Modal Spectral Analysis of Piping: Determination of the Significant Frequency Range

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This paper investigates the influence of the number of modes on the response of a piping system in a dynamic modal spectral analysis. It shows how the analysis can be limited to a specific frequency range of the pipe (independent of the frequency range of the response spectrum), allowing cost reduction without loss in accuracy. The "missing mass" is taken into account through an original technique.

In order to evaluate the response of a typical piping system as a function of the number of modes, a theoretical distribution of the modal mass with frequency is assumed, and upper and lower bounds are identified.
Good correlation is found with actual results from modal analysis of real piping systems. The generic response of the piping (displacements, forces, loads on supports) is analysed with this simplified model, and frequency bandwidths (i.e. yielding more than 90 % of the total response) are identified for the different parameters.

The results are compared with the corresponding values obtained from the modal spectral calculation of a true piping system.
Again, good correlation is found between the simplified approach and the actual values, allowing the definition of frequency bandwidths of the piping. Causes of possible discrepancies are identified : modal mass density discrete and not uniformly distributed, sophisticated combination rules of the individual modal contributions (closely-spaced modes) in the real analysis, deviation of the support span length from reference values.

The mass corresponding to higher modes behaves like a rigid body (the higher modes see the input motion as a quasi-static motion) and is uniformly submitted to the maximum floor acceleration (ZPA of the floor response spectrum); its contribution must be taken into account using a residual load ("missing mass") technique. None of the methods previously proposed to take this effect into account suggested a satisfactory combination rule of the missing mass effect with the dynamic ones.
A modified spectral technique is proposed here to this aim.

1. INTRODUCTION.

When compared with other MDOF systems, a piping system exhibits particular properties leading to specific conclusions :

- the geometrical shape is usually simple, yielding a dominant deformation process (bending)
- the number of support points is high : seismic restraining of the piping yields a large number of supports.

The selection of seismic supports is based on spacing criteria : the span length is limited in order to keep the stresses within allowable values. Using such criteria yields piping systems with a first natural frequency within a narrow range; in particular, this frequency depends mainly upon the mechanical properties and the design acceleration; the dependence upon the geometrical data (diameter, thickness) becomes less significant.

This frequency can be estimated a priori in each particular case, whence the cost of the dynamic modal spectral analysis of the piping depends only upon the number of modes.

This preferential situation however does not extend to "accessories" differently shaped like valve operators, etc..

2. RESPONSE OF A TYPICAL PIPING SYSTEM AS A FUNCTION OF THE NUMBER OF MODES.

In a modal spectral analysis, the displacement response A_{ij} of the i-th mode at point j is given by (CLOUGH [1])

$$A_{ij} = \mathcal{L}_i \, \phi_{ij} \, \mathcal{S}_i \quad , \qquad (1)$$

where one single direction is considered for simplification.

In this equation,

$\mathcal{L}_i$ is the modal participation factor of the i-th mode,

ϕ_{ij} is the amplitude at point j of the i-th eigenvertor,

$\mathcal{S}_i$ is the spectral acceleration.

Using the normalization convention,

$$\underset{\sim}{\Phi}^T \underset{\sim}{m} \underset{\sim}{\Phi} = 1 \quad , \qquad (2)$$

where $\underset{\sim}{m}$ is the mass matrix, and assuming that the modal amplitude (maximum value of ϕ_{ij} with j for each i) does not decrease significantly when the frequency increases, yields a dependence of the response as given by eq. (1) only w.r.t. the modal participation factors ($\mathcal{L}_i$) and the spectral amplitude ($\mathcal{S}_i$). The effective modal mass is given (CLOUGH [1]) by

$$\mu_i = \mathcal{L}_i^2 \quad . \qquad (3)$$

The first part of this paper consists of an analysis of the response of a system the mass of which is distributed in the modes within the frequency through an inverse power law.

Two different power laws are tested :

a) $\mathcal{L}_i^2 = A/\omega_i$ (option A); (4)

The $1/\omega_i$ series corresponds to the lower bound : the sum of this series diverges, and the $\mathcal{L}_i^2$ sum must converge as far as the mass is finite;

b) $\mathcal{L}_i^2 = B/(\omega_i)^2$ (option B). (5)

This rate of decrease is obtained in the single span simply supported beam.

