Transactions of the **14th International Conference on Structural Mechanics in Reactor Technology** (SMiRT 14), Lyon, France, August 17-22, 1997

K21/5

Application of residual mass correction in time history analysis

Subramanian K.V.(1), Sridhar Gopal P.(1), Kushwaha H.S.(2), Reddy G.R.(2)
(1) TATA Consulting Engineers, India
(2) Bhabha Atomic Research Centre, India

ABSTRACT

This paper describes the studies carried out to evolve a procedure for application of missing mass correction in time history analysis. Three methods for combining the responses obtained from time history analysis with the corrections from pseudo-static analysis using residual masses are investigated. It is concluded that the method of algebraic summation is most appropriate. The criteria for determining the cut off frequency can be either 33 Hz(rigid frequency) or 90% mass participation.

INTRODUCTION

For the solution of dynamic equilibrium equations in time history analysis, Direct Integration and Modal Superposition techniques are employed. For linear systems, Modal Superposition method is preferred from considerations of economics of computational time and system memory requirements. Damping is considered in the proportional damping formulation in Direct integration while Modal damping in Modal Superposition method. However, due to the inherent advantages of Modal Superposition method, it is a widely used procedure in linear dynamic analysis.

Modal Superposition method requires prior solution of the eigen value problem, to evaluate the natural frequencies and mode shapes of the structural system. A number of methods like vector iteration methods, transformation methods and subspace iteration methods are available for the solution of an eigen value problem. It is generally adequate to compute the lowest eigen values and corresponding mode shapes which constitute substantial portion of the dynamic response.

Section 3.7.2 of the USNRC Standard Review Plan[1] requires sufficient number of modes to be included in the dynamic response analysis to ensure that inclusion of additional modes does not result in more than 10% increase in the total responses. The implementation of this requirement may require the inclusion of modes in excess of 33 Hz in the response analysis.

Modal Superposition technique is an elegant method where it is sufficient to determine responses from the first few modes to arrive at the total response. Evaluation of the frequencies and mode shapes (higher modes) is a problem and time consuming as most of the eigen solution methods give accurate results upto a number of modes as indicated by the dynamic degrees of freedom of the system. For a system with 'n' degrees of freedom, the number of modes that can be correctly extracted is 'n/2'[2]. Thus satisfying the SRP criteria by evaluating the necessary modes takes out the advantages derived from the Modal Superposition technique.

One of the acceptable methods to achieve this is by determining the cut-off frequency such that the total modal mass considered in the response calculations is atleast 90% of the total system mass or the rigid frequency (33 Hz). To evaluate total response, the unparticipated mass needs to be appropriately included in the response calculations.

With this procedure, a correction is to be applied to the response components of interest such as axial forces, moments and shears which are used in subsequent design or detailed analysis using refined models. ASCE 4-86[2] and SRP 3.7.2 have indicated the determination of 'dof mass' and thus determine the unparticipated mass and have specified procedures for combination of rigid body responses in the case of Response Spectrum analysis. However, these procedures are not explicitly indicated with respect to Time History analysis.

On the the basis of the studies carried out, a procedure is evolved for including the missing mass correction (rigid body modes) in Time History analysis.

RESIDUAL MASS CORRECTION

Response calculations are performed by including all the modes upto the cutoff frequency which is defined by ZPA (Zero Period Acceleration) 33 Hz or 90% mass participation. A 'missing mass' correction is applied to account for the unparticipated mass.

For each degree of freedom(DOF) included in the analysis, determine the fraction of DOF mass included in the summation of the modes. This fraction d_i for each DOF 'i' is given by

$$d_i = \sum_{n=1}^{N} c_n * \phi_{n,i} \qquad \text{--------------------------(1)}$$

where

n is the order of the mode under consideration

N is the number of modes included in the response computation

$\phi_{n,i}$ is the n th natural mode of the system

c_n is the participation factor $\{\phi_n^T\}\ [1]\ /\ \{\phi^T\}[m]\{\phi_n\}$

Determine the fraction of DOF mass not included in the summation of modes (e_i)

