

ABSTRACT

SHEPPARD, PARKER WILSON. Collateral Constraints in DSGE Models: Theory and Empirics. (Under the direction of James Nason.)

This dissertation studies financial markets in macroeconomic models. Specifically, the focus is on the sources and causes of changes in leverage over the business cycle. The dissertation also includes an examination of the effect leverage has on business cycle fluctuations.

Leverage is the ratio of a firm's assets to its equity. A firm's leverage also determines its debt-to-assets ratio, which is referred to as "liquidity" in the literature on collateral constraints. The terminology can introduce some confusion, because there are many different definitions of "liquidity" in use in economics. In this dissertation, I define liquidity to be the relative value of debt that can be borrowed against the market value of an asset. The more liquid are assets, the higher is the maximum debt-to-asset ratio and the higher is maximum borrower leverage. This usage of the term "liquidity" is related to another common usage, in which a liquid asset is one that readily sells for its fundamental price. When an asset is posted as collateral, the amount of debt borrowed against it is the purchase price should the lender take possession of it. A decrease in liquidity could occur when lenders place a lower value on collateral than borrowers. A drop in the value of collateral to lenders reduces the amount of financing available to borrowers, which can affect their investment and output decisions. Fluctuations in liquidity are an additional pathway by which the financial market outcomes can affect outcomes in real markets.

The dissertation consists of two essays. The first essay develops a dynamic, stochastic general equilibrium (DSGE) model with a collateral constraint, which places a limit on the leverage of borrowers and creates a transmission mechanism for financial activity to affect real activity. The limit is a fraction of the market value of a borrowers assets. The first essay contributes to the literature on financial frictions by endogenizing the fraction in calculating the limit. The standard approach in the literature is to model the fraction using an exogenous stochastic process. Endogenizing the fraction in the collateral constraint partially resolves a puzzle in the literature that assumes liquidity is exogenous. In particular, models predict that an exogenous fall in liquidity produces an increase in asset prices, though asset prices and liquidity move together in the data. The first essay provides evidence for the transmission mechanism between liquidity in asset prices. Liquidity is endogenously determined, so it and asset prices fluctuate together with changes in productivity. The model finds that the effects of liquidity on asset prices are smaller than the effects of productivity on asset prices.

The DSGE model in the first essay assumes that the limits of the collateral constraint follow a stationary process. However, the sample data on the debt-to-assets ratio does not follow a

stationary process. The second essay is an empirical essay that investigates the co-movement of debt and real activity over the business cycle. It estimates vector autoregressions (VAR) that are subject to common trends and common cycles restrictions. The estimated models are then used to compute a permanent-transitory decomposition of macro and financial aggregates. The shocks to the permanent component constitute a significant portion of the total changes in the debt-to-assets ratio. The effects of the shocks to the permanent component are an important feature that models of the business cycle with collateral constraints need to incorporate.

© Copyright 2017 by Parker Wilson Sheppard

All Rights Reserved

Collateral Constraints in DSGE Models: Theory and Empirics

by
Parker Wilson Sheppard

A dissertation submitted to the Graduate Faculty of
North Carolina State University
in partial fulfillment of the
requirements for the Degree of
Doctor of Philosophy

Economics

Raleigh, North Carolina

2017

APPROVED BY:

Giuseppe Fiori

Douglas Pearce

Nora Traum

James Nason
Chair of Advisory Committee

DEDICATION

To my wife, Kelly. Without her support, I never would have been able to finish.

BIOGRAPHY

Parker Sheppard is a senior policy analyst at the Heritage Foundation. Previous to holding that position, he was the Jenkins Fellow at North Carolina State University, and a research assistant with the Bureau of Economic Analysis. He holds a master of science degree in mathematics and statistics from Georgetown University and a bachelor of arts degree in both economics and politics from Washington and Lee University.

ACKNOWLEDGEMENTS

I thank my advisors Jim Nason, Giuseppe Fiori, Doug Pearce, and Nora Traum, for many helpful discussions. I also thank Antje Berndt, Luigi Bocola, Luca Guerreri, Joeseoph Haslag, Mohammed Jahan-Pavar, Lutz Killian, Julia Thomas, Tao Zha, and macro seminar participants at N.C. State for helpful comments. Any remaining errors are my own.

TABLE OF CONTENTS

LIST OF TABLES	vi
LIST OF FIGURES	ix
Chapter 1 Production-Based Asset Pricing in a Real Business Cycle Model with Endogenous Liquidity	1
1.1 Introduction	1
1.2 Secured Debt Contract	6
1.3 Model	11
1.3.1 Production and Markets	12
1.3.2 Agents' Optimization Problems	14
1.3.3 Market-Clearing Conditions	16
1.3.4 Optimality Conditions	17
1.3.5 Equilibrium	20
1.4 Data and Calibration	20
1.4.1 Data	20
1.4.2 Calibration	23
1.5 Solution and Results	25
1.5.1 Solution	25
1.5.2 Results	27
1.6 Conclusion	31
Chapter 2 Common Credit Cycles and Business Cycles in the United States .	42
2.1 Introduction	42
2.2 Econometric Methods	43
2.3 Data	51
2.4 Results	52
2.4.1 Households	52
2.4.2 Firms	55
2.5 Conclusion	57
BIBLIOGRAPHY	70
APPENDICES	74
Appendix A Comparison of Collateral Constraints	75
Appendix B Deriving Optimality Conditions	78
Appendix C Stationary Equations	81
Appendix D Consolidated Optimality Conditions	83
Appendix E Cointegration Tests	85
Appendix F Solution Algorithm	88
Appendix G Euler Errors	91
Appendix H Data Description	93

LIST OF TABLES

Table 1.1	Calibrated Parameters. This table displays the parameters chosen to calibrate the model under three different cases. The cases differ in the amount of risk aversion exhibited by entrepreneurs and savers. Case 1 is the benchmark calibration, which uses the value in Shi (2015). Case 2 uses a high value for risk aversion and case 3 uses a low value for risk aversion.	23
Table 1.2	VAR Moments, Calib. 1. Rates are reported as percentages at an annual rate. Standard errors for the data are estimated by the delta method. Standard errors for the simulations are calculated by estimating the same regression on 1,000 simulations of each model and reporting the sample standard deviation of point estimates.	33
Table 1.3	VAR Moments, Calib. 2. Rates are reported as percentages at an annual rate. Standard errors for the data are estimated by the delta method. Standard errors for the simulations are calculated by estimating the same regression on 100 simulations of each model and reporting the sample standard deviation of point estimates.	34
Table 1.4	VAR Moments, Calib. 3. Rates are reported as percentages at an annual rate. Standard errors for the data are estimated by the delta method. Standard errors for the simulations are calculated by estimating the same regression on 100 simulations of each model and reporting the sample standard deviation of point estimates.	35
Table 2.1	Johansen Tests Regarding the Number of Cointegrating Relationships. Test statistics are reported allowing for a constant in the first differences (linear trend in the levels). P-values are computed by the Doornik (1998) gamma approximation with the command <code>coint2</code> using the statistical software <code>gretl</code>	58
Table 2.2	Cointegrating Vector Estimates, Households VECM. Standard errors for each term are reported in parentheses.	58
Table 2.3	Common Feature Test Results This table shows results for tests that there are s cofeature vectors in the data. $Cor.$ reports the canonical correlations of the data from smallest to largest. $C(p, s)$ is the test statistic. The test statistic has a χ^2 distribution with $DF = s^2 + snp + sr - sn$ degrees of freedom.	59
Table 2.4	Common Feature Vector Estimates, Household VECM. Test statistics are reported allowing for a constant in the first differences (linear trend in the levels). P-values are computed by the Doornik (1998) gamma approximation with the command <code>coint2</code> using the statistical software <code>gretl</code>	59

Table 2.5	Summary Statistics of the Permanent-Transitory Decomposition, Household Model. The data are reported in annualized growth rates as a percentage, i.e. 2.00 means 2% annual growth. The first two rows show the mean and standard deviation for each series. The next two rows show the standard deviation for the growth rate of the permanent component and the transitory component. The next two rows show the volatility of the permanent and transitory components relative to each series. The final two rows show the first autocorrelation of each transitory component and the contemporaneous correlation of the transitory component with the permanent component. The data for nondurable good and services consumption, durable goods consumption, residential investment, TFP, household debt, and household assets are sampled quarterly from 1952-2016. There are a total of 260 observations.	65
Table 2.6	Johansen Tests Regarding the Number of Cointegrating Relationships. Test statistics are reported allowing for a constant in the first differences (linear trend in the levels). P-values are computed by the Doornik (1998) gamma approximation with the command <code>coint2</code> using the statistical software <code>gretl</code>	65
Table 2.7	Estimated Cointegrating Vector, Business VECM. Standard errors are reported in parentheses.	67
Table 2.8	Common Feature Test Results. This table shows results for tests that there are s cofeature vectors in the data. <i>Cor.</i> reports the canonical correlations of the data from smallest to largest. $C(p, s)$ is the test statistic. The test statistic has a χ^2 distribution with $DF = s^2 + snp + sr - sn$ degrees of freedom.	67
Table 2.9	Summary Statistics of the Permanent-Transitory Decomposition, Business Model. The data are reported in annualized growth rates as a percentage, i.e. 2.00 means 2% annual growth. The first two rows show the mean and standard deviation for each series. The next two rows show the standard deviation for the growth rate of the permanent component and the transitory component. The next two rows show the volatility of the permanent and transitory components relative to each series. The final two rows show the first autocorrelation of each transitory component and the contemporaneous correlation of the transitory component with the permanent component. The data for non-residential investment, TFP, business debt, and business assets are sampled quarterly from 1952-2016. There are a total of 260 observations.	68
Table E.1	Engle-Granger Tests, Second Stage. Estimates are the coefficient on the lagged residual term in an augmented Engle-Granger test. Under the null hypothesis that there is no cointegration between the two series, the coefficient is 0.	87

Table G.1 Euler Errors. Euler errors are calculated as the absolute difference between the value implied by the Euler equation and the value set by the polynomial approximation as a percentage of the value set by the polynomial approximation. 91

LIST OF FIGURES

Figure 1.1	Structure of the Debt Contract. This figure shows how the prices P_t^k , R_t^i , and θ_t determine how corn and land exchange hands between borrowers and lenders in both repayment and failure.	7
Figure 1.2	Return to the Debt Contract in Partial Equilibrium. The two panels of this figure show how the return to the debt contract changes as the terms r_t^i and θ_t change. The left panel shows the effects of changing r_t^i , holding θ_t constant. The right panel shows the effects of changing θ_t , holding r_t^i constant.	8
Figure 1.3	Alternative Structure of the Debt Contract. This figure shows how the prices of the collateral and interest contracts map into the structure of the secured debt contract displayed in Figure 1.1	10
Figure 1.4	Data Series Plots. This figure show plots of real consumption per capita, TFP, and liquidity from 1952-2014.	22
Figure 1.5	Nonlinear Impulse Response Functions, Calib. 1. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 2$. The impulse responses are calculated for liquidity (θ_t), the price of land (p_t^k), entrepreneurs' share of capital $k_{e,t}$), and aggregate output ($y_{a,t}$ under the three specifications for liquidity, a constant (Param.), a stochastic process (Stoch.), and an endogenous process (Endog.). The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.	36
Figure 1.6	Nonlinear Impulse Response Functions, Calib. 1. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 2$. The impulse responses are calculated for the components of the collateralized debt contract when liquidity is endogenously determined. The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.	37
Figure 1.7	Nonlinear Impulse Response Functions, Calib. 2. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 4$. The impulse responses are calculated for liquidity (θ_t), the price of land (p_t^k), entrepreneurs' share of capital $k_{e,t}$), and aggregate output ($y_{a,t}$ under the three specifications for liquidity, a constant (Param.), a stochastic process (Stoch.), and an endogenous process (Endog.) The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.	38
Figure 1.8	Nonlinear Impulse Response Functions, Calib. 2. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 4$. The impulse responses are calculated for the components of the collateralized debt contract when liquidity is endogenously determined. The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.	39

Figure 1.9	Nonlinear Impulse Response Functions, Calib. 3. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 0.75$. The impulse responses are calculated for liquidity (θ_t), the price of land (p_t^k), entrepreneurs' share of capital ($k_{e,t}$), and aggregate output ($y_{a,t}$) under the three specifications for liquidity, a constant (Param.), a stochastic process (Stoch.), and an endogenous process (Endog.) The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.	40
Figure 1.10	Nonlinear Impulse Response Functions, Calib. 3. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 0.75$. The impulse responses are calculated for the components of the collateralized debt contract when liquidity is endogenously determined. The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.	41
Figure 2.1	Plot of the Data in Log Levels. The graphs show quarterly plots of the data in log levels over the sample period 1952:1-2016:4. Shaded portions represent NBER recessions. The series plotted are consumption of nondurable goods and services, consumption of durable goods, residential investment, non-residential investment, total factor productivity, corporate and non-corporate debt, household debt, corporate and non-corporate assets, and household assets. All variables are reported on a real, per capita basis.	60
Figure 2.2	Plot of the Data in Log Differences. The graphs show quarterly plots of the first differences of the data in log levels over the sample period 1952:1-2016:4. Shaded portions represent NBER recessions. The series plotted are consumption of nondurable goods and services, consumption of durable goods, residential investment, non-residential investment, total factor productivity, corporate and non-corporate debt, household debt, corporate and non-corporate assets, and household assets. All variables are reported on a real, per capita basis.	61
Figure 2.3	Debt-to-Asset Ratios. The graphs show quarterly plots of the log of the ratio of debts to assets for both households and businesses from 1952:1-2016:4. The value of debts and assets for businesses is the sum of corporate and non-corporate debt and assets, respectively.	62
Figure 2.4	Error Correction Term, Household VECM. This graph plots the error correction term, βy_t , for the single estimated cointegrating relationship in the household model.	63
Figure 2.5	Error Correction Term, Firms VECM. This graph plots the error correction term, βy_t , for the single estimated cointegrating relationship in the firm model.	64
Figure 2.6	Permanent-Transitory Decomposition, Household Model . These plots show the permanent-transitory decomposition for the household VECM. The top six panels plot the observed data and the calculated permanent component. The bottom six panels plot the transitory components. Shaded areas represent NBER recession periods.	66

Figure 2.7 Permanent-Transitory Decomposition, Business Model. These plots show the permanent-transitory decomposition for the firm VECM. The top four panels plot the observed data and the calculated permanent component. The bottom four panels plot the transitory components. Shaded areas represent NBER recession periods. 69

Figure E.1 Engle-Granger Tests, Stage 1. Levels for each series are in black. In the top row, the red, blue, and green lines are the fitted values. Changing the independent variables in the regression has little effect on the fitted values, as the time trend dominates the fit. In the bottom row, the red, blue, and green lines are the residuals from the separate first stage regressions. . . . 86

Chapter 1

Production-Based Asset Pricing in a Real Business Cycle Model with Endogenous Liquidity

1.1 Introduction

Following the 2007-2009 financial crisis, there has been renewed interest in the role that the financial sector plays in business cycle fluctuations and asset pricing. Collateral constraints are one mechanism commonly used in models that creates a financial friction. The canonical version of a collateral constraint, first introduced by Kiyotaki and Moore (1997), takes the form,

$$b_t \leq \theta_t p_t^k k_t,$$

where b_t is the quantity of debt, p_t^k is the price of capital assets, and k_t is the quantity of capital assets held by a borrower and θ_t is fraction that expresses the degree of the friction. The above equation limits the amount of debt borrowers can issue to a fraction (θ_t) of the market value of their assets. Kiyotaki and Moore (1997) endogenized the borrowing limit, the term on the right hand side, by finding equilibrium values for the price of capital. Because the price of capital depends on investors' ability to borrow to finance investment, their model featured dynamics

whereby shocks to the productivity of capital lowered the price of capital, which tightened the collateral constraint, and fed back into the price of capital, further tightening the collateral constraint.

A common line of inquiry in the literature is to posit that a financial friction exists between borrowers and lenders and then study the interaction of an exogenous shock and the friction on the aggregate economy. For example, Jermann and Quadrini (2012) assume that firms are subject to a debt-issuance constraint and study the effects of exogenous changes in the maximum amount of debt that firms can issue relative to their capital. However, financial markets themselves are part of the economy. It is likely that any debt limits created by a financial friction are determined endogenously within the financial system and are not simply an exogenous process.

Whether the potential misspecification of financial frictions presents a problem for business cycle models is not known. Shi (2015) presents an example of an issue with this type of model of financial frictions that may be due to misspecification. However, Shi finds that exogenous shocks to liquidity can match the behavior of investment, employment and consumption, but not the behavior of asset prices. The reason is that lower liquidity makes credit harder to come by. Because assets can be posted as collateral and used to issue debt, their price increases following a negative liquidity shock. This contradicts what has been seen over the U.S. business cycle, where asset prices and liquidity both fall during recessions.

This paper develops a procedure to endogenize changes in debt limits due to the degree of the financial friction and compares the moments generated by business cycle models with exogenous debt limits to those generated by models with endogenous debt limits. In this model, lenders have a limited ability to force borrowers to repay loans (Hart and Moore, 1994) . As a result, lenders ask that borrowers attach collateral when issuing debt. Collateral assets trade at a market price. The debt limit facing borrowers is endogenously determined by the market value of their collateral, which is related to but distinct from the market value of their capital.

A fall in liquidity in this model is a reduction in the price of collateral relative to the price of capital.

In the event that a borrower fails to repay the debt, a lender can repossess a certain quantity of collateral, but its resale value is uncertain. If the price of the collateral asset falls, a lender may not be able to resell it for the original value of the debt. In anticipation of this, lenders discount the collateral when making a loan both to reduce the probability that borrowers fail to repay the debt and to reduce losses should failure occur.

To facilitate borrowing and lending, I introduce a secured debt contract in which borrowers attach assets as collateral when issuing debt. The contract is able to determine liquidity endogenously by explicitly pricing the option that borrowers have not to repay the loan. Borrowers pay for that option at the time the contract is entered into. When the value of that option increases, borrowers can borrow less against the same amount of capital, and liquidity falls. The changes in liquidity reflect that the value of a collateral asset to lenders is due in part to its potential use in production, but more so in its use for ensuring that borrowers fulfill their obligations under the debt contract.

I choose to use secured debt in the model because it is an important source of funding for firms in the United States. Sixty-one percent of commercial and industrial loans are backed by collateral, with a total amount valued at \$72.0 billion according to the November 2015 Federal Reserve Survey of Terms of Business Lending. About 40 percent of commercial paper is backed by collateral, with a value of \$763 billion in 2008 (Anderson and Gascon, 2009). Repurchase agreements, a type of lending where borrowers sell securities, which serve as collateral, to a lender and agree to buy them back at a later date, had an average daily trading volume of \$7.6 trillion in 2008 Q1. At that time, major investment banks had \$1.4 trillion of assets, 47 percent of which were pledged as collateral (Hordahl and King, 2008; Gorton and Metrick, 2010). During the 2007-2009 financial crisis, debt issuance in the asset-backed commercial paper (ABCP) and repo markets declined as lenders applied higher discounts to collateral (Anderson and Gascon,

2009; Gorton, 2009).

