HANDY INCORPORATION OF BOND AND SINGULARITY ELEMENTS IN THE FINITE ELEMENT SOLUTION ROUTINE

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SUMMARY

Bond and singularity elements are outcomes of the efforts to cope with two peculiarities inherent to the mechanics of continua, one being the discontinuity of certain displacement and stress components along the interface of bodies in contact and the other the singularity in strain and stress distributions at the crack tip. Present paper introduces a mapping technique and it is shown that both elements can be treated in a unified manner and incorporated into the standard finite element solution routine with a minimum modification of the computer program.

Specifically in the case of contact problem, the rectangular or triangular element in the normalized parametric plane is mapped to the corresponding bond or interface element having vanishingly small thickness. On the other hand, an ingenious singularity transformation is applied to map the element in the parametric plane on the physical plane of the problem featuring singular strain and stress distributions. The mapped element thus obtained can be curvilinear, though the straight sided element is utilized conveniently in most of the applicaitons.

In each section discussing the bond and singularity element in order, relevant mapping or transformation technique is introduced first and then follow some details of the computer program modification connected with the incorporation of the element. Several numerical examples are included to illustrate the versatility and the effectiveness of the proposed method. They are (1) stress distribution in single and double lap joint, (2) Hertz elastic contact problem under nonfrictional and frictional conditions, (3) stress intensity factor at the tip of crack breaking out of a rivet-joint, and (4) notched compact tension specimen. Problem 3 is an example of combined use of bond and singularity elements. Problem 4 is an attempt to extend the application of the method as well as the finite element solution routine to three dimensions.

The paper concludes with a discussion of the elementology in general and emphasizes the easiness of the proposed technique to tie in the recently emerged concept that adopts the hierarchy ranking basis transformation and/or the adaptive finite element.

1. Introduction

Since the introduction of the finite element method to engineering societies, its development has been the repetition of sophistication and simplification. Quite a number of fruitful ideas, new and/or revived, have been brought in the finite element area, but the procedures with least departure from the standard routine remain standing in practical applications. This is because 'made simple' is the spirit of the finite element method by which ceaseless expansion of its scope has been attained. The elastic-plastic analysis based on the updated Lagrangian formulation and its extension to time dependent material behavior as well as large deformation regime are typical of such achievements, within knowledges and from experiences of the authors who have been involved in the development [1, 2].

Present paper aims to broaden the above view by taking part in the studies of two special type of finite elements, the bond and singularity elements. Bond element, which is often called alternatively by the name of joint or interface element, is evoked to handle the problem accompanying discontinuities in certain displacement and/or stress components [3, 4]. Its function is to replace rationally the displacement compatibility as well as the force equilibrium conditions along the interface of two bodies being in contact or bonded. It seems that the accomplishment is a difficult task and the incorporation of the element requires special techniques. This paper illustrates that it is not the case.

The second peculiality, i.e. the singularity in stress and strain distribution at crack tips, has been one of the major topics in fracutre mechanics. Of course, numerical analysts could not have been indifferent to the singularity problem. A variety of procedures have been proposed since fracture mechanics problems drew the attention of specialists in finite element method in the late 1960's and most popular is the formulation which incorporates the singular terms in the expression of displacement function [5-7]. An alternative procedure, which is simple but versatile, is to be introduced in the present paper.

The basic principle on which our unified formulation is being based is an ingenious use of mapping technique. In case of the bond element, the rectangular or triangular element in the normalized parametric plane is mapped to the corresponding bond element having vanishingly small thickness. On the other hand, appropriate singularity transformation is utilized to map the parent element defined in the parametric plane on the physical plane of the problem featuring singular strain and stress distributions.

