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Reliability analysis of reinforced concrete structures

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ABSTRACT: A reliability based analysis of reinforced concrete structures is the main objective of this paper. The analysis is performed using the Direct Monte Carlo Simulation Method with the Importance Sampling technique, taking advantage of previous deterministic works. The structure is discretized with 3D finite elements and the model deals with elasto-viscoplasticity, concrete cracking and crushing , reinforcing steel and long term effects.

1 INTRODUCTION

The design criteria of reinforced concrete structures are related to a well established theory, but owing to uncertainties of the involved variables, to the adopted mathematical model, to human errors in structural implementation and problems of phenomenological character, empirical formulations are also used, based in foregoing background and ensuring a well known structural performance. In structural design, it is important to known the safety level in which the designer is working. Experimental models may be used to forecast the structural behavior of such structures, but this choice is not always preferable due to its high cost and even so, there are not measures of the design safety level. Therefore, the main problem is to find a quantitative evaluation of the safety margin, leading toward a consistent project with an homogeneous safety level for the whole structure.

A reliability based analysis of reinforced concrete structures is presented in this work. This analysis is performed using the Direct Monte Carlo Simulation Method with the Importance Sampling technique. This approach takes advantage of previous deterministic works in order to predict reinforced concrete structural behavior. In this way, important effects such as material non-linearities, can be taken into account readily, with no computational difficulties.

Due to the large computational process time required by a deterministic analysis, the Importance Sampling technique is used, reducing significatively the amount of simulations, but conserving the accuracy of the numerical solutions.

It is assumed as basic random variables the intensities of live and death loads, and properties of steel and concrete such as the concrete compression strength(f_c,), the Young's mudulus (E) , the steel tensile strength (f_y) and the Poisson coefficient (ν). The CEB-FIP-90[4] recommendations are followed for assessment of material characteristics. Reliability results are compared with target indexes included in reference papers.

The effect of long term phenomena in the global reliability are also studied using this approach.

2 DETERMINISTIC MODEL FOR ANALYSIS OF REINFORCED CONCRETE STRUCTURES

The deterministic model for the analysis of reinforced concrete structures used in this work is based in the model presented in Ref.[3]. The adopted formulation provides the evaluation of some characteristics observed in concrete structures, such as elasto-viscoplasticity, cracking and crushing. The Ottosen's model[7], with four parameters, is employed as a yield and failure limit surface. This approach allows for the evaluation of simple elements, like beams and columns, or more realistic structures such as reactor containment vessels.

The concrete structure is discretized using three-dimensional isoparametric twenty-node brick elements (fig.(1)) of the Serendipity family, with 3 degree of freedom per node (translations), but eight-node brick elements may be used as an alternative. The analysis is performed using the iterative Newton-Raphson Method to take into account non-linear effects due to cracking, viscoplasticity and creep. The smeared crack model is used in this work for evaluation of discontinuities in concrete. The Young's modulus is reduced in the perpendicular direction to the crack. The shear transfer across the crack is estimated giving a reduced value to the shear modulus corresponding to the crack plane This value is a function of the fictitious tensile strain normal to the crack plane and the type of concrete .

The steel is modelled as a smeared two-dimensional layer of equivalent thickness "t" embodied in the brick element, coincident with the surface corresponding to two of the natural axes (fig.(2)). Perfect bond is assumed between concrete and steel. A classical one-dimensional elasto-plastic model with linear hardening is used as a constitutive model for the reinforcing steel.

3 DIRECT MONTE CARLO SIMULATION METHOD WITH THE IMPORTANCE SAMPLING THECNIQUE

Due to the large computer process time required for the deterministic analysis, the reliability index β is evaluated using the Direct Monte Carlo Simulation Method with the Importance Sampling technique[2],[6]. Instead to simulate the failure probability P_f as indicated in eq.(1), it is employed a importance sampling probability density function (eq.(2)), which carries the simulations to the failure region, providing more failure cases in the simulation process. This probability density function must be correctly chosen near the IFM-point(Iterative Fast Monte Carlo procedure)[2], to ensure a low variation coefficient .

