Creep Rupture of an Annular Plate

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Abstract

By use of Continuous Damage Mechanics /CDM/ theory, a study of creep rupture of annular metallic plates was carried out. The study was confined to the problems of i. evaluating the magnitude of the time-to-rupture, and ii. determining the location of the initial macroscopic crack appearance, both of which were found to be strongly influenced by plate geometry and load intensity factors.

A new damage cummulation law was proposed by which one may describe the transformation of isochronous creep rupture curves in plane stress for different times to rupture, offering a possible solution to both problems mentioned above.

1. Introduction

The progressive deterioration of a material which develops during the creep process leads to the formation of macroscopic cracks in structural members. As the fracture process continues, certain of these cracks propagate leading to ultimate failure of the member. This second stage can be dealt with by means of Fracture Mechanics, but the initial stage, due to the absence of one predominant crack, cannot. This problem gave rise to a new branch of mechanics, Continuous Damage Mechanics /CDM/, which has been widely used over the last two decades and employs the concept of the continuity parameter ψ proposed by Kachanov [1], or of the damage parameter $\omega=1-\psi$ as originated by Rabotnov [2].

For the uniaxial state of stress the law of continuity evolution was postulated by Kachanov [1]:

$$\frac{d\psi}{dt} = -A\left(\frac{\sigma}{\psi}\right)^n , \qquad /1/$$

where A , n are material constants. A simple interpretation of damage as the ratio of the remaining load carrying are of a specimen to its initial area was suggested by Odqvist and Hult [3].

In the multiaxial state of stress, however, an interpretation of damage parameters /dealt with as scalar, vector or tensor quantities/ becomes more complex. Intensive investigations in this field have been carried out in terms of material sciences as well as of mechanics. In the frame of mechanics the simplest, and perhaps the most effective idea is that of replacing stress σ in eq./1/ with an equivalent stress σ_e , determined in accordance with appropriate exertion theory for a given material and environmental conditions /load, temperature, etc./. Sdobyriev [4] has proposed equivalent stress as a combination of principal stress σ_1 and effective stress σ_i :

$$\sigma_e = \beta\sigma_1 + (1-\beta)\sigma_i \quad , \qquad /2/$$

where β is a material constant. As limiting cases we obtain:

i. If $\beta=1$, then $\sigma_e = \sigma_1$. This was originally proposed by Kachanov [1] for the multiaxial state of stress, and the corresponding limit curve for plane stress becomes that of Galileo's square.

ii. If $\beta=0$, then $\sigma_e = \sigma_i$. The limit curve in the form of an ellipse corresponds to the Huber-Mises-Hencky exertion theory.

These results also correspond well to classification of the so called "Δ" and "ϕ" materials sensitive, respectively, to principal stresses and to effective stress, as proposed by Hayhurst [5].

This paper will concentrate on the observed phenomenon /cf. e.g. Broberg [6]/that, even for the same material, limit curves depend on load level, approximating Huber's ellipse for higher loads and transitioning to Galileo's square for lower ones. In terms of Sdobyriev's proposal this means that the parameter β is an unknown function of time-to-rupture, $t^{\times}$, of a structure. This transition in the multiaxial state of stress is similar to the well known transition between pure ductile and pure brittle modes of failure in the uniaxial state of stress. One can expect that the load intensity will both influence the time to rupture and indicate the point where the first macrocrack will appear.

2. Isochronous Creep Rupture Curves

Since the function $\beta(t^{\times})$ remains unknown, another approach to accounting for the transformation of creep rupture curves is to modify the damage law, eq./1/. This has been done by the authors [7] in the form:

$$\frac{d\psi}{dt} = -A_o\left(\frac{\sigma_i}{\psi}\right)^{n_o}\frac{d\sigma_i}{dt} - A\left(\frac{\sigma_1}{\psi}\right)^{n} \quad , \qquad /3/$$

where A, A_o, n, n_o are material constants. It is seen that the first term on the right hand side of the above equation accounts for the time-independent

/instantaneous/ failure, whereas the second term describes the time-dependent deterioration /creep failure/. The former, which occurs at high load levels, is governed by effective stress /Huber's theory/; the latter by principal stress /Galileo's theory/. For a load level below that which causes instantaneous failure, intermaediate limit curves are obtained whose shape will depend on the time-to-rupture as shown in Fig. 1. These curves were obtained under the assumption that at the time $t=0$ a solid is loaded to a certain level of stress intensity, and then both principal stresses are held constant until rupture occurs. The formula for the time-to-rupture under these circumstances takes the form:

$$\tau^{x} = \frac{1}{s_1^{n}} \left[1 - \left(\frac{\sigma_{c7}}{R} \right)^{n_o+1} s_i^{n_o+1} \right]^{\frac{n+1}{n_o+1}} , \qquad /4/$$

where dimensionless variables were introduced as follows:

$$s_1 = \sigma_1 / \sigma_{c7} \quad , \quad s_i = \sigma_i / \sigma_{c7} \quad , \quad \tau^{x} = t^{x} / \bar{t}^{x} \quad ,$$

where

$$\bar{t}^{x} = \frac{1}{A(n+1)\, R^{n}} \quad , \qquad /5/$$

and R is a stress causing an instantaneous failure under uniaxial tension, σ_{c7} is a reference stress, and the material constant A_o was replaced by R^{-n_o-1} .