Higher modes are assumed to respond in phase with ZPA and hence these modes are combined algebraically. This is equivalent to a pseudostatic response to the inertial forces from the higher modes excited at ZPA. The pseudostatic inertial forces associated with the higher modes for each DOF 'i' is given by

$$P_i = ZPA * M_i * e_i \qquad \text{---------------------------- (2)}$$

where

P_i = force or moment to be applied at DOF i

M_i = Mass or moment of inertia associated with DOF i

The system is statically analysed for this set of pseudo-static inertial forces applied to all the degrees of freedom to determine the maximum responses associated with the high frequency modes. The pseudostatic responses are combined with the inertial forces in accordance with methods explicitly indicated in SRP 3.7.2 or ASCE 4-86. As stated above, these procedures are explicitly applicable to Response Spectrum Analysis.

COMBINATION OF RESPONSE COMPONENTS - TIME HISTORY

The effect of missing mass is accounted for by obtaining the combined response of all the rigid body modes in the form of pseudostatic analysis. This absolves the necessity of evaluation of rigid body modes and obtaining the modal responses.

In pseudo static analysis, the residual mass associated at each degree of freedom is calculated and applied as a force at that DOF with unit acceleration depending on the direction of excitation and a static analysis of the entire model is performed. The response components obtained from static analysis are then multiplied with the input ground motion to arrive at time history of response components (Rigid body Response - 'Corrections' ; Part-B). In the time history analysis, all the modes up to cutoff frequency are combined algebraically for various response components (These are called 'Uncorrected Values' ; Part-A). Part-A and Part-B are to be appropriately combined to obtain the final results.

ASCE 4-86 defines in Clause 3.2.3 (f) and (g) for Response Spectrum analysis that the residual response (for modal combination purposes) shall be considered as an additional mode having a frequency equal to the cutoff frequency or ZPA frequency whichever is higher. The individual modal responses are to be combined in accordance with standard procedures of modal combination. In Time History analysis, as the modal combination is performed algebraically among all the modes at each time step, this clause leads to a deduction of the procedure of algebraic combination of Part A and B.

SRP 3.7.2 Appendix A essentially referring to Response Spectrum Analysis has indicated an alternative procedure to SRSS between the inertial response and rigid body response. Modal responses are computed for enough modes to ensure that the inclusion of additional modes do not increase the total responses by more than 10%. Modes that have natural frequencies less than that at which the spectral acceleration approximately returns to ZPA (33 Hz) are combined in accordance with USNRC RG 1.92. [5] Higher modal responses are combined algebraically retaining signs with each other. This is equivalent to the combined response of high frequency modes. The 'absolute' value of the combined higher modes is to be added directly to the total responses from the combined lower modes.

The above stated procedure in SRP leaves an ambiguity on the appropriate method of combination of lower frequency modes and the high frequency modes viz 'algerbraic/ absolute' addition. Hence, both the above procedures (algebraic and absolute) are adopted to combine the inertial response with the rigid body response. In addition, a procedure in variance with both is also investigated. This leads to the following three methods for combination of Part A and B responses.

Method-1: Part-B is added only if its sign matches with Part-A. Otherwise Part-B is ignored. (Conservative assumption to ignore reduction due to rigid body response)

Method-2: Part-B is always added by assigning the sign of Part-A to Part-B. (Absolute addition. Most conservative assumption)

Method-3: Part-B is always added algebraically to Part-A. (Logically derived methodology).

VALIDATION OF METHODS

Suitable real life problems were chosen and all the above three methods are applied and the results are compared. To assess the importance of the methods of combination, Time History analysis is performed with a large number of modes resulting in almost 100% mass participation resulting in 'base' values. Results with less number of modes and missing mass correction performed using all the three methods of combination are compared with the base values.

A three dimensional stick model of a typical Reactor Building in a Nuclear Power Plant is considered as an example (Fig.-1). The Outer Containment Wall (OCW), Inner Containment Wall (ICW), Internal Structure (IS) and the Calendria Vault (CV) which are resting on the raft are modelled as beam elements (axial and flexural) as appropriate. The stiffnesses and masses are calculated and appropriately distributed. Soil is modelled using soil springs in all directions.

