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SAFETY FACTORS AND PROBABILITY IN LIMIT STATE DESIGN OF STRUCTURES

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ABSTRACT

1.1 Attempts to associate safety factors with probabilities of failure depend upon knowledge of the variability of all the parameters which affect the relationship between the stresses in the structure and the strength of its component materials. Whilst reasonable information can be obtained on the variability of materials there are few statistics available concerning the parameters causing variability of stress. The paper suggests a way round this difficulty by considering a well known type of structure such as a building and associating the design methods and safety factors commonly used with an assessment of the failure rate for buildings. It then becomes possible to deduce the variability of stresses which would be consistent with both this relationship and the known variability of the materials and so to establish a more general relationship between safety factors and probability of failure.

1.2 A reactor containment structure must cater for a whole series of accidents, each of which has a different probability of occurrence and for each of which the consequences of structural failure will differ. It would be desirable to have a design method which could enable safety factors to be chosen so that the combined probability of the accident happening and the structure failing can be matched to the acceptable probability of failure for each accident. This paper suggests a possible method of doing this.

2. INTRODUCTION

2.1 Engineering design has made considerable use of safety factors which are a ratio between best estimates of the load and best estimates of the strength. Satisfactory safety factors have evolved by tradition and more recently by examining the made-up of the factors. The use of such factors will not lead to a failure-proof design, rather it leads to an acceptably low failure rate. The safety factors with which engineers are familiar are generally associated with a high reliability (or low failure risk) and it is the intention here to examine methods whereby safety factors can be adjusted for different reliabilities.

2.2 One particular subject of interest is reactor containments in the form of large structures of pre-stressed concrete to contain vapour and radioactive materials under pressure in certain unlikely accidents. The risk of the containment ever being stressed by a release of vapour internally is very small and the further allowable risk of the containment failing its function can be defined by considering the consequential damage caused by such a failure.

3. LIMIT STATE DESIGN PHILOSOPHY

3.1 This design philosophy is being introduced into more and more structural standards and codes concerned with reinforced and prestressed concrete. The philosophy first defines the condition which the structure must reach to be considered as having failed. In some cases this may be cracking of the concrete in a particular part of the structure, in other cases general extensive cracking and deformation, and in others collapse, but whatever condition is defined as failure it becomes one of the limit states upon which the design is based. The parameters which may cause the structure to reach its limit state are next considered. These fall into two groups:-

a. Parameters which would make the actual strength of the materials in the structure be less than would be inferred from materials tests, such as variations in manufacture and construction, and

b. Parameters which would make the actual stresses sustained by the materials during the operation of the structure be greater than those predicted by the stress analysis. The most important of these are:-

   (i) Loads may be greater than allowed for and prestressing forces may be less.

   (ii) Errors in calculation.

   (iii) Errors in construction such as undersized scantlings, faulty tendon geometry and misplaced reinforcement.

   (iv) Corrosion of tendons and reinforcement.

3.2 The objective of the design criteria is to provide an acceptably low probability that the limit state will be reached in operation. To achieve this the strength of each material inferred from tests is divided by a partial safety factor in order to obtain a value for its design stress, and the applied loads are multiplied by a partial safety factor in order to obtain the design load. The values of these factors are such that, taking account of a. and b. the probability of the actual stresses in the structure exceeding the strength of the materials in the structure is acceptably low. Although the method is basically probabalistic the safety factors in all current design standards and codes are not associated with any quoted values for the probability of failure.

3.3 The design of the structure is satisfactory if the design loads for each limit state produce calculated stresses which do not exceed the design stresses appropriate to the relevant limit state. The stress analysis must take account of the structural mechanics by which loads are carried when the structure is in each of its limit states, ie any cracking of the concrete and any plasticity of the materials which is permitted as part of the limit state and any redundancies which must be removed before failure becomes possible must be taken into account when calculating the stresses at the cross-section which will determine the failure of the structure.

4. PROBABILITIES

4.1 Let

P_1 = Annual probability of a particular combination of loads occurring once.

P_2 = Probability of the stress exceeding the strength of the materials in the structure if the particular loading combination should be applied.

P_A = Acceptable annual probability of failure of the structure.

q = Number of alternative combinations of load which are considered in design and which could cause failure.

4.2 For any particular combination of loads the probability of failure is $(P_1)(P_2)$ and for all the load combinations $P_A \geqslant \sum (P_1)(P_2)$. However, if the failure probability is the same for each load combination then for any one load combination it would be satisfactory if $(P_1)(P_2) = \frac{P_A}{q}$ which is a conservative approximation since some load combinations will produce relatively low stresses and therefore a lower failure probability than allowed for that combination.

This gives $$P_2 = \frac{P_A}{q.P_1} \qquad (1)$$

4.3 The value to be given to P_A and the number of alternative loading cases considered will depend upon the type of structure and will be very strongly influenced by the consequences of failure. P_1 will depend upon the individual annual probabilities and durations of the various loads which make up the particular combination considered and some examples of the calculation of its value are dealt with later. The majority of this paper however concentrates upon relating P_2 to the safety factors used.

5. MATERIALS

5.1 It is reasonable to assume that variations in the strength of the materials follow a normal distribution, the properties of which are given in Ref 2. If x is the variate, the mean value is X, the standard deviation is s and the coefficient of variation is $\frac{s}{X}$, then for a range of values of x from X - ms to X + ms the probability P of x falling outside the range depends upon m. The probability of x being less than X - ms is $\frac{P}{2}$ and the probability of x being greater than X + ms is also $\frac{P}{2}$.

5.2 Fig 1 attached gives the relationship between m and the probability of the variate lying either above or below the range $\pm$ ms from the mean, the values given being taken from Ref 3.

5.3 Materials strength data, however, will be obtained from test specimens which, in the case of concrete, may not be identical with the actual concrete in the structure.