$$P_f = \int_{\{X | h(x)<0\}} f_X(x).dx \quad (1)$$

$$P_f = \int_{all\ domain} I[h(x) \le 0] \frac{f_X(x)}{l_v(x)} .l_v(x).dx \quad (2)$$

where, x is the vector of random basic variables, $h(x)$ is the limit state function, $f_X(x)$, is the joint probability density function, $l_v(x)$ is the importance sampling density function and $I[.]$ is the indicator function, which takes the following values:

$$I[h(x)] = \begin{cases} 0 \; for \; h(x) > 0 \\ 1 \; for \; h(x) \le 0 \end{cases} \quad (3)$$

The proposed model is outlined in fig.(8). First, a sample of basic uniformly distributed variables is generated and transformed to a normal Gaussian space. Then, the correlation matrix is imposed through the Nataf's Model[5]. After this stage, the samples are returned to the original space, and the inverse transformation is used to apply the corresponding probability density functions. The simulations are then made with the deterministic model for each of the generated samples. The mean value of the failure probability and variation coefficient are calculated as indicated in eq.(4). The reliability index β is calculated using the relation given by eq.(5).

$$\hat{P}_f = \frac{1}{N}\sum_{i=1}^{N} I[h(x_i) \le 0] \frac{f_X(x_i)}{l_v(x_i)} = \frac{1}{N}\sum_{i=1}^{N} I_W \quad ; \quad \hat{\delta}_{\hat{P}_f} = \frac{\hat{\sigma}_{\hat{P}_f}}{\hat{\mu}_{\hat{P}_f}} = \frac{1}{N\hat{P}_f}\sqrt{\sum_{i=1}^{N} I_W^2 - N\hat{P}_f^2} \quad (4)$$

$$\beta = -\phi^{-1}\left(\hat{P}_f\right) \quad (5)$$

where, N is the number of simulations, ϕ is the normal probability density function, $\hat{P}_f$ is an estimated value of the failure probabiility , $\hat{\sigma}_{\hat{P}_f}$ $\hat{\mu}_{\hat{P}_f}$ are estimated values of the standard deviation and the mean value of the failure probability, repectively, and $\hat{\delta}$ is the variation coefficient .

The parameters for the reliability analysis are summarized in tab.(1)

tab.1 Basic Variables and parameters for probability density function

Variable	*Probability Density Function*	*Unit*	*Mean value* μ	*Standard Deviation* σ
f_c	*Log-Normal*	*MPa*	***0.195x10²***	***0.500x10¹***
f_y^2	*Log-Normal*	*MPa*	***0.325x10³***	***0.300x10²***
f_y^1	*Log-Normal*	*MPa*	***0.320x10³***	***0.300x10²***
v	*Log-Normal*	-	***0.200x10⁰***	***0.200x10⁻¹***
G	*Normal*	-	***0.100x10¹***	***0.100x10⁰***
Q	*Normal*	-	***0.100x10¹***	***0.400x10⁰***

For sake of comparison, some reliability indexes cited in Ref.[8] are presented in tab.(2) below.

tab.2 Reliability indexes and security levels

	β index		
Security level	*1*	*2*	*3*
Ultimate limit state	*2.5*	*3.0*	*3.5*
Serviceability limit state	*4.2*	*4.7*	*5.2*

4 RHEOLOGIC MODEL FOR CONCRETE

Long Term effects in reinforced concrete structures are characterized by time dependent strains and stresses. Increase of strains under constant stress or decrease in stresses under constant strain are two faces of a same phenomenon, commonly called creep. This phenomenon is observed in the first months, but it prolongates through almost all the serviceability life of the structure, resulting, for example, in displacements of the same order than those due to service loads.

In this paper, the solidification theory, proposed by Bazant et al.[1], is used to evaluate long term effects in concrete. It is based in the micromechanical process of aging. The principal advantage in using this model lies in the nonaging characteristic of the rheologic model, i.e., its spring moduli and viscosities are independent of time, as in classical linear viscoelasticity. This fact makes the Kelvin model more advantageous than the Maxwell model, in which aging is described by considering the parameters of the chain as functions of time. A visco-elastic model based in a Kelvin's chain, with five elements, is used to evaluate the behavior under long term loads (fig.(3)). Using this theory, it is only necessary to evaluate the parameters of the chain at the beginning of the analysis. The Kelvin's parameters are adjusted to a known creep function, described in CEB-FIP90[4] through a non-linear least square fitting method. The influence of the temperature in concrete maturity and in subsequent days after loading (dilatational effects), type of cement and shrinkage are taken into account as imposed strains in this work. In all these cases, the CEB-FIP90[4] recommendations are also followed.

