

## A Local Damage Model for Creep Crack Growth

Zhengdong WANG, Dongdi WU  
East China University of Chemical Technology, Shanghai, China

### ABSTRACT

Based upon the local damage hypothesis and the principle of similarity, a theoretical model for the calculation of creep crack growth rate has been derived in this paper. Creep crack growth tests under constant load and under constant load-pin-displacement-rate have been carried out at 550°C and 565°C in a  $2\frac{1}{2}Cr-1Mo$  steel to verify the theory. A new concept of creep constraint has been established to meet the needs of the local damage model.

### 1 INTRODUCTION

In recent years, it has been widely noticed that the Fracture Mechanics to creep crack growth has many limitations because of blunt crack tip. Continuum Damage Mechanics seems to be more realistic, and its application in modelling creep crack initiation and propagation has been studied. Riedel<sup>[1]</sup> applied the continuous damage theory of Hayhurst<sup>[2]</sup> to study creep crack growth and pointed out that the  $C^*$  parameter is still valid under small scale damage.

A local damage model for predicting creep crack propagation rates will be presented in this paper and comparisons will be made with experimental data.

### 2 THEORETICAL ANALYSIS

#### 2.1 The local damage zone hypothesis

Assume a local damage zone next to the crack tip, encompassing several grains, with the size of  $r_\omega$ . This damage zone proceeds forward with the propagation of the crack and a narrow damage strip is formed. It is also assumed that the mechanical behaviour of the material inside the local damage zone follows the following constitutional equations:

$$\dot{\epsilon}_{ij} = \frac{3}{2} A \bar{\sigma}^{n-1} S_{ij} (1-\omega)^{-n} \quad (1)$$

$$\dot{\omega} = \frac{D(\bar{\sigma})^k}{(1+\psi)(1-\omega)^\psi} \quad (2)$$

where  $\dot{\epsilon}_{ij}$  is the creep strain-rate tensor,  $S_{ij}$  is the stress deviator,  $\bar{\sigma}$  is

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the Von Mises effective stress which is adopted in Eq.(2) for a creep ductile material,  $\omega$  is a damage variable,  $A$  and  $n$  are the material constants of the creep law and  $D$ ,  $k$  and  $\psi$  are the material constants of the damage law.

## 2.2 A theoretical model for creep crack growth

The asymptotic field characterized by the creep crack parameter  $C^*$  is a HRR-type stress field<sup>[1]</sup>:

$$\sigma_{ij} = \left[ \frac{C^*}{I_n A r} \right]^{\frac{1}{n+1}} \bar{\sigma}_{ij}(\theta, n) \quad (3)$$

where  $\sigma_{ij}$  is the stress tensor,  $I_n$  is a function of  $n$  usually expressed as 3 for plane stress and 4 for plane strain<sup>[1]</sup>,  $r$  and  $\theta$  are the polar coordinates in the crack tip field, and  $\bar{\sigma}_{ij}(\theta, n)$  is a dimensionless distribution function.

According to principle of similarity, through dimension analysis<sup>[1]</sup>, the crack tip strain-rate field related to local creep damage can be expressed as follows:

$$\dot{\epsilon}_{ij} = \frac{D}{1+\psi} \left[ \frac{C^*}{I_n A r} \right]^{\frac{k}{n+1}} \bar{\epsilon}_{ij}(\theta, n) \quad (4)$$

where  $\bar{\epsilon}_{ij}(\theta, n)$  is another dimensionless distribution function.

For an arbitrary point in the damage zone, located at distance  $r$  from the crack tip, the strain  $\epsilon_{ij}$  at this point is assumed to be the integration of strain-rate from the boundary of damage zone ( $r=r_0$ ) to the specified point  $r$ , i.e.  $\int_{r_0}^r \dot{\epsilon}_{ij} dt$  <sup>[4]</sup>, then we obtain (see Ref.[5] for detail)

$$\dot{\epsilon} = \frac{n+1}{n+1-k} \frac{D}{(1+\psi) \epsilon_f^*} \left[ \frac{C^*}{4A} \right]^{\frac{k}{n+1}} r_0^{1-\frac{k}{n+1}} \quad (5)$$

where  $\epsilon_f^*$  is local rupture strain. Once the material constants for creep and damage, the local creep ductility, and the  $C^*$  parameter are known, Eq.(5) can be used to predict creep crack growth rate.

