due to yielding in the opposite direction. These two taken together may be considered to represent the *cross-hardening (or softening) effect*, under general multiaxial loading, which refers to the change in yield characteristics in the direction of a shear component, for instance, as a result of prior yielding in the direction of a normal component (of the stress tensor).

The above requirements were sufficiently met by the formulation proposed earlier by the author (Garud, 1979) and shown to be promising for complex multiaxial loads (Garud, 1981 and 1982). These earlier applications dealt with the fully reversed type of loading, although, in principle, the (plasticity) formulation is also applicable to non-reversed loads as well. Therefore, the same theory is used in this work with implementation that allows for the presence of (multiaxial) mean stresses in addition to the cyclic loads.

Very briefly, the formulation incorporates Garud's hardening rule in the analytic framework of the classical (incremental) plasticity theory with three main components:

(1) An initial yield condition (as a multiaxial generalization of the elastic limit) that determines whether or not plastic strains will be induced during an increment of loading (starting from a stress-free state). The material is assumed to follow the elastic stress-strain relations prior to exceeding this condition. Here, von Mises yield criterion is used.

(2) A flow rule that relates the increment of plastic strain to the increment of stress. For metals, the most commonly used rule is the *normality condition* (or the *associative flow rule*) according to which the (tensor) components of plastic strain *increment* are proportional to the corresponding (tensor) components of the *unit normal* to the yield surface at the current state of stress. The constant of proportionality is, in general, a non-negative function of the stress, stress-increment, and the loading history. The normality condition is used in this work with the proportionality constant being defined by the concept of *field of plastic moduli* (Mróz, 1969) which is essentially a multilinear generalization of the uniaxial stress-strain curve.

(3) A hardening rule that specifies changes in the yield condition and in the field of plastic moduli as a result of the prior plastic deformation. Many such rules have been proposed over the years and this area of research can at best be characterized as *very dynamic*; selection and justification of a particular rule is beyond the scope and need of this paper. On the basis of familiarity and useful predictive capabilities of the earlier work (Garud, 1979, 1981, and 1982) the hardening rule as described in these papers is adopted here. It differs from the previous commonly known rules (e. g., Prager-Ziegler and Mróz) mainly in that the direction of (kinematic) translation of a yield surface is made to depend on the direction (in the pseudo-space of stress tensor) of the stress increment itself, in addition to being dependent on the current state of stress. Its implementation is such that the *consistency condition* is correctly satisfied for proportional as well as any *non-proportional* stressing (Garud, 1982; also see Nowack, 1988).

In this work it is assumed that only the generalized (multilinear) *kinematic* hardening is operative. Also, the usual assumptions of incompressibility of plastic flow and the initial isotropy of material properties are maintained.

Based on the above formulation a computer code named IPCRES (for Incremental Plasticity Calculations and Ratcheting Evaluation System) was developed. It performs the needed computations by following a specified loading (of steady stresses plus cyclic strains) in small increments simulating the *material point* response (i. e., for a homogeneous state of stress and strain). The code development and its implementation were carried out using an IBM-pc compatible computer.

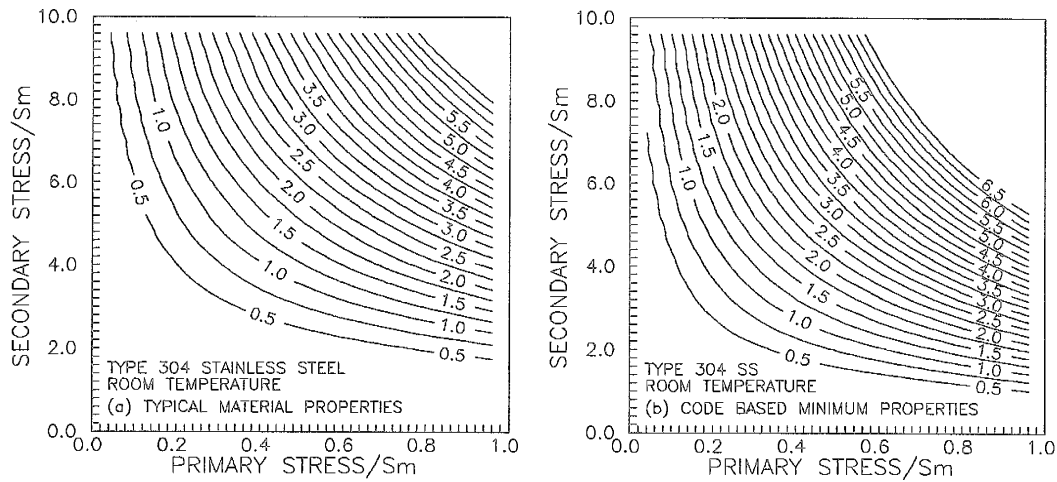


Fig. 2. IPCRES results for the ratchet assessment diagrams showing lines of constant accumulated % strain (10 cycles) for Type 304 stainless steel with typical and Code based minimum properties.

