Calculation Method of Floor Response Spectrum Based on Response Characteristic of Two-Degrees-of-Freedom Uncoupled System

A. Kurosaki, M. Kozeki, T. Ide
Nuclear Energy Systems Division, Mitsui Engineering and Shipbuilding Co., Ltd., 6-4, Tsukiji 5-chome, Chuo-ku, Tokyo 104, Japan

SUMMARY

Response spectrum method with floor response spectra is commonly used for seismic design of safety related systems such as nuclear power plant components and pipings which are appended to floor slabs of a supporting structure such as a reactor building. Therefore the reliability of the design of such appended systems depends on the floor response spectra.

To obtain floor response spectra, current design procedure employs the time-history computation using a few earthquake records or spectrum compatible time-histories. This procedure, though yields accurate results for a given time-history, is still unreliable for the earthquakes to which the structural system may be subject in the future when only a few time-histories are used. As this drawback is not only due to the variety in amplitude of a ground response spectrum but to the uncertain property of phase distribution and the time dependent unstationality, it is not avoided insofar as the procedure uses design earthquakes defined in the form of ground response spectra. An alternative definition of design earthquakes could be made in the form of the response spectrum of an oscilator which has sufficient but the least degrees-of-freedom to take account of the aforesaid uncertainty. For the purpose of obtaining floor response spectra, such response spectrum is clearly identified as that of appended systems of a two-degrees-of-freedom uncoupled system in which the upper mass and the lower mass correspond to the appended system and the supporting system respectively. If this new response spectrum is given as a design earthquake, the floor response spectra are obtained by sweeping the undamped natural frequency of the appended system, the total response of which is calculated by combining the maximum modal responses of the appended system in the two-degrees-of-freedom uncoupled system derived from solving the eigen value problem of the whole appendage and supporting strucutre system. To establish this new design approach,.it is required to find a method of model combination which would give a practical estimate of the total response.

This paper presents such a method. The method adopts a combination of algebraic summation and SRSS and is simple, yet reliable. The method pays a paticular attention to the phase relation between the modal responses of the appended systems. The validity and advantage of the proposed method is demonstrated by comparison with both the numerically exact solution derived from time-history computation and the more empirical SRSS method.

1. Introduction

To obtain floor response spectra, the current design procedure employs the time-hisotry computation using a few earthquake records or artificial time-histories. This procedure, though yields accurate results for a given time-history, is still unreliable for the earthquakes which a structural system may be subject to in the future when only a few time-histories are used [1-4]. With this drawback in mind, some authors have proposed alternative methods [1, 2, 4].

This study presents a more comprehensible approach in which the floor resopnse is predicted by combining the contributions of peak response acceleration of an oscillator appended to the modal systems of a suporting structure. To conduct the propoer modal combination is the major intrest of this paper. The input ground motion in the presented approach is not a ground response spectrum but a spectrum-wise reduction of the peak responses of appended oscillators in two-degrees-of-freedom uncoupled systems. Presenting such a design spectrum is beyond the scope of this paper.

2. Basic Equations

The equation of motion of a multi-degree-of-freedom system subject to base excitation, $\ddot{x}_g$, is given in matrix form by

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{1\}\ddot{x}_g \tag{1}$$

where [M], [C] and [K] are the mass, damping and stiffness matrices of the system respectively and {x} are the relative displacements to input motion. Assuming that normal modes exist, Eq.(1) is reduced to

$$\ddot{y}_j + 2H_j\Omega_j\dot{y}_j + \Omega_j^2 y_j = -\psi_j\ddot{x}_g \tag{2}$$

where y_j, H_j, Ω_j and ψ_j are repectively the j-th modal response of relative displacement, j-th modal damping, j-th modal frequency and j-th modal perticipation factor given by

$$\psi_j = (\{\phi_j\}^T[M]\{1\})/(\{\phi_j\}^T[M]\{\phi_j\}) \tag{3}$$

where $\{\phi_j\}$ is the j-th mode shape vecor. The response acceleration is given by

$$\ddot{y}_j + \psi_j\ddot{x}_g = \psi_j\int_0^t \ddot{I}_j(t-\tau)\ddot{x}_g(\tau)d\tau \tag{4}$$

where $\ddot{I}_j(t)$ is the impulse response function of acceleration. The response acceleration of i-th degree-of-freedom is written as

$$\ddot{x}_i = \sum_{j=1}^{n}\phi_{ij}\left(\psi_j\int_0^t \ddot{I}_j(t-\tau)\ddot{x}_g(\tau)d\tau - \psi_j\ddot{x}_g\right) \tag{5}$$

