EFFECT OF CRACK FORMATION PROCESS AND TENSION STIFFENING ON THERMAL STRESSES RELAXATION IN REINFORCED CONCRETE CONTAINMENTS

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ABSTRACT

Thermal stresses can be critical for the design of reinforced concrete thick containments; this can occur for consideration of steady state distributions of temperature across the section, in normal operating conditions, or of transient nonlinear distributions in upset conditions.
Serviceability criteria govern the design; nevertheless, such effects can frequently cause stresses above the permissible values established for normal structures, so that the assumption of a cracked reinforced concrete structure is taken into account; this considerably reduces the stiffness.
In such an analysis, the most simple assumptions can lead to overestimate the stress relaxation. This can occur if the reduced stiffness of the cracked section is simply adopted for the stiffness of the members; such an assumption is safe for the calculation of maximum deflections under loads, but is unsafe for the calculation of the maximum stresses in the materials under thermal effects. In fact, crack formation is a gradual process, and not all theoretical possible cracks are formed under a given thermal action.
Moreover, even when all the theoretical cracks are formed, the stiffness is higher than that of cracked section, for a sensible cooperation of concrete in tension between cracks (tension stiffening).
A satisfactory reliability of containment structures under thermal actions can be achieved by a design performed on the following principles:

- assumption of the cumulative distribution of crack formation as a linear distribution between two limits of tensile stress in concrete (according to experimental evidence);
- calculation of the average stiffness of members assuming the formation of the only cracks corresponding to the probable formation at the given stress level;
- consideration of the stiffening effect of concrete in tension between cracks, by means of a simplified empirical procedure;
- verification at the limit-state of crack width;
- verification at microcracking of concrete in compression.

1. Introduction

Consideration of the stress relaxation due to cracking of concrete is frequently necessary in the analysis of reinforced concrete containments or walls subjected to thermal actions in normal operating conditions or in upset conditions. Thermal effects are in fact seldom critical in incidental conditions, where the verification criterion is the ultimate-limit-state, and the possibility of developing large plastic strains diminishes the effect of imposed deformations such as the thermal strains.

Quite critical can be, on the contrary, the consideration of thermal effects (combined with pressures and service loads) in operating or upset conditions, where serviceability limit-states are governing; it can frequently happen that.under rather limited temperature differences (of the order of 30°C) the structure cannot meet the requirements imposed by codes or specifications based on the criterion of permissible stresses.

On the other hand, the verification at the ultimate-limit-state can in no way be considered sufficient, because a satisfactory behaviour of the structure in normal conditions has to be assured in addition to the overall safety in extreme conditions. It follows that cracking of concrete structures is frequently taken into account in the analysis in normal operating conditions, as it allows the consideration of a reduced stiffness of the wall, and therefore of reduced stresses.

Such a procedure is acceptable only if the tension stiffening of concrete in tension is taken into consideration, and suitably safe statistical account is taken of the process of crack formation; for both problems solutions have been suggested by Macchi in references /1/ and /4/ ; the here suggested procedure of verification under thermal actions has been first applied by the authors to the P.E.C. shield in 1977.

2. Deficiencies of cracked-section simple analyses

The assumption in the analysis of the reduced stiffness of the cracked section, that is with the "nacked" reinforcement in tension (as shown in Fig.1.1) seems to the authors to be twice on the unsafe side, having regard to the behaviour of the reinforced concrete members.

The first reason of inaccuracy of such an assumption is the fact of neglecting the considerable stiffening effect produced by the concrete in tension between adjacent cracks.

As shown in Fig.2, tension is transmitted by the reinforcement to the surrounding concrete through the bond stresses. This effect causes a considerable reduction of the mean stress in steel and therefore a reduction in curvatures, in comparison with the "nacked state", beyond the cracking moment M_c (Fig.3). Neglecting this effect is on the unsafe side, because the relaxation effect of cracking is overestimated.

The error is particularly important in the case of "small" percentages of reinforcement: this is often the case of containments or shielding walls, because of the large thickness of such structures.