2. Incorporation of Bond Element

The bond element concerned in this paper is either of quadrilateral or triangular shape as shown in Fig.1. The use of triangular bond element is rather new and it should be emphasized that its application does not necessarily require the coincidence of nodal point placement on the surfaces of opposing bodies. We assume naturally that both the element shape and the displacement are defined with respect to the common global coordinates, but the

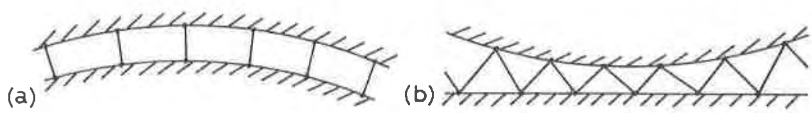

Fig. 1 Interface of two bodies and quadrilateral (a) and triangular (b) bond element

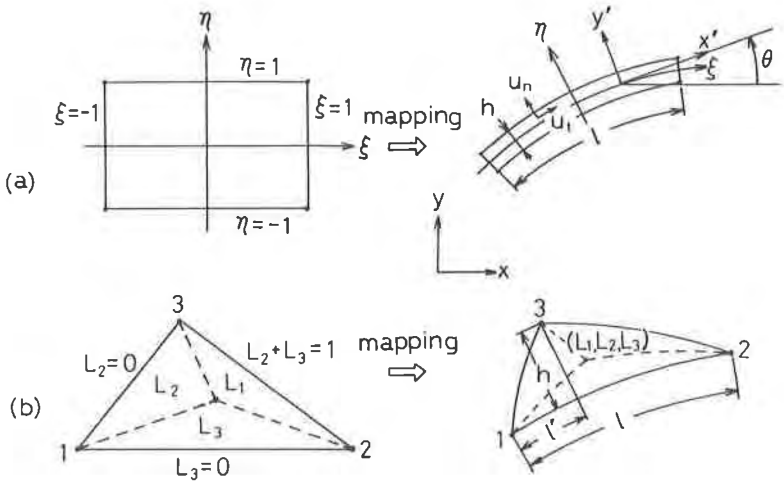

Fig.2 Parametric rectangular (a) and triangular (b) element and their mapping

introduction of local normal and tangent coordinates is convenient for the derivation of the stiffness equation relevant to the bond element.

Considering first the quadrilateral bond element in two dimensions, we assume that the mapping of the element from the normalized parametric ξ, η plane to the physical x, y plane in Fig.2(a) is

$$x = x(\xi, \eta), \quad y = y(\xi, \eta) \tag{1}$$

Further, the displacement or velocity field $\{u(x, y)\}$ is expressed by the nodal values $\{u_i\}$ and shape functions $\{N_i(\xi, \eta)\}$ as

$$\begin{aligned} u_x(x, y) &= N_1(\xi, \eta)u_{x1} + \cdots + N_m(\xi, \eta)u_{xm} \\ u_y(x, y) &= N_1(\xi, \eta)u_{y1} + \cdots + N_m(\xi, \eta)u_{ym} \end{aligned} \tag{2}$$

where m stands for the number of nodes.

Now introducing the tangential and normal displacement or velocity components u_t, u_n along local coordinates x', y', we note the following chain rule of differentiation

$$\begin{Bmatrix} \partial u_t/\partial\xi \\ \partial u_n/\partial\xi \\ \partial u_t/\partial\eta \\ \partial u_n/\partial\eta \end{Bmatrix} = \begin{bmatrix} \partial x'/\partial\xi & 0 & \partial y'/\partial\xi & 0 \\ 0 & \partial x'/\partial\xi & 0 & \partial y'/\partial\xi \\ \partial x'/\partial\eta & 0 & \partial y'/\partial\eta & 0 \\ 0 & \partial x'/\partial\eta & 0 & \partial y'/\partial\eta \end{bmatrix} \begin{Bmatrix} \partial u_t/\partial x' \\ \partial u_n/\partial x' \\ \partial u_t/\partial y' \\ \partial u_n/\partial y' \end{Bmatrix} \tag{3}$$

When the thickness h of the bond element is vanishingly small compared to its length ℓ, terms of the matrix in eq.(3) can be estimated from the relation of eq.(1) as

$$\begin{aligned} \partial x'/\partial\xi &= \ell_2/2 = [(\partial x/\partial\xi)^2+(\partial y/\partial\xi)^2]^{1/2}, \quad \partial y'/\partial\xi = 0 \\ \partial x'/\partial\eta &= \ell_3/2 = \pm[(\partial x/\partial\eta)^2+(\partial y/\partial\eta)^2]^{1/2}, \quad \partial y'/\partial\eta = h/2 \end{aligned} \tag{4}$$