To limit computational problems and focus the analysis on the link between debt financing, asset prices, and output, I work with a real business cycle (RBC) model. Non-trivial borrowing and lending requires heterogeneous agents, referred to here as entrepreneurs and savers. They use a single factor input and separate production functions to produce a single consumption good. The entrepreneurs are more productive on average and less patient than savers, so they borrow from savers to increase the scales of their operations. The limited enforcement of contracts leads to a collateral constraint that limits the amount of debt that entrepreneurs can issue. When borrowers can borrow more against capital, that capital is said to be more liquid, for reasons which will be explained later in the text. The terms of the secured debt contract, the interest rate and the liquidity of collateral, are determined endogenously.

I solve the model using global, non-linear methods in order to capture the dependence of liquidity on higher-order properties of the economy. I compare business cycle and asset pricing moments generated by the model to the data. I calibrate the model using quarterly asset price, consumption, and production data from 1952-2014. I also compare the model to benchmark models with an exogenously-specified liquidity shock to quantify the effects of endogenizing liquidity. To evaluate how endogenous liquidity fluctuations can propagate TFP shocks, I also present non-linear impulse response functions for each of the three models.

In the model, liquidity responds to the asymmetric returns to lending. That is, due to the limited enforcement of contracts, lenders may suffer losses when collateral values fall, but do not experience gains when collateral values rise. The model predicts that a positive TFP shock causes liquidity to fall, which limits the amount that borrowers can increase their capital holdings. The increase in borrowers' capital occurs more gradually than it does under exogenous liquidity.

The models fail to match the data on a substantial number of moments. However, the endogenous-liquidity model comes closer to the data on several key asset-pricing moments,

especially the mean return to capital and the mean interest rate on short-term debt. The impulse response functions suggest the predictions of the endogenous-liquidity RBC model are consistent with the LSH, but quantitative importance of fluctuations in liquidity is dominated by the effects of productivity shocks.

This paper contributes to the literature on financial frictions in general equilibrium models. Notable papers that examine the effect of exogenous shocks to the ability of borrowers to obtain financing are Gertler and Karadi (2011), Christiano, Rostagno, and Motto (2010), Kiyotaki and Moore (2012), Shi (2015), and Bigio (2012). However, the paper most similar to this one is Jermann and Quadrini (2012). In their real business cycle model, firms may issue debt and equity to obtain funding. Interest expense on debt may be deducted from income taxes, so firms prefer debt to equity. However, firms may default on their loans, so they are subject to a borrowing constraint. Jermann and Quadrini's model assumes that, in the event of a default by borrowers, lenders are able to capture either the full quantity of the collateral or nothing, with some exogenous probability. They show that the renegotiation process implies that the liquidity of collateral is the probability that lenders can recover the full quantity of capital. A contribution of this paper is to endogenize the parameter in the borrowing constraint that Jermann and Quadrini model as an exogenous process. It does so by allowing for lenders to recover a portion of the value of debt, where that portion depends on the realized price of the collateral asset.

The nature of changes in financial frictions is still an open area of research. Limited enforcement of contracts is only one potential motivation for the endogenous determination of liquidity. Other papers in the literature investigate endogenous liquidity using different modeling choices. Eisfeldt (2004) creates a model where long-term, risky assets are illiquid due to adverse selection. Cui and Radde (2016) build a model in which liquidity is related to search costs between borrowers and lenders. Kurlat (2013) and Bigio (2015) motivate liquidity by asymmetric information about the quality of collateral assets.

The remainder of the paper is organized as follows. Section 1.2 describes the terms of a secured debt contract with endogenous liquidity. Section 1.3 describes the RBC model that determines the terms of the debt contract in general equilibrium. Section 1.4 describes the data sources and calibration methodology. Section 1.5 describes how the numerical solution for the equilibrium is found and presents the results. Section 1.6 concludes.

1.2 Secured Debt Contract

Before outlining the parts of the real business cycle model used in this paper, this section explains the secured debt contract that is used to determine liquidity endogenously. For this section, the relevant parts of the model are that there is a single consumption good, called corn for concreteness, and a single productive asset, called land. Corn is the numeraire good. Land trades at price P_t^k at time t .

I assume that lenders are unable to force borrowers to repay a loan, as in Hart and Moore (1994). To facilitate lending, I introduce a secured debt contract in which land is attached to debt as collateral. If a borrower fails to repay the debt, a lender can take the collateral and either use it or sell it. The debt contract is not necessarily an optimal contract. Rather, it is a straightforward way to endogenize the determination of liquidity.

The contract works as follows: at time t , a borrower posts one unit of land as collateral for a lender. In return, the lender gives the borrower $\theta_t P_t^k$ units of corn. The term θ_t is referred to as the liquidity of land. Liquidity is the ratio of the amount of corn that can be borrowed against one unit of land to the market price of one unit of land.

At time $t + 1$, one of two events occurs. In “repayment,” the lender receives $R_t^i \theta_t P_t^k$ units of corn from the borrower, where R_t^i is the gross interest rate on the debt contract in repayment. In “failure,” the borrower fails to repay the debt, in which case no corn changes hands. The lender keeps the land posted as collateral, and may either use the land in production or sell it at the new market price, P_{t+1}^k . Figure 1.1 illustrates the structure of the debt contract.

Before continuing, a note on terminology is in order. The term “liquidity” appears frequently in the literature, often with slightly different uses in different papers. One commonly used definition of a liquid asset is an asset that can readily be sold for its fundamental price. Land is liquid in this model in the sense that if the borrower fails to repay the debt, he has effectively sold the land to the lender, and the sale price is the amount of corn loaned. The more liquid land is, the closer the “sale” price is to the fundamental value (in this case P_t^k), and the more corn a borrower can borrow against the land.

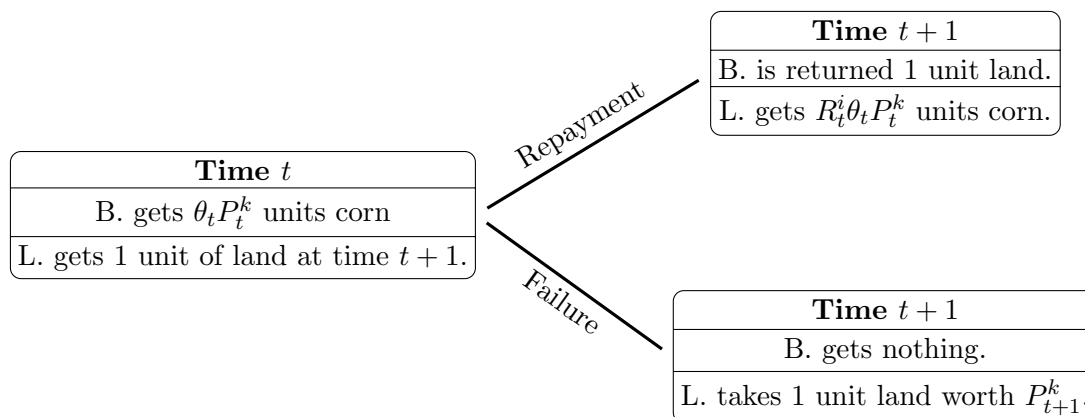


Figure 1.1 Structure of the Debt Contract. This figure shows how the prices P_t^k , R_t^i , and θ_t determine how corn and land exchange hands between borrowers and lenders in both repayment and failure.

At time $t + 1$, borrowers make the decision of whether or not to repay the debt. Whether repayment or failure occurs depends on the future price of land, P_{t+1}^k . If $P_{t+1}^k \geq R_t^i \theta_t P_t^k$, the market value of a unit of collateral is at least as large the principal of the debt plus interest. In this case, a borrower will repay the debt to obtain the collateral. If $P_{t+1}^k < R_t^i \theta_t P_t^k$, then borrowers can purchase land at the market price more cheaply than they could obtain it by repaying the loan, so they fail to repay the lenders.

Because borrowers choose whichever is cheaper from repayment or failure, the payoff to the

debt contract is $\min \{ R_t^i \theta_t P_t^k, P_{t+1}^k \}$. The return to the debt contract, R_{t+1}^b is the payoff divided by the principal, $\theta_t P_t^k$, or

$$R_{t+1}^b = \min \left\{ R_t^i, \frac{1}{\theta_t} \frac{P_{t+1}^k}{P_t^k} \right\}. \quad (1.1)$$

Figure 1.2 plots the return to the debt contract, expression (1.1), as a function of the gross appreciation of land, P_{t+1}^k/P_t^k , evaluated at different contract terms R_t^i and θ_t . The kink in each figure occurs at the break-even point where the return in repayment equals the return in failure. To the right of the kink in each curve, the appreciation of land is high, leading borrowers to repay the principal and interest on the debt. To the left of the kink in the figure, the appreciation of land is sufficiently low that borrowers fail to repay the debt contract.

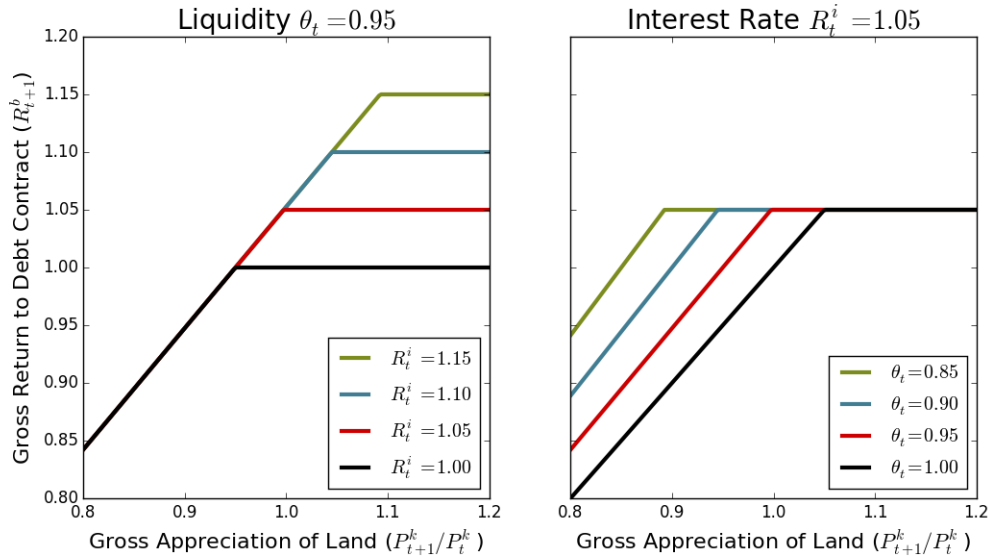


Figure 1.2 Return to the Debt Contract in Partial Equilibrium. The two panels of this figure show how the return to the debt contract changes as the terms r_t^i and θ_t change. The left panel shows the effects of changing r_t^i , holding θ_t constant. The right panel shows the effects of changing θ_t , holding r_t^i constant.

The left panel of Figure 1.2 shows that as the contracted interest rate on the debt contract

increases, the return to a debt contract increases, but only in states of the world where the debt contract is repaid. Higher interest rates also decrease the probability the borrower repays the debt because a higher return to land for is required for repayment to be the cheaper option for the borrower. The right panel of Figure 1.2 shows that as liquidity decreases, the return to the debt contract increases, but only in states of the world where the debt is not repaid. However, lower liquidity increases the probability that the borrower repays the debt because the land price must fall by a greater amount before failure is the cheaper option for the borrower.

The previous discussion shows that the interest rate and liquidity together determine the distribution of returns to the debt contract. However, the fact that a debt contract is one contract with two prices creates a problem: which price adjusts to clear the debt market, the interest rate or liquidity? To resolve this question, I express the debt contract as a pair of derivatives with land as the underlying asset. The pair of contracts exactly replicates the distribution of returns from the secured debt contract. Using two contracts creates two margins of adjustment that can determine two prices. An additional restriction on how the derivative contracts form the debt contract produces an extra equation that is required to solve for an equilibrium in a general equilibrium model.

The first derivative contract in the alternative representation is called a collateral contract. The terms of a collateral contract allow a borrower to sell one unit of land to a lender. The sale occurs at time t for price P_t^f (the mnemonic f is used because the lender receives the collateral in failure). The borrower receives corn at time t , retains the land for use in production, and at time $t + 1$ the lender takes possession of the land. The payoff to a collateral contract is the value of the land at time $t + 1$, P_{t+1}^k .

The second derivative contract in the alternative representation is called an interest contract. An interest contract makes explicit that the limited enforcement of contracts implies that borrowers always retain the option to repay the interest on the debt. Therefore, the interest contract is an option to repurchase the land at time $t + 1$ for the exercise price P_t^x . The exercise

price is equivalent to the principal and interest of the secured debt contract. The price of the option to repurchase the land is P_t^o (the mnemonic o is used because the borrower retains the option to repay the debt contract). Because the the interest contract enables a borrower to repurchase one unit of land worth P_{t+1}^k at a price of P_t^x , but the borrower will only do so if $P_{t+1}^k > P_t^x$, the payoff to an incentive contract is $\min\{P_{t+1}^k - P_t^x, 0\}$.

To execute a debt contract using the derivative contracts, a borrower writes a collateral contract and purchases an incentive contract. The borrower receives P_t^f units of corn from the sale of collateral, but spends P_t^o units of corn on the incentive contract. The net exchange is that the borrower receives $P_t^f - P_t^o$ units of corn, and the lender receives a claim on one unit of land to be delivered at time $t + 1$. At time $t + 1$, one of two events occurs. In “repayment,” the borrower repurchases the land from the saver for P_t^x units of corn, and the lender returns the land to the borrower. In “failure,” the borrower declines to exercise the option to repurchase the collateral, so the lender takes possession of the land, which is worth P_{t+1}^k , and no corn changes hands. Figure 1.3 illustrates the alternative structure of the debt contract.

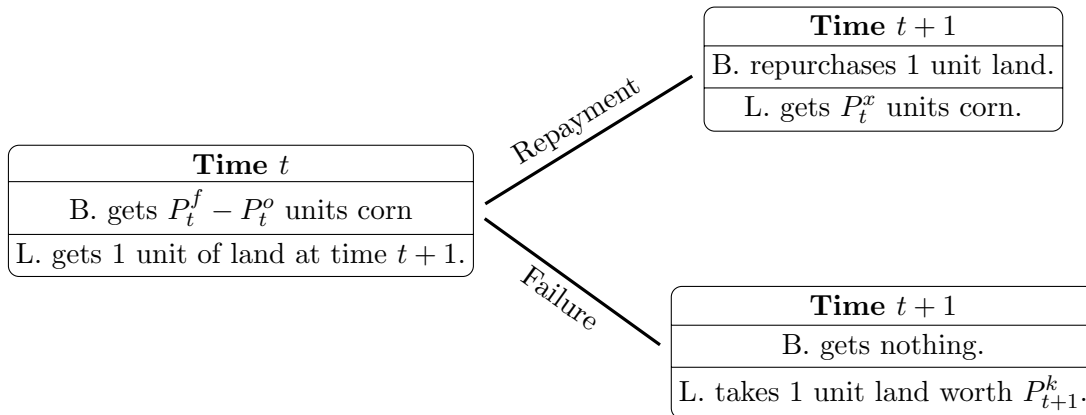


Figure 1.3 Alternative Structure of the Debt Contract. This figure shows how the prices of the collateral and interest contracts map into the structure of the secured debt contract displayed in Figure 1.1

Note that the prices in the alternative representation map neatly into the original representation. The gross interest rate on the debt contract in repayment is

$$R_t^i \equiv \frac{P_t^x}{P_t^f - P_t^o} = \frac{\text{Corn repaid}}{\text{Corn borrowed}}, \quad (1.2)$$

and liquidity is

$$\theta_t \equiv \frac{P_t^f - P_t^o}{P_t^k} = \frac{\text{Corn borrowed}}{\text{Collateral market value}}. \quad (1.3)$$

Liquidity is endogenously determined each period by a combination of three prices: the market value of land, P^k , the market value of collateral, P_t^f , and the price of the option entrepreneurs have to repay the debt, P_t^o .

The principal of the loan is the the price of the collateral less the price of the option to repurchase the collateral with interest, $P_t^f - P_t^o$. Therefore, the *ex post* return to a debt contract is

$$R_{t+1}^b = \min \left\{ \frac{P_t^x}{P_t^f - P_t^o}, \frac{P_{t+1}^k}{P_t^f - P_t^o} \right\},$$

The left-side argument of the min function is the return in repayment. Tthe other argument of the min function is the return in failure. The return to the debt contract depends on the promised interest rate, the liquidity of the collateral, and the realized appreciation in the value of land.

1.3 Model

This section describes the primitives of the model. It first discusses the production technology, the assets and goods, and the markets in which agents trade these objects. It next describes the agents' preferences and budget constraints. Finally, it derives the optimality conditions for each agent and equilibrium conditions for all markets.

1.3.1 Production and Markets

There is a unit mass of agents in this economy that is split between two types of agents, called entrepreneurs and savers. The fraction of the population that are entrepreneurs is $\xi \in [0, 1]$. Entrepreneurs and savers have different preferences and production technologies, but I assume there is a representative entrepreneur and a representative saver.

Entrepreneurs and savers use land and different technologies to produce corn. Each agent's technology is characterized by his total factor productivity (TFP) and elasticity of output with respect to land. The production functions are

$$Y_{e,t} = A_{e,t}k_{e,t-1}^{\alpha_e}, \quad (1.4)$$

$$\text{and } Y_{s,t} = A_{s,t}k_{s,t-1}^{\alpha_s}, \quad (1.5)$$

where $Y_{e,t}$ and $Y_{s,t}$ are the amounts of corn produced by entrepreneurs and savers, $k_{e,t}$ and $k_{s,t}$ are entrepreneur and saver land holdings, and $0 < \alpha_s \leq \alpha_e < 1$ are the elasticities of corn output with respect to land. The restriction on the elasticities of output captures the idea entrepreneurs will be able make better use of a marginal unit of land than savers. The difference between the two elasticities affects the relative value that savers and entrepreneurs place on a marginal unit of land, whether owned outright or as collateral. The aggregate output of corn is

$$Y_{a,t} = \xi Y_{e,t} + (1 - \xi)Y_{s,t}. \quad (1.6)$$

Entrepreneurs' and savers' TFP increase over time. The savers' TFP, $A_{s,t}$, increases deterministically at gross rate μ_s ,

$$\ln \mu_s = \ln A_{s,t} - \ln A_{s,t-1}. \quad (1.7)$$

Savers are not subject to shocks for simplicity. The entrepreneurs' TFP increases at the same

mean rate μ_s , but is subject to an exogenous shock,

$$\ln \mu_{e,t} = \ln A_{e,t} - \ln A_{e,t-1} = \ln \mu_s + (\rho_a - 1)(\ln A_{e,t-1} - \ln A_{s,t-1} - \ln \bar{a}) + \sigma_a v_{a,t}, \quad (1.8)$$

where $\bar{a} > 1$ is the mean ratio of entrepreneur TFP to saver TFP, $a_t = \frac{A_{e,t}}{A_{s,t}}$, ρ_a is the autocorrelation of the TFP ratio, $v_{a,t}$ follows a standard normal distribution, and σ_a is the standard deviation of TFP growth shocks. The second term on the right side of equation (1.8) is a cointegrating term that is necessary for the TFP ratio to be stationary. Combining equations (1.7) and (1.8), the TFP ratio follows an AR(1) process,

$$\ln a_t = (1 - \rho_a) \ln \bar{a} + \rho_a \ln a_{t-1} + \sigma v_t. \quad (1.9)$$

Because both TFP processes are non-stationary, I normalize all model equations relative to savers' TFP $A_{s,t}$ ¹.