As vertical frequencies are more rigid than horizontal frequencies, the total mass participation in the vertical direction is lower than the horizontal directions. Hence, the missing mass correction is expected to have a significant effect on vertical excitation. In the case of vertical excitation, as axial forces are most significant, the axial forces in the element are used for comparison. These axial forces correspond to vertical 'Z' direction excitation. Similarly, missing mass correction is performed for the horizontal directions with the shear force and bending moments tabulated for comparison.

Three analysis cases are considered for comparison. In Case-1, 50 modes are evaluated and used for Modal Superposition. 50th mode corresponds to a frequency of 34.77 Hz (> 33 Hz, the cutoff frequency.) The mass participation at this cutoff frequency is 82.4%, 82.87% and 76.91 % in the horizontal and vertical directions respectively. In Case-2, 70 modes are evaluated and used for Modal superposition. 70th mode corresponds to a total vertical mass participation of 90%. In Case-3, 100 modes are evaluated and used for modal superposition. In all the cases 1,2 and 3 above, missing mass correction is performed using the three alternatives (Methods 1,2,and 3). For the base values, 150 modes are evaluated and Modal Superposition used for Time History analysis .

DISCUSSION OF RESULTS

Table-1 indicates the last frequency value and total percentage mass participation in the three directions. With 150 modes, the participation in all directions is nearly 100%. Hence, the results corresponding to 150 modes are considered as the base values for comparison purposes.

Tables- 2 to 4 tabulates the results for five typical elements of different sub-structures of the given model for horizontal and vertical excitation. Results are tabulated for Case-1, Case-2 and Case-3 corresponding to without Missing Mass Correction (MMC), Method-1, Method-2 and Method-3. Percentage differences with respect to base values are indicated in the parenthesis.

It can be seen from the above tabulation that the results without Missing Mass Correction are unacceptable for vertical excitation. In the case of horizontal excitation, the results without MMC for 50 modes are unacceptable. It is observed that for horizontal excitation, the mass participation in the superstructure elements is greater than 90% while the raft participation is lower, thus reducing the overall mass participation to less than 90%. This is due to the behaviour of the raft as a rigid body. Even for 100 modes, the results are not correct in some cases Though, Method-1 shows good comparison in some cases (Case-2 and Case-3), it is only coincidental. It can be observed that the results obtained by Method-2 are generally not acceptable. Method-3 produces consistently close results with respect to base values for all the three cases .

CONCLUSIONS

Algebraic combination of the rigid body response compares well with the analysis using a large number of modes (the true response). Hence, the most appropriate method for missing mass correction in time history analysis is where the corrections are algebraically added to the uncorrected values (Method-3). With this procedure, the time consuming evaluation of higher modes in an eigen solution can be avoided.

Results of responses using the cutoff frequency as 33 Hz or the cutoff frequency at 90% mass participation and accounting for missing mass as per method-3 above when compared indicate practically the same responses. This leads to the conclusion that any one of the criteria can be used for determining the cutoff frequency.

ACKNOWLEDGEMENT

The authors thank Tata Consulting Engineers and Bhabha Atomic Research Centre for the constant encouragement received and having given permission to publish this paper.

REFERENCES

1. USNRC - *Standard Review Plan, SRP 3.7.2 , 'Seismic System Analysis'* - Rev .-1, July 1981.

2. ASCE Standard (4-86) - *Seismic analysis of safety related Nuclear Structures and commentary on Standard for Seismic analysis of safety related Nuclear Structures. , September1996.*

3. USNRC NUREG 1161 RD :*Recommended revisions to Nuclear Regulatory Commission Seismic Design criteria.*

4. Bathe, K. J., *Finite Element Procedures in Engineering Analysis.* Prentice Hall of India Private Limited., 1990.

5. USNRC - R G 1.92 : *Combining Modal responses and Spatial components in Seismic Analysis*

TABLE - 1 : EIGEN SOLUTION RESULTS

NO. OF MODES CONSIDERED	LAST FREQUENCY (Hz)	TOTAL PERCENTAGE MASS PARTICIPATION		
		NS(X)-DIRECTION	EW(Y) - DIRECTION	VERT(Z)-DIRECTION
50	34.77	82.40	82.87	76.91
70	46.69	96.27	96.22	90.31
100	68.68	99.48	99.34	98.60
150	103.30	99.68	99.69	99.86