The absolute acceleration is

$$\ddot{x}_i + \ddot{x}_g = \left(1-\sum_{j=1}^{n}\phi_{ij}\psi_j\right)\ddot{x}_g + \sum_{j=1}^{n}\phi_{ij}\psi_j\int_0^t \ddot{I}_j(t-\tau)\ddot{x}_g(\tau)d\tau \tag{6}$$

In Eq.(5) and Eq.(6), ϕ_{ij} is i-th degree-of-freedom component of j-th mode shape vector and n is the number of degrees of freedom. If the set of modes is complete, the following relation exists.

$$1-\sum_{j=1}^{n}\phi_{ij}\psi_j = 0 \tag{7}$$

Therefore, Eq.(6) is reduced to

$$\ddot{x}_i + \ddot{x}_g = \sum_{j=1}^{n}\phi_{ij}\psi_j\int_0^t \ddot{I}_j(t-\tau)\ddot{x}_g(\tau)d\tau \tag{8}$$

When a single-degree-of-freedom oscillator, k, is appended on the i-th mass and the appended mass is negligibly light compared to the supporting mass, the equation of motion is given as

$$\ddot{z}_k + 2h_k\omega_k\dot{z}_k + \omega_k^2 z_k = -(\ddot{x}_i + \ddot{x}_g) \tag{9}$$

where z_k, h_k and ω_k are respectively the relative displacement to the movement of the supporting mass, the damping ratio and the natural frequency of the appended oscillator. The response acceleration of the oscillator is given by

$$\ddot{z}_k + \ddot{x}_i + \ddot{x}_g = \sum_{j=1}^{m} \Phi_{ij}\Psi_j\ddot{X}_{kj} \tag{10}$$

where $\ddot{X}_{kj}$ is the response acceleration of an appended oscillator in a two-degrees-of-freedom uncouled system and is given as

$$\ddot{X}_{kj} = \int_0^t \ddot{I}_k(t-\tau)\int_0^t \ddot{I}_j(t-\xi)\ddot{x}_g(\xi)\,d\xi d\tau \tag{11}$$

Assuming the applicability of SRSS modal combination to obtain total response, the following equation can be derived from Eq.(10) for a floor response spectrum, S_F.

$$S_F(\omega_k, h_k)_i = \sqrt{\sum_{j=1}^{m} \Phi_{ij}^2\Psi_j^2 S_{ki}^2} \tag{12}$$

where $$S_{kj} = |\ddot{X}_{kj}|\max \tag{13}$$

Concepts of amplification curve used to determine S_{kj} have been introduced by some authors [1, 4, 5] and they may be integrated into Eq.(12) as discussed in the references. The emphasis in this paper, however, is on the modal combination and the use of such amplificatin curves is excluded to avoid clouding the issue with additional variables.

3. An example problem

An simple model is now given to illustrate some of the problems of the procedure outlined in the previous section. The model composed of two floor slabs has the inertia properties of the lumped mass and the stiffness properties as shown in Fig. 1, thus the mode shapes and associated eigenparameters are to be as in Fig. 2. The fundamental frequency of the model is arbitrarily selected by varying the ratio of stiffness to mass, k/m. Three cases in Table 1 are selected to evaluate the applicability of the method to variety of structures. All damping ratios are 5% for supporting structures and 1% for appended oscillators throughout the paper. The base exitation is El Centro (1940) NS. The periods at which the floor responses are computed are those arbitrarily selected from 0.03 seconds through 3 seconds besides those corresponding to the natural periods of the structure system. Fig. 3 shows the comparison of the floor response spectra predicted by SRSS (Eq.(12)), using the values of S_{kj} (Eq.(13)) which are known in advance, with those prediced by the time-history computation.

It should be noted that the SRSS method is too conservative for mass-point 1 and unconservative for mass-point 2 at the periods longer than the first modal period, but is otherwise very similer to the time-history computation. This tendency is observed in each of the three cases and it becomes more dominant as the system natural periods become shorter.