The range between options A and B covers the usual exponents obtained for real piping systems. In the example shown at fig. 1 (evolution of the effective mass in the three directions with the frequency for a RHR 12" line) the computed mean exponents were respectively

$$\varepsilon_x = -1.497 \quad , \quad \varepsilon_y = -1.523 \quad , \quad \varepsilon_z = -1.276 \; . \tag{6}$$

The floor response spectrum is assumed to be flat ($\mathcal{G}_i$ constant in eq. (1)) whence the response depends only upon the mass distribution.

The SRSS acceleration A_j and displacement D_j responses at point j are given by

$$A_j = \Big[\sum_{i=1}^{N} (A_{ij})^2 \Big]^{1/2} \quad , \quad D_j = \Big[\sum_{i=1}^{N} \{ A_{ij} / (\omega_i)^2 \}^2 \Big]^{1/2} \; . \tag{7}$$

Using the final assumption of a continuous and uniform spectral density of eigenmodes allows to devote to each frequency band $d\omega$ an effective mass dM , and to define a modal mass density $m(\omega)$ by

$$dM = m(\omega)\, d\omega . \tag{8}$$

The mass densities $m(\omega)$ corresponding to options A and B in eq. (4) and (5) are respectively

$$m_A(\omega) = M_A/\omega \quad , \quad m_B(\omega) = M_B/\omega^2 \; . \tag{9}$$

and eq. (7b) becomes

$$D_j = \Big[\int_{\omega_0}^{\infty} \{ a_j(\omega)/\omega^4 \}\, d\omega \Big]^{1/2} \quad , \tag{10}$$

where

$$a_j = m(\omega)\, \phi_j^2\, \mathcal{G}_j(\omega) \; . \tag{11}$$

It is well known (CLOUGH [1]) that the eigenvertors of a beam have the following form

$$X_i = A_i \cos\big[\sqrt{\omega_i}\, x\big] \cos \omega_i t , \tag{12}$$

whence double spatial derivation of the displacement to get moments and stresses (resp. triple derivation for shear forces) yields a coefficient ω (resp. $\omega^{3/2}$); eq. (10) becomes then

$$\Sigma_j = \left[\int \{ a_j(\omega) / \omega^2 \} \, d\omega \right]^{1/2} , \qquad (13a)$$

$$F_j = \left[\int \{ a_j(\omega) / \omega \} \, d\omega \right]^{1/2} , \qquad (13b)$$

where Σ_j and F_j are the bending moment and shear force at point j.

Introducing eq. (9) in eq. (10) and (13) yields for options A and B respectively

a) option A

$$D = d_A / \omega_o^2 \quad , \quad \Sigma = \frac{\sqrt{2}\, d_A}{\omega_o} \quad , \quad F = \frac{2 d_A}{\sqrt{\omega_o}} \quad , \qquad (14)$$

b) option B

$$D = \frac{d_B}{\omega_o^{5/2}} \quad , \quad \Sigma = \sqrt{\frac{5}{3}} \, \frac{d_B}{\omega_o^{3/2}} \quad , \quad F = \sqrt{\frac{5}{2}} \, \frac{d_B}{\omega} \quad . \qquad (15)$$

Defining as $d(\omega)$, $\sigma(\omega)$ and $f(\omega)$ the ratio of the cumulative response of all the modes up to frequency ω, to the total response, for the displacement D, bending moment Σ and shear force F yields.

a) option A

$$d(\omega) = \left[1 - \left(\frac{\omega_o}{\omega} \right)^4 \right]^{1/2} , \sigma(\omega) = \left[1 - \left(\frac{\omega_o}{\omega} \right)^2 \right]^{1/2} , \ f(\omega) = \left[1 - \frac{\omega_o}{\omega} \right) ; \qquad (16)$$

b) option B

$$d(\omega) = \left[1 - \left(\frac{\omega_o}{\omega} \right)^5 \right]^{1/2} , \sigma(\omega) = \left[1 - \left(\frac{\omega_o}{\omega} \right)^3 \right]^{1/2} , \ f(\omega) = \left[1 - \left(\frac{\omega_o}{\omega} \right)^2 \right]^{1/2} . \qquad (17)$$

Fig. 2 shows the evolution of $d(\omega)$, $\sigma(\omega)$ and $f(\omega)$ with the ratio $\frac{\omega}{\omega_o}$ for the two options : it is shown that

a) more than 90 % of the displacement and moment responses are obtained with a range $(\omega_o, 2.3\ \omega_o)$;

b) the computation beyond 5 ω_o becomes superfluous; the lowest convergent response is satisfactorily defined within this range, even with option A.

