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Elastoplastic Buckling Analysis of LMFBR Shell Structures: a Synthesis

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ABSTRACT

The analysis of the elastoplastic buckling of LMFBR shell structures is a difficult challenge that requires reliable, flexible, and efficient numerical methods. Some important issues are reviewed and discussed; namely, shell finite element models, integration of the constitutive equations, solution procedures for the buckling and postbuckling behavior, modelisation of the initial defects.

1. INTRODUCTION

The design LMFBR slender vessels involves complex and possibly stiffened shell structures that are submitted to seismic pressure loads and to large temperature gradients associated with the variation of the free level. The design requires an extended buckling analysis which accounts for geometrical and material nonlinearities as well as the influence of initial imperfections.

The purpose of this paper is the description of numerical methods that have been developped in the past few years in NOVATOME within the computer code NOVNL for the elastoplastic collapse and buckling analysis of LMFBR structures.

The first issue is the definition of shell finite elements able to sustain large temperature gradients and to model slightly stiffened shells. Moreover, the introduction of initial defects usually yields large displacements while the strains remain quite small up to buckling. For a general three-dimensional geometry, the biquadratic degenerated shell element with heterosis shape functions (1) gives a good compromise for most applications. When the geometry is axisymmetric even with non symmetric defects, the best choice consists in a two-node thin shell element expanded in Fourier series (2).

The next issue is an efficient algorithm for the integration of the constitutive equations. Up to buckling, the condition of radial loading is usually satisfied locally and large increments may be employed with accuracy (3). In the postbuckling behavior, the rotations of plastic strains and stress redistribution are more likely so that small increments have to be used. On the other hand, the rapid evolution of modal shapes also involves small steps in the initial postbuckling behavior, even with an elastic material.

Another issue consists in the solution procedures for the quasi-static buckling analysis. For the computation of the fundamental path, the arc-length method proposed by Riks and Wempner (4) can be combined with an incremental bifurcation analysis which allows the increment size to be defined in the load-displacement space (5). Possible bifurcation from the fundamental path can be investigated into an orthogonal subspace with different models of the elastoplastic behavior (differential or finite theory of plasticity). For multiple branching points, the initial postbuckling analysis is based upon an energy criterion that minimizes the work of internal forces for a given amplitude of the perturbation (6). This analysis employs a reduced basis technique with an automatic update of the initial modal basis.

The last important issue considered is the definition of the initial defects. In most cases, initial imperfections are not known except for their global order of magnitude that can be obtained from fabrication tolerances. For conservative design, the most critical defect should be assumed with the maximum allowable magnitude. This problem will be briefly investigated before concluding for future researches.

2. SHELL FINITE ELEMENT MODELS

A large number of papers has appeared in the litterature on shell finite elements (7), but we only want to emphasize some considerations that justified our choice. Depending on the geometry, two kinds of shell finite elements have been developed.

2.1. General shell element

The static buckling analysis of highly non linear structures requires implicit algorithms for wich high order elements are more efficient. This is specially true to catch the rapid evolution of membrane stresses that appear for large temperature gradients in the lamina, or in the buckling of slightly stiffened shell structures. On the other hand, cubic and higher order elements are difficult to use in practical applications and also exhibit a great sensitivity to the position of intermediate nodes. From these considerations, quadratic elements are the most suitable.

The usual flat shell elements do not have any coupling between bending and stretching at the element level, which reduces their rate of convergence with respect to curve elements. Moreover, for large displacement analysis, the update of these flat elements can be only partial and requires fine meshes. Some improvement is attained by using Marguerres's quadratic elements (8). However, the approximation of the geometry may introduce some gaps that destroy the accurate representation of the rigid body modes of the whole structure. These problems disappear by using isoparametric degenerated shell elements.

The normal integration rule on the lamina (3 x 3) produces shear locking in the thin-shell limit with the biquadratic degenerated shell elements. For curved elements, membrane locking may also occur because of a poor representation of the possible inextensional bending and torsion modes. The selective/reduced integration improves this situation but is very expensive in computer time (9). The uniform reduced integration (2 x 2) yields simple elements and is very attractive.