Even the model taking into account tension stiffening, and showed in Fig.1.2, is however unsafe in most cases; in fact, the formation of all the theoretical cracks (stabilized cracking) is an ideal limit-situation to which the structure tends for very high tensile stresses (and therefore near to the ultimate situation), or for a large number of repetitions of the loads, or after a long permanence of high stresses. The process of crack formation observed during the tests /4/ shows that under service conditions only a small part of the total number of cracks is normally present. This fact is shown in Fig.4, where the process of crack formation is plotted in function of the tensile stress in concrete at the bottom edge of the section.

Consequently, it seems necessary to take into account in the analysis only the reduction in stiffness due to the fraction of cracks probably formed at the stress level under consideration (scheme of Fig.1.3).

3. Suggested procedure of analysis

3.1 Principles of the proposal

A first approach to the solution of the above exposed problem can be proposed by taking advantage of the existing knowledge in the fields of the concerned phenomena. The proposed procedure is the following:

a) determination of the law of variation of stiffness for increasing moments, in stabilized cracking, account taken of tension stiffening (Fig.6);

b) assumption of a conventional cumulative distribution function of crack formation (Fig.5) and consequent correction of the stiffness function (Fig.7);

c) analysis of the structure under thermal effects and other actions, assuming for each element the mean stiffness corresponding to the reached stress level according to the function determined in b); iteration procedure;

d) verification of the limit-state of crack width (Figs.8 and 9);

e) verification of the limit-state of microcracking of concrete in compression.

3.2 Stiffness in stabilized cracking, taking into account tension stiffening

A refined idealization of tension stiffening is not yet available, but the experimental data allow the practical use of very simple conventional models. One of them is suggested in /4/ and shown in Fig.2; a constant distribution of tension stresses σ_t in the concrete fibers in the tensile part of the section, being:

$$\sigma_t = 0{,}05\ f_{cm} \qquad (1)$$

(The necessity of a safe assumption in the determination of the mean stiffness requires the reference to the mean value f_{cm} of the compressive strength of concrete or, better, to the upper characteristic strength).

More elaborated models give a false idea of higher accuracy: the results change very little with the various models, and the main feature of the phenomenon remains the randomness. Obvious equations of equilibrium (under moment M and axial load N) and compatibility of deformations (in linear field) applied to the scheme of Fig.2 yield the conventional depth $\underline{x}$ of neutral axis and the stress distribution; the obtained values of $\underline{x}$ and of the maximum concrete stress σ_c provide the conventional stiffness:

$$(EI)_2 = E_c M\ x / \sigma_c \qquad (2)$$

E_c being the modulus of elasticity related to f_{cm}.

Fig.6 shows an example of the laws of variation, in function of M and N, of the ratio:

$$(EI)_2/(EI)_0$$

between the conventional stiffness $(EI)_2$ and the stiffness $(EI)_0$ of the gross section of concrete.

3.3 Mean stiffness taking account of the process of crack formation

Several test results of the type shown in Fig.4 have been collected in order to suggest a cumulative distribution function of crack formation to be used in design (Fig.5). The assumed variable is the tensile stress σ_t at the edge of the section, calculated with the concrete gross section. The process is depending on many factors, the most important seem to be the percentage of reinforcement ρ and the concrete strength f_{cm}.

For small values of ρ (less than 1%), a relevant case for our problem, the linear distribution functions of Fig.5 can reasonably be adopted. The linear law is suggested by the experimental data; it is justified, lack of better statistical data, by the fact that its use is done for analyses in service conditions, that is in the mean and lower part of the distribution.

The suggested lines are obtained by shifting the mean lines to the right hand, in order to give lower fractiles, and consequently safe low values of the considered number of cracks. With reference to Fig.1.3, it is suggested to perform the structural analysis, at a given stress level (defined by σ_t or by the corresponding M) with the "design stiffness":

$$(EI)_3 = p\,(EI)_2 + (1-p)\,(EI)_0 \qquad (3)$$

taking in this way into consideration a mean stiffness of a member in which only a fraction p of the total number of cracks is formed.
An example of application is given in Fig.7, where this transformation has been applied to the curve of Fig.6.