Substituting eq.(4) into eq.(3) and inverting, we have

$$\begin{Bmatrix} \partial u_t/\partial x' \\ \partial u_n/\partial x' \\ \partial u_t/\partial y' \\ \partial u_n/\partial y' \end{Bmatrix} = \frac{2}{h\ell_2} \begin{bmatrix} h & 0 & 0 & 0 \\ 0 & h & 0 & 0 \\ -\ell_3 & 0 & \ell_2 & 0 \\ 0 & -\ell_3 & 0 & \ell_2 \end{bmatrix} \begin{Bmatrix} \partial u_t/\partial\xi \\ \partial u_n/\partial\xi \\ \partial u_t/\partial\eta \\ \partial u_n/\partial\eta \end{Bmatrix} \tag{5}$$

It is noted that ℓ_2 in eq.(4) is often constant and becomes equal to the length ℓ of the element. The sign of ℓ_3 corresponds to that of $\partial x'/\partial\eta$, ℓ_3 being zero as is often the case when the x' coordinates of the pair of opposing nodal points along the interface coincide.

Relation between components u_t, u_n in local coordinates and global ones u_x, u_y is

$$\begin{Bmatrix} u_t \\ u_n \end{Bmatrix} = [T]\begin{Bmatrix} u_x \\ u_y \end{Bmatrix}, \quad [T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \tag{6}$$

The local angle of inclination θ is obtainable by using eq.(1) as

$$\cos\theta = \frac{\partial x/\partial\xi}{[(\partial x/\partial\xi)^2+(\partial y/\partial\xi)^2]^{1/2}}, \quad \sin\theta = \frac{\partial y/\partial\xi}{[(\partial x/\partial\xi)^2+(\partial y/\partial\xi)^2]^{1/2}} \tag{7}$$

Introducing eq.(6) to eq.(5) and using eq.(2), we have

$$\begin{Bmatrix} \partial u_t/\partial x' \\ \partial u_n/\partial x' \\ \partial u_t/\partial y' \\ \partial u_n/\partial y' \end{Bmatrix} = [B]\begin{Bmatrix} u_{x1} \\ u_{y1} \\ \vdots \\ u_{xm} \\ u_{ym} \end{Bmatrix} \tag{8}$$

where

$$[B] = \begin{Bmatrix} [B_1] \\ [B_2] \end{Bmatrix} = \frac{2}{h\ell_2}\left[\begin{array}{cccc} h & 0 & 0 & 0 \\ 0 & h & 0 & 0 \\ \hline -\ell_3 & 0 & \ell_2 & 0 \\ 0 & -\ell_3 & 0 & \ell_2 \end{array}\right]\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}\times$$
$$\begin{bmatrix} \partial N_1/\partial\xi & 0 & \cdots & \partial N_m/\partial\xi & 0 \\ 0 & \partial N_1/\partial\xi & \cdots & 0 & \partial N_m/\partial\xi \\ \partial N_1/\partial\eta & 0 & \cdots & \partial N_m/\partial\eta & 0 \\ 0 & \partial N_1/\partial\eta & \cdots & 0 & \partial N_m/\partial\eta \end{bmatrix} \tag{9}$$

Here we concern merely with the gradient of displacement components in the normal direction y' and adopt the following constitutive relation for the material of bond element

$$\begin{Bmatrix} \sigma_{nt} \\ \sigma_{nn} \end{Bmatrix} = [D]\begin{Bmatrix} \partial u_t/\partial y' \\ \partial u_n/\partial y' \end{Bmatrix}, \quad [D] = \begin{bmatrix} G_S & 0 \\ 0 & G_E \end{bmatrix} \tag{10}$$

where σ_{nt} and σ_{nn} stand for the tangential and normal stress or stress-rate along the interface where the bond element is placed. G_E and G_S are the normal and shear modulus which represent the linear or nonlinear bond material behavior. Finally, by the standard procedure of the finite element method, the stiffness matrix of the bond element under consideration is determined as

$$[k] = \int_{-1}^{1}\int_{-1}^{1}[B_2]^T[D][B_2]|J'|\,t\,d\xi d\eta, \quad |J'| = \frac{1}{4}\begin{vmatrix} \ell_2 & 0 \\ \ell_3 & h \end{vmatrix} \tag{11}$$

where t is the width of the element normal to the x, y plane.