## 2.3 The concept of local creep constraint

It is widely known that a creep ductile material can present creep brittle behaviour under complex stress condition. This is due to the high constraint in the specimen or structure. In order to establish a comprehensive local damage model, a quantified concept of local creep constraint is defined as the 'creep constraint factor'  $\zeta$ , expressed by the following formula:

$$\zeta = \left( 1 - \frac{\epsilon_f^*}{\epsilon_f} \right)^{\frac{n-1}{n}} \quad (6)$$

where  $\epsilon_f$  is the creep ductility of the material under uniaxial tension condition, which means the creep rupture strain of the material under no constraint ( $\zeta=0$ ) and every element of the test specimen fulfils the condition  $\epsilon_f^* = \epsilon_f$ . For creep ductile material ( $n > 1$ ), local creep rupture strain value decreases with the increase of the degree of constraint and  $\epsilon_f^* \rightarrow 0$  when  $\zeta \rightarrow 1$ .

For creep brittle materials  $\zeta \approx 1$ , because  $n \rightarrow 1$ . The creep constraint factor defined here implies the local embrittlement of creep ductile materials.

In the local damage model proposed in this paper, the value of local creep rupture strain is needed for calculation. If the local constraint factor of the specimen or the element of the structure is known, then the creep rupture strain of a local element can be estimated, i.e.

$$\epsilon_r^* = \epsilon_r (1 - \zeta^{\frac{n}{n-1}}) \quad (7)$$

In the case of a compact tension (CT) specimen with side-grooves, the local constraint factor of an element next to the crack tip should be related to the geometry of side-groove and the creep index of the material. Hence, for CT specimen, the expression is

$$\zeta = f\left(\frac{a}{W}, \frac{2R}{B_n}, \frac{B_n}{B}, n\right) \quad (8)$$

where  $W$  is the width of the specimen,  $B$  the gross thickness of the specimen,  $B_n$  the thickness at the root of the side-groove or the net thickness of the specimen,  $R$  the radius of curvature of the root of the side-groove.

#### 2.4 Equations for $C^*$ calculation

Many equations for the calculation of  $C^*$  parameter can be found in the literatures, the most widely used ones are the Harper-Ellison equation<sup>[6]</sup> and the Kubo equation<sup>[7]</sup>. For convenience, the Kubo equation is adopted in this paper. For CT specimen, it takes the form:

$$C^* = \frac{2n}{n+1} \frac{P\dot{\Delta}}{B(W-a)} \quad (9)$$

where  $P$  is the applied load and  $\dot{\Delta}$  is the load point displacement rate.

### 3 EXPERIMENT

#### 3.1 Material

The material used in our experiment is a  $2\frac{1}{2}\text{Cr-1Mo}$  steel taken from a test section of a hydrogenation reactor. From creep rupture tests of this material at 550°C and 565°C, the following constants have been obtained<sup>[5]</sup>:

$$\begin{aligned} n &= 9.73, \quad k = 9.13, \quad \psi = 8.92, \\ A &= 7.88 \times 10^{-27} (550^\circ\text{C}), \quad 1.73 \times 10^{-26} (565^\circ\text{C}), \\ D &= 5.19 \times 10^{-24} (550^\circ\text{C}), \quad 1.07 \times 10^{-23} (565^\circ\text{C}). \end{aligned}$$

#### 3.2 Creep crack growth tests

Two types of tests have been conducted: a) constant load (on creep testing machine model RD2-3) and b) constant load-pin-displacement-rate (on servo-hydraulic testing machine model INSTRON 8032). DCPD method is used in the measurement of crack length.

### 3.2.1 Constant load creep crack growth tests

The tests were carried out under two temperatures, 550°C and 565°C. The test specimens are CT specimens with side-grooves. The size of specimen and test data are listed in Tab.1.

Tab.1 Data of creep crack growth tests (constant load)

Specimen No.	$B_n/B$	$R$ mm	$W$ mm	$B$ mm	$B_n$ mm	$a_o$ mm	$T$ °C	$P$ kN	$\sigma_{nom}^*$ MPa	$t_i$ h	$t_f$ h
CTA2-2	3/4	0.20	25.0	12.8	9.6	12.52	550	6.00	505	100	234
CTB1-3	1/2	0.20	25.0	12.8	6.4	12.54	550	5.00	633	10	57
CTB3-4	1/2	2.00	25.4	12.8	6.2	13.15	550	4.00	546	36	93
CTC1-2	1/4	0.20	25.8	12.8	3.2	12.25	550	2.00	494	120	308
CTA1-2	3/4	0.25	25.1	13.0	10.0	12.50	565	4.90	387	420	910
CTA1-3	3/4	0.25	25.1	13.0	10.6	12.20	565	5.88	416	210	522
CTA1-4	3/4	0.25	25.2	13.0	10.4	12.41	565	6.86	509	20	120
CTA1-6	3/4	0.25	25.0	13.0	10.5	12.45	565	7.85	596	15	49
CTA3-1	3/4	0.25	25.2	13.0	10.0	8.50	565	10.79	456	30	225
CTA3-2	3/4	0.25	25.3	13.0	10.1	8.50	565	9.81	407	100	370