The serendipity element with reduced integration does not have any mechanism (rank deficiency of the stiffness matrix) in an assemblage of two or more elements. However, shear locking still appears, even with a rectangular mesh, in the thin-shell limit, which reduces the allowable slenderness of the element. This drawback is avoided with the heterosis element, provided that the mesh is regular enough, but at the expense of two transmissible mechanisms. Although these mechanisms are usually fixed by the boundary conditions, they may destroy the solution when large rotations and stains occur (10).

The Lagrange element satisfies the patch test in the thin-shell limit, bul also has five transmissible mechanisms. These mechanisms could be stabilized by the so-called gamma-method (11) with additional computations. However, the choice of some internal parameters requires further researches, specially for highly non linear problems.

2.2. Quasi-axisymmetric geometry

By contrast with general shells, the case of quasi-axisymmetric geometry can be solved by an optimum shell element provided that the rotations remain moderate, which is usually true up to the initial postbuckling behavior. This is a two-node thin shell element expanded in Fourier series with one-point quadrature on the meridional line (2). It can be viewed as a generalization to Fourier expansion of Hughes' 2-D isoparametric element (12). The meridional rotation is relaxed by means of a penalty formulation and each node presents four degrees of freedom for each Fourier component.

The geometry has to be quasi-axisymmetric in the sense that an inextensional defect developed in Fourier series can be taken into account by a generalization of Marguerre's approach. The integration is performed on the perfect axisymmetric geometry. For an homogeneous linear material, the integration along the circumferencial direction could be performed analytically at the expense of the simplicity of the computations. However, numerical integration was preferred for its versatility, and also because vectorization of the computer code is easily implemented for such an element with a possible large number of degrees of freedom.

In comparison with an equivalent 3-D mesh, this approach usually reduces the computer time by a factor five. Moreover, the bifurcation analysis on an orthogonal subspace is greatly simplified within this approach. On the other hand, the choice of Fourier components to be retained in the analysis is crucial, depending on the initial defect and the possible harmonics of the loading system. For example, a large initial defect on a

Fourier component, N, requires the basis (0,N,2N) for an elastic material and the basis (0,N,2N,3N) for an elasto-plastic material, at least.

3. INTEGRATION OF THE CONSTITUTIVE EQUATIONS

In the prebuckling range, an efficient computation algorithm should be able to deal with large load increments, this requirement being attained without alteration of the solution accuracy and without significant increase of the number of iterations per step. The main difficulties, in case of elasto-plastic material, arise from the irreversibility of the plastic strains and from the discontinuous change of the material properties, possibly accentuated by temperature dependance.

The algorithm implemented by Nyssen (3) is based on an Euler forward multistep integration scheme coupled with a consistent correction to bring the stresses back to the yield surface at each subincrement. Within a Newton - Raphson iterative procedure, the assumption of incremental reversibility is advocated so that a point which deforms plastically during one increment is assumed to unload plastically until the plastic work done becomes again equal to its value at the beginning of the considered increment. This assumption allows to cumulate the corrections of stresses associated with the iterative corrections of strains, instead of the computation of the stress increment by integration on the total strain increment at each iteration. The number of subincrements quickly decreases as the iterations converge, and this procedure being more consistent with the Newton - Raphson method maintains good convergence properties, even with combined strong nonlinearities. Moreover, the solution becomes also reasonably independent of the particular computing strategy adopted for the increment.

Creep is also an essential parameter of LMFBR components operating under high temperature. Specially, the use of steels for short times with respect to their creep capacity requires a numerical model which integrates the primary creep in a reliable way. The implemented algorithms described in (13) allow plastic redistributions to be induced by creep. Possible buckling under creep may also be investigated.

4. SOLUTION PROCEDURES

The quasi-static buckling analysis of LMFBR structures requires efficient and reliable algorithms with the ability to cope with strong geometrical and material nonlinearities. Indeed, the amplification of initial geometrical defects usually yields sharp increases of plastic strains when local plastic hinges occur. The finite element mesh has to be fine enough, and the condition of convergence of the Euler bifurcation load with the mesh refinement may be not sufficient.

4.1. Fundamental path

An efficient procedure for computing the fundamental path consists in employing the modified arc-length method of Riks-Wempner in conjunction with an incremental bifurcation analysis (5). By giving a reasonably accurate estimation of the buckling load, the incremental bifurcation analysis allows optimum load increments to be defined so that the buckling load is attained within a few steps. By contrast with the usual constant arc-length method, this approach takes into account possible bifurcations and is able to cope with rapid evolutions of the load-displacement curve. When thermal loads are applied, this approach does not undergo the drawbacks of the constant arc-length method or of the Bergan's parameter. Moreover, the knowledge of the evolution of modal shapes yields a better understanding of the structural behavior and gives an efficient tool for design improvements.