In case of the triangular element shown in Fig.1(b), we assume that the mapping is expressed in terms of the area coordinates L_1, L_2, L_3 as

$$x = x(L_1, L_2, L_3), \quad y = y(L_1, L_2, L_3) \tag{12}$$

Similarly the expression for the displacement or velocity field is

$$\begin{aligned} u_x(L_1, L_2, L_3) &= N_1(L_1, L_2, L_3)u_{x1} + \cdots + N_m(L_1, L_2, L_3)u_{xm} \\ u_y(L_1, L_2, L_3) &= N_1(L_1, L_2, L_3)u_{y1} + \cdots + N_m(L_1, L_2, L_3)u_{ym} \end{aligned} \tag{13}$$

The area coordinates are not mutually independent and connected by the relation

$$L_1 + L_2 + L_3 = 1 \tag{14}$$

In the following, L_2 and L_3 are taken as the independent coordinate variables, so that

$$\left(\frac{d}{dL_2}\right)_{L_3} = \frac{\partial}{\partial L_1}\frac{dL_1}{dL_2} + \frac{\partial}{\partial L_2} = \frac{\partial}{\partial L_2} - \frac{\partial}{\partial L_1}, \quad \left(\frac{d}{dL_3}\right)_{L_2} = \frac{\partial}{\partial L_1}\frac{dL_1}{dL_3} + \frac{\partial}{\partial L_3} = \frac{\partial}{\partial L_3} - \frac{\partial}{\partial L_1} \tag{15}$$

It can be easily seen then that the equations relevant to the triangular bond element are obtainable by replacing the partial derivatives $\partial/\partial\xi$ and $\partial/\partial\eta$ in the relations pertaining to the quadrilateral element as follows

$$\begin{Bmatrix}\partial/\partial\xi \\ \partial/\partial\eta\end{Bmatrix} \rightarrow \frac{1}{2}\begin{Bmatrix}\partial/\partial L_2 - \partial/\partial L_1 \\ \partial/\partial L_3 - \partial/\partial L_1\end{Bmatrix} \tag{16}$$

Specifically, formulas for estimating ℓ_2 and ℓ_3 in eq.(4) are

$$\begin{aligned} \ell_2 &= \left[(\partial x/\partial L_2-\partial x/\partial L_1)^2+(\partial y/\partial L_2-\partial y/\partial L_1)^2\right]^{1/2} \\ \ell_3 &= \pm\left[(\partial x/\partial L_3-\partial x/\partial L_1)^2+(\partial y/\partial L_3-\partial y/\partial L_1)^2\right]^{1/2} \end{aligned} \tag{17}$$

Expressions for $\cos\theta$ and $\sin\theta$ of eq.(7) are modified in the similar manner. Furthermore, [B] matrix of eq.(9) becomes

$$[B] = \begin{Bmatrix}[B_1] \\ [B_2]\end{Bmatrix} = \frac{1}{h\ell_2}\left[\begin{array}{cccc} h & 0 & 0 & 0 \\ 0 & h & 0 & 0 \\ \hline -\ell_3 & 0 & \ell_2 & 0 \\ 0 & -\ell_3 & 0 & \ell_2 \end{array}\right]\begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \\ 0 & 0 & -\sin\theta & \cos\theta \end{bmatrix}\times$$

$$\begin{bmatrix} (\partial/\partial L_2-\partial/\partial L_1)N_1 & 0 & \cdots & (\partial/\partial L_2-\partial/\partial L_1)N_m & 0 \\ 0 & (\partial/\partial L_2-\partial/\partial L_1)N_1 & \cdots & 0 & (\partial/\partial L_2-\partial/\partial L_1)N_m \\ (\partial/\partial L_3-\partial/\partial L_1)N_1 & 0 & \cdots & (\partial/\partial L_3-\partial/\partial L_1)N_m & 0 \\ 0 & (\partial/\partial L_3-\partial/\partial L_1)N_1 & \cdots & 0 & (\partial/\partial L_3-\partial/\partial L_1)N_m \end{bmatrix} \tag{18}$$

The element stiffness associated with the constitutive relation of the bond material is evaluated as

$$[k] = \int_0^1\int_0^{1-L_3}[B_2]^T[D][B_2]h\ell_2 t\,dL_2 dL_3 \tag{19}$$

where t is the width of the element normal to the x, y plane. The length ℓ_3 is often equal to the difference ℓ' of the x' coordinate between nodes 1 and 3 as shown in Fig.2(b).