$$* \sigma_{nom} = \frac{P}{B_n(W-a)} \left(1 + 3 \frac{W+a}{W-a}\right)$$

### 3.2.2 Constant load-pin-displacement-rate creep crack growth tests

The tests were carried out under 550°C. The displacement rate varies from 0.001 to 1.0 mm/h. The specimens are CT specimens with side-grooves. The size of specimen and test data are shown in Tab.2.

### 3.2.3 Evaluation of creep constraint factor for CT specimen

Using Eqs.(7) and (8), the  $\zeta$  values and  $\epsilon_f^*$  values for the CT specimens used in our tests have been calculated and listed in Tab.3. The measured  $\epsilon_f^*$  values are obtained through the approximate equation:

$$\epsilon_f^* \approx -2 \ln \frac{B_n^*}{B_n} \quad (10)$$

where  $B_n^*$  is the net section thickness when local rupture occurs.

Tab.2 Data of creep crack growth tests (constant displacement rate)

Test No.	$B_n/B$	$r$ mm	$W$ mm	$B$ mm	$B_n$ mm	$a_o$ mm
CTA2-1	3/4	0.2	25.0	12.8	9.6	12.65
CTB2-1	1/2	0.2	25.0	12.8	6.5	12.50
CTB3-3	1/2	2.0	24.8	12.8	6.2	12.60
CTC1-1	1/4	0.2	24.9	12.8	3.3	12.60

Tab.3 Creep constraint of CT specimen

Type	$B_n/B$	$R$ mm	$\zeta$	$\epsilon_f^*$ (Cal.)	$\epsilon_f^*$ (Meas.)
CTA	3/4	0.2	0.85	0.32	0.33
CTB	1/2	0.2	0.88	0.27	0.28
CTB	1/2	2.0	0.83	0.38	0.36
CTC	1/4	0.2	0.90	0.21	0.20

## 4 DISCUSSIONS

### 4.1 Comparison with the NSW model

Nikbin, Smith and Webster<sup>(4)</sup> established a model for creep crack growth (so called NSW model) and derived a general expression for the prediction of creep crack growth rate:

$$\frac{da}{dt} = \frac{3}{\epsilon_f^*} (C^*)^{0.85} \quad (11)$$

where  $\epsilon_f^*$  is the available creep ductility at crack tip,  $\epsilon_f^* = \epsilon_f$  for plane stress and  $\epsilon_f^* = \epsilon_f/50$  for plane strain, where  $\epsilon_f$  is the creep ductility under uniaxial tension condition.

The differences between our model and the NSW model are:

1) In this paper, the concept of Damage Mechanics is the basis of analysis and has been extended to the analysis of local damage.  $\epsilon_f^*$  and  $\epsilon_{f,s}$  are treated as local parameters, thus, they are not the nominal strains ( $\epsilon_f$  and  $\epsilon_{f,s}$ ), but are the actual strains of the local element.

2) The concept of local creep constraint has been incorporated in this paper, setting up more strict rules for the estimation of the value of  $\epsilon_f^*$ .

Fig.1 to Fig.3 show the results of correlating creep crack growth data to  $C^*$ . Considering the fact that the specimens used in this paper are in a condition close to plane strain loading, prediction lines according to the NSW model (Eq.(11) and  $\epsilon_f^* = \epsilon_f/50$  for plane strain) are also included in those figures. It can be seen, the prediction lines according to our model ties in with the upper limit of the experimental data and agree with them reasonably well, whereas the NSW model prediction lines do not agree well with our experimental data.

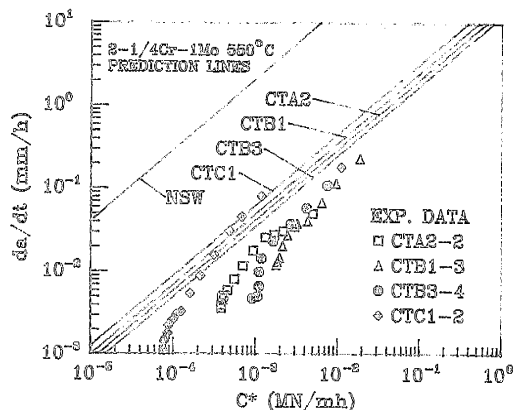


Fig.1 Prediction lines and experimental data (CL-CCG 550°C)