TABLE - 2 RESPONSE - EXCITATION - NS DIRECTION

EL NO.	BASE VALUE		METHOD OF	SHEAR FORCE (T)			BENDING MOMENT (T.m)		
	SHEAR (T)	B. M. (T.m)	COMBINATION	CASE 1 50 MODES	CASE 2 70 MODES	CASE3 100 MODES	CASE 1 50 MODES	CASE 2 70 MODES	CASE 3 100 MODES
1	11.648	24.146	WITHOUT MMC	11.744 (0.82)	12.075 (3.67)	11.621 -(0.23)	24.432 (1.18)	25.216 (4.43)	23.985 -(0.67)
			METHOD 1	11.744 (0.82)	12.075 (3.67)	11.647 -(0.01)	24.432 (1.18)	25.216 (4.43)	24.054 -(0.38)
			METHOD 2	11.806 (1.36)	12.441 (6.81)	11.647 -(0.01)	24.599 (1.88)	26.207 (8.54)	24.054 -(0.38)
			METHOD 3	11.682 (0.29)	11.71 (0.53)	11.647 -(0.01)	24.264 (0.49)	24.225 (0.33)	24.054 -(0.38)
24	35.157	71.557	WITHOUT MMC	34.949 -(0.45)	35.234 (0.22)	35.114 -(0.12)	71.055 -(0.7)	71.642 (0.12)	71.55 -(0.01)
			METHOD 1	35.135 -(0.06)	35.234 (0.22)	35.155 -(0.01)	71.283 -(0.38)	71.642 (0.12)	71.62 (0.09)
			METHOD 2	35.135 -(0.06)	35.313 (0.44)	35.115 -(0.01)	71.283 -(0.38)	71.776 (0.31)	71.62 (0.09)
			METHOD 3	35.135 -(0.06)	35.154 -(0.01)	35.115 -(0.01)	71.283 -(0.38)	71.508 -(0.07)	71.62 (0.09)
45	12.441	131.49	WITHOUT MMC	12.451 (0.08)	12.42 -(0.17)	12.441 (0)	131.48 -(0.01)	131.76 (0.21)	131.49 (0)
			METHOD 1	12.451 (0.08)	12.42 -(0.17)	12.441 (0)	131.48 -(0.01)	131.76 (0.21)	131.49 (0)
			METHOD 2	12.451 (0.08)	12.42 -(0.17)	12.441 (0)	131.48 -(0.01)	131.76 (0.21)	131.49 (0)
			METHOD 3	12.451 (0.08)	12.42 -(0.17)	12.441 (0)	131.48 -(0.01)	131.76 (0.21)	131.49 (0)
71	0.489	0.939	WITHOUT MMC	0.516 (5.5)	0.501 (2.41)	0.488 -(0.16)	1.037 (10.37)	1.108 (17.98)	0.937 -(0.16)
			METHOD 1	0.516 (5.5)	0.501 (2.41)	0.488 -(0.16)	1.037 (10.37)	1.108 (17.98)	0.938 -(0.16)
			METHOD 2	0.516 (5.5)	0.501 (2.41)	0.488 -(0.16)	1.037 (10.37)	1.108 (17.98)	0.938 -(0.16)
			METHOD 3	0.516 (5.5)	0.501 (2.41)	0.488 -(0.16)	1.037 (10.37)	1.108 (17.98)	0.938 -(0.16)
83	3666.9	128950	WITHOUT MMC	3409.2 -(7.03)	3622.1 -(1.22)	3666.1 -(0.02)	128940 -(0.01)	128940 -(0.01)	128950 (0)
			METHOD 1	3646.4 -(0.56)	3645.9 -(0.57)	3666.1 -(0.02)	128940 -(0.01)	128950 (0)	128950 (0)
			METHOD 2	3646.4 -(0.56)	3645.9 -(0.57)	3686.7 (0.54)	128950 (0)	128950 (0)	128950 (0)
			METHOD 3	3646.4 -(0.56)	3645.9 -(0.57)	3645.4 -(0.59)	128930 -(0.02)	128950 (0)	128950 (0)