3. Applications of Bond Element and Examples

The formulation of the bond element in the previous section is general and features that it is applicable to the problems involving curved interfaces. However, in cases where the bilinear rectangular and/or linear triangular bond element are usable, we can take advantage of the fact that stiffness equations pertaining to these elements are given in local coordinates as the following.

For bilinear rectangular element of Fig.3(a)

$$\frac{\ell t}{6h}\left[\begin{array}{cccc|cccc} 2G_S & 0 & G_S & 0 & -G_S & 0 & -2G_S & 0 \\ 0 & 2G_E & 0 & G_E & 0 & -G_E & 0 & -2G_E \\ G_S & 0 & 2G_S & 0 & -2G_S & 0 & -G_S & 0 \\ 0 & G_E & 0 & 2G_E & 0 & -2G_E & 0 & -G_E \\ \hline -G_S & 0 & -2G_S & 0 & 2G_S & 0 & G_S & 0 \\ 0 & -G_E & 0 & -2G_E & 0 & 2G_E & 0 & G_E \\ -2G_S & 0 & -G_S & 0 & G_S & 0 & 2G_S & 0 \\ 0 & -2G_E & 0 & -G_E & 0 & G_E & 0 & 2G_E \end{array}\right]\left\{\begin{array}{c} u_{t1} \\ u_{n1} \\ u_{t2} \\ u_{n2} \\ \hline u_{t3} \\ u_{n3} \\ u_{t4} \\ u_{n4} \end{array}\right\} = \left\{\begin{array}{c} P_{t1} \\ P_{n1} \\ P_{t2} \\ P_{n2} \\ \hline P_{t3} \\ P_{n3} \\ P_{t4} \\ P_{n4} \end{array}\right\} \tag{20}$$

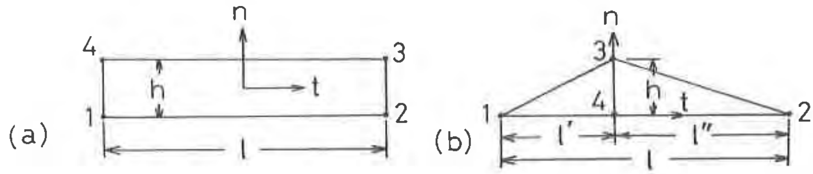

Fig.3 Bilinear rectangular (a) and linear triangular (b) bond element

For linear triangular element of Fig.3(b) where relevant notations are defined

$$\frac{t}{2h\ell}\left[\begin{array}{cc|cc|cc} G_S\ell''^2 & 0 & G_S\ell'\ell'' & 0 & -G_S\ell\ell'' & 0 \\ 0 & G_E\ell''^2 & 0 & G_E\ell'\ell'' & 0 & -G_E\ell\ell'' \\ \hline G_S\ell'\ell'' & 0 & G_S\ell'^2 & 0 & -G_S\ell\ell' & 0 \\ 0 & G_E\ell'\ell'' & 0 & G_E\ell'^2 & 0 & -G_E\ell\ell' \\ \hline -G_S\ell\ell'' & 0 & -G_S\ell\ell' & 0 & G_S\ell^2 & 0 \\ 0 & -G_E\ell\ell'' & 0 & -G_E\ell\ell' & 0 & G_E\ell^2 \end{array}\right]\left\{\begin{array}{c} u_{t_1} \\ u_{n1} \\ \hline u_{t_2} \\ u_{n2} \\ \hline u_{t_3} \\ u_{n3} \end{array}\right\} = \left\{\begin{array}{c} p_{t_1} \\ p_{n1} \\ \hline p_{t_2} \\ p_{n2} \\ \hline p_{t_3} \\ p_{n3} \end{array}\right\} \tag{21}$$

The stiffness matrices contained in eqs.(20) and (21) are the special cases of those obtainalbe by eqs.(11) and (19) respectively. It merits mentioning that these expressions are derivable as well by imposing the condition $h \to 0$ on the conventional stiffness equation of the bilinear rectangular and linear triangular element under plane strain condition.