4.1 Limit state of crack width

The results of the analysis of the structure (performed by an iteration procedure) provide values of load effects to be submitted to verification at the serviceability limit-state, (items d and e of 3.1).
The design is normally governed by the verification to the limit-state of crack width, that is by the control of the characteristic crack width (according to the CEB Model Code /6/):

$$W_k = 1.7\, s_{rm}\, \varepsilon_{sm} \qquad (4)$$

s_{rm} = average distance between cracks
ε_{sm} = mean elongation of steel
The crack width depends on the diameter, percentage, spacing, bond of the bars, on the effective embedment and the cover, on the stress in steel and the duration of loads. All the above variables are taken into account by the CEB formulas. The admissible W_k, on the other hand, has to be chosen according to the performances required by the structure, the environment, the sensitivity of the reinforcement to corrosion and the probable frequence or duration of the considered loading case.
If no concern of radiation penetration exists, the following values of permissible W_k can be suggested (for reinforced concrete):
W_k = 0,1 mm in severe conditions of exposure;
W_k = 0,2 mm under long-term or repeated actions;
W_k = 0,4 mm under rare and short-term actions.
Examples of relationships between steel stress and crack width W_k are given in Fig.9 (for a short time loading) and in Fig.8 (for a long duration or frequent loading).

4.2 Limit-state of microcracking of concrete in compression

In the case under consideration, the verification at the limit state of crack width can sometimes be insufficient.

High compression stresses in concrete can produce microcracking and excessive creep strains which, though being nearly ineffective on the safety at the ultimate limit state, can be inadmissible in service conditions for the long-term behaviour. Similarly to the case of prestressed concrete structures under initial maximum prestress, it is advisable to check that the compression stress is below the limit of microcracking:

$$\sigma_c \leqslant 0{,}6\ f_{ck} \qquad (5)$$

where f_{ck} is the characteristic strength.

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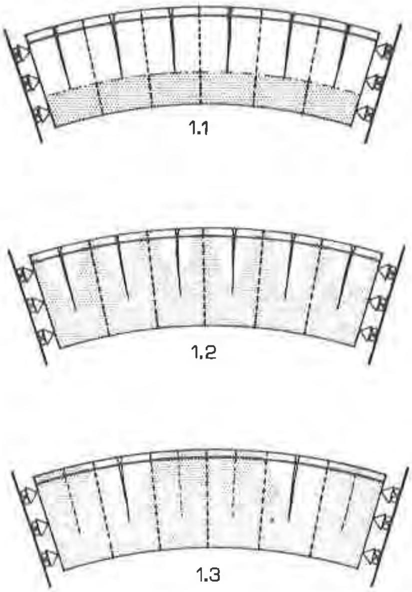

Fig. 1 Models of a cracked r.c. wall under temperature actions and other loads.
1.1. Stiffness of the cracked section, with nacked reinforcement.
1.2. Cracked member with tension stiffening.
1.3. The same, accounting for the process of crack formation.

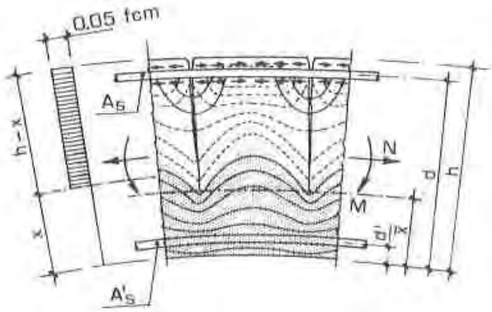

Fig. 2 Assumed model for tension stiffening

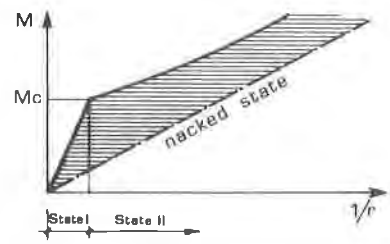

Fig. 3 Moment-curvature taking into account cracking and tension stiffening.