TABLE - 3 RESPONSE - EXCITATION - EW DIRECTION

EL NO.	BASE VALUE SHEAR (T)	BASE VALUE B.M (T m)	METHOD OF COMBINATION	SHEAR FORCE (T) CASE 1 50 MODES	SHEAR FORCE (T) CASE 2 70 MODES	SHEAR FORCE (T) CASE3 100 MODES	BENDING MOMENT (T.m) CASE 1 50 MODES	BENDING MOMENT (T.m) CASE 2 70 MODES	BENDING MOMENT (T.m) CASE 3 100 MODES
1	11.346	23.55	WITHOUT MMC	11.45 (0.92)	11.743 (3.5)	11.422 (0.67)	23.824 (1.16)	24.534 (4.18)	23.553 (0.01)
			METHOD 1	11.45 (0.92)	11.743 (3.5)	11.422 (0.67)	23.824 (1.16)	23.534 (4.18)	23.553 (0.01)
			METHOD 2	11.527 (1.6)	12.081 (6.48)	11.482 (1.2)	24.033 (2.05)	25.451 (8.07)	23.717 (0.71)
			METHOD 3	11.372 (0.23)	11.404 (0.51)	11.361 (0.13)	23.614 (0.27)	23.714 (0.7)	23.441 -(0.4)
24	28.315	56.775	WITHOUT MMC	28.169 -(0.52)	28.37 (0.19)	28.327 (0.04)	56.736 -(0.07)	56.767 -(0.01)	56.777 (0)
			METHOD 1	28.316 (0)	28.37 (0.19)	28.327 (0.04)	56.775 (0)	56.767 -(0.01)	56.777 (0)
			METHOD 2	28.316 (0)	28.441 (0.44)	28.305 (0.12)	56.775 (0)	56.786 (0.02)	56.783 (0.01)
			METHOD 3	28.316 (0)	28.299 -(0.06)	28.305 -(0.04)	56.775 (0)	56.749 -(0.05)	56.771 -(0.01)
45	4.024	55.541	WITHOUT MMC	4.044 (0.49)	4.02 -(0.12)	4.024 (0)	55.828 (0.52)	55.58 (0.07)	55.54 (0)
			METHOD 1	4.044 (0.49)	4.02 -(0.12)	4.024 (0)	55.828 (0.52)	55.58 (0.07)	55.54 (0)
			METHOD 2	4.044 (0.49)	4.02 -(0.12)	4.024 (0)	55.828 (0.52)	55.58 (0.07)	55.54 (0)
			METHOD 3	4.044 (0.49)	4.019 -(0.12)	4.024 (0)	55.828 (0.52)	55.58 (0.07)	55.54 (0)
71	0.477	0.841	WITHOUT MMC	0.483 (1.12)	0.474 -(0.78)	0.477 -(0.11)	0.853 (1.42)	0.835 -(0.65)	0.84 -(0.1)
			METHOD 1	0.483 (1.12)	0.474 -(0.78)	0.477 -(0.11)	0.853 (1.42)	0.835 -(0.65)	0.84 -(0.1)
			METHOD 2	0.483 (1.12)	0.474 -(0.78)	0.477 -(0.11)	0.853 (1.42)	0.835 -(0.65)	0.84 -(0.1)
			METHOD 3	0.483 (1.12)	0.474 -(0.78)	0.477 -(0.11)	0.853 (1.42)	0.835 -(0.65)	0.84 -(0.1)
83	3532.4	131860	WITHOUT MMC	3237.8 -(8.34)	3481.6 -(1.44)	3530.1 -(0.07)	131876 (0.01)	131990 (0.1)	131900 (0.03)
			METHOD 1	3535.3 (0.08)	3511.9 -(0.58)	3530.1 -(0.07)	131920 (0.05)	131990 (0.1)	131900 (0.03)
			METHOD 2	3535.3 (0.08)	3511.9 -(0.58)	3553.7 (0.6)	131920 (0.05)	132040 (0.14)	131940 (0.06)
			METHOD 3	3535.3 (0.08)	3511.9 -(0.58)	3506.4 -(0.74)	131920 (0.05)	131930 (0.05)	131860 (0)

