A Modified CCFS Approach for Multiply-Supported Secondary Systems

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ABSTRACT

A modified Cross-Cross Floor Spectra (CCFS) approach that accounts for the dynamic interaction, tuning, and non-classical damping is formulated and presented. The technique is based on attaching two fictitious oscillators to the primary system rather than only one oscillator as was previously suggested. Response results obtained by the proposed technique are compared with the values obtained using a coupled dynamic analysis. The proposed technique proved to be more accurate, especially in the case of tuned non-classically damped primary-secondary systems.

1 INTRODUCTION

A large number of critical pieces of equipment and components are housed inside major civil structures (e.g. nuclear power plants, dams, industrial facilities, offshore structures). For dynamic analysis purposes, these components can be modelled as a dynamic system in isolation of the supporting building structure and therefore receiving its motion from it. Alternatively the dynamic model of the equipment or component can be included with the building model and the combined Primary-Secondary (P-S) system model solved in either the time-domain or the frequency-domain.

Due to many practical difficulties in carrying out a coupled dynamic analysis, an uncoupled approach is customarily adopted, (e.g. Amin et al, 1971; Lee et al, 1983). Such approach ignores the dynamic interaction characteristics of the combined P-S system. Recently, the Cross-Cross Floor Spectrum (CCFS) technique has been developed based on the principles of random vibration (Asfura et al, 1986). Although the technique is based on an uncoupled analysis approach, it attempts to account for the dynamic interaction characteristics of the combined P-S system.

2 STATEMENT OF THE PROBLEM

Consider the model for the combined P-S system subjected to base excitation $u_g(t)$ as shown in Figure 1. It is assumed that each of the primary and the secondary systems are linear elastic, viscously and classically damped. The secondary system is attached to the primary system at several points (i.e. multiply-supported). The CCFS approach, as developed by Asfura and Der Kiureghian (1986), has employed the idea of attaching two fictitious oscillators that have two frequencies (ω_i, ω_j) equal to those of the secondary

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system at two support points (floors) of the primary system; i.e. K and L, Figure 2. Accordingly, using suitable modal combination rule, the mean of the peak acceleration at the r^{th} degree of freedom of the secondary system is given by:

$$E[\ddot{u}_{r,\max}] = [\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ri}a_{rj}\sum_{K=1}^{n_a}\sum_{L=1}^{n_a} b_{iK}b_{iL}S_{KL}^{a}(\omega_i,\xi_i;\omega_j,\xi_j)]^{\frac{1}{2}} \tag{1}$$

where

$$a_{ri} = \frac{\phi_{ri}}{m_i\omega_i^2} \tag{2}$$

and

$$b_{iK} = \sum_{l=1}^{n}\phi_{li}k_{c_{lK}} \tag{3}$$

$S_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)$ is the cross-cross floor response spectrum associated with floors K and L and is mathematically the mean of the peak response associated with the covariance of the responses of the two oscillators. In other words, it is the ordinate of a "cross-oscillator" and a "cross-floor" response spectrum associated with motions of the K^{th} and L^{th} floors of the primary system, Figure 3.ϕ_i, ω_i, ξ_i and m_i are the mode shape, modal circular frequency, modal damping factor and modal mass associated with the i^{th} mode of vibration of the secondary system. K_c is the conventional coupling stiffness matrix between the secondary and the primary systems.

The ordinates of the CCFS are evaluated directly in terms of the input ground response spectrum, and the modal properties of the primary system. The problem could be stated here as how to evaluate such ordinates so that the dynamic characteristics of the combined P-S system could be properly accounted for?