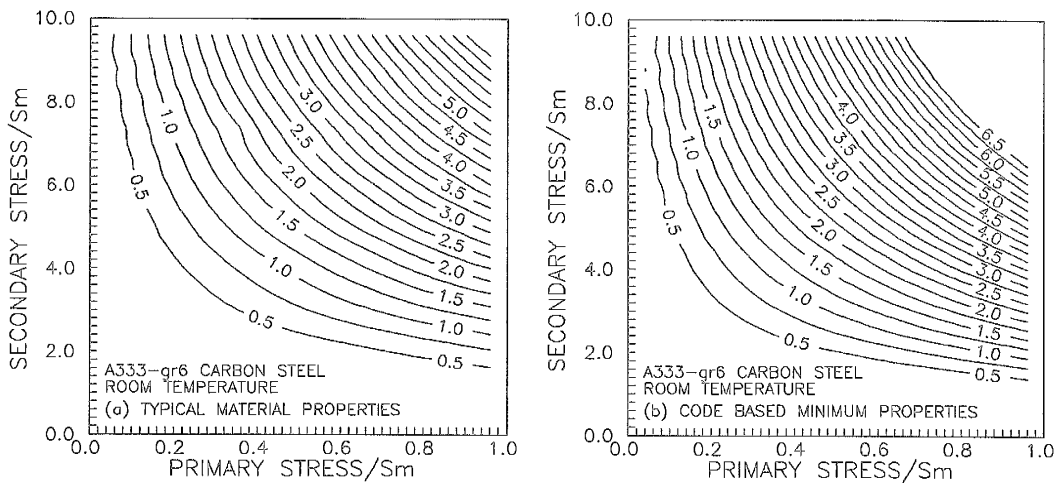


Fig. 3. IPCRES results for the ratchet assessment diagrams showing lines of constant accumulated % strain (10 cycles) for A333-gr6 carbon steel with typical and Code based minimum properties.

3.2 Code considerations

A few comments are offered in the following, with only a limited discussion, to interpret the above results from the viewpoint of Code considerations. Notwithstanding the assumption of time independent behavior, the above results are likely to be conservative on at least two accounts: (a) the possible (additional or transient) *isotropic hardening*, excluded from the formulation, is most likely to reduce the material strain accumulation and (b) *structural* aspects of real applications (involving stress gradients and constraint conditions) are also likely to resist the uniform strain accumulation.

In general, for large allowable cumulative strains (above 2 percent), much larger values of *secondary* stress (Q) can be traded with smaller values of the *primary* stress (P) (without exceeding the allowable total strain in a specified number of cycles). Thus, a simple (linear) criterion for limiting the total ratcheted strain can be expressed as follows:

$$P + \mathcal{F} \cdot Q \leq \mathcal{M} \cdot S_m$$

where $\mathcal{F} (\leq 1)$ and $\mathcal{M} (> 1)$ are constant multiples; here P is assumed to be less than S_m .

For example, from the ratchet diagram for Type 304 steel based on the Code minimum properties, for limiting total strain to 5 percent (maximum) in 10 (or more) cycles the following three linear relations between P and Q are suggested by the IPCRES results:

$$P + (1/7) \cdot Q = 1.5 \times S_m \quad (1)$$

$$P + (1/4) \cdot Q = 2.0 \times S_m \quad (2)$$

$$P + Q = 5.0 \times S_m \quad (3)$$

(To use the first 2 relations one has to identify P and Q as separate quantities). That is, if strain accumulation is the critical mode of failure then the corresponding allowable stresses are larger than those for the plastic collapse ($P + Q \leq 3 S_m$).

3.3 Comparisons with other analyses and experiments

Edmunds and Beer (1961) explored some theoretical aspects of *incremental collapse* in pressure vessels with simplifying assumptions of the *deformation theory* of plasticity, elastic-perfectly plastic material, and zero axial and radial steady stresses. Interestingly, for the case of internal pressure and axial cyclic straining, their results are qualitatively not too different from this study; however, there are differences of details. Both the studies suggest *continued* strain accumulation, even in presence of the generalized kinematic hardening as modelled in IPCRES, whenever the *elastic limit* is exceeded during any part of the cyclic loading.