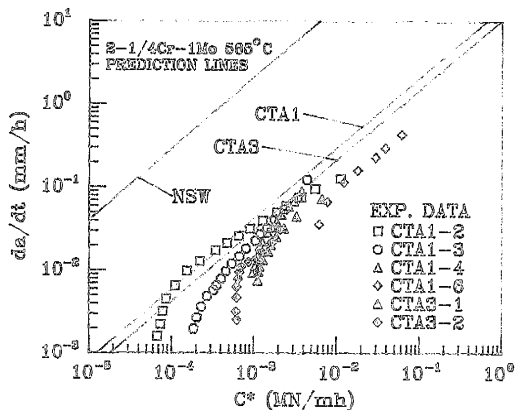


Fig.2 Prediction lines and experimental data (CL-CCG 565°C)

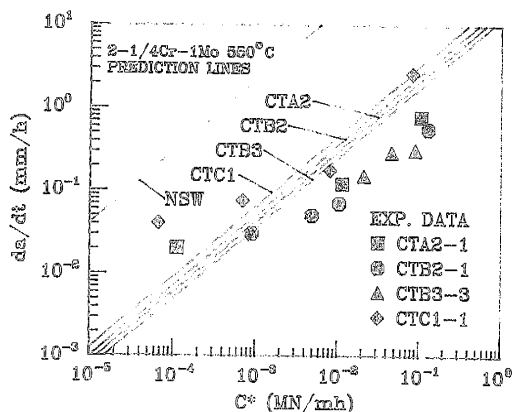


Fig.3 Prediction lines and experimental data (CSR-CCG)

#### 4.2 The influence of creep constraint factor on creep crack growth

In Fig.1 to Fig.3, several parallel lines are drawn to show the prediction of creep crack growth under different constraint factors. It can be seen that, with the increase of the constraint factor, the local creep ductility decreases and the creep crack growth rate increases. The same tendency is also shown in the test data. However, most of the data points lie lower than the prediction line, this is probably due to the fact that most tests are conducted under high stress levels or high load-pin-displacement-rates.

#### 4.3 The influence of test temperature on creep crack growth rate

Substituting the relevant data<sup>[5]</sup> into Eq.(5), the expressions for creep crack growth rate at different temperatures are as follows:

$$\begin{aligned} \dot{a} &= \frac{4.66}{\epsilon_f^*} (C^*)^{0.85} \quad (550^\circ C), \\ \dot{a} &= \frac{4.93}{\epsilon_f^*} (C^*)^{0.85} \quad (565^\circ C). \end{aligned} \quad (12)$$

It can be seen that creep crack growth rate increases with the rise of temperature. This point is also shown in the data plotted on Figs.1 and 2.

### 5 CONCLUSIONS

- 1) Based upon the local damage zone hypothesis, an equation for the calculation of creep crack growth rate, i.e. Eq.(5), has been derived. Predictions based upon this theoretical model agree reasonably well with experimental results.
- 2) The concept of local creep constraint is presented and the method for its calculation and measurement is given out in this paper. Creep crack growth experiments point out that the degree of creep constraint is a major factor influencing creep crack growth rate.
- 3) Creep crack growth tests under different temperatures show that the rates are higher at higher temperatures. This fact can be explained by the changes of material constants at different temperatures as reflected in the basic equation (Eq.(5)).

### 6 REFERENCES

- [1] Riedel, H.(1987), Fracture at High Temperatures, *Springer-Verlag*.
- [2] Hayhurst, D. R. and Leckie, F. A.(1983), Behaviour of Materials at High Temperatures, *ICM-4*, 4, pp.1195-1211.
- [3] Riedel, H. and Rice, J. R.(1980), Tensile Cracks in Creeping Solids, *ASTM, STP-700*, pp.112-130.
- [4] Nikbin, K. M., Smith, D. J. and Webster, G. A.(1983), Influence of Creep Ductility and State of Stress on Creep Crack Growth, *Int. Conf. Advances in Life Prediction Methods, ASME*, pp.249-258.
- [5] Wang, Z.(1990), A Local Damage Approach for Creep Rupture Study, Ph.D. Thesis, East China University of Chemical Technology, Shanghai, China.
- [6] Harper, M. P. and Ellison, E. G.(1977), The Use of the  $C^*$  Parameter in Predicting Creep Crack Propagation Rates, *J. Strain Anal.* 12(3), pp.13-31.
- [7] Kubo, S.(1975), Ph.D. Thesis, University of Osaka, Osaka, Japan.