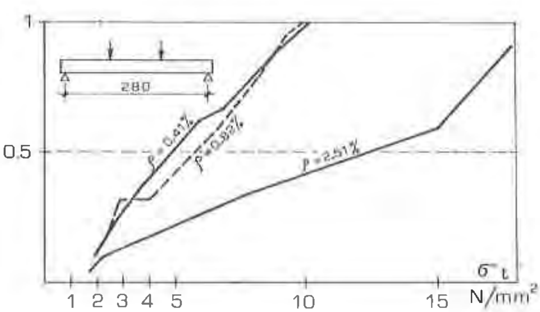

Fig. 4 Experimentally observed process of crack formation.

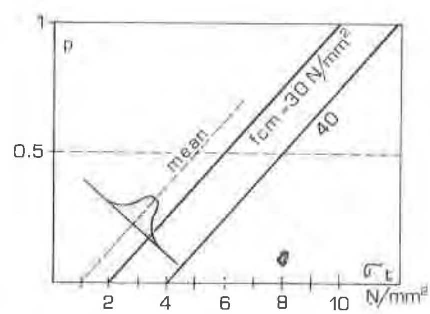

Fig. 5 Idealized cumulative distribution fuctions for crack formation

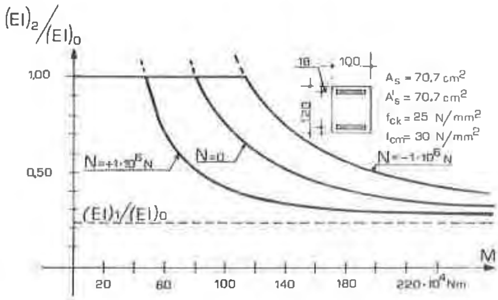

Fig. 6 Variation of stiffness $(EI)_2$ with moment and axial load. Example.

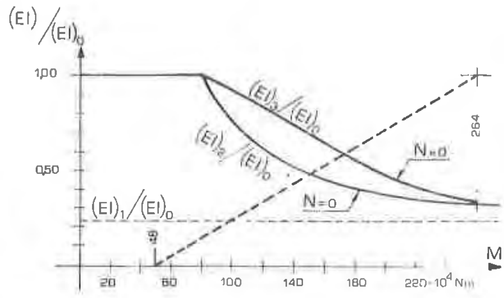

Fig. 7 Variation of stiffness $(EI)_3$ taking account of the crack formation (distribution function in dotted line). Example.

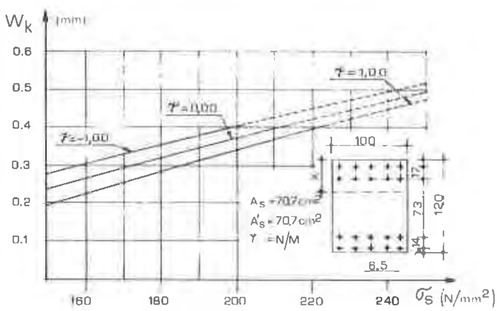

Fig. 8 Crack width in function of steel stress and axial load. Example for long-term actions.

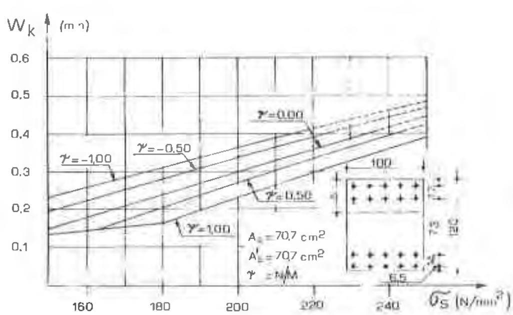

Fig. 9 Crack width in function of steel stress and axial load. Example for short-term actions.