The very stiff property of the bond element ascribed to its vanishingly small thickness h poses the problem of ill-condition of the assembled overall stiffness matrix. A technique suggested by Wilson [9] that utilizes the relative displacement is useful to overcome the difficulty. In case of the linear triangular element of Fig.3(b), the relative displacements

$$\Delta u_t = u_{t_3} - u_{t_4}, \quad \Delta u_n = u_{n_3} - u_{n_4} \tag{22}$$

between the corner node 3 and the fictious edge node 4 are conveniently used as independent variables. From eq.(22) and the postulate of linear displacement field, we have

$$u_{t_3} = \frac{\ell''}{\ell} u_{t_1} + \frac{\ell'}{\ell} u_{t_2} + \Delta u_t, \quad u_{n_3} = \frac{\ell''}{\ell} u_{n_1} + \frac{\ell'}{\ell} u_{n_2} + \Delta u_n \tag{23}$$

Introducing eq.(23) to the stiffness equation (21) and rearranging, we obtain

$$\frac{\ell t}{2h}\left[\begin{array}{cc|cc|cc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & G_S & 0 \\ 0 & 0 & 0 & 0 & 0 & G_E \end{array}\right]\left\{\begin{array}{c} u_{t1} \\ u_{n1} \\ \hline u_{t2} \\ u_{n2} \\ \hline \Delta u_t \\ \Delta u_n \end{array}\right\} = \left\{\begin{array}{c} p_{t_1} + \ell'' p_{t_3} / \ell \\ p_{n1} + \ell'' p_{n3} / \ell \\ \hline p_{t_1} + \ell' p_{t_3} / \ell \\ p_{n1} + \ell' p_{n3} / \ell \\ \hline p_{t_3} \\ p_{n_3} \end{array}\right\} \tag{24}$$

In the numerical examples shown below, we use eq.(21) or eq.(24) and apply the transformation to have the element stiffness equation in terms of the displacement and load components referring to the global coordinates. It should be noted that these equations are equally applicable to the circular triangle bond element, provided ℓ', ℓ'', ℓ are interpreted as the circular arc lengths.

Fig.4(a) depicts the single and double lap joint problem we have solved. Young's modulus and Poisson's ratio for the plate bonded and the adhesive are E_1, E_2 = 7000 or 21000,

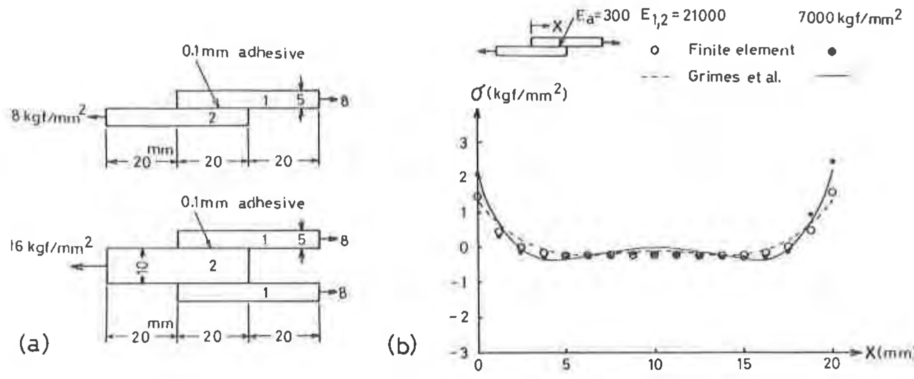

Fig.4 Single and double lap joint subjected to tangential shearing load

E_a = 300 kgf/mm^2 and ν_1, ν_2, ν_a = 0.3 respectively. Shearing force is applied tangentially to the joint line and the bond stress distribution is studied. Fig.4(b) shows the effect of the Young's modulus of the plate on the normal stress distribution along single lap joint line and compares the numerical result with the analytical solution of Grimes and his coworkers [9].

The second problem to which a brief comment might be in place is the Hertz elastic contact problem. We have treated the contacting of a circular cylinder and a plane block. Under the circumstances as such where the bodies in contact have different shapes and/or mechanical properties, the proper choise of bond element is triangular one introduced in the preceding section. Smooth and frictional conditions are assumed along the interface. It merits noting that incorporation of the Coulomb frictional condition necessitates the development of interactive solution procedure one way or the other in order to satisfy the relation $\tau=\mu\sigma$. Our solution of this problem has been reported in ref.[10].