Thus referring to Fig 2 if C = mean strength of test pieces

$$= \frac{1}{n} \sum_1^n C_1$$

$K_1.C$ = mean strength of concrete in structure

c = standard deviation of test pieces

$$= \sqrt{\frac{\sum_1^n (C_1 - C)^2}{n - 1}}$$

c_1 = standard deviation of concrete in structure

v_c = coefficient of variation of test pieces

$$= \frac{c}{C}$$

v_{1c} = coefficient of variation of concrete in structure

$$= \frac{c_1}{K_1.C}$$

Then according to Ref 1

Characteristic strength of test pieces = C - 1.64c

Design Stress = $S_{DC} = \frac{C - 1.64c}{F_C}$

5.4 The properties of the steel reinforcement and tendons in the structure will be obtained from tests on samples, some of which will fail acceptance criteria and be rejected. None of the material represented by the rejected samples will be installed in the structure. Thus if the steel statistics are based upon the samples which pass the acceptance criteria the figures will be directly representative of the steel in the structure.

If S = mean strength of test pieces which pass criteria

s = standard deviation of test pieces which pass criteria

v_s = coefficient of variation of test pieces

$= \frac{s}{S}$

Then Characteristic strength of test pieces $= S - 1.64s$

Design stress $= S_{DS} = \frac{S - 1.64s}{F_S}$

6. STRESSES

6.1 Consider the frequency diagram shown in Fig 3 representing the actual stress at a point in the structure. The calculated stress f is based on the mean value of the most unfavourable load combination and the stress $f(1 + v)$ is calculated in exactly the same way but using the characteristic loads. Because of errors in the mathematical descriptions of both materials behaviour and structural mechanics in the calculations, it is assumed that the mean value of the actual stress due to the mean most unfavourable load will be $K_2.f$. The stress frequency curve must also take account of the items mentioned in 1(b) (i) (iii) and (iv) as well as the fact that when cracking and plasticity have developed the stress is no longer proportional to the applied load. To allow for these variations it is convenient to assume a normal distribution for the stress frequency curve.

6.2 Let

f = Calculated stress at a point in structure due to the mean value of the most unfavourable loading combination.

W = Mean value of most unfavourable loading combination.

$v.W$ = Standard deviation of frequency curve of most unfavourable loading combination.

$K_2.f$ = Mean value of actual stress at the point.

v_1 = Coefficient of variation of frequency curve of actual stress at the point.

Then according to Ref 1

Characteristic load $= W(1 + v)$

Calculated stress due to characteristic load $= f(1 + v)$

Design load = $F_L.W.(1 + v)$
Calculated stress due to design load = $QF_L.f(1 + v)$, where Q allows for stress not being proportional to load after cracking and plasticity have developed.

7. PROBABILITY OF FAILURE

7.1 L S Lawrence shows in the discussion to Ref 4. that the probability of the stresses due to an applied load exceeding the strength of the materials in the structure is given by

$$P_2 = \int_m^{\infty} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}x^2} \, dx \qquad (2)$$

Where the distributions of stress and strength are shown in Fig 4 and

$$x = \frac{(R - R_o) - (L - L_o)}{\sqrt{s_L^2 + s_R^2}}$$

$$m = \frac{R_o - L_o}{\sqrt{s_L^2 + s_R^2}}$$

AND

R_o = Mean value of failure stress of material.

s_R = Standard deviation of frequency curve of failure stress of material, taken as a normal distribution.

L_o = Mean value of stress at a point in structure caused by applied loads.

s_L = Standard deviation of frequency curve of stress at a point in structure, taken as a normal distribution.

L = General value of stress at a point in structure, lying between $-\infty$ and $+\infty$.

R = Value of strength at a point in structure such that $-\infty < R - L < 0$.

P_2 is therefore the probability of the variate exceeding m standard deviations from the mean in a normal distribution and can be found from Fig 1.

7.2 From the relationships given in Figs 2 and 3

$R_o = K_1C$
$L_o = K_2f$

$$s_R = v_{1c}.K_1.C$$
$$s_L = v_1.K_2.f$$

giving

$$m = \frac{K_1C - K_2f}{\sqrt{(v_{1c}.K_1C)^2 + (v_1.K_2.f)^2}} \qquad (3)$$

7.3 If the calculated stress due to the design load is equal to the design stress then

$$S_{DC} = QF_L.f(1 + v) = \frac{C - 1.64C.v_c}{F_C}$$

$$\text{ie } f = \frac{C(1 - 1.64v_c)}{QF_L.F_C(1 + v)}$$

Substituting in (3), putting $F_L.F_C = F_{GC}$ and re-arranging gives

$$m = \frac{F_{GC} - \dfrac{K_2(1 - 1.64v_c)}{QK_1(1 + v)}}{\sqrt{F_{GC}^2 v_{1c}^2 + \left\{\dfrac{K_2(1 - 1.64v_c)}{QK_1(1 + v)}\right\}^2 v_1^2}}$$

7.4 Putting $A = \dfrac{K_2(1 - 1.64v_c)}{QK_1(1 + v)}$

gives

$$m = \frac{\dfrac{F_{GC}}{A} - 1}{\sqrt{\left(\dfrac{F_{GC}.v_{1c}}{A}\right)^2 + v_1^2}} \qquad (4)$$

This may be re-written

$$F_{GC}^2 (m.^2v_{1c}^2 - 1) + 2AF_{GC} + A^2(m^2v_1^2 - 1) = 0$$

Solving and taking the +vc root to give the highest value of F_{GC} we get

$$F_{GC} = A \left[\frac{1 + m\sqrt{v_{1c}^2 + (1 - m^2v_{1c}^2)\, v_1^2}}{1 - m^2v_{1c}^2}\right] \qquad (5)$$

7.5 If the above is repeated for steel tendons instead of concrete we get

$$m = \frac{\dfrac{F_{GS}}{B} - 1}{\sqrt{\left(\dfrac{F_{GS}.\ v_s}{B}\right)^2 + v_1^2}}$$

$$\text{and} \quad F_{GS} = B\left[\frac{1 + m\sqrt{v_s^2 + (1 - m^2 v_s^2)\ v_1^2}}{1 - m^2 v_s^2}\right] \qquad (6)$$

$$\text{where } B = K_2 \ \frac{(1 - 1.64v_s)}{Q(1 + v)}$$

7.6 Equations 5 and 6 relate global safety factors to probability of failure but to make use of them we must have values for all the various terms in the equations. Reasonable judgements can perhaps be made for those associated with materials such as v_c, v_{1c} and v_s but no statistical information is available to give values to the remaining terms. One way round this difficulty is to consider a well known type of structure such as a building designed to a well defined code of practice and by judgement associate a maximum failure rate with these methods of design and construction. It is shown below that this enables values to be derived for the missing quantities in the equations.

