A Simplified Approach for Ratcheting Analysis in Structures with Elastic Follow-up

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ABSTRACT

In the framework of an elastic analysis, the RCC-MR design code uses the concept of the efficiency diagram to assess the behaviour of a structure relatively to ratcheting.

This diagram was obtained from a lot of experimental results and allows to cover many reactor situations. However this approach needs to classify stresses between primary and secondary stresses and for a few cases, in particular for structures with significant elastic follow-up, this classification is not obvious.

After a recall of elastic follow-up definition and a few considerations on the way to evaluate it, an approach is proposed to take it into account in an elastic analysis verifying the avoidance of ratcheting.

An experimental program has been developed to study this interaction between elastic follow-up and ratcheting. The first results are presented together with interpretations with the proposed method.

1 INTRODUCTION

Structures working at high temperature must be designed relatively to creep-fatigue interaction. The current practice in design codes such as RCC-MR is to use the results of an elastic analysis for a first approach. In the classic context, the creep-fatigue analysis method is available only if it is possible to demonstrate the absence of ratcheting beforehand.

In the RCC-MR code and in the context of an elastic analysis, the checking for the absence of ratcheting is based on the concept of efficiency diagram. The utilization of this method relies on the concepts of primary and secondary stresses and is, thus, very simple as long as the classification of stresses is simple itself.

Difficulties arise for structures in which an elastic follow-up effect is encountered. In such a structure submitted to global strain controlled loading, stress redistributions will occur resulting in strain concentrations in the weakest sections. The strains so obtained are greater than those estimated in an elastic analysis which considers that the loading is fully kinematically determined. Consequently a more accurate evaluation of loading must be made.

After a recall on the definition used for the elastic follow-up effect and a few observations on its evaluation, a method is proposed to integrate this into an elastic analysis in order to check the ratcheting rule using the efficiency diagram.

This method was developped in the framework of piping studies and is available for beam type structures in plastic field. The extension for creep behaviour will be made later.

Finally specific ratcheting tests were carried out on structures with elastic follow-up. These tests are used to verify the method proposed.

2 ELASTIC FOLLOW-UP EFFECT

2.1 Definition

A definition of elastic follow up was given in ASME code : "When only a small portion of the structure undergoes inelastic strains while the major portion of the structural system behaves in an elastic manner ..... certain areas must be subjected to strain concentrations due to the elastic follow up of the rest of the connected structure".

Mr ROCHE proposed a way to take account of this effect in the field of plasticity (Ref. 1).

The results of an elastic calculation ($\Delta\sigma_o$, $\Delta\varepsilon_o$) give no indication on the characteristics of the loading.

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If this later is locally statically determined $\Delta\sigma o$ will give a good value for the true stress but the strain will be underestimated.

If it is kinematically determined $\Delta\varepsilon o$ will be a good representation of the strain but the stress will be overestimated.

The general case is somewhere between the two previous ones and corresponds to the elastic follow-up situations. This is characterized by the slope θ of AO line in figure 1.

More precisely the elastic follow-up coefficient r is defined as $- \sigma/E = tg\ \theta$.

2.2 Evaluation

An evaluation of the elastic follow-up coefficient, r, on the basis of elastic calculation is proposed in reference 1 in the following form :

$$r = E\,\frac{\Delta\varepsilon - \Delta\varepsilon o}{\Delta\sigma o - \Delta\sigma} = \frac{T}{t} - 1 \qquad (1)$$

with $T = \dfrac{\int_v \Delta\sigma o^2\, dv}{\int_v \dfrac{\Delta\sigma o^2}{t}\, dV}$ characteristic parameter of the structure of volume, V, and of the loading

and $t = \dfrac{\Delta\varepsilon o^e}{\Delta\varepsilon o^P}$ locally defined parameter (see figure 2)

2.3 Example

If we consider a bar with a variable cross-section, S(x) (see figure 3) subjected to cyclic elongation, Δu, and if we apply the hypothesis of a Ramberg-Ostgood law for the deformation :

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + B\,\sigma^n$$

We obtain :

$$T = \frac{\int_v \sigma^2\, dv}{\int_v \dfrac{\sigma^2}{t}\, dv} = \frac{1}{EBF^{n-1}}\ \frac{\int_x \dfrac{1}{S(x)}\, dx}{\int_x \dfrac{1}{S^n(x)}\, dx}$$

$$t = \frac{\Delta\varepsilon o^e}{\Delta\varepsilon o^P} = \frac{S^{n-1}(x)}{EBF^{n-1}} \qquad \text{for section } x$$

$$\text{whence } \frac{T}{t} = \frac{1}{S^{n-1}(x)}\ \frac{\int_x \dfrac{1}{S(x)}\, dx}{\int_x \dfrac{1}{S^n(x)}\, dx} \qquad \text{for section } x \qquad (2)$$

It is thus noted that in this case, elastic follow-up coefficient is dependent only on the geometry.