Entrepreneurs and savers trade in competitive markets for corn and land. The quantity of land is fixed at \bar{k} . There is no aggregate investment in land. At each time period, the total amount of land is held by entrepreneurs and savers,

$$\xi k_{e,t} + (1 - \xi) k_{s,t} = \bar{k}. \quad (1.10)$$

At the beginning of time, land is divided evenly between each entrepreneur and saver.

¹The normalization is necessary to find a stationary, recursive representation of the economy. The non-stationary equations are presented in the main text and the stationary equations are presented in an appendix. Non-stationary variables are in upper-case, while stationary variables are in lower-case.

1.3.2 Agents' Optimization Problems

Entrepreneurs and savers have lifetime preferences defined by

$$U_{e,t} = E_t \left[\sum_{j=0}^{\infty} \beta_e^j \frac{C_{e,t+j}^{1-\eta_e}}{1-\eta_e} \right], \quad (1.11)$$

$$\text{and } U_{s,t} = E_t \left[\sum_{j=0}^{\infty} \beta_s^j \frac{C_{s,t+j}^{1-\eta_s}}{1-\eta_s} \right]. \quad (1.12)$$

where $C_{e,t}$ and $C_{s,t}$ are entrepreneurs' and savers' consumption at time t , η_e and η_s are the coefficients of relative risk aversion, and β_e and β_s are the discount rate of time preference. I assume $0 < \beta_e < \beta_s < 1$, so that entrepreneurs are less patient than savers. I also assume that $1 < \eta_e \leq \eta_s$, so that savers are as risk-averse or more so than entrepreneurs. Differences in risk aversion will affect how entrepreneurs and savers prefer to hold collateral and interest contracts, and thus the liquidity of land in the secured debt contract.

Entrepreneurs and savers are subject to the budget constraints

$$C_{e,t} + P_t^k k_{e,t} + P_t^f f_{e,t} + P_t^o o_{e,t} \leq Y_{e,t} + P_t^k (k_{e,t-1} + f_{e,t-1}) + \max \{ P_t^k - P_{t-1}^x, 0 \} o_{e,t-1}, \quad (1.13)$$

$$\text{and } C_{s,t} + P_t^k k_{s,t} + P_t^f f_{s,t} + P_t^o o_{s,t} \leq Y_{s,t} + P_t^k (k_{s,t-1} + f_{s,t-1}) + \max \{ P_t^k - P_{t-1}^x, 0 \} o_{s,t-1}, \quad (1.14)$$

Sources of funds for entrepreneurs and savers are the amount of corn produced in the current period, $Y_{i,t}$, the market value of land owned and the market value of collateral to be delivered, $P_t^k (k_{i,t-1} + f_{i,t-1})$, and the market value of interest contracts due, $\max \{ P_t^k - P_{t-1}^x, 0 \} o_{i,t-1}$, for $i = e, s$. Uses of funds are consumption of corn this period, $C_{i,t}$, the market value of land held for production next period $P_t^k k_{i,t}$, the market value of collateral contracts written or held due next period, $P_t^f f_{i,t}$, and the market value of interest contracts written or held due next period, $P_t^o o_{i,t}$, for $i = e, s$.

In addition to the budget constraint, entrepreneurs are subject to the collateral constraint,

which can be written as a short-sale constraint. Due to of limited enforcement, the amount of land that an entrepreneur can post as collateral at time t is limited to the amount of land he purchases at time t .

$$-f_{e,t} \leq k_{e,t}, \tag{1.15}$$

where $f_{e,t}$ is the number of collateral contracts held by entrepreneurs and $f_{e,t} < 0$ represents that entrepreneurs are selling collateral. This short-sale constraint is algebraically equivalent to a Kiyotaki-Moore collateral constraint. The difference is that Kiyotaki-Moore constraints are typically written in units of corn as

$$B_{e,t} \leq \theta_t P_t^k k_{e,t},$$

where $B_{e,t}$ is the units of corn borrowed. Given the definitions of the collateral an interest contracts, the units of corn borrowed in this model are

$$B_{e,t} = -(P_t^f - P_t^o) f_t.$$

The short sale constraint (1.15) follows from the definition of liquidity (1.3).

Let $z_t = [k_{e,t-1}, p_{t-1}^x, a_t]$ be the vector of state variables. The preferences, budget constraints, and collateral constraints for entrepreneurs and savers can be combined into the following recursive problems:

Problem 1 (Entrepreneurs' Problem). Entrepreneurs enter time t with portfolios of $k_{e,t-1}$, $f_{e,t-1}$, and $o_{e,t-1}$ and take market prices p_t^k , p_t^f , p_t^o , and p_t^x as given. They choose consumption $c_{e,t}$, land $k_{e,t}$, collateral contracts $f_{e,t}$, and interest contracts $o_{e,t}$ to maximize lifetime utility (1.11) subject to the budget constraint (1.13) and the short-sale constraint (1.15). The solution to the entrepreneurs' problem is a set of policy functions $c_e(z_t)$, $k_e(z_t)$, $f_e(z_t)$, and $o_e(z_t)$ that describe the optimal quantity choices as functions of the state variables.

Problem 2 (Savers' Problem). Savers enter time t with portfolios of $k_{s,t-1}$, $f_{s,t-1}$, and $o_{s,t-1}$ and take market prices p_t^k , p_t^f , p_t^o , and p_t^x as given. They choose consumption $c_{s,t}$, land $k_{s,t}$, collateral contracts $f_{s,t}$, and interest contracts $o_{s,t}$ to maximize lifetime utility (1.12) subject to the budget constraint (1.14). The solution to the savers' problem is a set of policy functions $c_s(z_t)$, $k_s(z_t)$, $f_s(z_t)$, and $o_s(z_t)$. that describe the optimal quantity choices as functions of the state variables.

1.3.3 Market-Clearing Conditions

The market for corn clears when the total amount of corn consumed equals the total amount of it produced,

$$\xi C_{e,t} + (1 - \xi)C_{s,t} = \xi Y_{e,t} + (1 - \xi)Y_{s,t}, \quad (1.16)$$

The market for land clears when the total amount of land purchased equals the fixed amount available,

$$\xi k_{e,t} + (1 - \xi)k_{s,t} = \bar{k}. \quad (1.17)$$

The debt market clears when the number of collateral contracts written equals the number of collateral contracts held, when the number of interest contracts written equals the number of interest contracts held, and entrepreneurs buy a sufficient number of interest contracts to repurchase exactly the amount of land sold using collateral contracts,

$$\xi f_{e,t} + (1 - \xi)f_{s,t} = 0, \quad (1.18)$$

$$\xi o_{e,t} + (1 - \xi)o_{s,t} = 0, \quad (1.19)$$

$$\text{and } f_{e,t} + o_{e,t} = 0. \quad (1.20)$$

The four prices P_t^k , P_t^f , P_t^o , and P_t^s are sufficient to satisfy the five market-clearing conditions (1.16)-(1.20) The price of land clears the market for land, the price of a collateral contract

clears the market for collateral contracts, the price of interest contracts clears the market for interest contracts and the exercise price adjusts to satisfy the formation requirement (1.20).

1.3.4 Optimality Conditions

The optimality conditions for the savers' problem are the Euler equations

$$E_t \left[\beta_s C_{s,t+1}^{-\eta_s} (P_{t+1}^k + A_{s,t} \alpha_s k_{s,t}^{\alpha_s - 1}) \right] = C_{s,t}^{-\eta_s} P_t^k \quad (1.21)$$

$$E_t \left[\beta_s C_{s,t+1}^{-\eta_s} P_{t+1}^k \right] = C_{s,t}^{-\eta_s} P_t^f, \quad (1.22)$$

$$\text{and } E_t \left[\beta_s C_{s,t+1}^{-\eta_s} \max \left\{ P_{t+1}^k - P_t^x, 0 \right\} \right] = C_{s,t}^{-\eta_s} P_t^o. \quad (1.23)$$

Equation (1.21) sets the expected discounted marginal benefit of land to savers equal to the marginal cost of land. Equation (1.22) sets the expected discounted marginal benefit of a collateral contract to savers equal to the marginal cost of a collateral contract. Equation (1.23) sets the expected discounted marginal cost of an interest contract equal to the marginal benefit of an interest contract.

Similarly, the optimality conditions for the entrepreneurs' problem are the Euler equations

$$E_t \left[\beta_e C_{e,t+1}^{-\eta_e} (P_{t+1}^k + A_{e,t+1} \alpha_e k_{e,t}^{\alpha_e - 1}) \right] + \nu_{e,t} = C_{e,t}^{-\eta_e} P_t^k, \quad (1.24)$$

$$E_t \left[\beta_e C_{e,t+1}^{-\eta_e} P_{t+1}^k \right] + \nu_{e,t} = C_{e,t}^{-\eta_e} P_t^f, \quad (1.25)$$

$$\text{and } E_t \left[\beta_e C_{e,t+1}^{-\eta_e} \max \left\{ P_{t+1}^k - P_t^x, 0 \right\} \right] = C_{e,t}^{-\eta_e} P_t^o, \quad (1.26)$$

where $\nu_{e,t}$ is the Lagrange multiplier on the entrepreneurs' collateral constraint. The Lagrange multiplier is the shadow price of the collateral constraint and represents the additional utility that would accrue to an entrepreneur if he were to have one additional unit of land or need to post one fewer unit of land as collateral. Equation (1.24) sets the marginal benefit of holding land equal to the marginal cost of land. The marginal benefit of holding land is the expected discounted value of the land next period, P_{t+1}^k , plus the expected discounted value of the marginal product of land, $\alpha_e a_{t+1} k_{e,t}^{\alpha_e - 1}$, plus the value of being able to post land as collateral today, $\nu_{e,t}$.

Equation (1.25) sets the marginal cost of a collateral contract equal to the marginal benefit of a collateral contract. The marginal cost of a collateral contract is the expected discounted market price of the collateral next period, P_{t+1}^k , plus the cost from posting land as collateral and tightening the collateral constraint, $\nu_{e,t}$. Equation (1.26) sets the expected discounted marginal benefit of an interest contract, $\max\{P_{t+1}^k - P_t^x, 0\}$ equal to the marginal cost of an interest contract.

The two optimality conditions for the interest contract together imply a condition that determines P_t^x :

$$E_t \left[\underbrace{(M_{s,t+1} - M_{e,t+1})}_{\text{SDF difference}} \underbrace{\max\{P_{t+1}^k - P_t^x, 0\}}_{\text{Payoff to incentive contract}} \right] = 0. \quad (1.27)$$

This equation describes risk sharing between entrepreneurs and savers. Across states of the world, including both repayment and failure, savers may value the payoff more than entrepreneurs (positive SDF difference), or savers may value the payoff less than entrepreneurs (negative SDF difference). The arbitrage condition requires the average difference across states weighted by the payout to the incentive contract be zero. As P_t^x adjusts to satisfy the above arbitrage condition, it changes which states of the world are mapped into repayment and failure. The limited enforcement of repayment prevents complete sharing of risk between entrepreneurs and savers. Risk is shared up to the difference between the future price of land and the repurchase price of land specified in the debt contract.

The collateral constraint creates a wedge between the price that entrepreneurs are willing to pay for land and the price that savers are willing to pay for land, which is captured by $\frac{\nu_{e,t}}{c_{e,t}^{-\eta_e}}$, the Lagrange multiplier on the collateral constraint deflated by the entrepreneurs' marginal utility. This wedge appears in both the optimality conditions for land and for the collateral contract.

Combining those equations produces a consolidated land and financing optimality condition²,

$$\begin{aligned}
& E_t \left[M_{s,t+1} \left(\underbrace{\alpha_s A_{s,t} k_{s,t}^{\alpha_s - 1}}_{\text{Saver MPK}} + \underbrace{\min \{ P_{t+1}^k - P_t^x, 0 \}}_{\text{'Payoff' to limited enforcement}} \right) \right] \\
&= E_t \left[M_{e,t+1} \left(\underbrace{\alpha_e A_{e,t+1} k_{e,t}^{\alpha_e - 1}}_{\text{Entrepreneur MPK}} + \underbrace{\min \{ P_{t+1}^k - P_t^x, 0 \}}_{\text{'Payoff' to limited enforcement}} \right) \right]. \quad (1.28)
\end{aligned}$$

The limited enforcement of contracts introduced the possibility that savers might incur losses due to the entrepreneurs' failure to repay debt. An entrepreneur who issues debt to purchase land earns the marginal product of land plus any potential gains from failing to repay the debt on the land. Similarly, a saver who declines to purchase debt and instead buys land outright gains the marginal product of land and avoids having to bear potential losses due to failure. If there were perfect enforcement, then the market for land would equalize the discounted values of the marginal products of land.

Recall that liquidity is endogenously determined by the prices of land, the collateral contract, and the interest contract, $\theta_t = \frac{P_t^f - P_t^o}{P_t^k}$. The savers' optimality conditions can be solved for these prices and substituted into the above expression to show how liquidity is related to the fundamentals of production,

$$\theta_t = \frac{E_t \left[\beta_s \left(\frac{C_{s,t+1}}{C_{s,t}} \right)^{-\eta_s} \min \{ P_t^x, P_{t+1}^k \} \right]}{E_t \left[\beta_s \left(\frac{C_{s,t+1}}{C_{s,t}} \right)^{-\eta_s} (P_{t+1}^k + A_{s,t} \alpha_s k_{s,t}^{\alpha_s - 1}) \right]}. \quad (1.29)$$

The above equation repeats that liquidity is the ratio of the value of the debt contract to the value of land. Equation (1.29) connects liquidity to the distribution of land through $k_{s,t}$ and to the amount of risk that entrepreneurs and savers are able to share through P_t^x .

²Details of the derivation are provided in the appendix.

1.3.5 Equilibrium

A stationary recursive competitive equilibrium is given by

1. Initial conditions $z_0 = [k_{e,0}, p_0^x, a_0]$,
2. Pricing functions $p^k(z_t)$, $p^f(z_t)$, $p^o(z_t)$, and $p^x(z_t)$, and
3. Policy functions $c_e(z_t)$, $c_s(z_t)$, $k_e(z_t)$, $k_s(z_t)$, $f_e(z_t)$, $f_s(z_t)$, $o_e(z_t)$, and $o_s(z_t)$,

such that

4. Given positive prices $p^k(z_t)$, $p^f(z_t)$, $p^o(z_t)$, $p^x(z_t)$, the optimal quantities $c_e(z_t)$, $k_e(z_t)$, $f_e(z_t)$, and $o_e(z_t)$ solve the entrepreneurs' Problem 1, satisfying equations (1.24) (1.25), and (1.26),
5. Given positive prices $p^k(z_t)$, $p^f(z_t)$, $p^o(z_t)$, $p^x(z_t)$, the optimal quantities $c_s(z_t)$, $k_s(z_t)$, $f_s(z_t)$, and $o_s(z_t)$ solve the savers' Problem 2, satisfying equations (1.21), (1.22), and (1.23), and
6. Given positive prices $p^k(z_t)$, $p^f(z_t)$, $p^o(z_t)$, $p^x(z_t)$, markets for the corn, land, collateral contracts, and interest contracts clear and the number of collateral contracts written by entrepreneurs equals the number of interest contracts held by entrepreneurs, satisfying equations (1.16), (1.17), (1.18), (1.19), and (1.20).

1.4 Data and Calibration

This section describes the sources of the data used to calibrate the model and the values to which the model parameters are calibrated.

1.4.1 Data

The sample data is quarterly, beginning in 1952 and ending in 2014. I follow the same process as in Mehra and Prescott (1985) in constructing the data, but obtain the data from different

sources.

Consumption is measured by Personal Consumption Expenditures as reported by the Bureau of Economic Analysis. Real PCE is calculated by dividing the nominal value of nondurables and services (NIPA Table 1.1.5) by the implicit price deflator (NIPA 1.1.9) to get a measure of real PCE in chained dollars. I calculate a Fisher ideal index for PCE less expenditures on durable goods (Whelan, 2000). I divide the calculated real PCE series by the population in each quarter to get real personal consumption expenditures per capita, because the model does not feature population growth.

Asset price data are taken from Shiller (2015). The price of equity and dividends are associated with Standard and Poor's Composite Stock Price Index. Prices and dividends are available at monthly and annual intervals. Quarterly return data are calculated by taking the product of monthly returns at monthly rates in each quarter.

The interest rate on short-term debt is taken from a number of data sources on commercial paper rates. From 1952-1971, I use three-month commercial paper rates from the NBER Macrohistory Database. From 1972-2014, I use three-month commercial paper rates available from the Federal Reserve. From 1997 onward, the Federal Reserve reports separate series for the rates of commercial paper issued by financial and nonfinancial firms. I combine the series weighting the financial rates by 0.75, which is approximately the average share of commercial paper outstanding issued by the financial sector from 2001-2013. That date range is chosen because it is the longest available from the Federal Reserve.

Nominal interest rates are converted to *ex post* real rates according to

$$\text{Real rate}_t = \text{Nominal rate}_t - \text{Inflation rate}_t.$$

The inflation rate is the percentage change in the Fisher ideal price index for services and nondurable consumption described above.

The TFP series used in this paper is the utilization-adjusted TFP series taken from Fernald

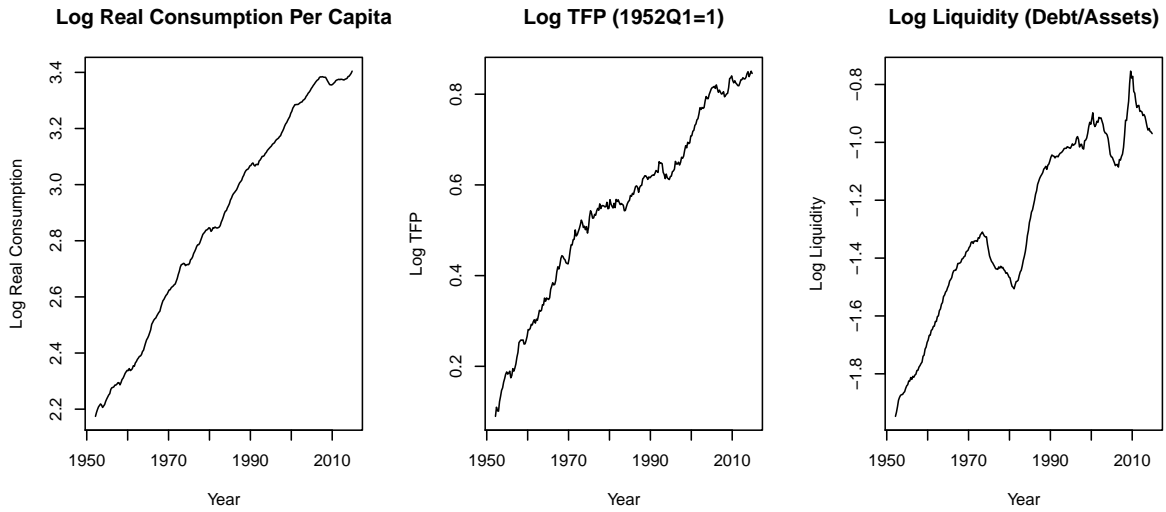


Figure 1.4 Data Series Plots. This figure show plots of real consumption per capita, TFP, and liquidity from 1952-2014.