In a structural member s_1 and s_i are functions of space coordinates, and to evaluate time-to-rupture of a structure, one must find the point at which values of s_1 and s_i give the smallest value of τ^{x} according to eq. /4/.

3. Stress Field in a Creeping Annular Plate

Assume that an annular plate is subjected to a uniformly distributed load q with axi-symmetrical boundary conditions. The material of the plate obeys Norton's law of stationary creep:

$$s_{ij} = \frac{2}{3} \tau\, \dot{\varepsilon}^{\frac{1}{m}} \dot{\varepsilon}_{ij} \quad ,$$

and

$$\dot{\varepsilon}_i = (s_i)^{m} \frac{1}{\tau} \quad , \qquad /6/$$

where s_{ij} , $\dot{\varepsilon}_{ij}$ are deviator of stress tensor and tensor of creep rate, respectively, $\dot{\varepsilon}_i$ is effective creep rate, and τ and m are material constants. With the set of dimensionless variables:

$$\varphi = \tau \frac{\dot{w}}{h} \;,\quad \rho = \frac{r}{10^{c} h} ,\quad \zeta = \frac{z}{h} ,\quad p = \frac{q}{\sigma_{c7}} \quad ,$$

$$\xi_r = \tau \dot{\varkappa}_r h \;,\; \xi_\theta = \tau \dot{\varkappa}_\theta h \;,\; s = \frac{\sigma_r}{\sigma_{c7}} ,\quad s = \frac{\sigma_\theta}{\sigma_{c7}} \quad , \qquad /7/$$

where $\dot{w}$ is plate displacement rate, h is plate thickness, r, θ, z are cylindrical coordinates, σ_r, σ_θ are radial and circumferential stresses, $\varkappa_r$, $\varkappa_\theta$ are radial and circumferential curvature rates, and c is a scaling factor, the Kirchoff-Love theory of moderate thickness plates yields:

and
$$\dot{\varepsilon}_r = \frac{1}{\tau}\xi_r\zeta \quad , \quad \dot{\varepsilon} = \frac{1}{\tau}\xi_\theta\zeta \quad , \quad \dot{\varepsilon}_r + \dot{\varepsilon}_\theta = -\dot{\varepsilon}_z \quad . \qquad /8/$$

Equilibrium equation takes the form:

$$\frac{d}{d\rho}\left\{\frac{d}{d\rho}\left[\xi^{\frac{1}{m}-1}\rho\left(\frac{d^2\varphi}{d\rho^2} + \frac{1}{2\rho}\frac{d\varphi}{d\rho}\right)\right] - \xi^{\frac{1}{m}-1}\left(\frac{1}{2}\frac{d^2\varphi}{d\rho^2} + \frac{1}{\rho}\frac{d\varphi}{d\rho}\right)\right\} = \left(2+\frac{1}{m}\right)10^{4c}p\rho \quad , \qquad /9/$$

where:

$$\xi = \left[\left(\frac{d^2\varphi}{d\rho^2}\right)^2 + \frac{1}{\rho}\frac{d\varphi}{d\rho}\frac{d^2\varphi}{d\rho^2} + \frac{1}{\rho^2}\left(\frac{d\varphi}{d\rho}\right)^2\right]^{\frac{1}{2}} \quad . \qquad /10/$$

The formulae for bending moments are:

$$m_r = -\xi^{\frac{1}{m}-1}\left(\frac{d^2\varphi}{d\rho^2} + \frac{1}{2\rho}\frac{d\varphi}{d\rho}\right) \quad , \quad m_\theta = -\xi^{\frac{1}{m}-1}\left(\frac{1}{2}\frac{d^2\varphi}{d\rho^2} + \frac{1}{\rho}\frac{d\varphi}{d\rho}\right) \quad , \qquad /11/$$

and for stresses:

$$s_r = 2^{\frac{1}{m}+1}m_r\rho^{\frac{1}{m}}10^{-2c} \quad , \qquad s_\theta = 2^{\frac{1}{m}+1}m_\theta\rho^{\frac{1}{m}}10^{-2c} . \qquad /12/$$

The governing function φ can be found by integration of eq./9/ with appropriate boundary conditions for the plates shown in Fig. 2.

Equation /9/ is a non-linear differential equation of fourth order, and it was integrated numerically with a specific iterative procedure. Initially, m=1 was used, and the linear equation solved to obtain the first approximation of the φ function. Then the value of the ξ function was found by using eq./10/. This function was introduced to eq./9/ which was solved numerically by means of the Finite Difference Method. New values of the function ξ were calculated and again eq./9/ was solved until the values of φ function in two successive iterations remained the same to the desired degree of accuracy. Eqs /11/ and /12/ enable one to calculate the stress field in a creeping plate.