If the geometry is as shown in figure 3b (two different sections), the preceding equation is written as follows (for the weak section) :

$$\frac{T}{t} = \frac{\dfrac{l_1}{S_1} + \dfrac{l_2}{S_2}}{\dfrac{l_1}{S_1{}^n} + \dfrac{l_2}{S_2{}^n}} \cdot \frac{1}{S_1^{n-1}} \quad \text{and thus } r = \frac{T}{t} - 1 = \frac{\dfrac{S_1}{S_2}\quad 1 - (\dfrac{S_1}{S_2})^{n-1}}{\dfrac{l_1}{l_2} + (\dfrac{S_1}{S_2})^n} \qquad (3)$$

3 ALLOWANCE FOR THE ELASTIC FOLLOW-UP EFFECT IN THE PREVENTION OF RATCHETING

3.1 Reminders on the efficiency diagram

The efficiency diagram is used in the context of an elastic analysis. Elastic stresses are separated in : maximum value of the intensity of primary stresses P during the cycle and secondary stresses range ΔQ.

The secondary ratio is then evaluated : $SR = \Delta Q/P$.

The efficiency diagram (see figure 4) gives the efficiency index V. From this we can deduce an effective primary stress Peff = P/V which must be compared with allowable values in order to check the absence of ratcheting.

In this method the roles of stresses P and ΔQ are very different. In particular, an increase in P of a few MPa is far more severe than the same increase in ΔQ.

3.2 Method proposed for the evaluation of elastic follow-up situations

This method is based on the current efficiency diagram constructed according to null or negligible elastic follow-up tests. It should be noted that, to avoid being unnecessarily conservative, detailed confirmation of this point is required. The maximum value of the primary stresses on the cycle resulting from the structure equilibrium equations will be referred to as P. The elastic stresses related to the strain controlled loadings will be referred to as ΔQ. Among these displacements, those which are not imposed locally are liable to have consequences related to ratcheting which should be specifically evaluated. For this purpose, we will therefore use the efficiency diagram and replace the couple (P, ΔQ) by another couple (P', ΔQ') and state :

$P' = P + Q_p$

$\Delta Q' = \Delta Q - Q_p$

The problem is thus reduced to seeking Q_p for which the following two formulations are proposed :

3.2.1 Formulation 1

The effect of a slight variation ΔQ in the secondary stress range ΔQ is studied (see figure 5). The actual stress will vary between σ and σ + Δσ.

Q_S represents the actual stress in the case where loading is fully strain controlled locally.

We obtain : $\Delta Q \;= \Delta Q \frac{E_t}{E}$

$k_1 = \Delta Q_p/\Delta Q$ must be zero when there is no elastic follow-up effect (in this case $\Delta Q_S = \Delta\sigma$) and equal to 1 when the elastic follow-up effect is ∞.($\Delta Q = \Delta\sigma$).

The most simple formulation is written as follows :

$$k_1 = \frac{\Delta Q_P}{\Delta Q} = \frac{\Delta\sigma - \Delta Q_S}{\Delta Q - \Delta Q_S} = \frac{1}{1 + \frac{E}{r\,E_t}} \qquad (4)$$

E_t is the tangent modulus in the vicinity of σ.

3.2.2 Formulation 2

The formulation of coefficient K_1 is local. In fact, it incorporates the value of the tangent modulus in the vicinity of the point considered. It is possible to integrate the previous formula.

$$k_2 = \frac{Q_P}{\Delta Q} = \frac{1}{\Delta Q}\;{}_{o}^{Q_P}\,\Delta Q_P$$

In the case where the material behaviour curve is written in the following form :

$$\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{1/m}$$

The previous formula becomes :

$$k_2 = \frac{Q_P}{\Delta Q} = \frac{r}{r+1} \cdot \frac{\sigma}{\Delta Q} \qquad (5)$$

3.2.3 Extension to the case where primary stresses are null

In the hypothesis where the load controlled stresses are null, it is still possible to calculate Q_P if there is elastic follow-up effect.

In this case, the efficiency diagram is applied with the couple $\{Q_P, \Delta Q - Q_P\}$ and we deduce, from that, an effective primary stress : $Peff = Q_P/V$.

4 EXPERIMENTAL PROGRAMME

To study the influence of the elastic follow-up effect on ratcheting, tests were carried out on structures in which the elastic follow-up effect is encountered.

The specimens tested were three bar specimens inspired by the UGA specimens (ref. 2).

An elastic follow up effect is created on the cold bars by varying their cross-sections (see figure 6).