(2012). That measure corrects the Solow residual measure of TFP for variation in the capital utilization rate, which facilitates comparison to this model because it does not contain a decision about capital utilization for either entrepreneurs or savers. The data is available at the website of the Federal Reserve Bank of San Francisco.

A data series for liquidity is constructed following a process similar to that used by Jermann and Quadrini (2012). The empirical measure of liquidity is calculated to match the model analogue, the ratio of the value of entrepreneurs' debt to the market value of their capital assets. For the value of assets, I use the sum of nonfinancial, noncorporate business nonfinancial assets (FL11201000) and the market value of nonfinancial corporate equity (LM103164103) from Table B.1 in the Financial Accounts of the United States published by the Federal Reserve. For the value of debt, I use the sum of debt securities (FL104122005) and loans (FL144123005) for nonfinancial business, published in Table L.102 of the Financial Accounts of the United States.

Figure 1.4 plots the log levels of consumption, TFP, and liquidity over the sample period. The

model hypothesizes that liquidity follows a stationary distribution, but a plot of the liquidity sample suggests that the mean could be rising over time. Cointegration tests fail to reject the null hypothesis that the series are not cointegrated. That is, the data is consistent with the hypothesis that consumption, TFP, and liquidity are three independent, $I(0)$ series. More detail on the cointegration tests is provided in the appendix.

1.4.2 Calibration

Table 1.1 displays the calibration of the model parameters.

Table 1.1 Calibrated Parameters. This table displays the parameters chosen to calibrate the model under three different cases. The cases differ in the amount of risk aversion exhibited by entrepreneurs and savers. Case 1 is the benchmark calibration, which uses the value in Shi (2015). Case 2 uses a high value for risk aversion and case 3 uses a low value for risk aversion.

Symbol	Description	Calib. 1	Calib. 2	Calib. 3
β_e	Entrepreneur discount rate	0.996	0.996	0.996
β_s	Saver discount rate	0.998	0.998	0.998
η_e	Entrepreneur risk aversion	2.000	4.000	0.750
η_s	Saver risk aversion	2.000	4.000	0.750
ξ	Entrepreneur pop. share	0.400	0.400	0.400
\bar{k}	Land supply	1.000	1.000	1.000
α_e	Entrepreneur output elasticity	0.300	0.300	0.300
α_s	Saver output elasticity	0.300	0.300	0.300
μ	Mean TFP growth	1.003	1.003	1.003
\bar{a}	Mean TFP ratio	2.000	2.000	2.000
ρ_a	TFP ratio autocorrelation	0.980	0.980	0.980
σ_a	TFP ratio shock st. dev.	0.008	0.008	0.008
ρ_θ	Liquidity shock autocorrelation	0.950	0.950	0.950
σ_θ	Liquidity shock relative std. dev.	0.003	0.003	0.003

The preference parameters for time discounting and risk aversion are chosen to facilitate a comparison with Shi (2015). The risk aversion parameters in the CRRA preferences are set at $\eta_e = \eta_s = 2$, the same value that Shi uses. I also estimate the model with two alternative

calibrations that raise and lower risk aversion for entrepreneurs and savers ³. The discount factors β_e and β_s , must vary slightly from Shi in order to satisfy the assumption that $\beta_e < \beta_s$. The discount factors, the risk aversion parameter and the mean growth rate of TFP imply an annual internal rate of 2.84% for entrepreneurs and 2.01% for savers.

The production parameters are calibrated so that entrepreneurs correspond to firms and savers correspond to households. I calibrate the population share of entrepreneurs to $\xi = 0.4$, which is roughly consistent with the average share of households who own stock as reported in the Federal Reserve's Survey of Consumer Finances. I calibrate the elasticity of output with respect to capital for both entrepreneurs and savers at $\alpha_e = \alpha_s = 0.3$ to match the average of capital's share of income as reported in Fernald (2012). The size of the stock of capital, \bar{k} , is irrelevant to the model, so I normalize $\bar{k} = 1$.

The mean TFP ratio \bar{a} is calibrated to be consistent with the mean ratio of nonfinancial assets held by businesses to nonfinancial assets held by households, which is around 1.5. This value is chosen making the entrepreneurs analogous to businesses and savers analogous to households. The parameters for the TFP process are chosen to be consistent with the process estimated by VAR from the sample data. I estimate a VAR on the return to capital, the interest rate on short-term debt, liquidity growth, consumption growth, and TFP growth. Four lag periods are used, which was chosen by AIC.

I solve three versions of the model. The first assumes liquidity is constant, $\theta_t = \bar{\theta}$. The second sets θ_t as following an exogenous stochastic process,

$$\theta_t = (1 - \rho_\theta)\bar{\theta} + \rho_\theta\theta_{t-1} + \sigma_\theta v_{\theta,t},$$

where, ρ_θ is the autocorrelation of liquidity and σ_θ is the standard deviation of liquidity. The

³ Mehra and Prescott (1985) show that models with CRRA preferences have difficulty simultaneously matching the equity premium and risk-free rate observed in the data. This is because CRRA preferences have one parameter that governs both risk aversion and the elasticity of intertemporal substitution. Other papers in the literature have used a preference structure that allows for risk aversion and the elasticity of intertemporal substitution to be raised simultaneously in order to resolve this problem, see Kreps and Porteus (1978) and Epstein and Zin (1989, 1991)

third sets $\theta_t = \frac{P_t^d - P_t^o}{P_t^k}$ according to the procedure outlined in Section 1.3. I set the parameters of the parameterized-liquidity model and the stochastic-liquidity to match the mean, standard deviation, and autocorrelation of the simulated liquidity series in the endogenous-liquidity model.

1.5 Solution and Results

This section discusses the methods applied to find approximate solutions to the models as outlined in Section 1.3.5. I give a brief overview of the approximation method and provide discussion of my choices for the parameters of the solution algorithm. I then present simulated moments and impulse response functions for the three models.

1.5.1 Solution

I solve the model using projection methods (Judd, 1998; Heer and Maussner, 2009). I use projection methods because the return to the interest contract introduces a discontinuity into the derivatives of the system of equations that characterize equilibrium. The discontinuity occurs at the kink point where the payoff to the debt contract changes between repayment and failure. Methods such as log-linearization or perturbation rely on continuous derivatives of the equilibrium equations. Taking derivatives of the equilibrium equations at an approximation point would capture the behavior of the economy in either repayment or failure, but not both across all possible future states. Projection is a global solution method that gives a good approximation over a range of the state space, not just around a particular approximation point.

Projection methods use a linear combination from a family of polynomials $P = \{\varphi_i\}_{i=0}^{\infty}$ to approximate a function f with

$$\hat{f}(\gamma, z) = \sum_{i=0}^p \gamma_i \varphi_i(z),$$

where the γ_i are coefficients of the polynomial. The polynomials are functions of the state

variables, z_t . The chosen degree of approximation is p . The closeness of the polynomial function to the true function is measured with a residual

$$R(\gamma, z) = F(\hat{f}(\gamma, z)),$$

where $F(\cdot)$ contains the vector of Euler equations (1.24), (1.21), (1.23), (1.22), and (1.26).

I choose five functions to approximate with polynomials: $c_e(z)^{-\eta_e}$, $p^k(z)$, $p^x(z)$, $p^f(z)$, and $p^o(z)$. The five functions correspond to the five Euler equations (1.24), (1.21), (1.23), (1.22), and (1.26). Each Euler equation can be solved for one of the approximated variables. The remaining policy functions are found by substituting the approximated variables into the remaining Euler equation (1.25); the market-clearing conditions (1.16), (1.17), (1.18), (1.19), (1.20); the budget constraint (1.13); and the collateral constraint (1.15).

I take the family of polynomials to be simple monomials in the log of the state variables, i.e. $1, \ln x, (\ln x)^2, (\ln x)^3, \dots$. Chebyshev polynomials are another alternative that, due to their orthogonality, should make it easier to identify higher-order coefficients. I choose to work with monomials here because Chebyshev polynomials are defined only on the interval $[-1, 1]$. To use Chebyshev polynomials, one must first define a compact subset of the state space to map into $[-1, 1]$. The polynomial coefficients are sensitive to the choice of subset. By using monomials, I reduce the number of parameters needed to solve the model.

If the approximating functions are close to the true functions, then the policy functions can be plugged into the Euler equations to produce a small residual. The residual is evaluated over a set of points in the state space that approximates the ergodic distribution of the state variables, following (Maliar and Maliar, 2015). This procedure ensures that the residual is evaluated at points in state that are relevant to the model and reduces the computational burden required to find a solution.

The solution algorithm follows an iterative process, alternating between simulating the model and solving for the projection coefficients. Given a candidate set of projection coeffi-

cients, I simulate the model. I then sample a subset of simulated points that approximates the ergodic distribution. Given the sample of points in state space, I find the projection coefficients that minimize the residuals of the Euler equations. This process iterates until the coefficients converge. To generate the initial projection coefficients, I use the first-order perturbation solution to a version of the model without failure.

Euler errors calculated from simulations suggest that the fit for all three models is good. Average Euler errors are less than 0.6%, which means that for an asset that would have a true value of \$1,000, the model sets prices within \$6 of the true value. The table of Euler errors is presented in the appendix, along with a complete description of the solution algorithm.

1.5.2 Results

1.5.2.1 VAR Moments

To estimate empirical moments, I first remove a linear trend from the data, then fit a VAR to detrended quarterly data on the return to capital, the interest rate on short-term debt, liquidity growth, consumption growth, and TFP growth, with a lag length of four chosen by AIC. Table 1.2 reports estimates of the mean, standard deviation, and first-order autocorrelation, along with their standard errors, as calculated from the sample data. Standard errors are calculated using the delta method.

Table 1.2 reports the average of the same statistics estimated on 1,000 simulations of each variation of the model using the benchmark Calibration 1. The simulated sample size matches the actual sample size. Standard errors for simulated data are taken to be the sample standard deviation of the estimates.

All three versions of the model fail to match the data, which is not surprising given the limited number of elements in the model and the well-known difficulty of RBC models to explain asset price behavior. The mean returns to capital are too low, the standard deviation of the return to capital is too low, the mean interest rate is too high and the standard deviation

of interest rates is too low. Additionally, consumption growth across all three models shows an average that is close to TFP growth, whereas the average consumption growth rate in the data is a little higher than the average TFP growth rate. Consumption growth in the data is substantially more persistent than it is in any of the models.

However, despite the difficulty the models have matching the data, the simulations are still useful for examining the marginal effect of endogenizing liquidity. The endogenous-liquidity model produces a higher mean return to capital and a lower mean interest rate than the other two models, which are both closer to the data. Additionally, introducing liquidity as another margin of adjustment in the debt market produces interest rates that are highly correlated, well above what is seen in the data. However, the sign is in the right direction, unlike the estimates for the exogenous-liquidity models. In sum, it appears that the endogenous-liquidity model generates asset pricing moments that are closer to the data without producing substantially worse moments for other model variables.

Table 1.3, Table 1.4, and Table ?? show the same results calculated under Calibrations 2-4. Calibration 2 increases risk aversion in both entrepreneurs and saver to bring the mean of the return to capital in line with that of the data. Calibration 3 decreases risk aversion to bring the mean interest rate in line with that of the data. Calibration 4 increases risk aversion for the savers and decreases it for the entrepreneurs, which seems plausible when thinking of entrepreneurs and stock-owning households as more people more comfortable holding a larger portion of their wealth in riskier assets.

The general pattern observed in the benchmark calibration tends to hold for the alternative calibrations. It is worth noting that when risk aversion is increased, the endogenous-liquidity model matches the mean return to capital while the exogenous-liquidity models have means that are just below the data. Alternatively, when risk aversion is decreased, the endogenous-liquidity model does worse, posting a mean return to capital that is lower and a mean interest rate that is higher than those in the exogenous-liquidity models. This effect is understandable

given that the key mechanism in the model depends on second-order properties of the returns to assets.

1.5.2.2 Impulse Response Functions

The nonlinear nature of the economic system means that the response to a shock depends on the state of the system when the shock occurs. Using methods developed by Gallant, Rossi and Tauchen (1993), I calculate non-linear impulse response functions.

To calculate the impulse responses, I find a discrete approximation to the ergodic distribution of the state space called an Epsilon-Distinguishable Set (EDS) (Maliar and Maliar, 2015). I choose an EDS of size M_{irf} , where the number of points M_{irf} is between 95 and 105. At each point in the EDS, I find the TFP ratio that is one standard deviation above and below the value of a_m , $m = 1, \dots, M_{irf}$. For each reference point, as well as the points one standard deviation above and below, I simulate the model for $T_{irf} = 20$ quarters a total of $N_{irf} = 200$ times. The impulse response function at each point is the difference between simulation paths averaged of the N_{irf} , and the reported impulse response function is the average difference averaged across M_{irf} points.

Figure ?? plots the impulse response functions to a positive one-standard-deviation productivity shock at each point in an EDS for all three models. The impulse responses to a one-standard deviation negative shock are similar in magnitude but opposite in sign.

A positive one-standard-deviation TFP shock has no effect on liquidity in the parameterized- and stochastic-liquidity models, as expected. A positive one-standard-deviation TFP shock causes liquidity to fall by about 0.6%. Liquidity then rises, returning to slightly higher than it was before the shock after about 10 quarters.

Across all three models, a positive one-standard-deviation TFP shock raises asset prices by about 0.6-0.7%. The persistence of the TFP process is evident, as prices are still about 0.4% higher after 20 quarters. However, the fall in liquidity associated with the TFP shock in the endogenous-liquidity model results in slightly lower asset prices than would otherwise occur.

In the exogenous-liquidity models, the positive TFP shock results in a rapid reallocation of land to entrepreneurs. The effect takes place immediately and persists with no subsequent change over 20 quarters. However, in the endogenous-liquidity model the reduction in liquidity limits the funding available for entrepreneurs to purchase additional land. Entrepreneurs eventually increase their share of land by the same 0.6% as in the exogenous-liquidity models, but the reallocation occurs over 15-20 quarters.

Similarly for output, a positive one-standard-deviation TFP shock raises output by about 0.5%, and output remains elevated over the following 20 quarters. Again, the reduction in liquidity following the TFP shock limits the amount that entrepreneurs increase their share of land held, so output increases by a slightly smaller amount in the endogenous-liquidity model. The effect of liquidity on land distribution is cumulative, so that the response of output to the TFP shock tends to die out more rapidly than in the other models.

Figure 1.6 breaks down the changes in liquidity by the components of the debt contract. Both land and collateral increase in value following a positive TFP shock. However, the exercise price shows a delayed increase. This delayed increase ensures increases the benefit to entrepreneurs from repaying the loan. The extra incentive is necessary to prevent entrepreneurs from borrowing to purchase newly-productive assets, then failing to repay the loans if the change in productivity reverses. The drop in liquidity delays the entrepreneurs' purchase of land until their wealth can increase to support the expansion. Corresponding with the delayed increase in the exercise price, the price of the option rises. The effect on the interest rate is small, between -0.02% and 0.05% initially and declining thereafter.

It is difficult to evaluate the liquidity shock hypothesis directly using the endogenous-liquidity model because there is no such thing as an exogenous shock to liquidity. However, the impulse response functions show that a decrease in liquidity occurs with a decrease in asset prices and a decrease in output for the endogenous-liquidity model relative to the change in asset prices that would occur in the parameterized- or stochastic-liquidity model. This lends

support to the mechanism put forth in the liquidity shock hypothesis. However, the effect of liquidity on asset prices is dominated by the effect of the productivity shock, which fits with Bigio's conclusion that liquidity shocks are not quantitatively important for explaining the comovement of asset prices and aggregate fluctuations.

Figures 1.7-?? show the same impulse responses for the alternative calibrations. Increasing risk aversion increases both the magnitude of the impulse responses and the variation in the impulse responses across initial states. Decreasing risk aversion has the opposite effect. The impulse response functions of the alternative calibrations reinforce the importance of risk to the model and of global, non-linear methods to the solution.

1.6 Conclusion

A real business cycle model with endogenous liquidity generates asset pricing moments that are closer to the data than models with exogenous liquidity. This suggests that modeling how firms obtain funds to purchase assets could be important in modeling asset prices. Nonlinear impulse response functions suggest that liquidity and asset prices move together, which supports the liquidity shock hypothesis. However, the quantitative effects of liquidity on asset prices are dominated by the response to TFP shocks. Under the model presented here, movements in liquidity are not quantitatively important for the comovement of asset prices and aggregate fluctuations. This result is in line with those of Shi (2015) and Bigio (2012).

The key mechanism in the model works by assigning a price to the option of whether or not to repay debt that borrowers are assumed to have due to limited enforcement of contracts. Following a positive productivity shock, liquidity falls in order to provide borrowers a stronger incentive to repay and to limit the losses to lenders in the event of failure. The endogenous liquidity mechanism slows the change in capital allocation to keep pace with changes in borrowers' wealth, which increases the variation in how the model responds to shocks due to the interaction between credit and asset markets. The additional variation is a source of risk that

factors into the return to capital and could contribute to explaining the equity premium puzzle.

Further research is needed to evaluate the effects of endogenous liquidity in DSGE models. The model considered here studies the effect of aggregate shocks to borrowers on liquidity. While aggregate shocks are important for setting liquidity, it is possible that shocks affecting individual borrowers play a role in determining liquidity, too. In the model presented here, all borrowers either repay debt or fail to repay at the same time. A more plausible scenario is that some borrowers repay debt while others do not, and lenders are concerned about the proportion of borrowers who repay. Future research should incorporate individual shocks in addition to aggregate shocks.

In this model, liquidity varies with changes in the state variables of the model. An additional source of variation in liquidity may be shocks to the volatility of TFP shocks. Greater volatility of productivity shocks may increase the spread of land price changes, which may increase the value of the interest contract, i.e. the value to borrowers of the option not to repay the debt.

Additionally, the alternative calibrations with varying parameters show the influence that agents' attitudes towards risk has on the model. Using time-separable constant relative risk aversion preferences simplifies the model equations, but it comes at the cost of restricting the relationship between risk aversion and the elasticity of intertemporal substitution. Future work should consider preference specifications that allow for more flexibility in setting preferences.

Incorporating these additional elements into the model presented here could produce interesting and useful research. The existing literature on liquidity shocks thinks of changes in debt limits due to collateral constraints as unforeseen and uncontrollable events. Those models are useful for analyzing how monetary policy can respond to financial shocks. The model presented here thinks of changes in liquidity as endogenous responses to the state of the economy. Future models that incorporate endogenous liquidity could be used to analyze policy designed to reduce fluctuations in liquidity.

Table 1.2 VAR Moments, Calib. 1. Rates are reported as percentages at an annual rate. Standard errors for the data are estimated by the delta method. Standard errors for the simulations are calculated by estimating the same regression on 1,000 simulations of each model and reporting the sample standard deviation of point estimates.