In the original CCFS approach, a technique was suggested to evaluate the ordinates of the CCFS which account for the interaction and tuning effects. A mass value has been assigned to each oscillator. The mass value is calculated to bring about a shift in the nearest frequency of the primary system similar to that which actually takes place in the combined P-S system. Thus, according to a formula which is based on a tuning criterion, (Igusa et al, 1983), the mass values for the different oscillators are determined depending on the attachment point of each oscillator. Suppose that "N" is the number of degrees of freedom of the primary system, "n" is the number of degrees of freedom of the secondary system and "n_a" is the number of the attachment points supporting the secondary system. In order to evaluate a CCFS ordinate, ($n \times n_a$) systems are analyzed. Each of these systems consists of the original primary system and an oscillator representing one of the secondary system modes. Thus, each system is an (N+1) DOF system. The effect of the non-classical damping character has been approximately accounted for based on another tuning formula, (Igusa et al, 1983). Finally, a modal combination rule for evaluating the CCFS ordinates $S_{KL}(\omega_i,\xi_i;\omega_j,\xi_j)$ is developed. In this rule, a correlation coefficient, that accounts for the cross modal correlation between the two (N+1) DOF systems is employed.

3 MODIFIED TECHNIQUE FOR EVALUATING CCFS ORDINATES

The major sources of error arising in case of adopting the original technique to analyze tuned P-S systems could be attributed to the negligence of the dynamic interaction in case of tuning. Although, the interaction effect is approximated by assigning mass values to the oscillator in each (N+1) DOF system, it is believed that, still in cases of tuned P-S systems, that effect is not considered properly. The alternative technique presented here is based on the fact that the (N+2) DOF system that has two oscillators with equal frequency can not be replaced with two similar (N+1) DOF systems. It is clear that the dynamic interaction between the two oscillators themselves is completely neglected if the technique of the (N+1) DOF system is followed. Moreover, the multiple tuning situation which arises due to coincidence of frequencies of the two oscillators and one (or more) of the frequencies of the primary system has also been ignored. It is believed that to account for those neglected effects, the original (N+2) DOF system rather than the two (N+1) DOF systems has to be adopted in evaluating the CCFS ordinates.

To account for both the interaction and tuning effects, the idea of assigning equivalent mass values to the oscillators is again adopted. For the case of two oscillators with tuned frequencies, each mass value is related to that of the corresponding (N+1) DOF system with a reduction factor (α). The factor (α) is introduced to account for the tuning effect between the two oscillators and the multiple tuning between the two oscillators and the primary system in case of tuned P-S systems. Several cases were analyzed to quantify the reduction factor (α). It was found that a value that ranges between 0.9 and 1.0 is suitable for the cases of tuned P-S combined systems. For the cases of detuned P-S systems, a value of 0.75 could be reasonably assumed.

Accordingly, the proposed technique for evaluating the CCFS ordinates can be summarized in the following two steps:

a) Determination of the dynamic modal properties of the (N+2) DOF systems. The modal frequencies and mode shapes are determined through direct analysis rather than employing perturbation techniques in order to achieve better accuracy. A reduction factor (α) is applied to the equivalent masses assigned to the two oscillators when they have equal frequencies.

b) Determination of the cross-cross floor spectrum (CCFS) ordinates utilizing a modal combination rule to combine the modal responses of the $(N+1)^{th}$ and $(N+2)^{th}$ degrees of freedom in each (N+2) DOF system.

4 NUMERICAL EXAMPLE

A model for the combined P-S systems as shown in Figure 4 is selected for analysis in order to examine the validity of the proposed (N+2) technique. For comparison purposes, the theoretically "exact" responses have been obtained as well as the responses determined following the original (N+1) technique. The "exact" responses are determined by a coupled analysis of the combined P-S system. Both an idealised acceleration ground response spectrum as well as the N-S component of ElCentro earthquake are employed as seismic inputs. Four cases are considered. These cases are detuned, classically damped; detuned, non-classically damped; tuned, classically damped and tuned non-classically damped. Two different sets of mass and stiffness ratios were adopted to achieve the tuned and detuned cases. The sets of mass and stiffness ratios, the frequencies of the primary system and the modal damping factors for the two subsystems are tabulated in Table 1 for the all four cases. Table 2 summarizes the estimated peak acceleration responses at the nodes of the secondary system subjected to the ElCentro earthquake.