Two different test specimens were made corresponding to two different thickness values, e_2 (e_2 = 7 mm and e_2 = 9 mm) with e_1 remaining equal to 6 mm).

The elastic follow-up effect coefficient is calculated using formula (2) and we obtain the following values for the two test specimens respectively : r = 0.44 and r = 1.06.

The test specimens are subjected to constant primary loading. The secondary loading is obtained by cyclic heating of the central bar. Thermocouples are fitted along the hot bar and also along the cold bars. In this way, it is possible to calculate the mean temperature of the hot bar, on the one hand, and to check that the temperature on the cold bars never exceeds 50°C, on the other hand. The temperature of the cold bars is controlled by air cooling throughout the duration of the tests.

Finally, strain gauges are fixed on the cold bars at the minimum cross-section, on the one hand, and on the thick section, on the other.

The tests are carried on until the cycles are stabilized (45 cycles) and final strains of 1.2 % for the first test specimen and 2.2 % for the second test specimen are recorded. The test results are used both to check the ratcheting rule and also to position the points in the efficiency diagram.

The strains are measured on the cold bars and it is also on these bars that the elastic follow-up effect was created. Analysis is thus performed by considering the characteristics of these bars. In particular, the behaviour stress-strain curves are taken at 50°C (maximum temperature of the cold bars).

Using the formula (5), we evaluate k_2 and thus Q_P from which, we deduce :

$SR = (\Delta Q - Q_P)/(P + Q_P)$

The diagram then allows us to obtain v and thus Peff = P/V.

The RCC-MR indicates that a limit value for Peff of 1.2 Sm guarantees a saturation of less than 0.45 %.

We obtain :

Test	P_{MPa}	ΔQ_{MPa}	QP_{MPa}	SR	V	$Peff_{MPa}$
1	103	477	106.9	1.76	0.7	299.9
2	111	533	206.8	1.03	0.84	378.3

These two values for Peff are far greater than 1.2 Sm which is consistent with the local measurements of saturation strains which greatly exceed 0.45 %. The method thus allows the experimental results to be predicted locally.

Another way of interpreting the experimental results consists in using the tests to place them in the efficiency diagram.

It should be remembered that this diagram is built up on the basis of a great number of ratcheting tests.

Knowing the loadings imposed on the structure, we deduce values P and ΔQ.

On the basis of saturation strain that was obtained experimentally, we evaluate an effective primary stress. This is the stress which, when applied alone and for the same test duration, will give the same final strain.

We can then place the point in a plane $(\frac{\Delta Q}{P}, \frac{P}{Peff})$.

The efficiency diagram is the minimum envelope curve containing all the experimental points.

Using our experimental results with the proposed method in order to make allowance for the elastic follow-up effect, we obtain :

Test	Peff exp. MPA	P/Peff exp.	SR
1	295	0.71	1.76
2	321	0.99	0.84

Experimental Peff is determined on the basis of the monotonous curve at 50°C.

We observe that these two points are located above the efficiency diagram.

This shows that the utilization of the efficiency diagram is correct even for tests on structures in which a considerable elastic follow-up effect is encountered.

Other ratcheting tests were used to verify this approach (ref. 3).

The device used is close to a three bar system : the test samples are twin tubes (two concentric tubes) subjected to constant primary loading with the secondary loading being obtained by cyclic heating of the outer tube. The inner tube is cooled throughout the duration of the test (see figure 8). The strain gages allow continuous monitoring of the strains of the cold tubes at both their narrow and thick sections.

On account of their geometry (a narrow section and a thick section), these test specimens show an elastic follow-up effect.

By using equations (2) we can evaluate a coefficient r = 0.354.

Various combinations of primary and secondary loadings were tested. All the results are given in reference 3.

The tests are of interest to check the proposed method as they correspond to various values of the secondary ratio, SR. With no correction due to the elastic follow-up effect SR varies between 1.58 and 4.73.

Among the tests described in reference 3, we use only those that are performed without overloading of the primary stress. The tests are processed in the same way as for the previous tests. Figure 9 shows their positioning in the efficiency diagram.

It is thus noted that, thanks to proposed method, the tests in which an elastic follow-up effect is encountered are correctly positioned in the efficiency diagram.

5 CONCLUSIONS

The checking of the ratcheting rule in RCC-MR is based on the efficiency diagram.

This method is extremely simple to use once we are able to separate the stresses into primary stresses and secondary stresses.

If there is an elastic follow-up effect, the partition of stresses may be not obvious.

A method is proposed allowing the RCC-MR efficiency diagram to be used with input data taking this specific situation into account by using a quantified definition of the elastic follow-up effect based on an elastic calculation followed by the estimation of stress redistribution by plasticity.

This method can be extended to the case of structures for which load controlled stresses are null.