7.7 The new UK draft code for the design of buildings adopts the following safety factors for the collapse limit state

For concrete	F_c	=	1.5
For steel tendons and reinforcement	F_s	=	1.15
For superimposed loads	F_L	=	1.6

Taking account of the number of concrete structures and buildings which have been designed to earlier codes with similar safety factors and the extreme rarity of such structures collapsing during use it would be reasonable to assume that the above safety factors in conjunction with the other requirements of the code provide an annual probability of failure no higher than 1 in 10,000.

7.8 We must now deduce from this an estimate of the probability of failure each time the structure is loaded. The safety factor caters for variations in materials, construction and the accuracy of design methods as well as variations in load but only the latter is relevant to the number of loadings/yr. If the loading is assumed to follow a normal distribution with a co-efficient of variation v and if the number of loadings per year were known then from Fig 1 the annual chance of exceeding a given load could be found in terms of standard deviations above the mean value. The characteristic load to which the safety factors relate is taken as 1.1 times the mean load giving the following relationships

for 100 loadings/yr max load $= \dfrac{1 + 2.3v}{1.1}$ (characteristic load)

for 1,000 loadings/yr max load $= \dfrac{1 + 3.1v}{1.1}$ (characteristic load)

for 10,000 loadings/yr max load $= \dfrac{1 + 3.75v}{1.1}$ (characteristic load)

7.9 Ref 1 recommends that if the coefficient of variation of the loads is not known it should be taken as 0.1 but this is clearly intended to be its highest reasonable value. We wish to adjust the annual probability of failure to the probability of failure each time the structure is loaded. For this purpose we should take the smallest reasonable value for v, perhaps a half or a third of its highest reasonable value. In these circumstances it can be seen that it would be reasonable and in most cases conservative to make the probability of failure per loading the same as the annual probability of failure for loadings applied at least once per year.

7.10 This assumption would associate $F_{GC} = (1.6)(1.5) = 2.4$ with $m = 3.75$ in eqn 5 giving

$$2.4 = A\left[\frac{1 + 3.75\sqrt{v_{1c}^2 + (1 - 3.75^2 v_{1c}^2).v_1^2}}{1 - 3.75^2 v_{1c}^2}\right] \qquad (7)$$

We would also associate $F_{GS} = (1.6)(1.15) = 1.84$ with $m = 3.75$ in eqn 6 giving

$$1.84 = B\left[\frac{1 + 3.75\sqrt{v_s^2 + (1 - 3.75^2 v_s^2).v_1^2}}{1 - 3.75^2 v_s^2}\right] \qquad (8)$$

Equation 7 and 8 establish relationships between A, B, v_{1c}, v_s and v_1 which must be maintained to satisfy the assumption that the building code safety factors together with the other requirements of the code provide a probability of failure of 10^{-4} per loading.

7.11 The relationships of eqn 4 are shown graphically in Fig 5 which can be used to survey the effect of varying the values of A, B, v_{1c}, v_s and v_1 whilst maintaining the relationships of eqn 7. In equation 5, A will have its maximum reasonable value when v_{1c} and v_1 have their minimum reasonable values. v_1 (stress) cannot be less than v(load) which has a recommended value of 0.1 in Ref 1. v_{1c} is unlikely to be less than 0.08 or exceed 0.12. Putting $v_{1c} = 0.08$ and $v_1 = 0.1$ in eqn 7 gives $A = 1.48$ which is its highest reasonable value. The lowest reasonable value for A would be unity and a choice of $A = 1.2$ would be an approximate average value

For $A = 1.2$ and $v_{1c} = 0.08$ eqn 7 gives $v_1 = 0.212$

For $A = 1.2$ and $v_{1c} = 0.12$ eqn 7 gives $v_1 = 0.114$

7.12 These two sets of values represent the extremes of the range into which v_{1c} and v_1 should fall. If we now consider a range of structures in which the same values of A, B, v_{1c}, v_s and v_1 as used in the building structure are valid and at one extreme a failure probability of 10^{-1} is satisfactory whereas at the other extreme a failure probability of 10^{-7} is required we can plot F_{GC} against probability of failure using eqn 5 for each of the two extreme sets of values of v_{1c} and v_1 as shown in Fig 6. It can be seen that only at the very low probabilities is there any significant difference between the curves and this can be catered for by choosing a mean value of $v_{lo} = 0.1$ which gives $v_l = 0.175$ from eqn 7

Eqn 5 now becomes

$$F_{GC} = 1.2 \left(\frac{1 + m\sqrt{0.0410 - 0.00031m^2}}{1 - 0.01m^2} \right) \tag{9}$$

7.13 In eqn 7 v_l will have the value already chosen ie $v_l = 0.175$, and v_s will lie in the range 0.04 to 0.08. Taking the mean value $v_s = 0.06$ in eqn 8 gives B = 1.03 and eqn 6 becomes

$$F_{GS} = 1.03 \left(\frac{1 + m\sqrt{0.0346 - 0.000112m^2}}{1 - 0.0036m^2} \right) \tag{10}$$

7.14 Equations 9 and 10 give global safety factors in terms of failure probabilities that differ only marginally from any which can be calculated using the most extreme values of v_{lc}, v_s and v_l in equations 5 and 6. Equations 9 and 10 therefore are virtually independent of the values of these quantities provided that their relationships depend only upon the assumption that concrete buildings designed and constructed in accordance with the building code have a probability of failure of 10^{-4} per loading.