Fig. 4 illustrates some points of comparison for the case of primary (hoop) stress equal to S_m and 10 cycles. Fig. 4(a) shows that the Edmunds and Beer analysis is sensitive to the choice of elastic stress value needed for their ratcheting evaluation and that an arbitrary choice (here, flow stress = average of the yield and ultimate strengths) can underpredict the strain accumulation (compared to IPCRES). At the very low stress levels, even the choice of 0.2 percent yield stress can underpredict the strains. Fig. 4(b) compares the predictions of allowable stress combinations for 5 percent strain accumulation in 10 cycles. Since IPCRES (almost) fully accounts for the material strain hardening, the IPCRES allowables are higher than the corresponding Edmunds-Beer prediction based on $\sigma_y = \text{elastic limit}$ (not shown) or on the 0.2 percent yield stress (as shown in Fig. 4(b)).

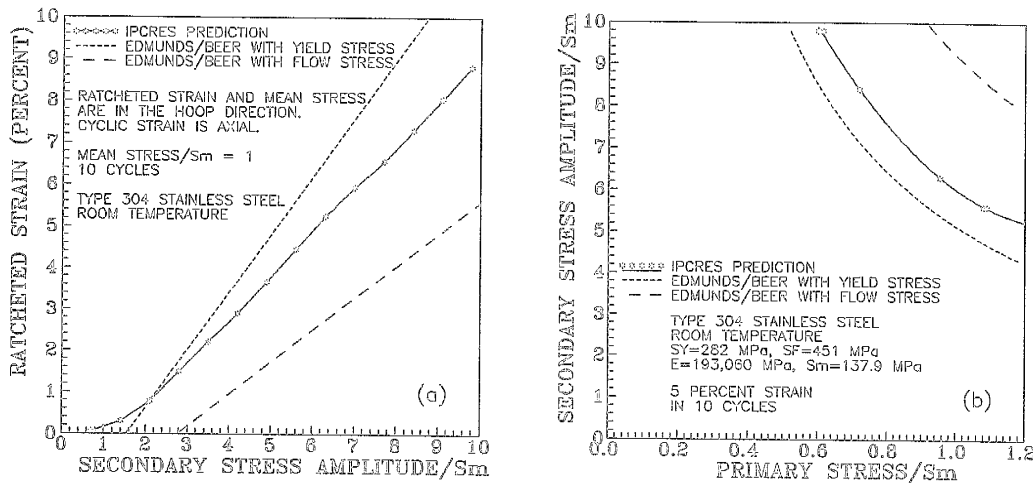


Fig. 4. Comparison of IPCRES results with the analysis of Edmunds and Beer: (a) ratcheted strain at the end of 10th cycle as a function of the secondary stress amplitude (for steady hoop stress equal to S_m) and (b) allowable stress combinations for 5 % strain accumulation in 10 cycles.

In an earlier survey of ratcheting related works, Burgreen (1973) expressed the need to review and interpret experimental results that are already available, and to do so with (needed) new information on effects of strain hardening and multiaxial state of stress. He made reference to some earlier work stating that *relatively uniform cumulative axial extensional deformation, up to about 20 percent, can be obtained before local instabilities disrupt the uniform strain accumulation* in the case of a hollow tube subjected to steady axial load with superimposed cyclic torsion. Also, Yamanouchi et al. (1976) have reported significant (progressive) diametral change of thin tube specimens under steady pressure when cyclic axial strain is superimposed. Clément et al. (1989) have recently reported that steady compressive—as opposed to tensile—axial loading when applied with cyclic torsion (on thin tubes of austenitic Type 316L steel and of annealed copper) resulted in nearly *uniform progressive shortening* of tubular specimens (before reaching instability in the buckling mode). Lebey and Roche (1979) observed axial elongation of thin tubes (of Type 304 stainless steel) under steady axial stress when cyclic torsion was applied. Likewise, a recent experimental work by Asada (1989) on carbon steel tubes shows considerable (progressive) diametral growth from ratcheting under cyclic axial or torsional strain with a steady internal pressure. (He has also reported some reduction in the *fatigue life* due to the ratcheting). Also, Williams (1988) reported *ten-fold* strain-accumulation in the circumferential direction (as compared to the axial strain accumulation) in pressurized specimens when cyclic (axial) bending was superimposed.

All the above observations (on ratcheting with steady internal pressure or axial stress) are in good agreement, at least qualitatively, with the IPCRES predictions. Fig. 5 is added here to illustrate IPCRES results for the case of steady internal pressure stress and cyclic torsional strain; the continued growth in the hoop direction is seen to far exceed that in the axial direction.