4. Location and Time of First Crack Appearance

The stress field found in the previous section was investigated to minimize the values of the time to first crack appearance given by eq./4/.

The results are summarized in Fig. 3 where τ^* is plotted against the creep exponent m for two given values of n /n=2 and n=4/, different boundary conditions, different plate geometry / ρ_z=4.25, ρ_w=0.25, and ρ_z=5.00, ρ_w=1.00/, and, finally, for different ratios of material constants n_o/n involved in eq./3/. Solid dots correspond to the location of the first crack on the upper surface of a plate, crosses - on the lower one. The following observations can be made:

i. Higher material constants m increase the values of time τ . Slightly greater values of τ^x correspond to greater values of the ratio n /n, although the difference is insignificant. The time τ^x is much more influenced by the value of material constant n. Usually, the higher the value of n, the longer the time τ^x. The opposite situation can be encountered for very high load intensity /i.e. for small time τ^x/.

ii. The increase of the outer radius ρ_z has the opposite influence for two cases of boundary conditions: For a freely supported plate it increases the time τ whereas for a plate with one edge fixed, the time τ^x decreases.

iii. The location of the first crack depends also on boundary conditions. For the freely supported plate, increase of ρ_z causes relocation of the point where the first crack appears from the inner edge and upper surface /point 2/ to a mid-span cross-section and lower surface of the plate /point 20/. The case m=2 is an exception. For the fixed plate the increase of ρ_z relocates the point of failure from the inner to the outer edge of the plate, but it always appears on the upper surface.

iv. The shortest times to failure τ^x correspond to the smallest values of m and n. On the other hand it is known /cf. e.g. [8]/ that these values decrease with an increase in temperature. Thus the proposed theory is in good qualitative agreement with the experimental observations of reductions of time to failure with growth in temperature.

References

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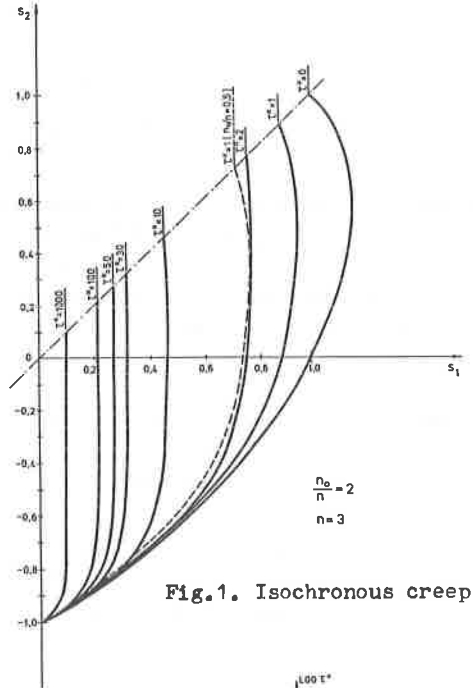

Fig.1. Isochronous creep rupture curves

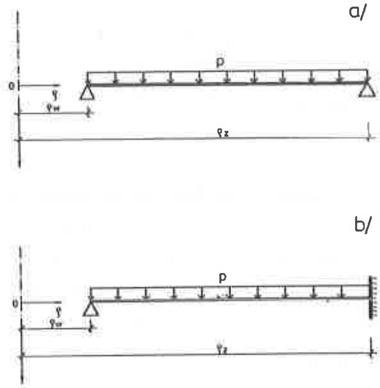

Fig. 2. Boundary conditions and geometry of plates

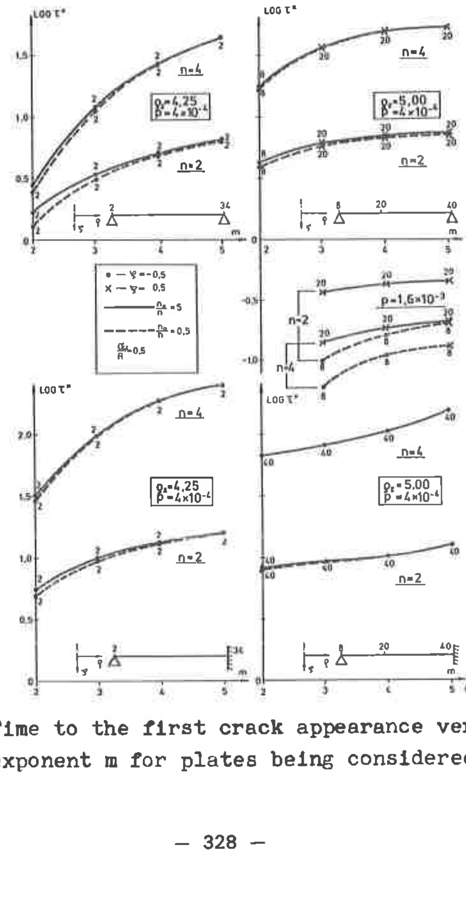

Fig.3. Time to the first crack appearance versus creep exponent m for plates being considered