TABLE - 4 RESPONSE - EXCITATION - VERTICAL DIRECTION

EL NO.	BASE VALUE AXIAL FORCE (T)	METHOD OF COMBINATION	AXIAL FORCE (T) CASE 1 50 MODES	AXIAL FORCE (T) CASE 2 70 MODES	AXIAL FORCE (T) CASE3 100 MODES
1	45.337	WITHOUT MMC	48.939 (7.94)	45.08 -(0.57)	45.283 -(0.12)
		METHOD 1	48.939 (7.94)	45.249 -(0.19)	45.283 -(0.12)
		METHOD 2	54.409 (20.01)	45.249 -(0.19)	45.675 (0.75)
		METHOD 3	45.062 -(0.61)	45.249 -(0.19)	45.097 -(0.53)
24	112.61	WITHOUT MMC	119.35 (5.99)	106.67 -(5.27)	114.8 (1.94)
		METHOD 1	119.35 (5.99)	114.25 (1.46)	114.8 (1.94)
		METHOD 2	125.59 (11.53)	114.25 (1.46)	116.3 (3.28)
		METHOD 3	113.11 (0.44)	114.25 (1.46)	113.3 (0.61)
45	2321.4	WITHOUT MMC	2286.6 -(1.5)	2337.2 (0.68)	2323.5 (0.09)
		METHOD 1	2325.4 (0.17)	2337.2 (0.68)	2323.5 (0.09)
		METHOD 2	2325.4 (0.17)	2353 (1.36)	2325.7 (0.19)
		METHOD 3	2325.4 (0.17)	2321.4 (0)	2321.4 (0)
71	85.748	WITHOUT MMC	138.93 (62.02)	89.208 (4.04)	76.574 -(10.7)
		METHOD 1	138.93 (62.02)	89.208 (4.04)	86.445 (0.81)
		METHOD 2	200.1 (133.36)	92.972 (8.42)	86.445 (0.81)
		METHOD 3	89.211 (4.04)	86.409 (0.77)	86.445 (0.81)
83	26564	WITHOUT MMC	23162 -(12.81)	25271 -(4.87)	26457 -(0.4)
		METHOD 1	26460 -(0.39)	26467 -(0.37)	26459 -(0.4)
		METHOD 2	26460 -(0.39)	26467 -(0.37)	26459 -(0.4)
		METHOD 3	26460 -(0.39)	26467 -(0.37)	26459 -(0.4)

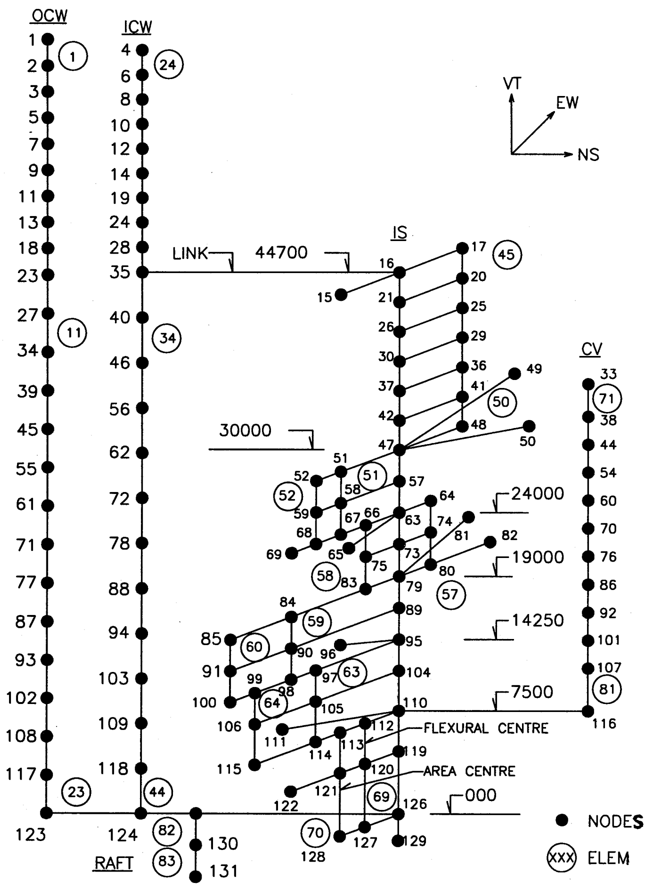

FIG – 1 3-D STICK MODEL FOR A REACTOR BUILDING