The problem shown in Fig.5(a) concerns with the stress intensity factor at the crack tip breaking out of a rivet joint. It is considered that the insertion of the bond element along the rivet-plate interface is an actual simulation of the physical situations in this problem. We have solved the problem by employing circular triangle bond elements and compared the

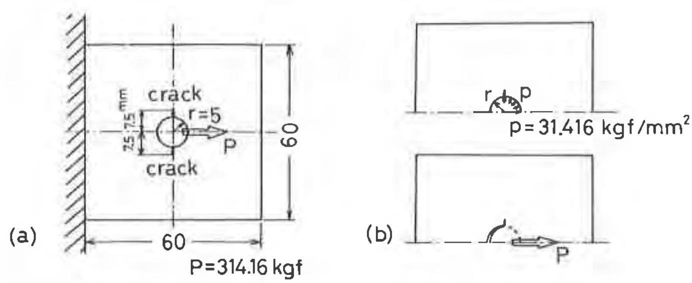

Fig.5 Crack breaking out of a rivet joint and idealizations of contact condition

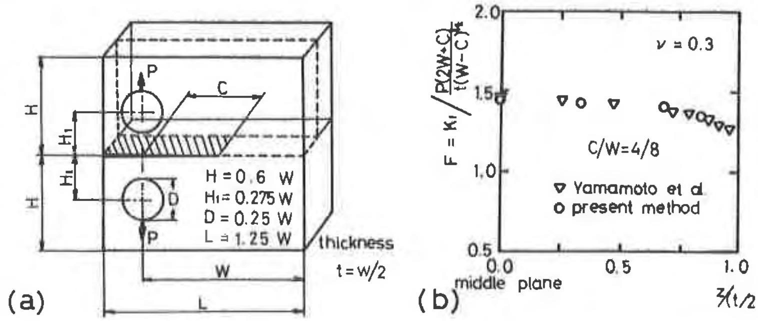

Fig.6 Stress intensity factor in compact tension specimen

stress intensity factor K thus obtained with those evaluated under two simpler auxiliary conditions depicted in Fig.5(b). The idealization of the interfacial condition by a uniform hydrostatic pressure p was found to give an underestimate of K, while the assumption of sticking condition yielded an overestimate. The results have been also compiled in ref.[10] and indicate the importance of the use of the bond element.

4. Singularity Element

A quite number of numerical analysts have been zealous to incorporate singularities into the standard finite element analysis routine since the late 1960's. A variaty of methods or techniques have been proposed and summarized in a beautiful review article of Gallagher [11]. The method of authors in this direction is principally based on the mapping such as employed for the development of bond element in section 2.

As detailed in ref.[12], the authors have taken notice that the mapping of the conventional linear triangular element from the parametric plane to the physical one is represented by

$$x - x_1 = [\frac{1-\eta}{2}(x_2-x_1)+\frac{1+\eta}{2}(x_3-x_1)]\rho, \quad y - y_1 = [\frac{1-\eta}{2}(y_2-y_1)+\frac{1+\eta}{2}(y_3-y_1)]\rho \tag{25}$$

where

$$\rho = L_2 + L_3 = 1 - L_1, \quad \eta = (L_3-L_2)/(L_3+L_2)$$

x_1, y_1 denote coordinates of nodal point and L_1, L_2, L_3 are area coordinates as shown in Fig. 2(b). Singular transformation replaces ρ in eq.(25) by $\rho^{1/\lambda}$ and it can be shown that the mapping brings forth the ρ^{λ} singularity of the strain and stress distribution in the region adjacent to a generic node 1.

As an example of the application of singular transformation, the stress intensity factor at the crack tip of compact tension specimen of Fig.6(a) is determined. Fig.6(b) gives the stress intensity factor in nondimensional form and shows how it depends on the distance from the center of the specimen. It is found that the results correlate satisfactorily well with the solutions of Yamamoto et al. obtained by a more or less sophisticated procedure [13].

CONCLUDING REMARKS

Present paper has shown that the bond and singularity elements can be easily incorporated into the standard finite element computation routine by the mapping technique. The result is a considerable expansion of the scope of the finite element structural analysis. The future direction is the unification of the technique with the hierarchy ranking basis transformation outlined in ref.[12] by the authors as well as the concept of adaptive finite element [14]. By virtue of the development, we expect that not only the accuracy of finite elemenet solutions would be significantly improved but also the solution of huge three dimensional problems would be within our reach in the near future.

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