The solidification theory associated to a visco-elastic model with a Kelvin chain, as explained in Bazant et al.[1], allows the use of the exponential algorithm, with gradually increasing time steps until values greatly exceeding the shortest retardation time of a Kelvin unit, while, at the same time, conserving numerical stability and good accuracy. The theory presented by Bazant et al.[1] was generalized to triaxial stress states assuming isotropy.

5 NUMERICAL RESULTS

First, it was accomplished a deterministic analysis for the beam sketched in fig.(4). The beam was loaded with a uniformly distributed load of 3,04 KN/m (22,36% dead load - 77,63% live load). At the loading age (16th day) the temperature and humidity were 20°C and 70%, and at age of 90th day the temperature and humidity were 25°C and 65%, respectively. The results presented here are related to the midspan deflection of the beam. This analysis was performed for short term loading, and compared with experimental results (fig.(5)). A reliability analysis was performed for short term load, using the maximum deflection at the central cross section (<1.9x10^{-2} m), crushing of any integration point or divergence in the solution process as limit state functions. In fig.(6) it is shown a comparison between numerical and experimental results for long term effects. Results of a deterministic analysis for the outlined beam for 10.000 days (~27 years) are shown in fig.(7). The reliability analysis was performed for this age. For long term loads, the calculated reliability index was β=*0.71* with a variation coefficient of δ=*0.21*, while for short term loads, the computed reliability index was β=*1.96* with a variation coefficient of δ=*0.20*.

6 CONCLUSIONS

It is observed that long terms effects are decisive to evaluate the reliability index in reinforced concrete structures. A simple analysis of short term load could lead to different reliability indexes than those obtained considering long term effects.

In both cases, the reliability indexes calculated are quite below the target indexes cited in Ref.[8] for any security level.

7 ACKNOWLEDGMENT

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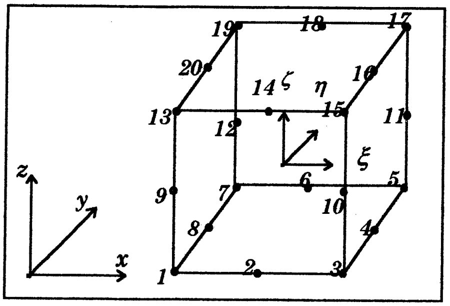