7.15 In deriving eqns 9 and 10 and plotting the curves referred to above on Fig 5 it has been assumed that keeping the coefficients of variation v_{lc}, v_s and v_l constant over the range of safety factors properly expresses the variability of the materials strengths and the stresses due to the applied loads. It could perhaps be argued that instead the standard deviations should have been kept constant although the authors believe that their choice is the better of the two. The effect of keeping the standard deviations constant is shown in Fig 5 and produces a less conservative value for the safety factors at the lower probabilities than the chosen approach.

8. TYPE OF PROBABILITY DISTRIBUTION

8.1 So far the normal distribution has been considered but many others are possible. For example Freudenthal (4) when discussing structural failure considers the logarithmic-normal distribution and suggests it is more useful than the normal distribution because it has more realistic extreme value distribution.

8.2 L S Lawrence in the discussions on Freudenthal's paper shows that the probability of failure on a log-normal distribution can be obtained from normal distribution tables provided the limits of integration are adjusted. D Roberts (in a private communication) has described the effect of varying the parameters for log-normal distributions and has shown that the relationship between safety factors and probability of failure is not closely dependent on the relationship adopted between the standard deviations of the stress and materials strength. Associating $\frac{F_{GC}}{A} = 2$ with a failure probability per loading of 10^{-4} provides the curves shown in Fig 6 for the log-normal distribution and illustrates how little the assumed relationship between the co-efficients of variation for stress and materials strength affects the result.

8.3 Other forms of extreme value distributions have been discussed by several writers but as Weibull pointed out when discussing the strength of materials in 1939(5), an impossibly large number of failure tests would be needed to distinguish between, say, log-normal and extremal distributions. Besides this important point that no particular distribution can be proved, it is likely that all theoretical distributions will have regions where they do not describe the real situation as we believe it to exist. Fortunately most of the contribution to the failure probability occurs with the stress-resistance equality between the mean values, and the extreme values of the distributions are relatively unimportant.

8.4 With both a normal and a log-normal distribution the safety factors over a range of failure probabilities three orders of magnitude either side of the chosen relationship are dominated by the assumption that a particular safety factor provides a particular probability of failure. The choice of a normal distribution is more conservative and more convenient than the log-normal and for these reasons is preferred by the authors.

9. PARTIAL SAFETY FACTORS

9.1 Eqns 9 and 10 give global safety factors which can be split in any arbitrary manner desired into partial factors provided their products always equal the global factor. It is obviously convenient to have the same load factor for all the materials in the structure enabling a single stress analysis to be prepared. It would be convenient to let F_c and F_s be constant over the whole range of failure probabilities but with a common F_L this would mean that $\frac{F_{GC}}{F_{GS}}$ would have to be constant over the whole range of failure probabilities which does not conform to equations 9 and 10.

9.2 If F_{LO}, F_{SO}, F_{CO} are the factors for a building and F_L, F_S, F_C are the factors for a structure with a different failure probability and we let

$$\frac{F_{LO}}{F_{SO}} = \frac{F_L}{F_S}$$

then we get $F_L = \sqrt{\frac{F_{LO}}{F_{SO}}} \cdot \sqrt{F_{GS}} = \sqrt{\frac{1.6}{1.15}} \sqrt{F_{GS}} = 1.18 \sqrt{F_{GS}}$ (11)

$$F_S = \sqrt{\frac{F_{SO}}{F_{LO}}} \cdot \sqrt{F_{GS}} = \sqrt{\frac{1.15}{1.6}} \cdot \sqrt{F_{GS}} = 0.85\sqrt{F_{GS}} \qquad (12)$$

$$F_C = \sqrt{\frac{F_{SO}}{F_{LO}}} \; \frac{F_{GC}}{\sqrt{F_{GS}}} = 0.85 \; \frac{F_{GC}}{\sqrt{F_{GS}}} \qquad (13)$$

10. PROBABILITY OF LOAD COMBINATIONS

10.1 Let 10^{-n1} = Annual probability of a particular load

10^{-n2} = Duration in years of peak value of load each time it occurs.

The probability of a number of independent loads occurring once in the same year is the product of their individual probabilities ie $10^{-\Sigma n1}$.

10.2 Now consider two of these loads occurring in the same year. The probability of their peak values coinciding depends upon the one with the shortest duration occurring during the period when the one with the longest duration exists. If the one with the longest duration has a duration of $10^{-\check{n}2}$ then this is the annual probability of the peak values of the loads coinciding if both loads occur in the same year. This probability can be rewritten as $10^{(\hat{n}2 - \Sigma n2)}$ where $10^{-\hat{n}2}$ is the duration of the one with the shortest duration and $-\Sigma n_2$ is the sum of the duration indices for both loads. The duration of the loading combination is the same as that of the original load with the shortest duration ie $10^{-\hat{n}2}$. These same expressions will also cater for any number of loads.

10.3 Since the probability of the loads all occurring in the same year is $10^{-\Sigma n1}$ the overall annual probability of them co-inciding is

$$10^{\left[\hat{n}_2 \; - \; \Sigma(n_1 \; + \; n_2)\right]}$$

To be consistent with earlier assumptions we would take $n_1 + n_2 = 0$ for any type of loading applied at least once per year.

10.4 Max wind load may have an annual probability of 10^{-2} and a duration per occurrence of 10^{-3} years. Whilst design earthquake loading may have a similar annual probability and a duration per occurrence of minutes. Clearly each load should separately be combined with prestress, dead and superimposed loads but provided they originate independently the annual probability of max wind and earthquake co-inciding with these is

$$10^{\left[\hat{n}_2 \; - \; (2 + 2 + 3 + \hat{n}_2)\right]} = 10^{-7}$$

and could probably be ignored for all but the most important structures.