Moment	Variable	Data	Param.	Stoch.	Endog.
Return to Capital	Mean	6.8455 (7.8203)	3.4374 (0.1830)	3.4380 (0.0361)	3.8997 (4.2482)
	Std. Dev.	29.7597 (74.2922)	2.4978 (0.1122)	2.4826 (0.1124)	2.5297 (0.1157)
	First Autocorr.	0.1609 (0.1193)	0.0217 (0.0660)	0.0223 (0.0665)	-0.0306 (0.0658)
Interest Rate	Mean	0.2072 (2.6645)	3.2507 (0.4949)	3.2308 (0.1148)	2.7924 (2.2525)
	Std. Dev.	3.2111 (0.2850)	0.0819 (0.0038)	0.1872 (0.0089)	0.0823 (0.0120)
	First Autocorr.	0.2850 (0.2850)	-0.8380 (0.0658)	-0.0965 (0.0663)	0.7359 (0.0982)
Liquidity Growth	Mean	0.0338 (1.0347)	– –	0.0139 (0.0672)	0.0175 (0.1772)
	Std. Dev.	1.3376 (2.4773)	– –	2.5142 (0.1200)	2.1832 (0.1006)
	First Autocorr.	0.4238 (0.2683)	– –	-0.0288 (0.0650)	-0.1928 (0.0663)
Consumption Growth	Mean	0.7151 (0.9128)	1.2049 (0.0894)	1.2133 (0.0324)	1.2341 (1.0705)
	Std. Dev.	1.8351 (2.7371)	1.9128 (0.0947)	1.8998 (0.0944)	2.0610 (0.1101)
	First Autocorr.	0.4516 (0.2694)	0.0086 (0.0675)	0.0121 (0.0683)	-0.0517 (0.0668)
TFP Growth	Mean	0.9998 (0.0020)	1.2123 (0.1118)	1.2218 (0.0443)	1.2614 (1.6505)
	Std. Dev.	0.0314 (0.0153)	3.0662 (0.1374)	3.0451 (0.1372)	3.0415 (0.1389)
	First Autocorr.	-0.0166 (0.1147)	-0.0292 (0.0658)	-0.0259 (0.0663)	-0.0352 (0.0658)

Table 1.3 VAR Moments, Calib. 2. Rates are reported as percentages at an annual rate. Standard errors for the data are estimated by the delta method. Standard errors for the simulations are calculated by estimating the same regression on 100 simulations of each model and reporting the sample standard deviation of point estimates.

Moment	Variable	Data	Param.	Stoch.	Endog.
Return to Capital	Mean	6.8455 (1.9551)	4.6702 (0.2772)	4.6753 (0.199)	6.0390 (0.5675)
	Std. Dev.	29.7597 (18.5731)	4.3176 (0.2375)	4.2741 (0.2392)	4.4333 (0.2518)
	First Autocorr.	0.1609 (0.1193)	0.0138 (0.068)	0.0158 (0.0670)	-0.0417 (0.0634)
Interest Rate	Mean	0.2072 (0.6661)	4.434 (0.0779)	4.4293 (0.1006)	4.4345 (0.1623)
	Std. Dev.	3.2111 (1.5231)	0.1946 (0.0131)	0.295 (0.0150)	0.1621 (0.0148)
	First Autocorr.	0.285 (0.2884)	-0.5327 (0.0697)	-0.0968 (0.0791)	0.333 (0.1188)
Liquidity Growth	Mean	0.1353 (1.0485)	– –	0.0106 (0.0555)	0.0185 (0.0139)
	Std. Dev.	5.3506 (2.4773)	– –	2.5695 (0.1236)	2.9982 (0.1604)
	First Autocorr.	0.4238 (0.2683)	– –	-0.0333 (0.0695)	-0.4374 (0.0725)
Cons. Growth	Mean	0.7151 (0.2282)	0.0098 (0.0526)	0.0096 (0.0344)	0.0123 (0.0822)
	Std. Dev.	1.8351 (0.6843)	2.1588 (0.1189)	2.1396 (0.1223)	2.2235 (0.1267)
	First Autocorr.	0.4516 (0.2694)	-0.0148 (0.0744)	-0.013 (0.0743)	-0.09 (0.0700)
TFP Growth	Mean	-0.0173 (0.198)	0.0175 (0.0615)	0.0175 (0.0465)	0.0199 (0.1028)
	Std. Dev.	3.1369 (1.5329)	3.0580 (0.1595)	3.0299 (0.161)	3.0272 (0.1607)
	First Autocorr.	-0.0166 (0.1147)	-0.0382 (0.0640)	-0.0364 (0.0638)	-0.0436 (0.0638)

Table 1.4 VAR Moments, Calib. 3. Rates are reported as percentages at an annual rate. Standard errors for the data are estimated by the delta method. Standard errors for the simulations are calculated by estimating the same regression on 100 simulations of each model and reporting the sample standard deviation of point estimates.

Moment	Variable	Data	Param.	Stoch.	Endog.
Return to Capital	Mean	6.8455 (1.9551)	0.5847 (0.0109)	0.5833 (0.0222)	0.5308 (0.0321)
	Std. Dev.	29.7597 (18.5731)	1.1477 (0.0482)	1.142 (0.0496)	1.1589 (0.0500)
	First Autocorr.	0.1609 (0.1193)	0.0141 (0.0673)	0.0205 (0.0682)	-0.0504 (0.0706)
Interest Rate	Mean	0.2072 (0.6661)	0.4891 (0.1469)	0.4821 (0.1503)	0.6881 (0.8184)
	Std. Dev.	3.2111 (1.5231)	0.0441 (0.0019)	0.1705 (0.0075)	0.0501 (0.0041)
	First Autocorr.	0.285 (0.2884)	-0.7472 (0.0674)	-0.0499 (0.0608)	0.9046 (0.0746)
Liquidity Growth	Mean	0.1353 (1.0485)	– –	0.0233 (0.2053)	-7e-04 (0.0187)
	Std. Dev.	5.3506 (2.4773)	– –	2.4211 (0.1093)	1.0105 (0.0433)
	First Autocorr.	0.4238 (0.2683)	– –	-0.0248 (0.0609)	-0.1389 (0.0707)
Cons. Growth	Mean	0.7151 (0.2282)	0.0081 (0.0265)	0.0136 (0.0626)	-0.0128 (0.2625)
	Std. Dev.	1.8351 (0.6843)	2.0092 (0.0937)	1.9986 (0.0969)	1.941 (0.0931)
	First Autocorr.	0.4516 (0.2694)	0.007 (0.0668)	0.0182 (0.0674)	-0.035 (0.0703)
TFP Growth	Mean	-0.0173 (0.1980)	0.0153 (0.0241)	0.022 (0.0764)	-0.0123 (0.2358)
	Std. Dev.	3.1369 (1.5329)	3.0467 (0.1275)	3.03 (0.1305)	3.0293 (0.1306)
	First Autocorr.	-0.0166 (0.1147)	-0.0448 (0.0674)	-0.0339 (0.0687)	-0.044 (0.0706)

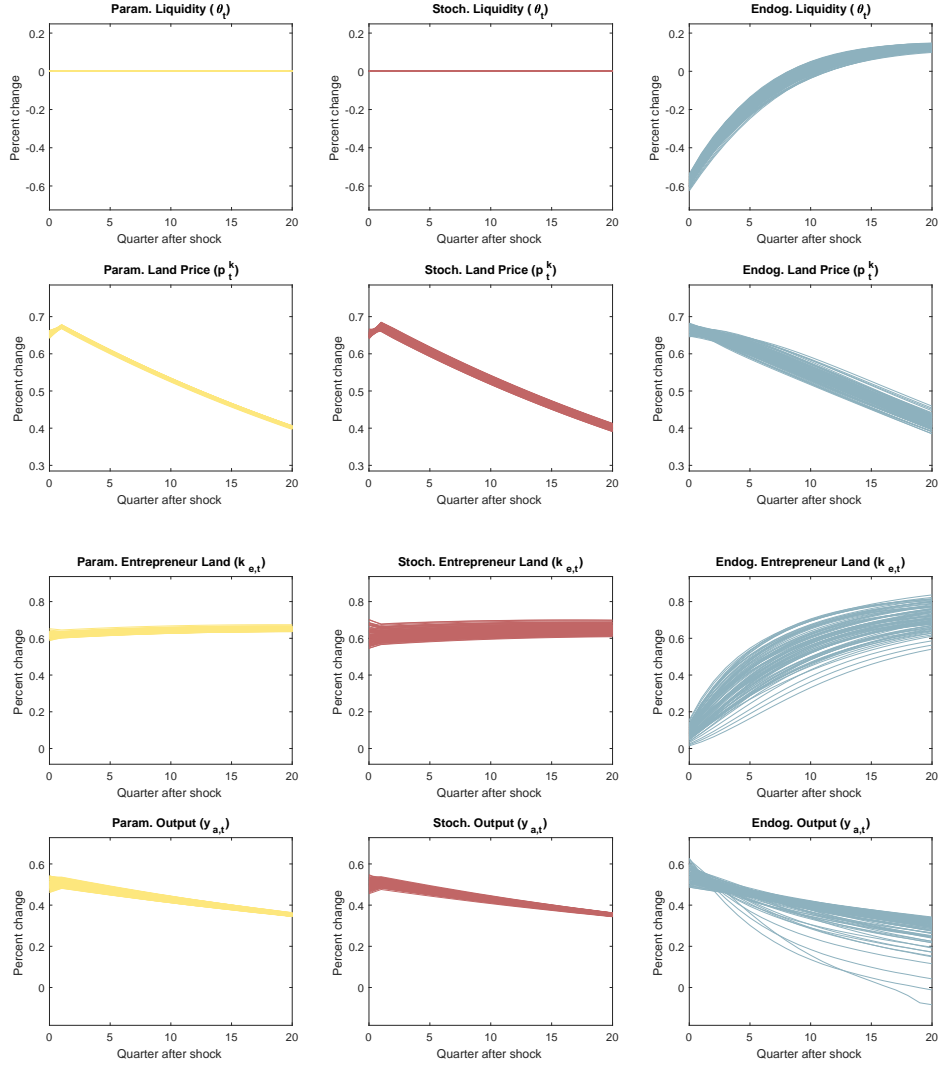


Figure 1.5 Nonlinear Impulse Response Functions, Calib. 1. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 2$. The impulse responses are calculated for liquidity (θ_t), the price of land (p_t^k), entrepreneurs' share of capital $k_{e,t}$, and aggregate output ($y_{a,t}$) under the three specifications for liquidity, a constant (Param.), a stochastic process (Stoch.), and an endogenous process (Endog.). The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.

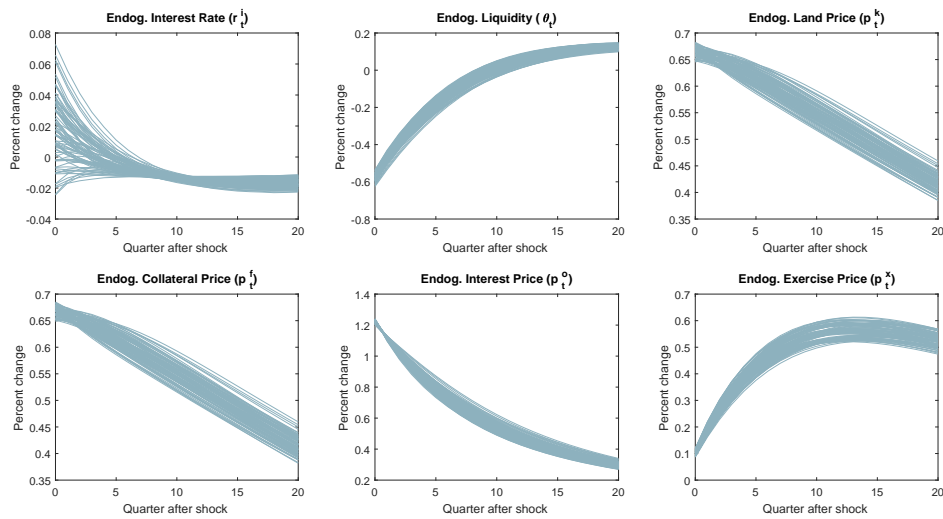


Figure 1.6 Nonlinear Impulse Response Functions, Calib. 1. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 2$. The impulse responses are calculated for the components of the collateralized debt contract when liquidity is endogenously determined. The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.

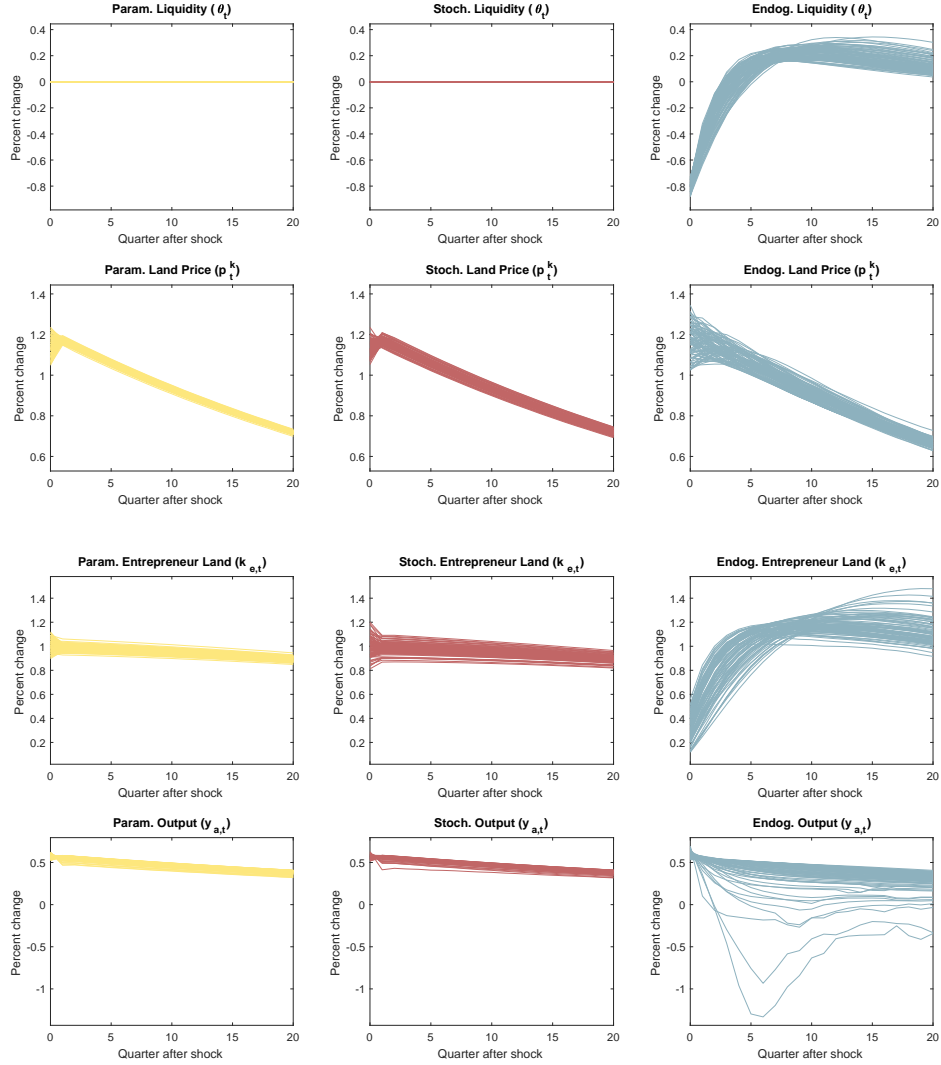


Figure 1.7 Nonlinear Impulse Response Functions, Calib. 2. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 4$. The impulse responses are calculated for liquidity (θ_t), the price of land (p_t^k), entrepreneurs' share of capital $k_{e,t}$, and aggregate output ($y_{a,t}$) under the three specifications for liquidity, a constant (Param.), a stochastic process (Stoch.), and an endogenous process (Endog.) The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.

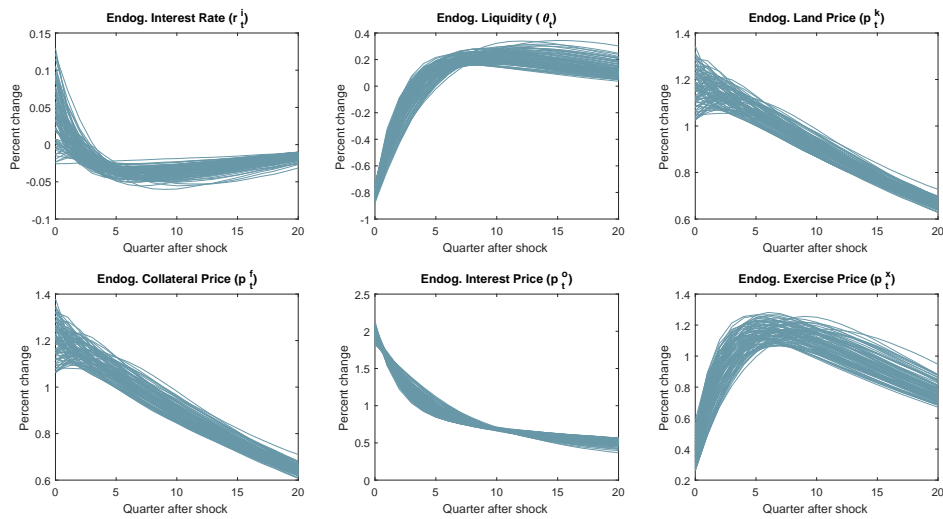


Figure 1.8 Nonlinear Impulse Response Functions, Calib. 2. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 4$. The impulse responses are calculated for the components of the collateralized debt contract when liquidity is endogenously determined. The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.

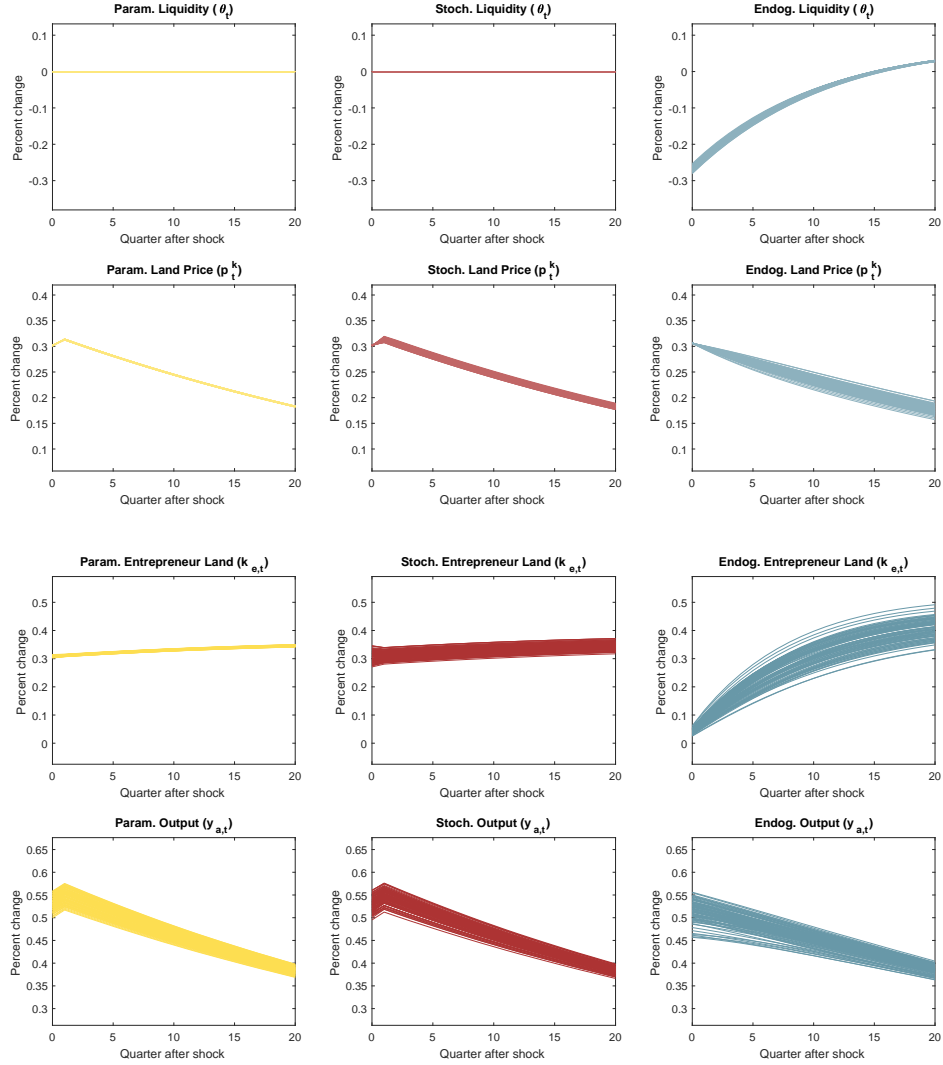


Figure 1.9 Nonlinear Impulse Response Functions, Calib. 3. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 0.75$. The impulse responses are calculated for liquidity (θ_t), the price of land (p_t^k), entrepreneurs' share of capital $k_{e,t}$, and aggregate output ($y_{a,t}$) under the three specifications for liquidity, a constant (Param.), a stochastic process (Stoch.), and an endogenous process (Endog.) The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.