fig.1 Isoparametric element with 20 nodes

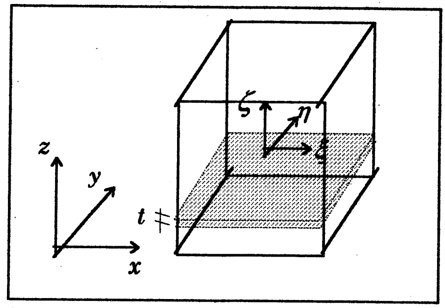

fig.2 Reinforcing steel model

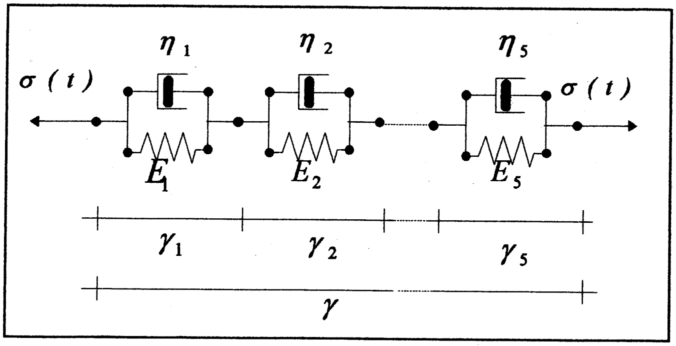

fig.3 Kelvin chain

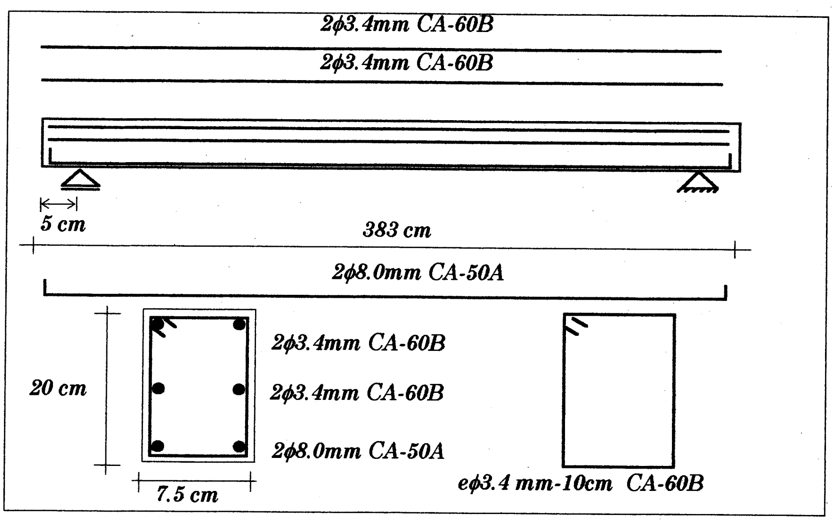

fig.4 Geometry data

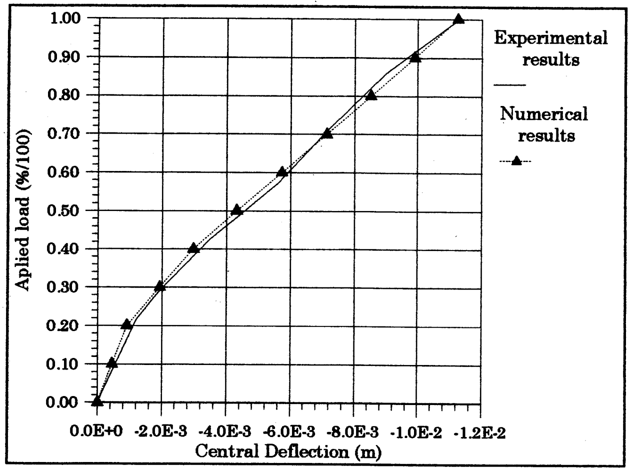

fig.5 Short term load

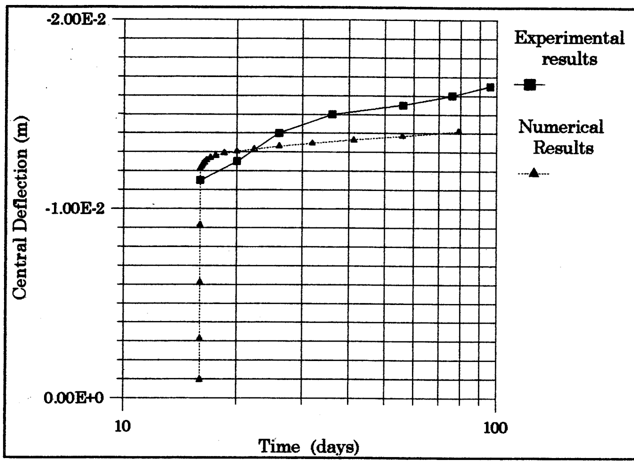

fig.6 Long term effects (100 days)

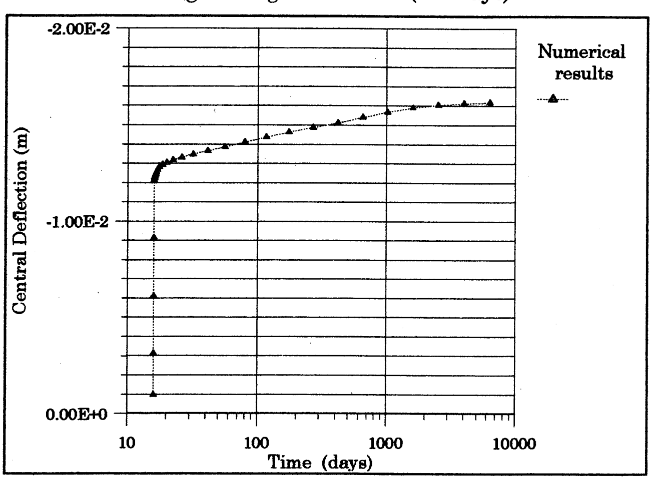

fig.7 Long term effects (10.000 days)