11. REACTOR CONTAINMENT STRUCTURES

11.1 These structures are designed to withstand the loads and temperatures arising from reactor accidents and to contain the fission products released into the building as a result of the reactor accident. If the structure were to fail these fission products would be released to the atmosphere and their quantity would define the consequences of failure. Farmer[7] has suggested a criterion which in its simplest form defines the acceptable annual probability of such a release as the reciprocal of the number of curies of I_{131} which would be released to atmosphere if the structure should fail. Each different kind of accident will, in general, release different amounts of fission products as well as having a different probability of occurrence, providing different values for P_A and P_1 in eqn 1. It is primarily to cater for this situation that the authors have investigated the possibility of linking P_2 to a scale of failure probabilities enabling the structure to be designed for a whole series of accidents rather than just for one which must be chosen as the maximum credible accident.

11.2 A fairly large power reactor is likely to have a max. fission product inventory of approx 4×10^7 curies I_{131} not all of which would reach the atmosphere if the reactor and containment structure failed. This tends to set an upper limit of about 10^{-7} for P_A which would be associated with a major accident having a relatively low probability P_1. The required value of P_2 therefore is unlikely to be lower than the corresponding figure for a building even for the most serious reactor accident that can be envisaged. For many accidents which do not release so many fission products P_2 can be orders of magnitude higher permitting smaller safety factors. It also follows from the earlier argument that although it will never be necessary to combine earthquake and wind together with the accident and rarely necessary to combine either one with the accident when they all occur independently, the exception will be when the earthquake causes the reactor accident.

11.3 The failure of a reactor containment structure to contain fission products will occur when it develops a major leak which will usually be when the condition of the concrete supporting the lining is such that the lining is in danger of tearing. This condition can differ from the collapse condition in varying degrees depending on whether tendons are bonded or unbonded, reinforcement is included or not, and the tearing strain of the type of lining used. Since failure is defined as lining failure the structure should be designed to have safety factors against lining failure linked with the probability P_2 even though the scale of safety factors and failure probabilities is derived from the collapse limit state of a building.

11.4 The design of a structure to cater for a whole range of accidents requires careful consideration to be given to testing it at completion of construction. It may well turn out that the safety factors used for the accident providing the greatest pressure are low enough to make it imprudent to test at this pressure. In these circumstances the test pressure should be the highest which would not do unacceptable damage to the structure, such as cracking, during the test. The behaviour of the structure can be measured in terms of strains and deflections of the concrete which can then be used to confirm the validity of the design calculations. A leak test can also be conducted at the same pressure.

11.5 Difficulties may arise with repeat pressure tests if the reactor has accumulated its full inventory of fission products and collapse of the building during the test would breach the reactor circuit. If the test is to be repeated every 5 years then $P_1 = 2 \times 10^{-1}$ which will be a much higher probability than that for most of the accidents. Because the reactor will be shut down, cool and depressurised the breach of the circuit by the collapsing building would not necessarily release the entire fission products. Although P_A should be more than 10^{-7} it may still be of a low order however, giving a P_2 value requiring fairly high safety factors against collapse during the test. To be logical the test pressure should be correspondingly low so that the risk of testing the structure is consistent with the risk of operating. It would be a pity, however, to forego the advantages of repeat pressure tests which need not be for structural checks but to measure leak-tightness and could therefore be at the lowest pressure suitable for this purpose.

12. CONCLUSIONS

12.1 Other papers which have examined the general question of structural failure and safety factors in statistical terms have all approached the subject in similar terms. They have decided on probability distribution functions and on the necessary coefficients for the stress and the resistance. From these, the relationship between safety factor and the failure probability is derived. Arguments arise over the type of distribution and the coefficients because neither can be proved correct by actual experience of failures. The alternative method proposed here has similarities but appears easier to use; it assumes a distribution and determines the necessary coefficients by assuming that some particular safety factor leads to a particular risk of failure. All other values of safety factor and risk can then be derived using the same coefficients and distribution.

12.2 Using the normal distribution it has been shown that safety factors are not seriously influenced by the values chosen for the coefficients of variation and the other factors which define the probability of failure provided these are related to one another by the assumption that a particular safety factor leads to a particular risk of failure.

12.3 The proposed method of varying safety factors has its greatest use in the economical design of reactor containment structures where it enables the structure to be designed for a whole series of reactor accidents using different safety factors for each.