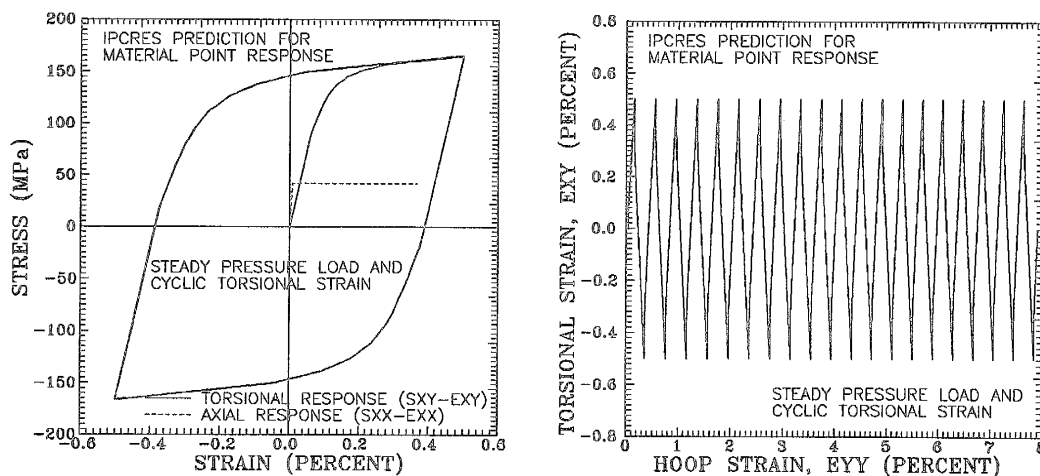


Fig. 5. Stress-strain-ratcheting response predicted by IPCRES for the case of steady internal pressure (hoop stress = $0.6 S_m$) with superimposed cyclic torsion ($\pm 1\%$ engineering shear strain).

A few other related points of interest are: (a) For equivalent loading, the axial strain cycling produced more ratcheting, 0.547 percent, than the torsional cycling, 0.389 percent (during 20th cycle). This is in agreement with the earlier reported experimental observation (Hancell and Harvey, 1979) for which the authors have already offered the suggestion that *in some situations the sustained stresses may share common slip planes with the cyclic stresses and in others they may not*. (b) When only hoop stress is used the IPCRES predictions of strain accumulation were *less* than if *all* stress components were present in the case of superimposed axial cycling; however, the opposite was found with torsional cycling (at least for Fig. 5 specifics). (c) In agreement with other reported observations and unlike the earlier implementations of kinematic hardening (Hancell and Harvey, 1979), the IPCRES results suggest, at least for a homogeneous stress condition, the continued accumulation of strain in certain complex multiaxial loadings.

4 CONCLUDING REMARKS

Results of a theoretical exploration of cycle dependent material ratcheting response under multi-axial steady and cyclic loads (similar to those anticipated in pressure retaining components) were presented. The theory used was an implementation of the incremental plasticity based on Garud's hardening model limited to the kinematic type hardening. This allowed for the inclusion of strain hardening and Bauschinger effects and the complete set of multi-axial state of stress in estimating the ratcheting.

The relatively large and uniform strain accumulation in the circumferential direction, as observed by many, for a tube (or a thin pipe) under internal pressure and cyclic axial or torsional strain, was predicted by the model. Also, progressive elongation (or shortening) in the axial direction, as observed for a tube under steady axial tension (or compression) and cyclic torsion, was predicted. For a few cases examined, the IPCRES results appear to confirm the effect of *directionality* of steady stresses relative to the direction of cyclic straining (for equivalent load intensities) in determining the strain accumulation.

Compared to the simplified analysis based on perfectly plastic response, it is shown that the use of strain hardening characteristics reduces the over-conservatism related to strain accumulation criteria. The IPCRES results were presented in the form of ratchet assessment diagrams for two typical steels and also expressed in terms of simple linear relations for allowable stresses which may prove to be useful estimates; in particular, since only a *homogeneous state of stress* was considered in these results, the *structural* considerations of stress gradients and constraints are likely to provide enough additional margins. For better utilization of the formulation and more accurate comparison with component/specimen tests data it is preferable that the theory be incorporated in the framework of a finite element code. Preliminary work in this direction (Garud, 1990) has shown some encouraging results.

The main focus of this work was to investigate the *time independent* material response to *multi-axial* load combinations likely to cause *cycle dependent* strain accumulation; as such, several time dependent (visco-plasticity) phenomena were not included: stress relaxation, creep, strain-aging (Pellissier-Tanon, 1982; Chaboche and Nouailhas, 1989; Ruggles and Krempl, 1989). Significance of these phenomena, as observed for certain experimental loadings and material conditions, needs further critical examination, especially in relation to the *type, intensities, and rates* of loadings and material conditions employed in service applications. The dynamic/seismic loads with short duration of application may not allow enough time for significant impact. For other applications the *lower bound* response predictions based on the use of Code minimum stress-strain data may be considered to reflect some margin against additional strain accumulation from such phenomena; however, this aspect needs further evaluation.

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