The results of the "exact" coupled analysis as well as the two CCFS techniques are also shown in Table 2. The percentages of error in estimating the peak responses by the (N+1) and (N+2) techniques are also tabulated. For the detuned cases, the reduction factor (α) is assumed 0.75 while for the tuned cases, (α) is assigned to 0.9 and 1.0 for classical and non-classical damping respectively.

It can be observed that the percentages of error in estimating the peak responses of the secondary system following the (N+1) technique is large, specially for the tuned, non-classically damped cases. The error percentage exceeds 40% when subjected to the ElCentro earthquake. It is also noticed that a more accurate response could be achieved by following the proposed (N+2) technique. Employing the reduction factor (α) in the tuned, non-classically damped cases greatly improves the predicted peak responses. In other words, using the (N+2) technique leads to more refined ordinates of the CCFS. In the mean time, the detuned cases indicate that a reduction factor (α) of approximately 0.75 is essential to get accurate predictions of the peak response by the (N+2) technique. The percentages of error in those cases are comparable to those in the corresponding cases analyzed by the (N+1) technique. Similar findings and trends were observed for other ground motions and idealized ground response spectra.

5 CONCLUSIONS

A modified CCFS approach that accounts for the dynamic interaction, tuning, non-classical damping, cross correlation between the secondary system modal responses and the cross correlation between the different support motions was presented. An alternative technique for evaluating the ordinates of the cross-cross floor spectra has been developed. While the original technique is based on analyzing a number of (N+1) DOF systems, the proposed technique is based on the analysis of a number of (N+2) DOF systems. Neglecting the tuning between the two oscillators themselves and the multiple tuning situation between the two oscillators and the primary system were found to be the major sources of error in estimating the response of tuned, non-classically damped combined P-S systems. The proposed technique, using a reduction factor (α) applied to the equivalent masses assigned to the two oscillators, proved to be more accurate in estimating the peak response of the secondary system, especially in the case of the tuned non-classically damped primary-secondary system.

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Table 1. Properties of P-S Model Investigated

	Primary System Properties	Secondary System Properties			
		Detuned Classical Damped	Detuned Non-Classical Damped	Tuned Classical Damped	Tuned Non-Classical Damped
Frequencies in (rad/s)	4.025	16.106	16.106	4.025	4.025
	11.750	22.361	22.361	5.588	5.588
	18.522	33.993	33.993	8.494	8.494
	23.794	38.730	38.730	9.678	9.678
	27.139	45.662	45.662	11.410	11.410
Modal Damping Factor	0.05	0.05	0.02	0.05	0.02
m/M	-----	0.02	0.02	0.03203	0.03203
k/K	-----	0.05	0.05	0.005	0.005

Table 2. Estimated Peak Acceleration Response of Model Subjected to ElCentro earthquake

	DOF	Coupled	CCFS			
			(N÷1)		(N÷2)	
		Acc. (g)	Acc. (g)	%Error	Acc. (g)	%Error
Detuned Classical Damped	1	0.2592	0.246	-5.09	0.253	-2.39
	2	0.2217	0.211	-4.83	0.218	-1.67
	3	0.2149	0.221	2.84	0.218	1.44
	4	0.2675	0.267	-0.19	0.271	1.31
	5	0.2685	0.265	-1.30	0.270	0.56
Detuned Non-Classical Damped	1	0.2821	0.261	-7.48	0.273	-3.23
	2	0.2473	0.229	-7.40	0.245	-0.93
	3	0.2184	0.224	2.56	0.224	2.56
	4	0.2874	0.280	-2.57	0.291	1.25
	5	0.2873	0.276	-3.93	0.285	-0.80
Tuned Classical Damped	1	0.3825	0.509	33.07	0.420	9.80
	2	0.4854	0.693	42.77	0.560	15.37
	3	0.3782	0.548	44.90	0.446	17.93
	4	0.4901	0.680	38.75	0.565	15.28
	5	0.3529	0.475	34.60	0.402	13.91
Tuned Non-Classical Damped	1	0.4841	0.598	23.53	0.473	-2.29
	2	0.6870	0.833	21.25	0.665	-3.20
	3	0.5648	0.670	18.63	0.546	-3.33
	4	0.7464	0.836	12.00	0.707	-5.28
	5	0.5313	0.586	10.30	0.506	-4.76