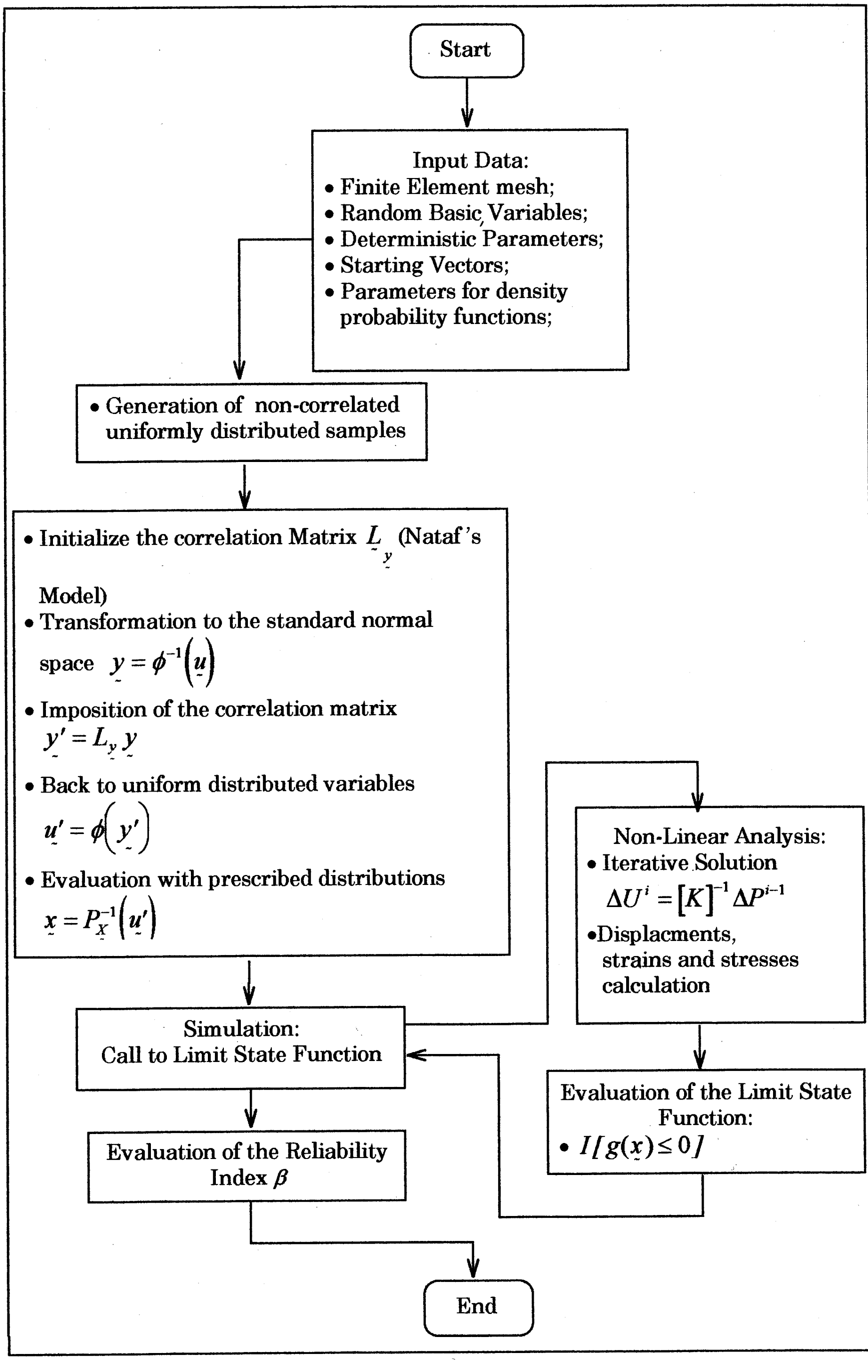

fig.8 Outline of the proposed model for reliability analysis of reinforced concrete structures