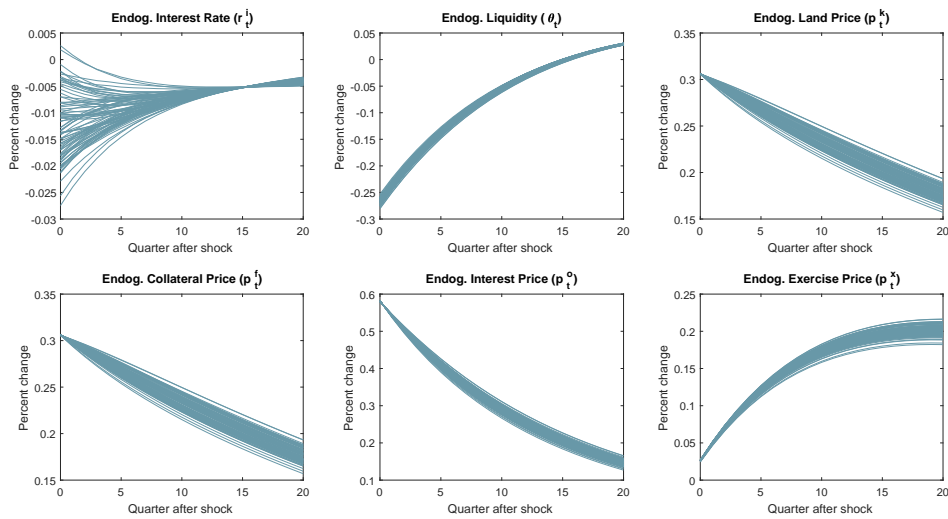


Figure 1.10 Nonlinear Impulse Response Functions, Calib. 3. This plot displays nonlinear impulse functions for the benchmark calibration with $\eta_e = \eta_s = 0.75$. The impulse responses are calculated for the components of the collateralized debt contract when liquidity is endogenously determined. The multiple plots show how the impulse responses change with variation in the state of the economy at the time of the shock.

Chapter 2

Common Credit Cycles and Business Cycles in the United States

2.1 Introduction

The preceding chapter developed a real business cycle model in which the ratio of debt to the market value of assets, referred to as liquidity, is endogenously determined. The model implies that though liquidity may fluctuate with changes in the state of the economy, there is no trend to liquidity. A firm with a debt-to-assets ratio above 1 is insolvent and due to shut down. This suggests that aggregate liquidity will not exceed one and that it cannot contain an indefinite trend. However, the plots of the data on liquidity used in the previous chapter suggest that a trend may be an important feature of the data during the sample period.

This chapter provides analysis of the co-movement between consumption, total factor productivity, and liquidity in U.S. data. The goal is to develop stylized facts about the co-movement of consumption, TFP, assets, and debt at the business cycle and growth frequencies. This paper pursues this goal by testing for the existence of common trends and common cycles in credit and output, and total factor productivity data in the United States. I estimate of a vector error-correction model (VECM) subject to restrictions on the number of common trends and cycles.

The VECM produces decompositions of the observed series into permanent and transitory components. A permanent-transitory decomposition allows tests of whether the changes in liquidity observed over the sample are predicted by permanent shocks or transitory shocks. If the trend dominates movements in actual assets and debt, DSGE models with financial frictions need to add elements that account for these observed low-frequency fluctuations.

This paper contributes to the broader literature documenting properties of the co-movement between credit markets and the real economy. The financial crisis of 2007-2009 and the ensuing recession has renewed interest in understanding how the two interact. Gourinchas and Obstfeld (2012) document that an increase in firm leverage is a precursor to financial crises. Mendoza and Terrones (2008) document a strong association between credit booms, firm leverage, rising asset prices, and economic expansions using both macro- and micro-level data. Schularick and Taylor (2012) construct a long data set across 140 years and 14 developed countries. They find that firm leverage increases after 1940 along with a more developed financial system. However, they find that financial stability risks increase and larger boom-and-bust cycles in asset prices occur in more financialized economies. Claessens, Kose, and Terrones (2011) and Bussiere and Fratzscher (2006) show that increases in asset prices and lending are leading predictors of recessions. Bordo and Haubrich (2010) examine cycles in money, credit, and output and find evidence that suggests more severe financial events are associated with more severe recessions.

The rest of the paper is organized as follows: Section 2.2 describes the econometric models used and establishes notation. Section 2.4 reports the results of the estimation. Section 2.5 concludes.

2.2 Econometric Methods

This section describes the econometric methods used to estimate common trends and common cycles in credit and real activity in the United States.

Suppose that an n -dimensional vector time series, y_t , follows a VAR with p lags,

$$y_t = m_t + \sum_{i=1}^p \Pi_i y_{t-i} + \epsilon_t \quad (2.1)$$

where m_t is the deterministic component of y_t , Π_i are $n \times n$ matrices, and ϵ_t is a vector of forecast innovations. Suppose further that y_t is integrated of order one and that the first difference in y_t , Δy_t , is integrated of order zero. We say that the variables in y_t are cointegrated if there is a linear combination of them, $\beta' y_t$, that is $I(0)$, where β is an $n \times r$ matrix. The columns of β are referred to as cointegrating vectors (Engle and Granger, 1987). If there are r cointegrating relationships among the n variables, then there are $n - r$ independent random walks in the process y_t .

The existence of cointegrating and cofeature vectors impose restrictions on the parameters of the VAR representation. These restrictions capture short-run and long-run economic relationships that might exist between the variables in the model. Imposing these restrictions when estimating the model limits the degrees of freedom and improves the efficiency of the coefficient estimates.

Tests for the number of cointegrating vectors have been developed by Johansen (1988, 1991). The Johansen tests are a series of likelihood ratio tests for nested models. The most general model includes $r = n$ cointegrating relationships and a linear time trend in the first differences of the data. The idea is to sequentially test the rank of the $\Pi(1)$ matrix against a model with the rank reduced by one until there is sufficient evidence against reducing the rank. The Johansen tests contain two test statistics. The trace test statistic tests the null hypothesis that the rank is r against the alternative hypothesis that the rank is n . The λ_{\max} test statistic tests the null hypothesis that the rank is r against the alternative hypothesis that the rank is $r + 1$. The tests proceed sequentially from the most general model to the most specific model, i.e. testing the smallest eigenvalue is zero up to testing that the largest eigenvalue is zero. The procedure starts by testing $H_0 : r = n - 1$ (there is at least one common trend and at most $n - 1$ cointegrating

relationships) against $H_1 : r = n$ (all processes are $I(0)$, i.e. there are no common trends). If the null hypothesis is not rejected, the null hypothesis is decremented by 1 and the tests are repeated. Once the null hypothesis is rejected, the process is stopped.

However, the distribution of those tests are conditional on assumptions about the deterministic term in equation (2.3), namely whether and how the deterministic term varies with time (Osterwald-Lenum, 1992). If the deterministic term is written as the sum of a constant m_0 and a time trend, m_1t ,

$$m_t = m_0 + m_1t, \quad (2.2)$$

For α_\perp , the $n \times (n - r)$ matrix that is orthogonal to the $n \times r$ matrix of loading coefficients α , define the projections

$$\begin{aligned} \beta_0 &= \begin{pmatrix} \alpha' & \alpha \end{pmatrix}_{(r \times n) \ (n \times r)}^{-1} \begin{pmatrix} \alpha' & m_0 \end{pmatrix}_{(r \times n) \ (n \times 1)}, \\ \gamma_0 &= \begin{pmatrix} \alpha_\perp' & \alpha_\perp \end{pmatrix}_{((n-r) \times n) \ (n \times (n-r))}^{-1} \begin{pmatrix} \alpha_\perp' & m_0 \end{pmatrix}_{((n-r) \times n) \ (n \times 1)}, \\ \beta_1 &= \begin{pmatrix} \alpha' & \alpha \end{pmatrix}_{(r \times n) \ (n \times r)}^{-1} \begin{pmatrix} \alpha' & m_1 \end{pmatrix}_{(r \times n) \ (n \times 1)}, \\ \text{and } \gamma_1 &= \begin{pmatrix} \alpha_\perp' & \alpha_\perp \end{pmatrix}_{((n-r) \times n) \ (n \times (n-r))}^{-1} \begin{pmatrix} \alpha_\perp' & m_1 \end{pmatrix}_{((n-r) \times n) \ (n \times 1)}. \end{aligned}$$

Restrictions on these projection matrices are important for determining the distribution of the test statistics. Johansen (1994) defines five potential cases nested within the most general model, $H(r)$:

- Unrestricted trend: $H(r)$. Estimate equation 2.2 as shown. Deterministic term is $m_0 + m_1t$. Cointegrating equations are trend-stationary.
- Restricted trend, unrestricted constant: $H^*(r)$. Set $\gamma_1 = 0$ ($n - r$ restrictions). Deterministic term is $m_t = m_0 + \alpha\beta_1t$. Cointegrating equations are trend

stationary. Trends in levels are linear, but not quadratic.

- No trend, unrestricted constant: $H_1(r)$. Set $\beta_1, \gamma_1 = 0$ (an additional r restrictions). Deterministic term is $m_t = m_0$. Cointegrating equations are stationary around constant means. Trends in levels are linear.
- Restricted constant: $H_1^*(r)$. Set $\gamma_0, \beta_1, \gamma_1 = 0$ (an additional $n - r$) restrictions. Deterministic term is $m_t = \alpha\beta_0$. Cointegrating equations are stationary around constant means. No time trend in the data.
- No constant: $H_2(r)$. Set $\beta_0, \gamma_0, \beta_1, \gamma_1 = 0$ (an additional r restrictions). Deterministic term is $m_t = 0$. Cointegrating equations, levels, and differences have means of zero.

Additionally, we say that the variables have a serial correlation common feature if there is a linear combination of the first differences, $\tilde{\beta}'\Delta y_t$, that is unpredictable with respect to the information available prior to time t , where $\tilde{\beta}$ is an $n \times s$ matrix. That is, the linear combinations $\tilde{\beta}'\Delta y_t$ are uncorrelated with the past. The columns of $\tilde{\beta}$ are referred to as cofeature vectors (Vahid and Engle, 1993). If there are s common feature vectors, among the n variables, then there are $n - s$ independent cycles in the process y_t .

Tests for the number of cofeature vectors have been developed by Vahid and Engle (1993). The tests for common cycles proceed in a similar manner as the tests for common trends. By definition, a common feature vector creates a linear combination of the first differences that is uncorrelated with the past. The tests proceed by computing the canonical correlations of the first differences Δy_t with the lagged first differences $\Delta y_{t-1}, \dots, \Delta y_{t-p+1}$ and the error-correction terms βy_{t-1} . The number of common features corresponds to the number of canonical correlations that are equal to zero, and the number of common cycles corresponds to the number of canonical correlations that are not equal to zero. The test statistic of a null hypothesis that

there are at most $n - s$ common cycles when the number of lags in the VAR is p is

$$C(p, s) = -(T - p - 1) \sum_{i=1}^s \log(1 - \lambda_i^2),$$

where λ_i ($i = 1, \dots, s$) are the s smallest canonical correlations. The first test compares the null hypothesis that the $s = n$ smallest canonical correlations are zero against the alternative hypothesis that the $s = n - 1$ smallest canonical correlations are zero. That is, it tests the null hypothesis that there are at most $n - s$ common cycles against the alternative hypothesis that there are at most $n - s + 1$ common cycles. If the null is rejected, the tests are repeated with the hypothesized canonical correlations decremented by one. The tests stop when the null fails to be rejected.

To capture the cointegrating restrictions implied by the results Johansen tests, the VAR in levels can equivalently be represented as a VECM in first differences with order $p - 1$. Let L be the lag operator, $\Delta = 1 - L$ be the first difference operator, and define the matrix polynomial $\Pi(L) = I_n - \sum_{i=1}^p \Pi_i L^i$. Separate the matrix polynomial as $\Pi(L) = \Pi(1)L + \Gamma(L)(1 - L)$, where $\Gamma(L) = I_n - \sum_{j=1}^{p-1} \Gamma_j L^j$, $\Gamma_j = \sum_{k=j+1}^p \Pi_k$, $j = 1, \dots, p - 1$. The VECM representation of the VAR model is then

$$\Delta y_t = m_t - \Pi(1)y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \epsilon_t, \quad (2.3)$$

where m_t , $\Pi(1)$, and ϵ_t are as defined above.

The behavior of the system depends on the rank of the error correction matrix $-\Pi(1)$. Engle and Granger (1987) show that $-\Pi(1)$ can be written as $\alpha\beta'$, where β is the $n \times r$ matrix of cointegrating vectors and α is an $n \times r$ matrix of error-correction loadings. This decomposition is possible because β spans the cointegration space. The vector $\beta'y_{t-1}$ captures the disequilibrium of the cointegrating relationships, and α captures how quickly the system adjusts to the disequilibrium. Johansen (1988, 1991) describes methods to estimate the parameters of equation (2.3) by maximum likelihood.

The existence of common features imposes restrictions on the VECM in equation (2.3). These restrictions imply the reduced structural form,

$$\Delta y_t = \begin{bmatrix} -(\tilde{\beta}^*)' \\ I_{n-s} \end{bmatrix} \left[m_t^* + \sum_{i=1}^{p-1} \Gamma_i^* \Delta y_{t-i} + \alpha^* \beta' y_{t-1} \right] + \epsilon_t \quad (2.4)$$

where m_t^* , the Γ_i^* matrices, and α^* are the elements of the unconstrained reduced-form equations for the $n - s$ variables not subject to the cofeature restrictions, and the cofeature vectors are normalized such that $\tilde{\beta} = \begin{bmatrix} I_s \\ \tilde{\beta}^* \end{bmatrix}$. The cofeature vectors imply that there are only $n - s$ independent cycles in the n time series, therefore only $n - s$ independent equations should be estimated. The elements of the restricted reduced form are related to the reduced form by

$$m_t = \begin{bmatrix} -(\tilde{\beta}^*)' \\ m_t^* \end{bmatrix} \quad \Gamma_i = \begin{bmatrix} -(\tilde{\beta}^*)' \\ \Gamma_i^* \end{bmatrix} \quad \alpha = \begin{bmatrix} -(\tilde{\beta}^*)' \\ \alpha^* \end{bmatrix} \quad (2.5)$$

By definition, Δy_t has the Wold representation $\Delta y_t = m_t + C(L)\epsilon_t$. Stock and Watson (1988) develop a multivariate version of the Beveridge and Nelson (1981) decomposition that rewrites the system as the sum of a vector random walk and a vector variance-stationary stochastic process. The random walk is referred to as the trend and the stationary part is referred to as the cycle. The matrices β and $\tilde{\beta}$ are also sometimes referred to as cycle generators and trend generators, respectively (Issler and Vahid, 2001). That is, pre-multiplying y_t by β' produces series without any trend component, leaving only cyclical components. Additionally, pre-multiplying Δy_t by $\tilde{\beta}'$ eliminates the cyclical movements, leaving only trend components.

Hecq, Palm, and Urbain (2000) summarize the permanent and transitory restrictions with the following definition:

Definition 1 (Permanent-Transitory Decomposition). Let y_t be an n -dimensional integrated process of order 1 that follows equation (2.3). A permanent-transitory decomposition is a pair of processes (μ_t, ψ_t) such that

1. μ_t is a random walk process,
2. ψ_t is a covariance-stationary process,

3. $\text{Var}(\Delta\mu_t)$ and $\text{Var}(\Delta\psi_t)$ are strictly positive,
4. $y_t = \mu_t + \psi_t$,
5. both μ_t and ψ_t are functions of the observable variables, and
6. if there exist a cointegrating matrix β and a cofeature matrix $\tilde{\beta}$, then $\beta'\mu_t = 0$ and $\tilde{\beta}'\psi_t = 0$.

It is straightforward to produce a common trends/common cycles representation of y_t if $r + s = n$, i.e. the number of cointegrating vectors plus the number of cofeature vectors equals the dimension of the series. Collect the cointegrating and cofeature vectors into the matrix

$$\begin{bmatrix} \Psi_{\cdot,s} & \Psi_{\cdot,r} \end{bmatrix} = \begin{bmatrix} \tilde{\alpha}' \\ \alpha' \end{bmatrix}^{-1}.$$

The inverse exists because the cointegrating and cofeature vectors are linearly independent by definition. The common trends/common cycles permanent-transitory decomposition is given by

$$y_t = \begin{bmatrix} \Psi_{\cdot,s} & \Psi_{\cdot,r} \end{bmatrix} \begin{bmatrix} \Psi_{\cdot,s} & \Psi_{\cdot,r} \end{bmatrix}^{-1} = [\Psi_{\cdot,s}\tilde{\alpha}' + \Psi_{\cdot,r}\alpha'] y_t,$$

where $\Psi_{\cdot,s}\tilde{\alpha}'y_t$ is the cycle component and $\Psi_{\cdot,r}\alpha'y_t$ is the trend component. This decomposition makes use of the cycle generator $\tilde{\alpha}'$ and the trend generator α' .