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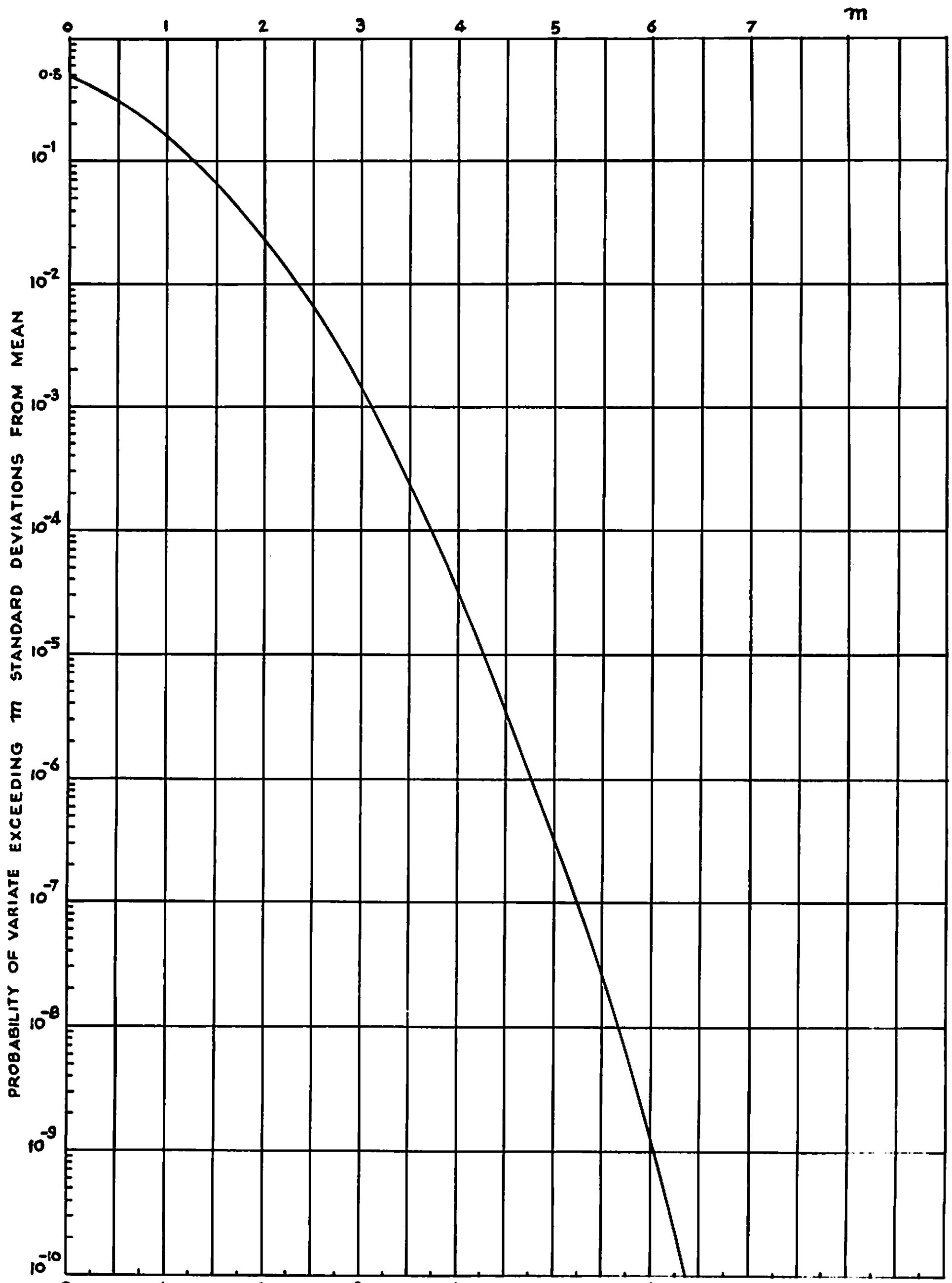

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10^{-9}
10^{-10}
PROBABILITY OF VARIATE EXCEEDING m STANDARD DEVIATIONS FROM MEAN
0 1 2 3 4 5 6 7