Experimental results (ratcheting on structures with elastic follow-up) are used to verify this approach.

It is noted that results are satisfactory.

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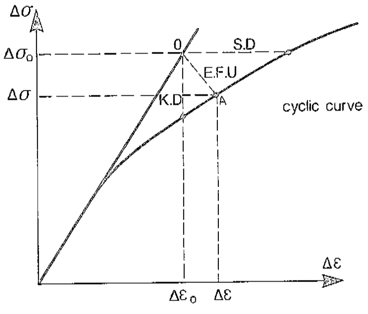


S.D. : statically determined (primary)
K.D. : kinematically determined (secondary)
E.F.U. : elastic follow up (general case)

FIGURE 1

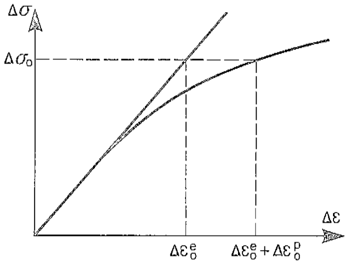


FIGURE 2

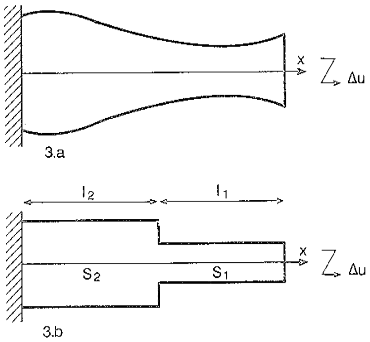


FIGURE 3

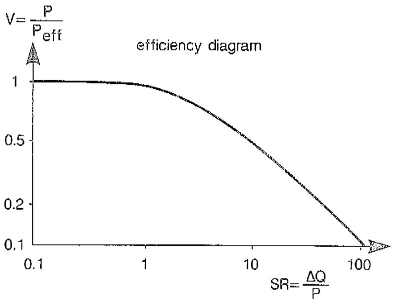


FIGURE 4

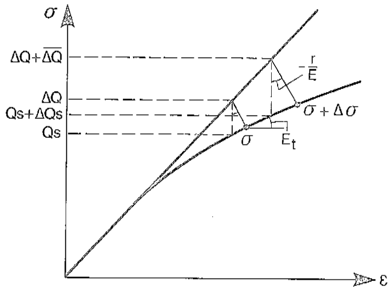


FIGURE 5

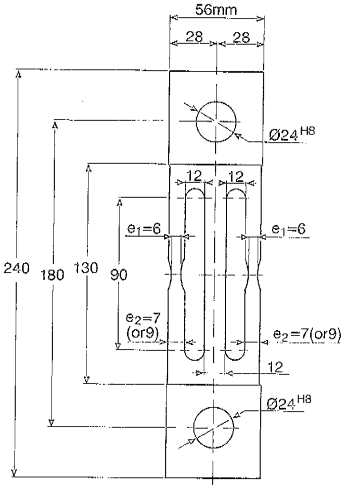
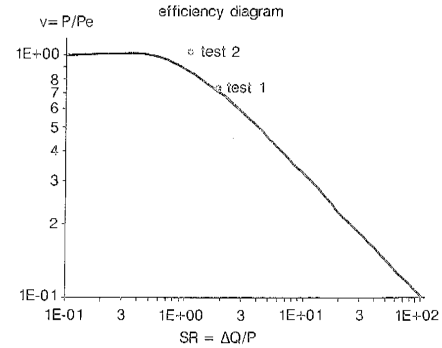


FIGURE 6



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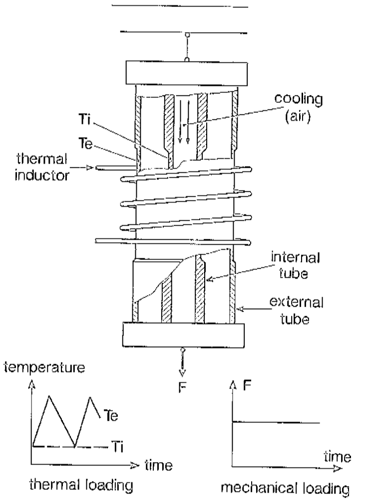

FIGURE 8

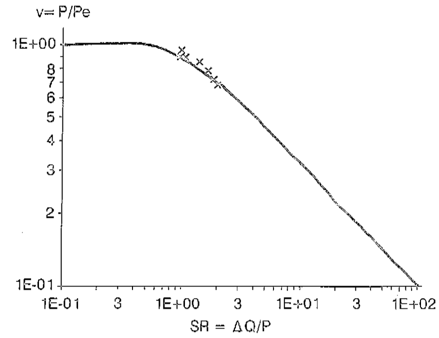

FIGURE 9