If $r + s < n$, then a permanent-transitory decomposition is still possible even though the matrix $\begin{bmatrix} \tilde{\alpha}' \\ \alpha' \end{bmatrix}^{-1}$ does not exist. Proietti (1997) and Hecq, Palm, and Urbain (2000) develop methods to decompose the system into a permanent component and a transitory component that satisfy Definition 2.2. This paper computes the permanent-transitory decomposition following the method in Hecq, Palm, and Urbain (2000). Rewrite equation (2.3) in state space form as

$$\Delta y_t = Z f_t, \tag{2.6}$$

$$f_t = c + T f_{t-1} + v_t \tag{2.7}$$

where

$$f = \begin{bmatrix} \Delta y_t \\ \vdots \\ \Delta y_{t-p+2} \\ \beta' y_{t-1} \end{bmatrix}, T = \begin{bmatrix} \Gamma_1 + \alpha\beta' & \Gamma_2 & \cdots & \Gamma_{p-1} & \alpha \\ I_n & 0_{n \times n} & \cdots & 0_{n \times n} & 0_{n \times r} \\ 0_{n \times n} & \ddots & \ddots & \vdots & \vdots \\ \vdots & \cdots & \cdots & I_n & \vdots \\ \beta' & 0_{n \times n} & \cdots & \cdots & I_r \end{bmatrix},$$

$$Z = \begin{bmatrix} I_n & 0_{n \times n(p-2)+r} \end{bmatrix}, c_t = \begin{bmatrix} m_t \\ 0_{n \times 1} \\ \vdots \\ 0_{n \times 1} \\ 0_{r \times 1} \end{bmatrix}, \text{ and } v_t = \begin{bmatrix} \epsilon_t \\ 0_{n \times 1} \\ \vdots \\ 0_{n \times 1} \\ 0_{r \times 1} \end{bmatrix}.$$

The expected value of the drift term is $c^* = (I - T)^{-1}c$. Transfer the constant term to the measurement equation and rewrite the state space representation as

$$\Delta y_t = Z f_t^* + Z c^*, \quad (2.8)$$

$$f_t^* = T f_{t-1}^* + v_t, \quad (2.9)$$

where $f_t^* = f^t - c^*$. This representation provides an expression for the forecastable changes in y_t , $\Delta y_{t+j|t}$. The trend component of y_t is defined to be

$$\mu_t = y_t + \left[\lim_{k \rightarrow \infty} \sum_{j=1}^k \Delta y_{t+j|t} - E(\Delta y_t) \right] \quad (2.10)$$

$$= y_t + \left[\lim_{k \rightarrow \infty} \sum_{j=1}^k Z T^j f_t^* - Z c^* \right], \quad (2.11)$$

which is the current value of the series plus any forecastable changes in the series. The trend component represents the value the series would take if it were on its long-run path. As k goes to infinity, the expression $\sum_{j=1}^k T^j$ converges to $(I - T)^{-1}T$, so that the sum of the forecastable

changes converges to $\psi_t = -Z(I - T)^{-1}Tf_t^*$. The permanent-transitory decomposition is then

$$\psi_t = -Z(I - T)^{-1}Tf_t^*, \quad (2.12)$$

$$\mu_t = y_t - \psi_t. \quad (2.13)$$

The permanent-transitory decomposition allows for the extraction of cyclical fluctuations in economic data. Models of the business cycle describe how the economy fluctuates around an equilibrium. The permanent component extracted from the above decomposition can be interpreted as the equilibrium value of a series changing over time. Therefore, the transitory component is the object whose behavior business cycle models should attempt to replicate. The estimated permanent-transitory decompositions provide empirical moments that should inform the calibration and estimation of models of the business cycle.

2.3 Data

Consumption and investment are measured by personal consumption expenditures and gross private domestic investment as reported by the Bureau of Economic Analysis. The TFP series used is the utilization-adjusted TFP series from Fernald (2012) and the Federal Reserve Bank of San Francisco. Data on the debt securities and loans and the nonfinancial assets of corporate and noncorporate businesses come from the Federal Reserve's Flow of Funds data. Liquidity is measured as the ratio of the total debt securities and loans of both corporate and noncorporate business to their total nonfinancial assets.

All quantity variables used in this study are reported on a per capita basis. Data are converted to logs before estimating the coefficients. No other transformations are made. The sample period is 1952:1-2016:4. The Flow of Funds data are not available on a quarterly basis prior to 1952. Detail on the construction of the data set is presented in the appendix.

Figure 2.1 shows plots of the data in levels over the sample period. All four series show a positive trend and are highly persistent, all having first autocorrelations over 0.98. The debt-

to-asset ratio shows the most persistence and the largest swings, with long periods above trend during the 60's and 80's and distinct periods below trend in the 70's and 2000's. TFP also has long swings above and below trend that vaguely correspond to the changes in liquidity. Consumption is the smoothest of the four series. Figure 2.2 shows the same data in first differences.

Augmented Dickey-Fuller tests fail to reject the null hypothesis of a unit root in the levels each time series and reject the same hypothesis in the first difference of each time series. This is consistent with visual evidence that suggest that all series are integrated of order 1. Plots of the levels display a clear upward trend, while plots of the differences appear to be stationary.

Additionally, though both debt and assets in all three sectors grow over the sample period, debt grows faster than assets, which causes the debt-to-assets ratio for all three sectors to rise over the sample period as well. The debt to asset ratios for households and businesses are plotted in Figure 2.3

2.4 Results

This section estimates common trends and cycles from two VAR models. One model contains variables pertinent to a households' decisions: consumption of nondurable goods and services, consumption of durable goods, residential investment, total factor productivity, household debt, and household assets. The other model contains variables pertinent to firms' decisions: nonresidential investment, total factor productivity, business debt, and business assets.

2.4.1 Households

The household VECM includes nondurable goods and services consumption, durable goods consumption, residential investment, total factor productivity, household debt, and household assets.

The first step is to determine the lag length of the VAR. I choose the lag length that minimizes the Aikake Information Criterion (AIC), which is $p = 5$. Aikake's Final Prediction

Error also suggests a lag length of 5. However, the Schwartz Criterion and the Hannan-Quinn Criterion suggest a lag length of 2. I estimated the model with both lag lengths and found that $p = 5$ gave estimates of the constant growth terms that were close to the mean of the data, which produced trend components that fit approximately through the data, as opposed to trend components that are significantly above or below the data.

On the whole, I judged that the model should be estimated with a constant, but no time trend (case H_1). The results of the trace and eigenvalue statistics for this case are reported in Table 2.1. Taking the results of the trace and λ_{\max} tests together along with implications for the VECM, I judged that a model with a constant term and one cointegrating relationship fits the data best.

The trace and λ_{\max} tests are unable to reject the null hypothesis at the 5% level for cointegrating ranks of 2, 1, and 0. However, the trace test is statistically significant at the 10% level and nearly so at the 5% level, while the λ_{\max} test is nearly statistically significant at the 10% level. Given the closeness of the test results, I plot the cointegration vector estimated in figure 2.4. The plot appears stationary during the sample period. Taking all of this information together, I conclude that the rank of the error correction matrix is $r = 1$. The implication is that there is 1 cointegrating relationship in the data and $n - r = 2$ independent random walks. The estimated cointegrating vector is reported in Table 2.2. The cointegrating vector emphasizes the relationship between TFP and households' stock of assets.

Both the Vahid-Engle and the Rao tests reject $s = 3$ and $s \geq 2$, and both are unable to reject $s \geq 1$. Therefore, I conclude that there is one common feature relationship in the data, i.e. there are two common cycles. The common feature vectors emphasize the relationship between consumption of nodurable goods and services and TFP and the relationship between consumption of durable goods and TFP.

The preceding tests suggest that $r + s < n$. Therefore, I use equations (2.12) and (2.13) to compute a permanent-transitory decomposition of the data. Summary statistics of the decom-

position are shown in Table 2.5 and plots of the trend and cycle are shown in Figure 2.6.

The first two rows of Table 2.5 contain the means and standard deviations of the growth rates of the data. Durable consumption grows the fastest on average, over the period, followed by household debt, then household assets. Household debt grows at a rate one and a half times as fast as business assets, 3.80% per year compared to 2.47% per year. Residential investment and durable goods consumption are the most volatile, while TFP and nondurable goods and services consumption are the least volatile.

The third and fourth rows of Table 2.5 contain the means and standard deviations of the growth rates of the permanent component. The mean growth rates of the permanent components are all similar to the mean growth rates in the data and the mean growth rates of the transitory components are all close to zero. The ratios of the growth rate means for the permanent and transitory components to the growth rate mean of the data are reported in the seventh and eighth rows of the table.

What is interesting here is to compare the standard deviations of the permanent and transitory components with the the standard deviation. The fifth and sixth rows contain the means and standard deviations of the transitory component, and the ratios of the growth rate standard deviations for the permanent and transitory components to the growth rate standard deviation of the data are reported in the seventh and eighth rows of the table. Residential investment has the most volatile permanent component, followed by debt, durable goods consumption, assets, nondurable goods and services consumption, and, lastly, total factor productivity. The standard deviations of the transitory components follow a similar pattern.

The first autocorrelation coefficients of the transitory components are reported in the eleventh line of Table 2.5). Durable goods consumption and residential investment are the most persistent, while nondurable goods and services consumption assets and debt all still have relatively strong persistence. The coefficient on TFP is close to zero (and slightly negative), suggesting that trend shocks to TFP die out quickly.

The effects of the Great Recession can be clearly seen from the plots of the permanent-transitory decomposition. A reduction in trend debt precedes the crisis period, which includes large drops in assets, residential investment, and durable goods consumption relative to trend. By the end of the sample, durable goods consumption and debt are back near trend, while assets and residential investment are below trend, which suggests that further increases in asset prices and residential investment can be expected in the near future.

It is notable that the standard deviation of the permanent and transitory components of household debt are approximately three times as large as the standard deviation of the data. This is due to the high negative correlation between the two series. Inspecting the permanent-transitory decompositions, one can see a pattern in the behavior of debt. A negative shock to the permanent component of debt levels leads recessions, and debt declines back to the trend over a number of quarters following the permanent shock. This pattern occurs prior to the 1969, 1973, 1980 and 1981, and the 2007 recessions. The sequence of events indicated is similar to the specification of the model in the first essay of this dissertation, where debt is a state variable that slowly adjusts over time to TFP shocks.

2.4.2 Firms

The firm VECM includes non-residential investment, total factor productivity, business debt, and business assets.

I start the estimation of the firms model by choosing the lag length. Aikake's information criterion and final prediction error both suggest a lag length of $p = 3$, while the Hannon-Quinn criterion and the Schwartz criterion suggest a lag length of $p = 2$. I choose to follow AIC and set $p = 3$.

The Johansen tests regarding the trend and a visual assesment of the fitted models suggest that the VECM should be estimated with a constant term and no trend. The results of the Johansen tests regarding the number of cointegrating relationships are presented in Table 2.6. The trace test is able to reject $r = 0$ at the 5% level. I conclude from the results of the test

that there is $r = 1$ cointegrating relationship in the data.

The estimated cointegrating vector and standard errors are presented in Table 2.7. The cointegrating vector emphasizes the relationship between TFP and business debt. The signs of the coefficients for TFP and debt are opposite, suggesting that they lie on opposite sides of the equation representing the equilibrium relationship. The magnitude of the coefficients suggests that a 1% higher level of TFP implies a 0.3% higher level of debt, with a negligible increase in assets. This fits with the observation that the debt-to-asset ratio has increased over the sample period.

The results of the Vahid-Engle tests for common features are presented in Table 2.8. The tests reject all hypotheses of the existence of a common feature at the 5% level or less. From the results, I conclude that there are $s = 0$ common features in the business data.

Summary statistics of the resulting permanent-transitory decomposition are presented in Table 2.9. The table presents the mean and standard deviations of the data, the permanent component, and the transitory component, the ratios of those values of the permanent and transitory components to the data, the first autocorrelation of the transitory component, and the correlation between changes in the permanent and transitory components.

Business debt and non-residential investment grow the fastest, at similar rates of 3.3 and 3.2 percent per year, respectively. Business assets and TFP grow more slowly, at 1.8 and 1.2 percent per year, respectively. Non-residential investment is the most volatile, followed by business assets. Business debt and total factor productivity have similar low values of volatility, which matches the link implied by the cointegrating relationship.

Non-residential investment has the most volatile permanent component, followed by business debt and assets. All three series are significantly more volatile than total factor productivity. Similar to that of households, business debt has permanent and transitory components that are more volatile than the overall series. The correlation between innovations to the permanent component and the transitory component of business debt are -0.9, showing a tendency for the

actual series to be smoother than changes in the trend.

Business debt grows on average nearly twice as fast as business assets, 3.33% per year compared to 1.79% per year. The mean rate of debt growth is closer to the mean growth of nonresidential investment, while the mean rate of asset growth is closer to the rate of TFP growth. The differing growth rates are reflecting in the permanent components of both series, not the transitory components. Again, this suggests that there has been a persistent increase in firms' leverage over the sample period

All of the transitory components are similarly persistent, with first autocorrelation coefficients around 0.9. Figure 2.7 shows that the transitory components of business assets and non-residential investment move closely together. Business debt follows the same general pattern, but deviates for substantial periods, particularly after 1980.

2.5 Conclusion

This paper studies the relationship between activity in credit markets and real markets in the United States. It estimates a VAR with common trends and common cycles restrictions and uses the models to calculate a permanent-transitory decomposition.

The model suggests that consumption and productivity share a common trend and a common cycle. The debt-to-assets ratio of firms appear to move separately from real activity.

The estimates suggest that for both households and businesses, debt does not share a common trend with either assets or TFP. This suggests that there is an additional factor associated with firms' financing decisions that is unrelated to the level of assets.

The results of this paper provide useful evidence that should guide the construction of DSGE models that pay special attention to the financial sector. The results of this analysis suggest that trend shocks dominate the observed fluctuations in debt and assets. The permanent effects of trend shocks to debt should be separated from models that written in stationary terms mean to explain the cyclical behavior of debt and assets.

Table 2.1 Johansen Tests Regarding the Number of Cointegrating Relationships. Test statistics are reported allowing for a constant in the first differences (linear trend in the levels). P-values are computed by the Doornik (1998) gamma approximation with the command `coint2` using the statistical software gretl.

Null Hyp.	Eigenvalue	Trace test	p-value	λ_{\max} test	p-value
$r \leq 5$	0.0077498	1.9839	0.1590	1.9839	0.1590
$r \leq 4$	0.039196	12.180	0.1498	10.196	0.2031
$r \leq 3$	0.052453	25.919	0.1348	13.739	0.4011
$r \leq 2$	0.10064	52.968	0.0140**	27.049	0.0556
$r \leq 1$	0.10732	81.916	0.0033***	28.948	0.1768
$r = 0$	0.19353	136.76	0.0000***	54.848	0.0002***

Table 2.2 Cointegrating Vector Estimates, Households VECM. Standard errors for each term are reported in parentheses.

Nondurables and Services	Durables	Residential Investment	TFP	Debt	Assets
1.0000	-0.077772	-0.71175	1.5862	0.52953	-1.5795
(0.00000)	(0.20335)	(0.17684)	(0.72261)	(0.24509)	(0.29870)

Table 2.3 Common Feature Test Results This table shows results for tests that there are s cofeature vectors in the data. Cor. reports the canonical correlations of the data from smallest to largest. $C(p, s)$ is the test statistic. The test statistic has a χ^2 distribution with $DF = s^2 + snp + sr - sn$ degrees of freedom.

Null Hyp.	Cor.	χ^2 Test			Rao's F Test		
		$C(p, s)$	DF	p-value	F	DF	p-value
$s = 6$	0.8379	707.6164	186	0.0000***	5.75281	144	0.0000***
$s \geq 5$	0.7325	406.1462	150	0.0000***	3.81908	115	0.0000***
$s \geq 4$	0.5716	214.6231	116	0.0000***	2.49120	88	0.0000***
$s \geq 3$	0.4493	116.1010	84	0.0117**	1.82864	63	0.0002***
$s \geq 2$	0.3740	59.9627	54	0.2684	1.46510	40	0.0367**
$s \geq 1$	0.2936	22.4436	26	0.6642	1.14179	19	0.3106

Table 2.4 Common Feature Vector Estimates, Household VECM. Test statistics are reported allowing for a constant in the first differences (linear trend in the levels). P-values are computed by the Doornik (1998) gamma approximation with the command `coint2` using the statistical software `gretl`.

Nondurables and Services	Durables	Residential Investment	TFP	Debt	Assets
1.0000	0.0000	-0.1425	-1.8888	-0.0532	0.0233
0.0000	1.0000	-0.4781	-5.1323	0.1713	0.0698

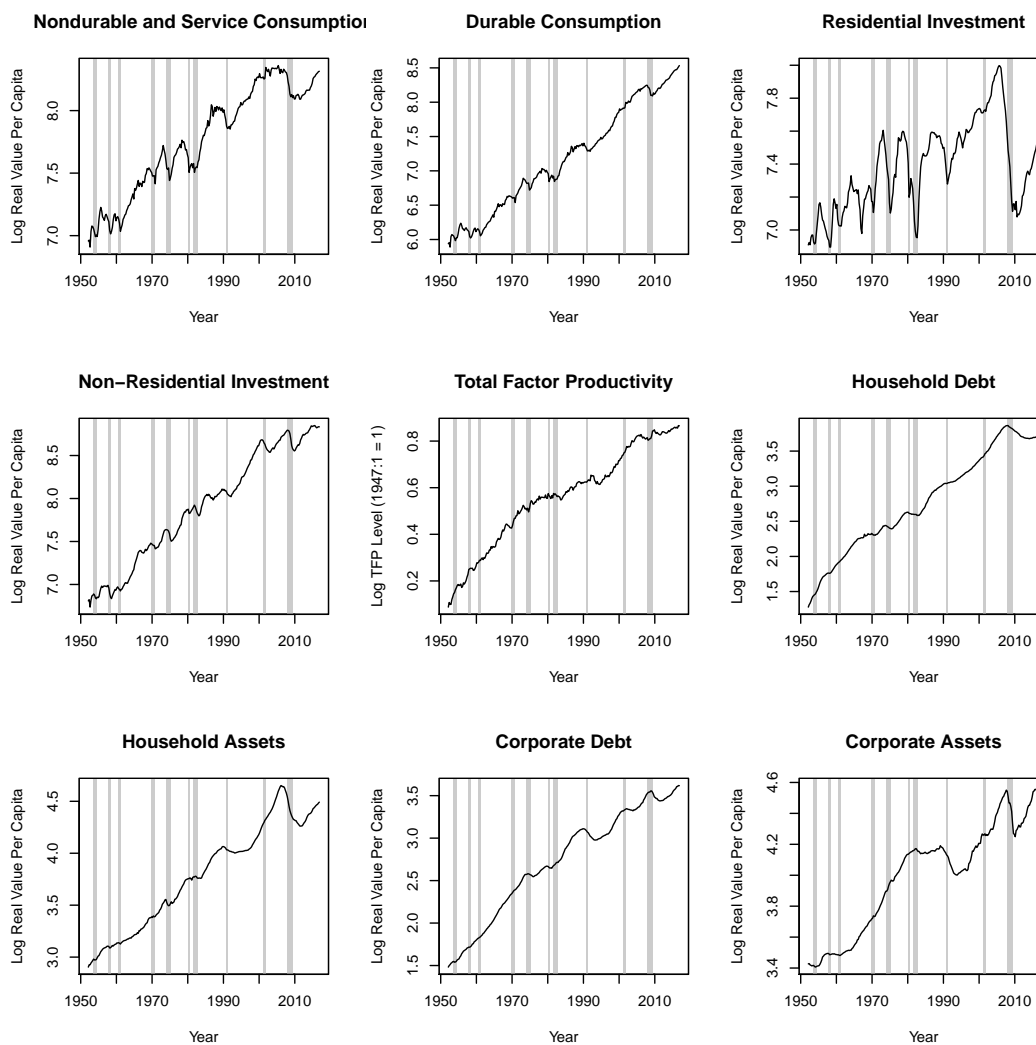


Figure 2.1 Plot of the Data in Log Levels. The graphs show quarterly plots of the data in log levels over the sample period 1952:1-2016:4. Shaded portions represent NBER recessions. The series plotted are consumption of nondurable goods and services, consumption of durable goods, residential investment, non-residential investment, total factor productivity, corporate and non-corporate debt, household debt, corporate and non-corporate assets, and household assets. All variables are reported on a real, per capita basis.