4. Consideration on phasing of modal responses

The problem of the SRSS noted in the previous section can be connected with the phasing of the modal resopnses where the naural frequencies, ω_j, of those modes are greater than the frequency, ω_k, at which the floor response is to be calculated ($\omega_k < \omega_j$). When ω_j is greater than ω_k and they are not closely spaced to each other, the frequency contents at ω of $\omega_j < \omega$ are filtered out through the supporting system and the appended system of the modal system under consideration. Whereas, the frequency contents at ω of $\omega < \omega_j$ is amplified through the supporting system subject to the relation between ω and ω_j and, thereafter, the contents around ω_k are amplified again through the appended system, but the contents around ω_j are filtered out. Therefore, the appended system responds essentially at the frequency ω_k irrespective of the natural frequency of the supporting system. This frequency component of ω_k is likely to be in phase among those modes because of the relation of $\omega_k < \omega_j$, and thus the modal responses are to be additive.

This relationship is true for harmonic exciation of the frequency ω_k. It is also true for the modes of rigid body vibration excited by an earthquake type motion since the appendage responds as though it were supported directly on the ground. For flexural modes, refer to Table 2 which shows the time (t) to yield maximum response acceleration in the first row of each box and the maximum resopnse acceleration in the second row of each box, both of which are calculated for the frequency ratio, $R = \omega_j/\omega_k$, and the natural period of the appended system, Ta. The bottom row of the table shows the time (t_o) to yield the maximum resopnse acceleration of the appendage on a very rigid supporting system ($R = \infty$), and the third row of each box shows the response at the time (t_o). Except when R is very close to 1.0, the time, t, coincide the time, t_o, or at least the maximum response acceleration is similar to the response acceleration at t_o. Fig. 4 shows the difference between t and t_o in the form of phase angle defined by

$$\theta = \omega_k \ (t - t_o) \tag{14}$$

This phase angle is generally some number times π radians even for the case where t does not coincide to t_o. This means that the aforesaid relationship is applicable for flexural modes except when R is too close to 1.0.

Now it can be concluded that the problems encountered in the example of the previous section stem from the use of SRSS modal combination in Eq.(12) for the cases where the modal resonses must be additively combined.

5. Proposed method of modal combination

What is required for the combination of modes where ω_k is lower than ω_j is to take direct sum of those modal results. The following equation is based on the more correct algebraic inter-relation of the modes than SRSS modal combination expessed as Eq.(12).

$$S_F(\omega_k, \ h_k)_i = \sqrt{\sum_{j=1}^{\ell} \phi_{ij}^2 \psi_j^2 S_{kj}^2 + \left(\sum_{j=\ell+1}^{n} \phi_{ij} \psi_j S_{kj} \right)^2} \tag{15}$$

Where $\varepsilon\omega_k > \omega_j$ for $j = 1 \text{ --- } \ell$, $\varepsilon\omega_k < \omega_j$ for $j = \ell + 1 \text{ --- } n$ and ε is recommended to be 1.1 or 1.2. Instead of taking all modes up to n-th mode into account, one can separate higher modes of rigid body vibration from frexural modes up to m-th mode by using the following relation derived from Eq.(7).

$$\sum_{j=\ell+1}^{n} \phi_{ij} \psi_j S_{kj} = \sum_{j=\ell+1}^{m} \phi_{ij} \psi_j S_{kj} + S_{kg} \left(1 - \sum_{j=1}^{m} \phi_{ij} \psi_j\right) \tag{16}$$

where S_{kg} is the ground response spectrum at ω_k for h_k. Then, Eq.(15) is replaced by the more practical equation as Eq.(17) using only the frexural modes.

$$S_F(\omega_k, h_k) = \sqrt{\sum_{j=1}^{\ell} \Phi_{ij}^2 \Psi_j^2 S_{kj}^2 + \{\sum_{j=\ell+1}^{m} \Phi_{ij}\Psi_j S_{kj} + S_{kg}(1 - \sum_{j=1}^{m} \Phi_{ij}\Psi_j)\}^2} \quad (17)$$

The floor response spectra predicted by Eq.(17) are displayed in Fig. 3 where they are compared to those by the time-history computation and those by SRSS (Eq.(12)). The distinct difference is observed between Eq.(17) and SRSS. The results predicted by Eq.(17) are very similar to those by the time-history computation throughout the whole frequency range.

6. Application to an multiple-degrees-of-freedom system

The proposed method is applied to a simply modeled reactor building of a typical BWR power plant. The model and its modal characteristics are shown in Fig. 5. The lowest seven modes are involved in the calculation. The input motions are El Centro (1940) NS and Miyagi-ken-oki (1978) NS scaled to 1.0G. The results shown in Fig. 6 are the floor response spectra on mass-point 1, 5 and 10.