The evolution of the variables $d(\omega)$, $\sigma(\omega)$ and $f(\omega)$ has been studied for real piping cases. Typical results are shown on fig. 3; the main conclusions are

- as expected, the convergence rate decreases from $d(\omega)$ to $\sigma(\omega)$ and $f(\omega)$;
- option B is closer to the real cases than option A;
- the selection of a frequency bandwidth $(\omega_o, 4\ \omega_o)$ is acceptable.

3. EVALUATION OF THE DISPERSION.

The behaviour $d(\omega)$, $\sigma(\omega)$, $f(\omega)$ described above (eq. (16, 17)) is of course typical; the evolution of the real response with the number of modes may be different; the following reasons have been identified :

- modal density discontinuous and non uniform
- product $\mathcal{L}_i \phi_{ij}$ not constant
- combination of individual modal contributions through a "corrected" SRSS rule when closely spaced modes exist (R.G. 1.92)
- variations in the span length.

Cumulating those effects yields a "dispersion range" which has been evaluated from several dynamic analysis on real piping configurations to 25 % of the bandwidth.

4. MISSING MASS CORRECTION.

We consider a structure with eigenfrequencies both in the "flexible" ($\omega_i \leq \omega_{ZPA}$) and in the "rigid" ($\omega_i > \omega_{ZPA}$) range of the spectrum; this applies in particular to piping like structures, wherein the number of modes is high and the modal participation factors do not decrease notably with increasing frequencies.

The response of piping systems to dynamic events, computed by the use of modal spectral techniques, has been shown to have a specific property, due to the very high number of support points : the fraction of effective mass taken into account in the dynamic analysis (i.e. in the frequency range lying in the amplified zone of the response spectrum) is very low (typically in the range 20 to 40 %), in contrast with other structures where the small number of constraints yields usually ratios higher than 90 %

We consider for simplification

a) a one-dimensional earthquake

b) a tri-linear spectrum (fig. 4)

$$\begin{aligned}
\varphi(\omega) &= \varphi_1 && , \omega \leq \omega_p , \\
\varphi(\omega) &= \varphi_2 && , \omega > \omega_{ZPA} , \\
\varphi(\omega) &= \varphi_1 \exp \{ [\ln (\omega/\omega_p) / \ln (\omega_{ZPA}/\omega_p)] \ln (\varphi_2/\varphi_1) \} && , \omega_p < \omega \leq \omega_{ZPA} . \quad (18)
\end{aligned}$$

c) the effect of closely spaced modes is not considered (pure SRSS combination rule)

4.1. The tradional modal analysis yields the following response,

$$R = \left\{ \sum_{j=1}^{n_1} [\Phi_j \mathcal{L}_j \mathcal{S}_1]^2 + \sum_{k=1}^{n_2} [\Phi_k \mathcal{L}_k \mathcal{S}(\omega_k)]^2 + \sum_{l=1}^{n_3} [\Phi_l \mathcal{L}_l \mathcal{S}_2]^2 \right\}^{1/2} \tag{19}$$

where

$$n_1 + n_2 + n_3 = n ,$$
$$\omega_j \leq \omega_p \; (j=1, n_1) , \quad \omega_p < \omega_k \leq \omega_{ZPA} \; (k=1,n_2), \quad \omega_l > \omega_{ZPA} \; (l=1, n_3). \tag{20}$$

The influence of the higher modes ($\omega > \omega_{ZPA}$) is negligible ($<$ 0.05) as far as $\mathcal{S}_2 / \mathcal{S}_1 \leq$ 0.3.

4.2. Modal analysis and classical missing mass correction.

The spectral analysis is performed up to the ZPA frequency and a static correction is made on the higher modes (POWELL [2] , LE GUERN [3]).