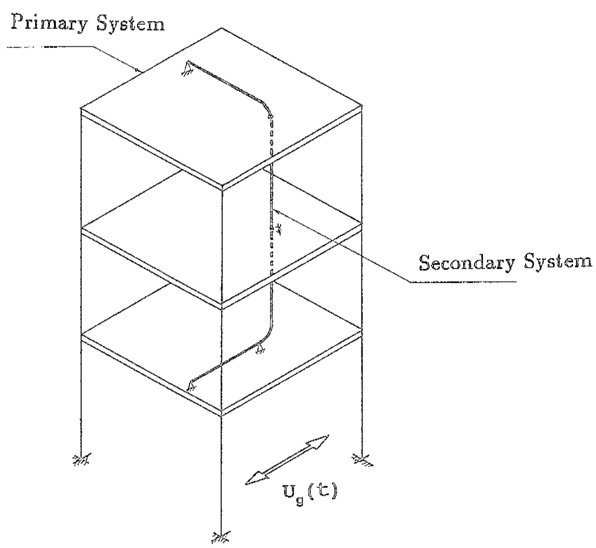

Figure 1 Combined Primary-Secondary (P-S) System

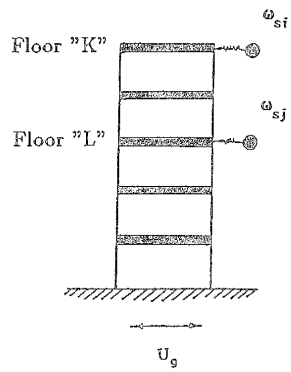

Figure 2 Primary-Double Oscillator System

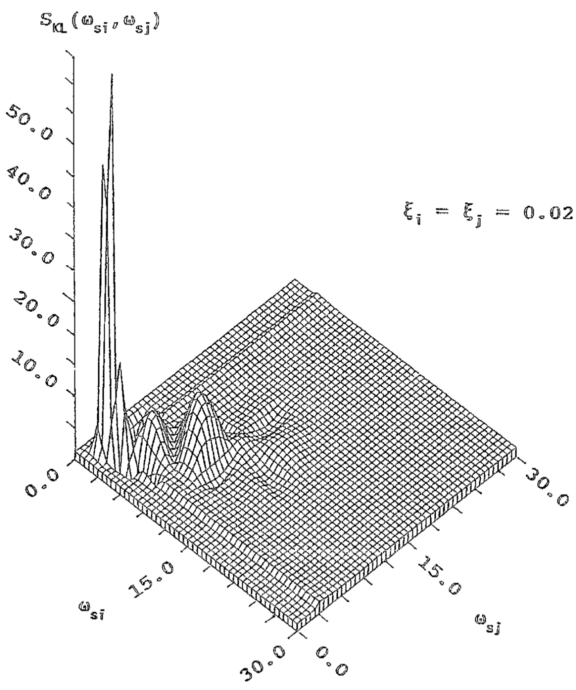

Figure 3 Cross Cross Floor Response Spectrum (CCFS)

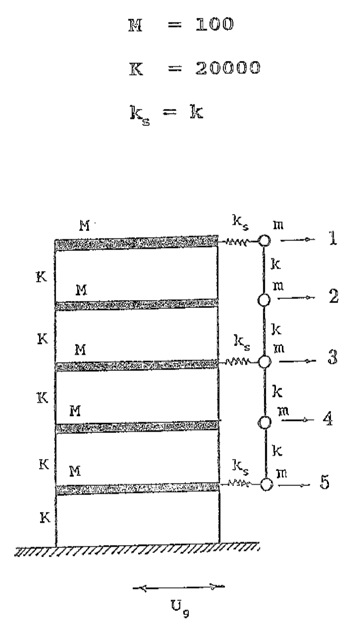

Figure 4 Model Considered in The Analysis