The formulation of the incremental bifurcation analysis gives a generalized eigenvalue problem with two matrices. The first matrix corresponds to the tangent stiffness matrix associated with the current equilibrium state. The second matrix is the stability matrix defined as the first order term of the expansion of the tangent stiffness matrix along the current tangent approximation of the fundamental path. The stability matrix may involve up to four contributions; namely, the geometric stiffness matrix defined as in the Euler bifurcation problem, the load stiffness matrix associated with a lively load such as a fluid pressure (14), the so-called material stiffness matrix which accounts for the change of the tangent modulus in the case of an elastoplastic material (15), and finally, the linear displacement stiffness matrix that takes into account the change of geometry along the current tangent approximation (16). Nevertheless, this last contribution should be neglected in the case of slender structures with large initial defects and undergoing large displacements.

For the solution of the eigenvalue problem, the tangent stiffness matrix is already triangularized and the algorithm does not require the stability matrix to be assembled. Moreover, the number of iterations may be reduced by employing the first few eigenmodes of the preceeding increment as starting vectors. A suitable algorithm is given by the Lanczos method (17), or even better, the block Lanczos method (18).

4.2. Orthogonal bifurcation

The technique of the modal defect and the possible symmetries of the geometry and the loading system make often possible to model a part of the whole structure for the computation of the fundamental path. Then, it is necessary to check the stability of the fundamental path with respect to possible orthogonal bifurcation. In the case of a general three-dimensional geometry, this analysis is usually restricted to a change of boundary conditions from symmetry to antisymmetry. In the case of a quasi-axisymmetric geometry, the range of application of the orthogonal bifurcation analysis is much more important, specially with the use of the two-dimensional shell element expanded in Fourier series.

For example, an initial defect on the Fourier component, N, with an axisymmetric loading system may require a bifurcation analysis on a Fourier mode, M, with the orthogonal basis (M, N +/- M, 2N +/- M, ...) (19). Out of phase bifurcation may also be investigated by using cosine and sine modes simultaneously. Another example is given by a loading system in mode 1 (fundamental path : 0, 1) and a bifurcation on the basis (N +/- 1, N +/- 2, N +/- 3, ...). The advantage of this approach is the reduction of the size of the eigenvalue problem associated with each bifurcation analysis, which reduces the computer time and also improves the conditionning of the eigenvalue problem.

Along the fundamental path, the classical J2 flow theory is employed to model the plastic behavior. However, for the bifurcation analysis, finite theories of plasticity usually yield better agreement with experiments, specially for highly plastic buckling. The reason is that the finite theory of plasticity is less sensitive to small initial imperfections than the differential theory, and anticipates, in some way, the occurence of such imperfections. While more sophisticated theories may be employed (20), the classical finite theory was implemented in which the Young modulus and the Poisson ratio are replaced by their secant values in the tangent Hooke matrix.

4.3. Postbuckling analysis

In the postbuckling range, the modified Riks-Wempner method can be employed, possibly with an adaptive step size depending on the number of iterations required to get convergence (21), provided that only simple limit or branching points are encountered. For a simple bifurcation point, the arc-length method is reinitialized by choosing a perturbation of displacements parallel to the eigenmode with a suitable amplitude.

For multiple branching points, the problem is much more complex since, depending on the initial perturbation, several postbuckling paths can be obtained. Moreover, the elastic criterion of the minimum of the total potential energy has to be extended to non-conservative systems, such as elastoplastic materials. Although a quasi-static approach is employed, the mass matrix is the only physical norm that can be used for defining the perturbation amplitude (22). Then, considering a small but finite perturbation, the critical postbuckling path is defined by the perturbation that minimizes the work of internal forces.

In the diagram of the load factor as a function of the work done by the unit applied load system, the critical postbuckling path corresponds to the closest equilibrium path with respect to the fundamental path. In fact, when the postbuckling path is unstable, the structure behaves dynamically and the actual trajectory in this diagram may be very complicate and is usually not known. It depends on the way the loads are applied during buckling, and generally on the complete system. However, the critical postbuckling path as previously defined will maximize the kinetic energy whatever the actual trajectory is. In this sense, it is the most dramatic behavior that the structure could undergo.