The prediction by Eq.(17) is again very similar to that by the time-history computation. The only problem remains at mass-point 10 where underestimates are liable to occur at short periods. This problem stems from the use of SRSS modal combination and is common for Eq.(17) and SRSS.

7. Conclusions

To easily obtain reliable floor response spectra, a new approach was cnducted. In this approach, the floor response is calculated by combining the maximum responce accelerations of appendage in two-degrees-of-freedom uncoupled modal systems. The modes are combined by taking algebraic summation and then SRSS as proposed by Eq.(17).

The validity and advantage of this approach was demonstrated by comparing it both with the time-history computation and the case employing the empirical SRSS method in the modal combination.

8. Acknowledgement

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Table 1 Natural Frequencies

	CASE 1	CASE 2	CASE 3
1ST MODE	5.00	10.00	20.00
2ND MODE	8.09	16.18	32.36

Table 2 Maximum Response Acceleration and Time of the Occurrence

FREQ. RATIO R	NATURAL PERIOD OF APPENDAGE Ta (SEC)				
	0.05	0.1	0.2	0.5	1.0
1.1	2.605 -6.291 -5.070	5.210 -12.596 -9.948	3.505 -15.469 14.041	3.180 14.435 -7.106	5.455 10.882 5.776
1.2	2.530 4.140 -3.295	5.010 -8.832 -8.557	3.495 -9.874 8.958	3.140 10.455 -7.084	4.925 -8.404 7.140
1.3	2.505 -3.577 -2.870	5.005 -6.489 -6.489	3.490 -7.796 6.652	2.880 -8.138 -6.264	4.390 5.625 5.625
1.4	2.505 -3.061 -2.658	5.005 -5.229 -5.229	3.205 6.469 6.404	2.870 -6.700 -5.473	4.905 -4.395 3.857
1.5	2.505 -2.665 -2.390	5.005 -4.636 -4.636	2.695 -5.952 5.523	2.635 5.905 -5.108	4.890 -4.461 3.608
1.6	2.500 -2.374 -2.333	5.005 -4.170 -4.170	2.685 -5.130 4.918	2.410 -5.397 -5.058	4.865 -3.805 3.803
1.8	-	5.005 -4.140 -4.140	3.200 4.513 4.513	2.405 -4.972 -4.790	4.890 -3.652 2.491
2.0	-	5.005 -3.848 -3.848	3.200 4.145 4.145	2.395 -4.815 -4.765	4.860 -3.265 2.698
2.5	-	5.005 -3.430 -3.430	3.200 3.773 3.773	2.385 -4.027 -4.027	4.835 -2.744 2.446
3.0	-	5.005 -3.242 -3.242	3.200 3.589 3.589	2.375 -3.530 -3.514	4.415 2.331 2.312
4.0	-	-	3.200 3.420 3.420	2.390 -3.377 -3.374	4.400 2.312 2.304
5.0	-	-	3.200 3.344 3.344	2.385 -3.302 -3.302	4.850 -2.237 2.221
7.0	-	-	-	2.385 -3.224 -3.224	4.395 2.202 2.201
10.0	-	-	-	2.385 -3.180 -3.180	4.390 2.170 2.170
∞	2.455	5.005	3.200	2.385	4.390