FIG. 1

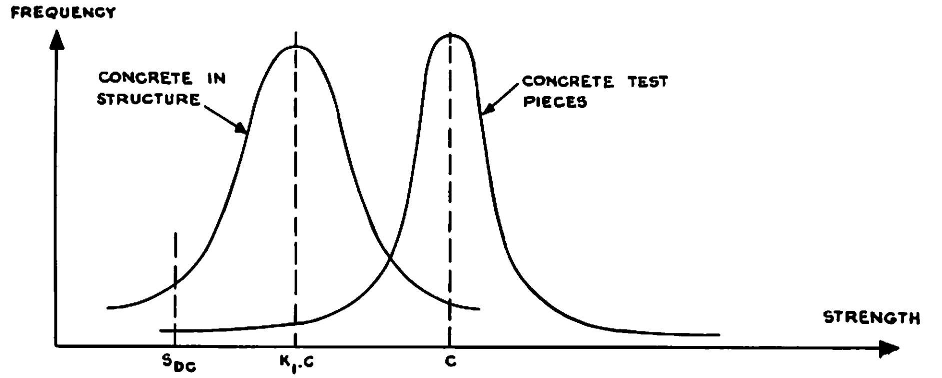

FIG. 2. STRENGTH FREQUENCY CURVES FOR CONCRETE

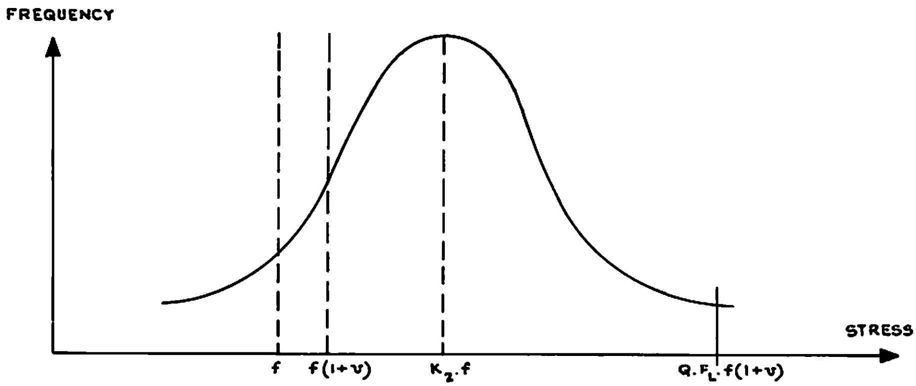

FIG. 3 FREQUENCY OF ACTUAL STRESS AT A POINT IN STRUCTURE

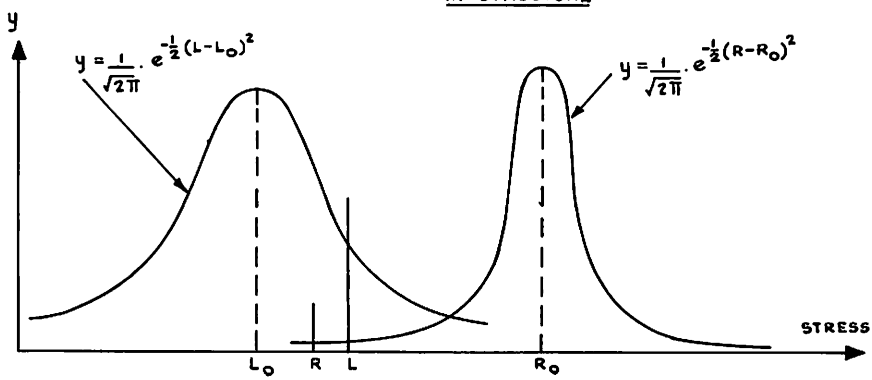

FIG. 4. STRESS AND STRENGTH FREQUENCY CURVES

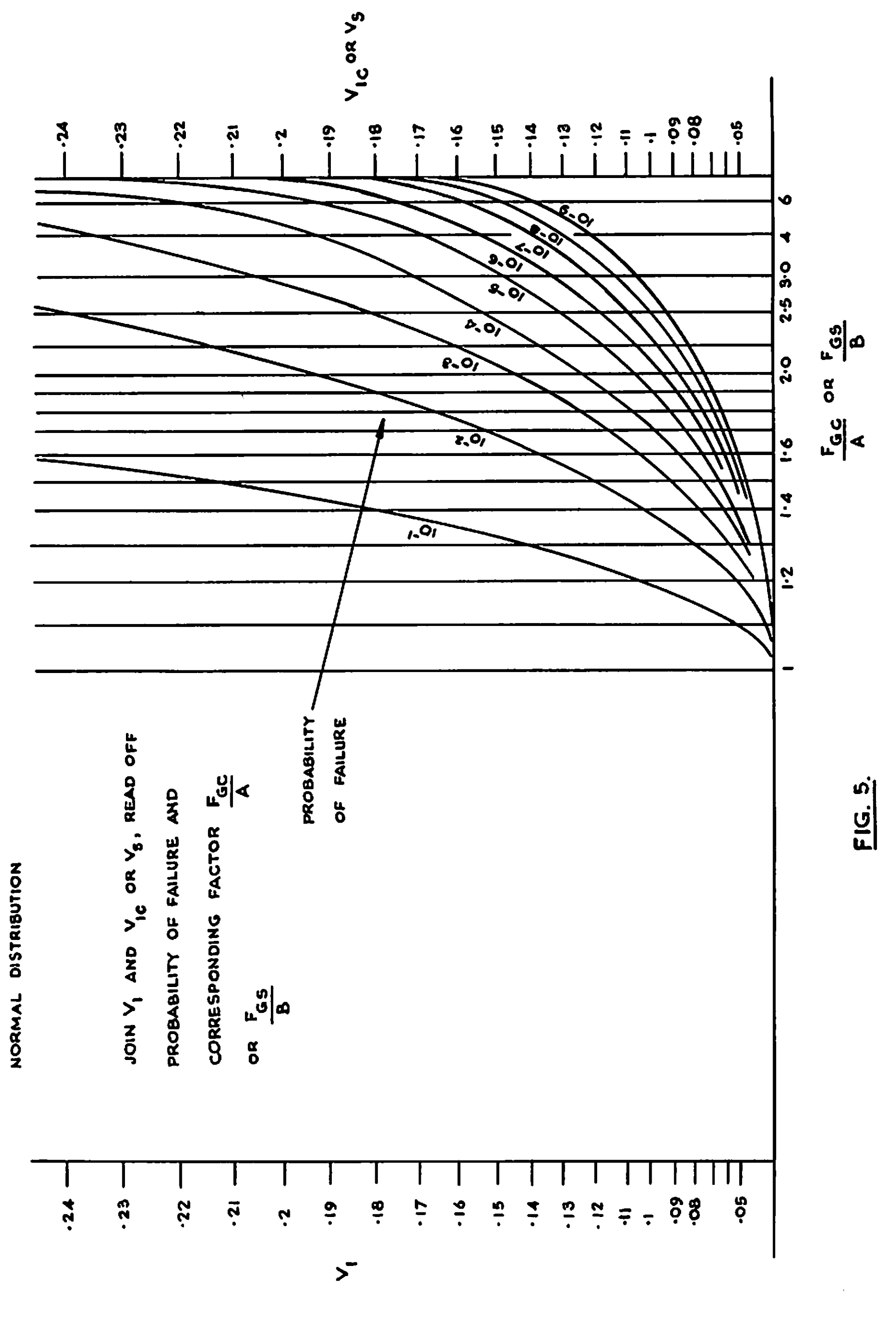
NORMAL DISTRIBUTION
JOIN V_1 AND V_{1C} OR V_S, READ OFF PROBABILITY OF FAILURE AND CORRESPONDING FACTOR $\frac{F_{GC}}{A}$ OR $\frac{F_{GS}}{B}$
PROBABILITY OF FAILURE
V_1
V_{1C} OR V_S
$\frac{F_{GC}}{A}$ OR $\frac{F_{GS}}{B}$
·24 ·23 ·22 ·21 ·2 ·19 ·18 ·17 ·16 ·15 ·14 ·13 ·12 ·11 ·1 ·09 ·08 ·05
1 1·2 1·4 1·6 2·0 2·5 3·0 4 6
10^{-1} 10^{-2} 10^{-3} 10^{-4} 10^{-5} 10^{-6} 10^{-7} 10^{-8} 10^{-9}

FIG. 5.