Two options exist :

$$R^* = \left\{ \sum_{j=1}^{n_1} [\Phi_j \mathcal{L}_j \mathcal{S}_1]^2 + \sum_{k=1}^{n_2} [\Phi_k \mathcal{L}_k \mathcal{S}(\omega_k)]^2 \right\}^{1/2} + \sum_{l=1}^{n_3} |\Phi_l \mathcal{L}_l \mathcal{S}_2| , \tag{21}$$

$$R^{**} = \left\{ \sum_{j=1}^{n_1} [\Phi_j \mathcal{L}_j \mathcal{S}_1]^2 + \sum_{k=1}^{n_2} [\Phi_k \mathcal{L}_k \mathcal{S}(\omega_k)]^2 \right\}^{1/2} + \mathcal{S}_2 \, [1 - \mathcal{M}_{eff}/\mathcal{M}_{tot}] \, R_s , \tag{22}$$

where R_s is the static response to a unit excitation, $\mathcal{M}_{eff}$ is the effective mass taken into account in the analysis and $\mathcal{M}_{tot}$ is the total mass.

Eq. (21) requires the determination of all the significant modes (i.e. all the modes up to obtaining a sufficient fraction of the effective mass, e.g. 90 %); eq. (22) requires a combination a posteriori (calculation of $\mathcal{M}_{eff}$); none of those methods solves satisfactorily the case of modes "k" ($\omega_p < \omega_k \leq \omega_{ZPA}$) where quadratic combination will be applied, while a transition from quadratic to absolute sum should be applied : the lack of transition yields results which can change significantly with the - rather arbitrary - selection of ω_{ZPA} .

4.3. This paper proposes the following rule :

Combine through absolute sum the results of a static analysis using ZPA ($\mathcal{S}_2$) on the total mass, with SRSS (or SRSS corrected for closely-spaced modes) of individual modal results of the spectral analysis on the difference between the spectral acceleration and the ZPA $(\mathcal{S}(\omega) - \mathcal{S}_2)$.

$$R_{corr} = \Sigma \left\{ \Phi_i \mathcal{L}_i \, (\mathcal{S}(\omega_i) - \mathcal{S}_{ZPA}) \right\} + R_s \, \mathcal{S}_{ZPA} , \tag{23}$$

where Σ holds for the selected SRSS rule. In our example (eq.(18)), eq.(23) becomes

$$R_{corr} = \left\{ \sum_{i=1}^{n_1+n_2} [\Phi_i \mathcal{L}_i \, (\mathcal{S}(\omega_i) - \mathcal{S}_2)]^2 \right\}^{1/2} + \mathcal{S}_2 \, R_s . \tag{24}$$

It can be shown that

$$R_{corr} \geq R \quad (eq.\ 18) \quad , \tag{25}$$

$$R_{corr} = R_s \quad \text{if} \quad \varphi(\omega_i) = \varphi_2 \quad \text{for any } \omega_i \quad , \tag{26}$$

$$R_{corr} \simeq R \quad \text{if} \quad \varphi_2 << \varphi_1 \text{ and } \varphi(\omega_i) = \varphi_1 \text{ for any } \omega_i . \tag{27}$$

This rule solves the inadequacy of the SRSS rule for "rigid" modes : the SRSS combination, justified on a probabilistic basis (CHU et al [4]) assumes that the likelihood of simultaneous occurrence of two modal maxima is low. This assumption becomes incorrect for modes lying in the "rigid" part of the spectrum ($\omega > \omega_{ZPA}$) :

- all those modes see the input motion as a "quasi static" signal, and the maximum of the response occurs at maximum of input
- the maximum response corresponds to maximum of the excitation in a certain frequency range : a mode the frequency of which lies in this range will have its maximum response at about the same time, concomitant with the former ones.

With the alternative rule proposed hereabove :

- the "rigid" contributions of all the modes are summed absolutely,
- the result is added to the quadratic combination of the flexible contributions of the flexible modes.

Applying the static calculation to the total mass is justified for

- simplicity : no mass calculation is needed
- conservatism
- mathematical coherence : the result becomes rather independent of the selection of the cut-off frequency corresponding to the ZPA

5. CONCLUSION.

The dynamic modal spectral analysis of piping systems may be limited to a frequency range (ω_o, $\alpha\ \omega_o$) where ω_o is the first natural frequency and α is less than 5. An important part of the total dynamic response is obtained for all significant points(90 % for the support loads, about 98 % for stresses and 100 % for displacements).
The accuracy of the results is not affected by this procedure which allows significant cost reductions.