If we assume that it could be possible to control the load system in order to follow this critical path, the structure would behave statically, while it is not the case for another postbuckling paths.

The algorithm used for solving the complex minimum problem is described in (6). It employs Crisfield's method (23) to get convergence in the postbuckling range for a given perturbation : the additional constraint consist in keeping constant the norm of the incremental displacement with respect to the mass matrix. Then, an automatic procedure allows the critical perturbation to be defined on the basis of the energy criterion. Moreover, this algorithm uses a reduced basis technique in order to save computer time. The initial basis contains the first few vibration eigenmodes computed at the reference equilibrium state from which successive perturbations are applied. This basis also contains the deformation mode tangent to the fundamental path. In order to cope with the rapid change of modal shapes in the postbuckling range, this modal basis is continuously updated by the addition of the correction of displacements associated with the residual forces in the complete system.

5. GEOMETRICAL DEFECTS

In order to assess the influence of geometrical defects on the buckling load, an important parameter has been defined that characterizes the structural behavior; namely, the quotient of the Euler bifurcation load over the plastic bifurcation load for the perfect structure. An accurate prediction of the plastic bifurcation load requires an incremental analysis because of the nonlinearity of the material law. However, a good estimation of this plastic bifurcation load can simply be obtained by adding the material stiffness matrix to the stability matrix of the Euler bifurcation problem (15).

If this parameter is much larger than unity, the structure is stiff and probably not very sensitive to initial defects. The plastic bifurcation load yields a good approximation of the actual buckling load.

If the parameter is close to unity, the structure is soft and usually sensitive to the initial defects. The choice of the geometrical imperfection to be included in the finite element model for a complete nonlinear analysis may be very difficult, specially in the case of multiple bifurcation loads. The current practice employs the so-called modal defect parallel to the lowest bifurcation eigenmode of the perfect structure with the maximum allowable magnitude. This practive should be conservative in the sense that more realistic defects, manufacturing imperfections, have been shown less critical than the modal defect, experimentally (24).

However, the modal defect technique may be not conservative for several cases and further researches are necessary for defining the critical defect. For example, when the lowest bifurcation eigenmode yields a stable postbuckling behavior, the modal defect does not reduce the buckling load significantly, while possible higher modes are unstable and imperfection sensitive. This is the case for some large self-stiffened vessels where the lowest bifurcation eigenmode is associated with a local deformation pattern and higher modes yield global instability. Moreover, for multiple branching points, the critical defect usually corresponds to some combination of the lowest bifurcation eigenmodes. Unfortunately, simple Euler bifurcation analysis or initial plastic bifurcation analysis performed on imperfect structures are unable to define such a combination accurately, specially for large defects of about twice the thickness. On the other hand, it would be prohibitive in computer time to try several combinations within a complete nonlinear analysis.

For small imperfections, asymptotic theories describe the critical defect as parallel to the initial postbuckling behavior of the actual structure (25,26,27). This fact combined with the energy criterion defined beforehand for the critical postbuckling path should enable the definition of the critical defect, at least for sufficiently small amplitudes. Preliminary results show this assertion is verified if the amplitude is measured with respect to the mass matrix.

Another approach, appropriate for elastic and possibly non linear elastic material, consists in applying finite perturbations along the fundamental path, but far away from the buckling point of the perfect structure (28). In this case, the critical defect could be defined by the perturbation that requires the lowest energy increase to get a perturbated configuration of equilibrium, if one exists, at the same load level (29).

6. CONCLUSIONS

Several aspects of the quasi-static buckling analysis of elastoplastic thin-shell structures have been discussed and the development of numerical methods have been presented, that make the computer code NOVNL very efficient for solving these problems, based upon an intensive industrial practice and experimental correlations : namely, the implementation of suitable shell finite elements, the development of reliable algorithms for the integration of the constitutive equations, the working out of efficient solution procedures for the buckling and postbuckling analysis.

Nevertheless, several issues are currently studied, mainly concerning the initial imperfections. In addition to the critical defect for geometric imperfections, other kinds of imperfections need to be considered such as material heterogeneity and residual stresses, and specially imperfect knowledge of the loading system. This is the most important issue for the next few years, since the loading system comes from complex fluid-structure interactions during seismic event.

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