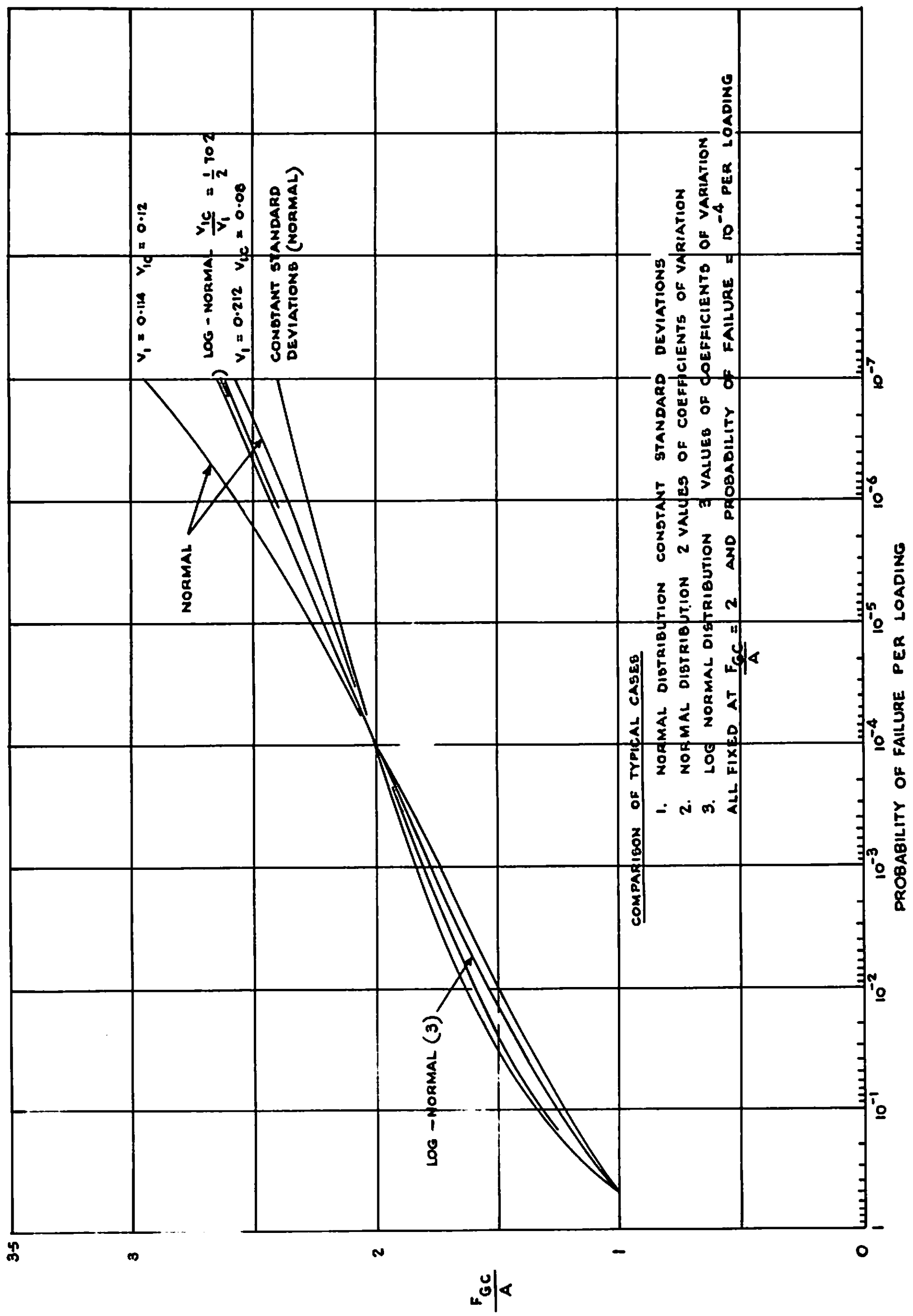
$V_1 = 0{\cdot}114 \quad V_{1C} = 0{\cdot}12$
LOG - NORMAL $\frac{V_{1C}}{V_1} = \frac{1}{2}$ TO 2
$V_1 = 0{\cdot}212 \quad V_{1C} = 0{\cdot}06$
CONSTANT STANDARD DEVIATIONS (NORMAL)
NORMAL
LOG - NORMAL (3)
COMPARISON OF TYPICAL CASES
1. NORMAL DISTRIBUTION CONSTANT STANDARD DEVIATIONS
2. NORMAL DISTRIBUTION 2 VALUES OF COEFFICIENTS OF VARIATION
3. LOG NORMAL DISTRIBUTION 3 VALUES OF COEFFICIENTS OF VARIATION
ALL FIXED AT $\frac{F_{GC}}{A} = 2$ AND PROBABILITY OF FAILURE = 10^{-4} PER LOADING
$\frac{F_{GC}}{A}$
3·5
3
2
1
0
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
PROBABILITY OF FAILURE PER LOADING

FIG.6.

DISCUSSION

Q

F. ELLYIN, Canada

It appears to me that the probability of failure of the order of 10^{-4} is rather low for civil engineering structures designed according to the "limit state design". We have shown in a paper by F. Ellyin and E. Ghannoum, "An assessment of reliability of structures", paper presented at the 85th National meeting of the Engineering Institute of Canada, Québec, Sept. 15-18, 1971, that the proper probability of failure of real structures designed by the limit state design code of Russia is much higher.
I may also add that the nature of assumed distribution for variables contained in the resistance and strength calculation may effect the probability of failure (see above mentioned reference).

A

J. H. BOWEN, U. K.

The reference quoted by Dr. Ellyin is not available to us, nor are the values of safety factor specified by the Russian code. Our judgment of a conservative value for the collapse rate of buildings in service is based upon the assumption that there must be several hundred buildings in the UK which, broadly speaking, can be said to have been designed with safety factors similar to those of the latest building code. Collapses during construction are special cases, and discounting these if the annual collapse rate in service were as high as 10^{-3} we would, therefore, hear of a building collapsing in service every few years which does not seem to be the case. An annual collapse rate of 10^{-4} therefore seems a reasonable assumption. Moreover, to be consistent with this and the known variability of the materials requires a higher value for the coefficient of variation of the stresses and this is further indication that the assumed collapse rate is reasonable. Both of these judgments are subjective as they must be in the absence of statistics on the collapse of buildings.
The nature of the assumed distribution of variables affects the safety factor against failure probability relationship and Fig. 6 compares normal and log-normal distributions under various conditions. Provided all curves pick up the same relationship between the safety factors and failure probability of buildings the effect is shown to be relatively small over a practical range.

C

D. KECECIOGLU, U. S. A.

I belong to the school of thought that once the strength distribution is determined it should not be mutilated by pacing some arbitrary number of standard deviations from the mean of strength and then from the mean of stress in a similar way and then using these two new values to arrive at the reliability involved. I believe in using the true stress and strength distributions and calculating the associated true reliability and also confidence limit as shown in my paper M 1/4*.