Those conclusions are valid only for piping-like structures and may not be extended to other structures without careful verification.

The missing mass may be fairly important (up to 70 %) and is adequately taken into account through a modified response spectrum technique which provides a smooth and logical transition from the rigid to flexible behaviour.

References

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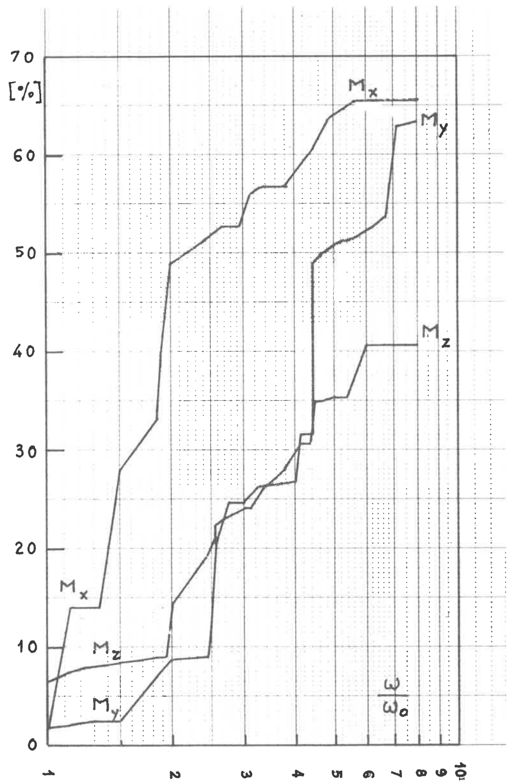

Fig.1 Evolution of the effective mass in three directions with the number of modes for a 12" RHR line

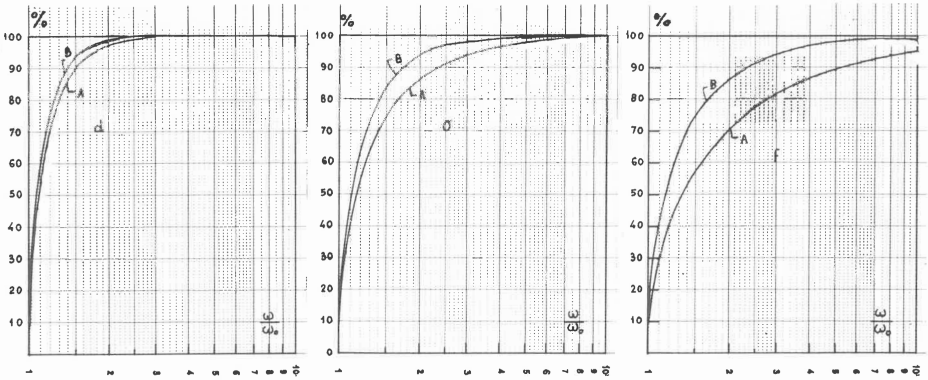

Fig.2 Evolution of reduced variables (displacement,stress, support load) with the number of modes for an ideal piping model

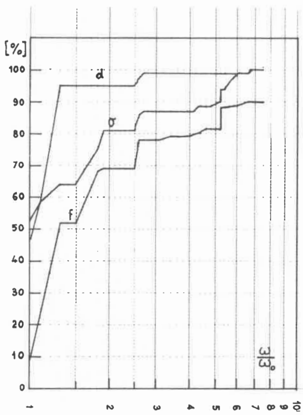

Fig.3 Evolution of reduced variables (displacement, stress, support load) with the number of modes for a 12" RHR line.

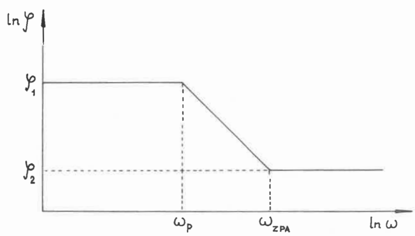

Fig.4 Tri linear Response Spectrum