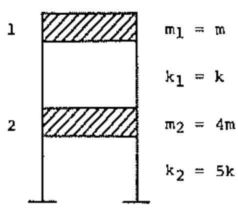


Fig. 1 A Two-degrees-of-freedom Model

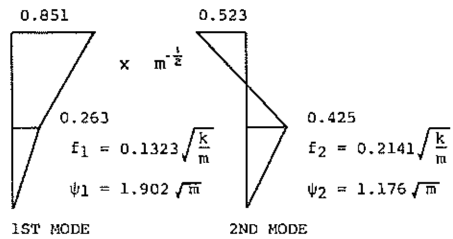


Fig. 2 Mode Shapes and Eigenparameters

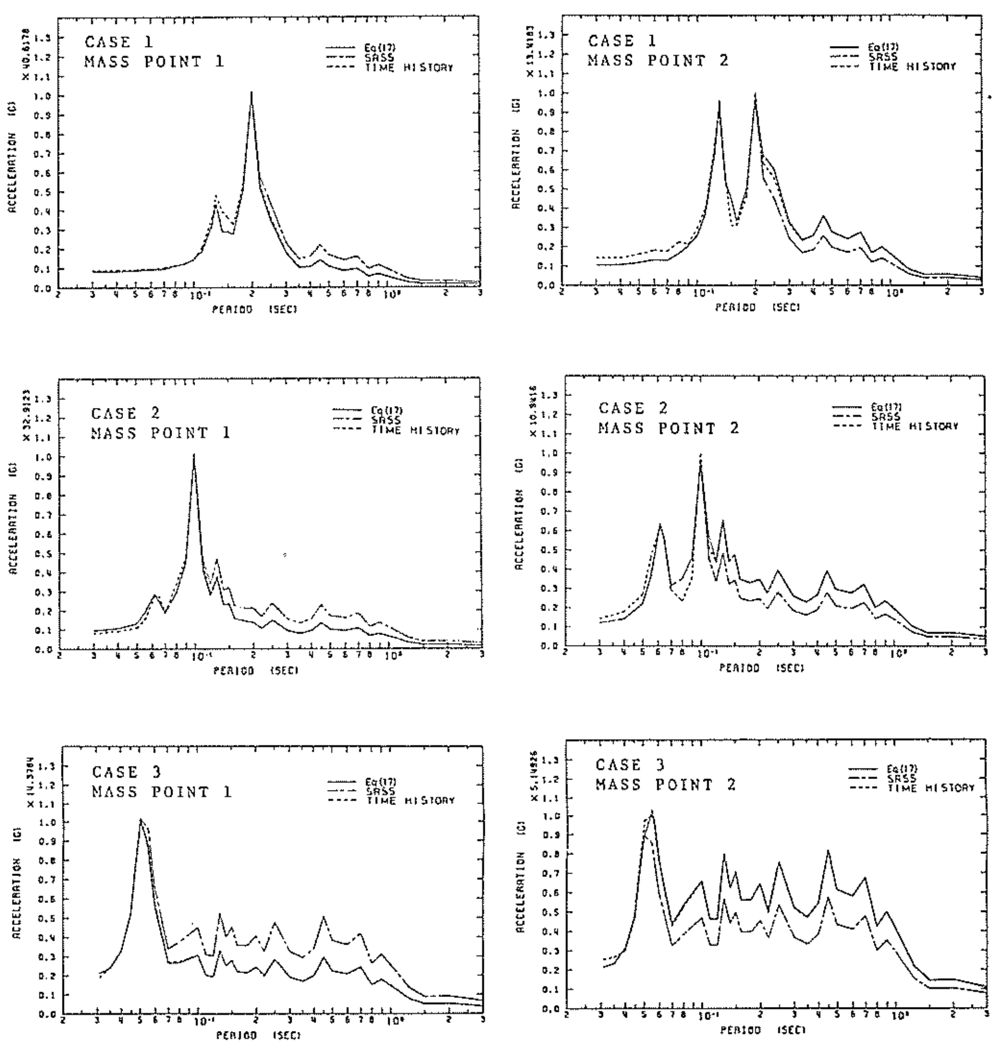


Fig. 3 Floor Response Spectra of the Two-degrees-of-freedom Model

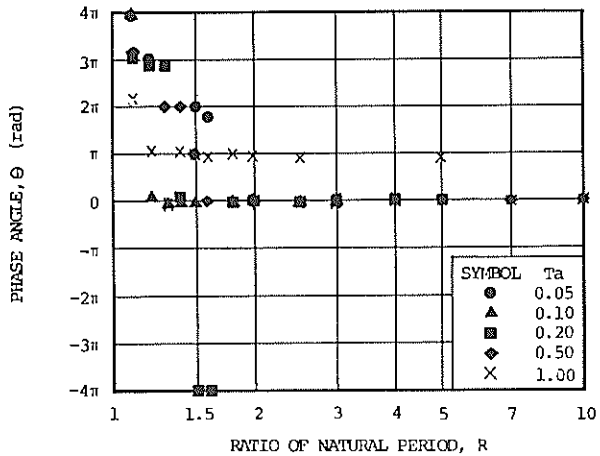
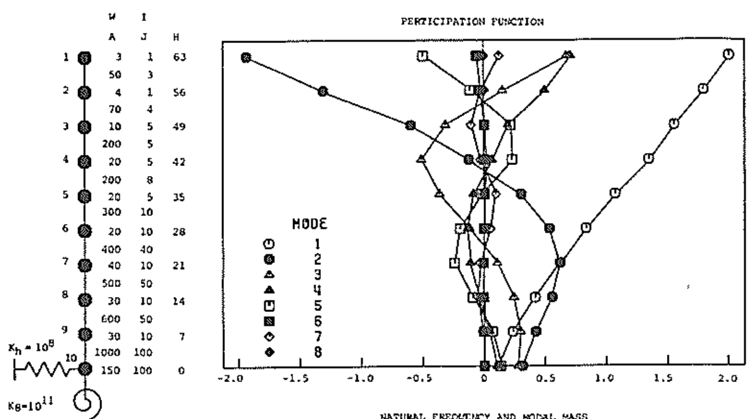
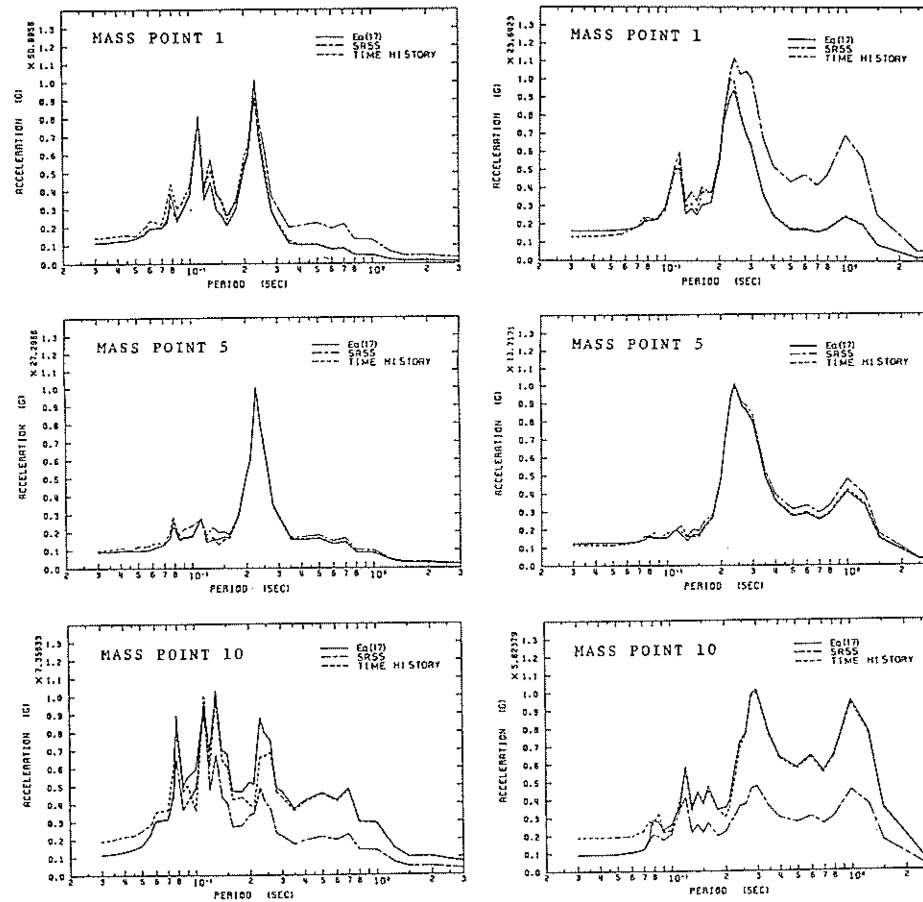


Fig. 4 Phase Angle



MODE	1	2	3	4	5	6	7	8	9	10
NAT. PERIOD sec	0.2275	0.1122	0.0783	0.0752	0.0571	0.0556	0.0408	0.0360	0.0319	0.0298
NAT. FREQ. Hz	4.39	8.91	12.77	13.30	17.52	17.97	24.51	27.75	31.38	33.50
MODAL MASS %	48.15	30.47	13.08	4.54	3.46	0.01	0.20	0.00	0.04	0.04

Fig. 5 A Multiple-degrees-of-freedom Model and the Dynamic Characteristics



(1) El Centro NS (2) Miyagi-ken-oki NS

Fig. 6 Floor Response Spectra of the Multiple-degrees-of-freedom Model