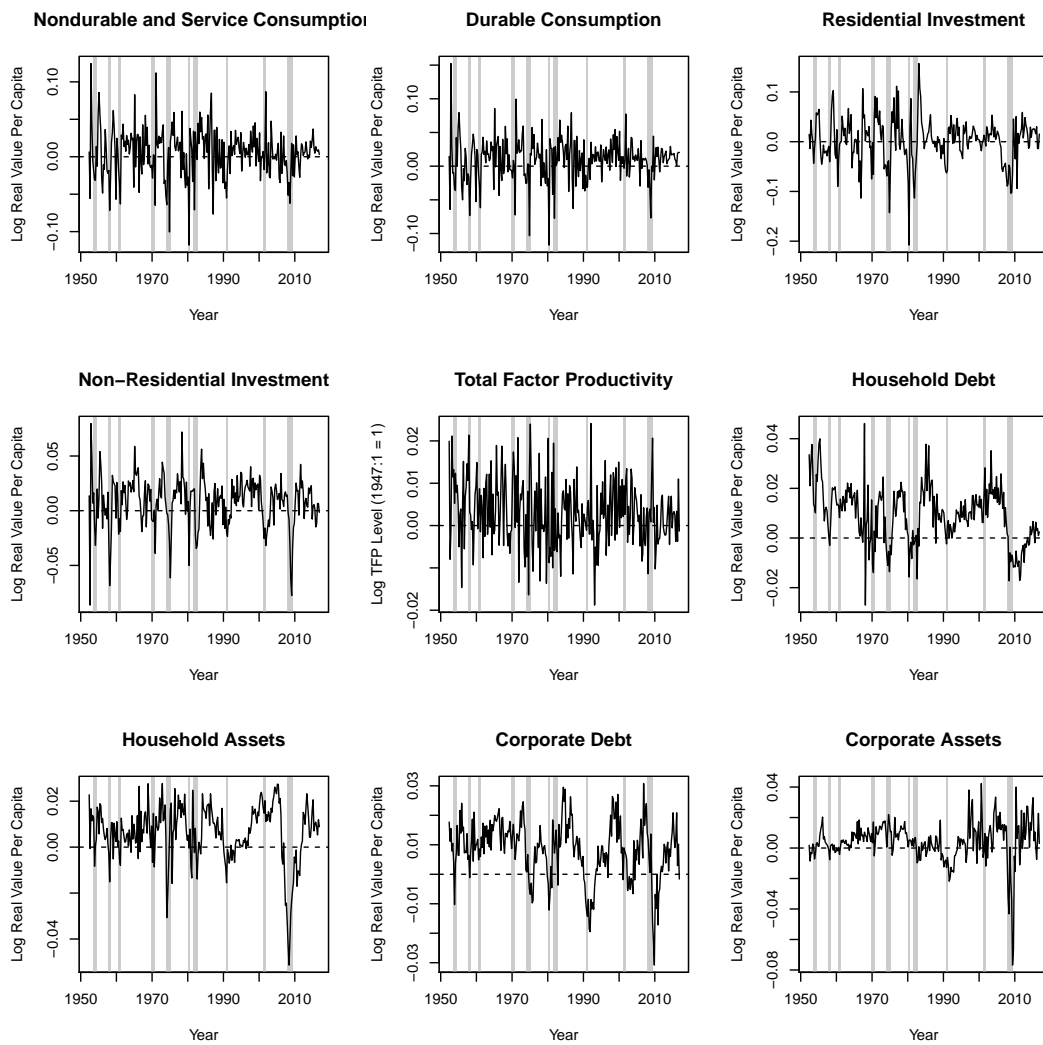


Figure 2.2 Plot of the Data in Log Differences. The graphs show quarterly plots of the first differences of the data in log levels over the sample period 1952:1-2016:4. Shaded portions represent NBER recessions. The series plotted are consumption of nondurable goods and services, consumption of durable goods, residential investment, non-residential investment, total factor productivity, corporate and non-corporate debt, household debt, corporate and non-corporate assets, and household assets. All variables are reported on a real, per capita basis.

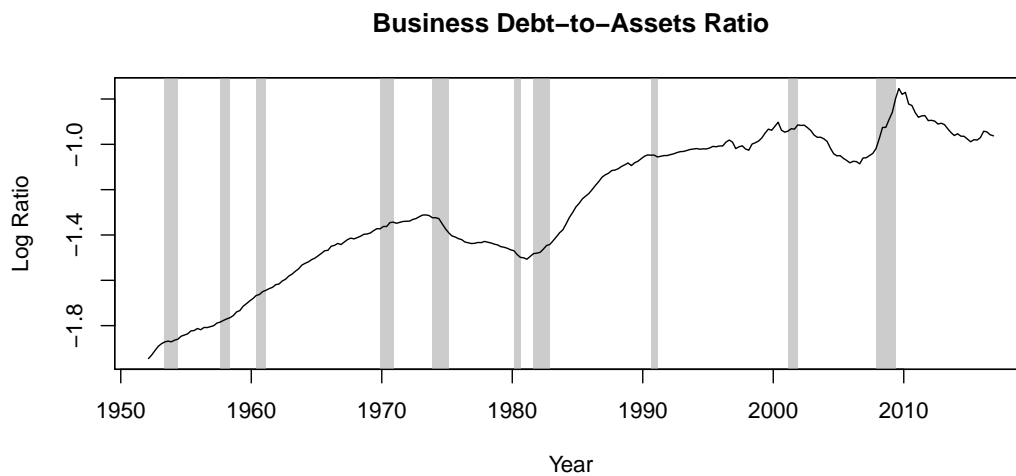
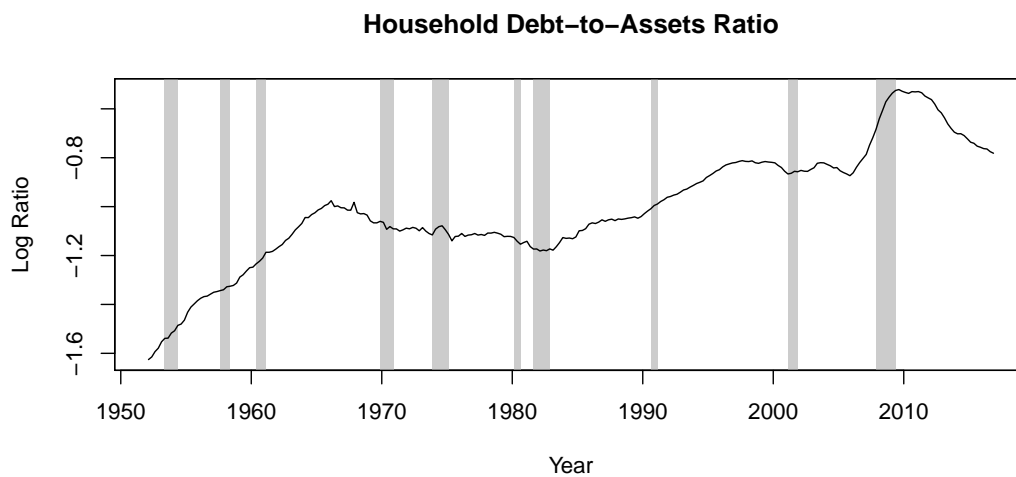


Figure 2.3 Debt-to-Asset Ratios. The graphs show quarterly plots of the log of the ratio of debts to assets for both households and businesses from 1952:1-2016:4. The value of debts and assets for businesses is the sum of corporate and non-corporate debt and assets, respectively.

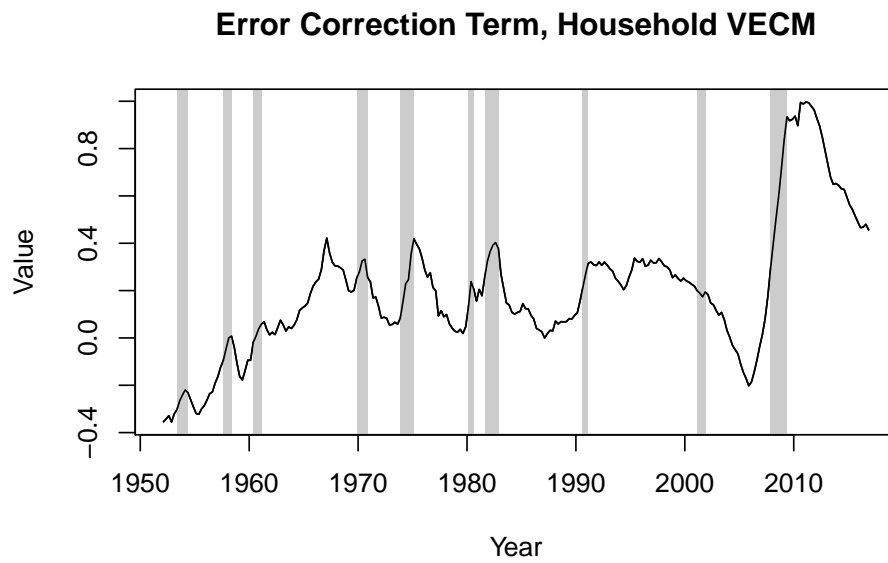


Figure 2.4 Error Correction Term, Household VECM. This graph plots the error correction term, βy_t , for the single estimated cointegrating relationship in the household model.

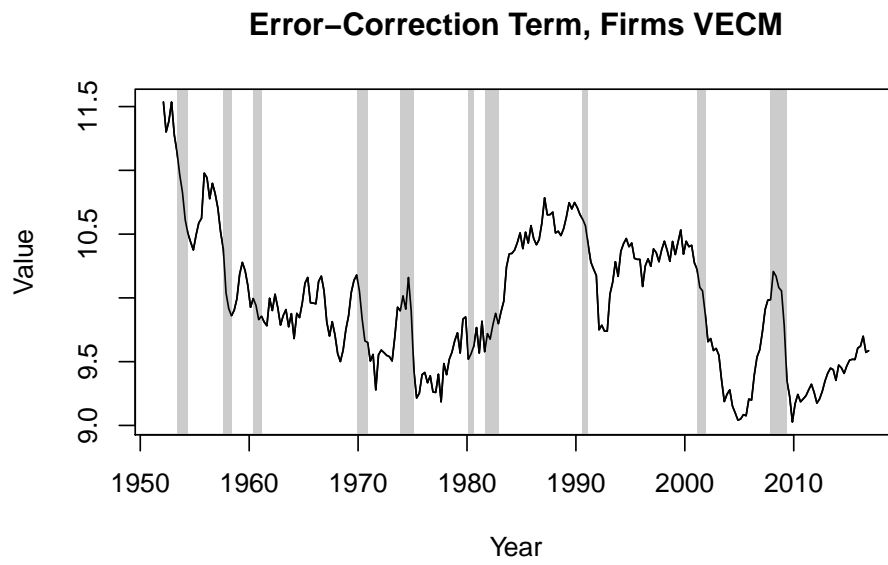


Figure 2.5 Error Correction Term, Firms VECM. This graph plots the error correction term, βy_t , for the single estimated cointegrating relationship in the firm model.

Table 2.5 Summary Statistics of the Permanent-Transitory Decomposition, Household Model. The data are reported in annualized growth rates as a percentage, i.e. 2.00 means 2% annual growth. The first two rows show the mean and standard deviation for each series. The next two rows show the standard deviation for the growth rate of the permanent component and the transitory component. The next two rows show the volatility of the permanent and transitory components relative to each series. The final two rows show the first autocorrelation of each transitory component and the contemporaneous correlation of the transitory component with the permanent component. The data for nondurable good and services consumption, durable goods consumption, residential investment, TFP, household debt, and household assets are sampled quarterly from 1952-2016. There are a total of 260 observations.

	NDSC	DC	RI	TFP	Debt	Assets
Mean(Δy_t)	2.2170	4.0674	0.9244	1.2062	3.8084	2.4746
Sd(Δy_t)	2.9116	12.6571	18.0118	3.1613	4.6402	4.9857
Mean($\Delta \mu_t$)	2.1765	4.1026	1.6370	1.1252	3.4849	2.7410
Sd($\Delta \mu_t$)	5.1895	11.1744	15.4982	3.5171	14.8354	7.6775
Mean($\Delta \psi_t$)	-0.0038	-0.1792	-0.7827	0.0484	0.1812	-0.3014
Sd($\Delta \psi_t$)	4.9909	12.3341	15.7588	1.9200	13.2628	6.8786
$\frac{\text{Mean}(\Delta \mu_t)}{\text{Mean}(\Delta y_t)}$	0.9818	1.0085	1.7662	0.9331	0.9161	1.1066
$\frac{\text{Mean}(\Delta \psi_t)}{\text{Mean}(\Delta y_t)}$	-0.0018	-0.0447	-0.8521	0.0403	0.0482	-0.1231
$\frac{\text{Sd}(\Delta \mu_t)}{\text{Sd}(\Delta y_t)}$	1.7823	0.8829	0.8605	1.1126	3.1972	1.5399
$\frac{\text{Sd}(\Delta \psi_t)}{\text{Sd}(\Delta y_t)}$	1.7142	0.9745	0.8749	0.6073	2.8583	1.3797
AR1(ψ_t)	0.8940	0.9430	0.9670	-0.0340	0.7370	0.8630
Corr($\Delta \mu, \Delta \psi$)	-0.8418	-0.4745	-0.3284	-0.4723	-0.9539	-0.7698

Table 2.6 Johansen Tests Regarding the Number of Cointegrating Relationships. Test statistics are reported allowing for a constant in the first differences (linear trend in the levels). P-values are computed by the Doornik (1998) gamma approximation with the command `coint2` using the statistical software gretl.

Null Hyp.	Eigenvalue	Trace test	p-value	λ_{\max} test	p-value
$r \leq 3$	0.013921	3.6028	0.0577	3.6028	0.0577
$r \leq 2$	0.029342	11.257	0.1991	7.6538	0.4238
$r \leq 1$	0.064504	28.393	0.0730	17.136	0.1715
$r = 0$	0.088382	52.174	0.0171*	23.781	0.1447

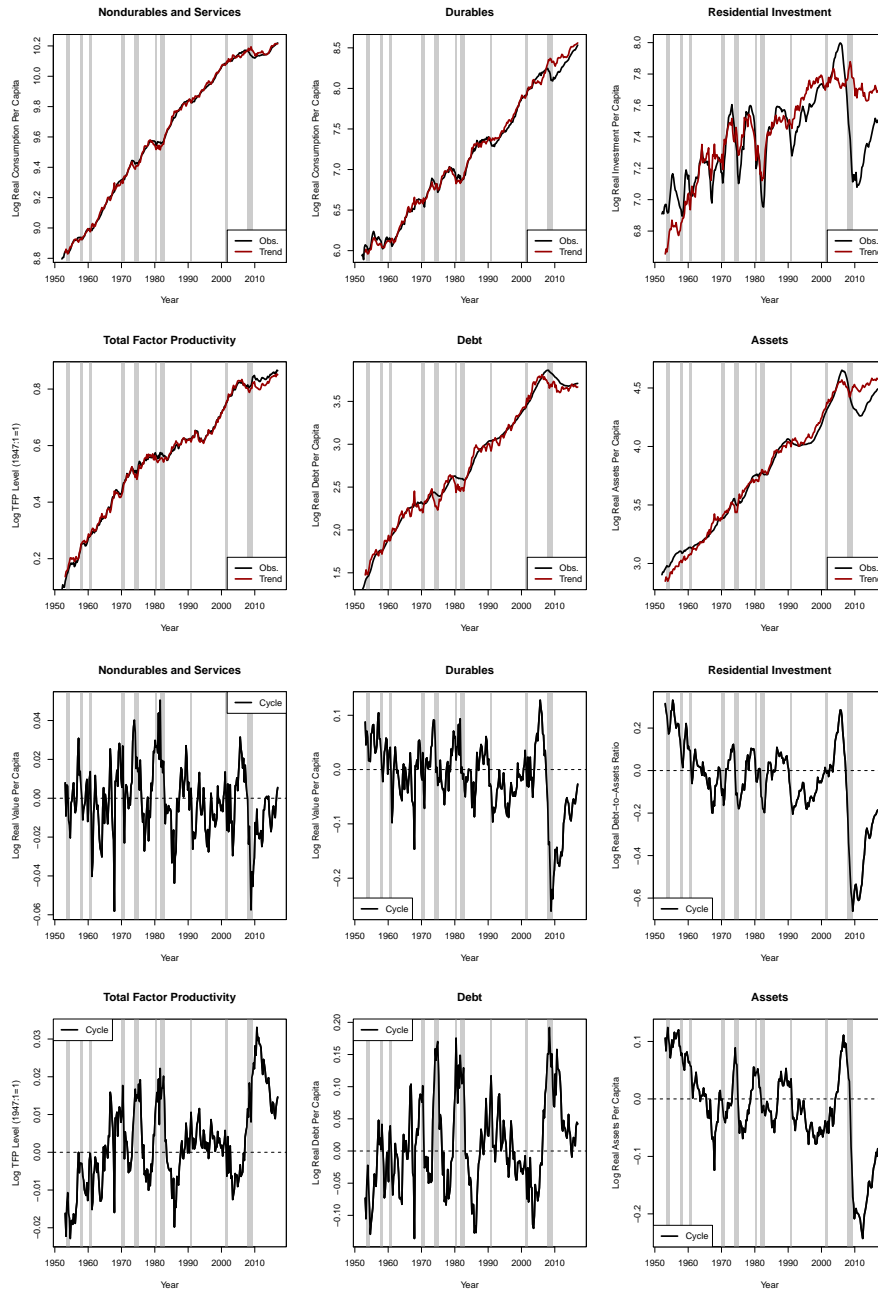


Figure 2.6 Permanent-Transitory Decomposition, Household Model . These plots show the permanent-transitory decomposition for the household VECM. The top six panels plot the observed data and the calculated permanent component. The bottom six panels plot the transitory components. Shaded areas represent NBER recession periods.

Table 2.7 Estimated Cointegrating Vector, Business VECM. Standard errors are reported in parentheses.

Nonresidential Investment	TFP	Debt	Assets
1.0000	7.0508	-2.1361	-0.45873
(0.00000)	(1.4495)	(0.50638)	(0.44193)

Table 2.8 Common Feature Test Results. This table shows results for tests that there are s cofeature vectors in the data. Cor. reports the canonical correlations of the data from smallest to largest. $C(p, s)$ is the test statistic. The test statistic has a χ^2 distribution with $DF = s^2 + snp + sr - sn$ degrees of freedom.

Null Hyp.	Cor.	$C(p, s)$	χ^2 Test		Rao's F Test		
			DF	p-value	F	DF	p-value
$s = 4$	0.7843	420.2013	52	0.0000***	14.19178	36	0.0000***
$s \geq 3$	0.6242	178.5756	36	0.0000***	8.16472	24	0.0000***
$s \geq 2$	0.3493	53.6951	22	0.0002***	3.93431	14	0.0000***
$s \geq 1$	0.2807	20.7708	10	0.0227*	3.52232	6	0.0002**

Table 2.9 Summary Statistics of the Permanent-Transitory Decomposition, Business Model. The data are reported in annualized growth rates as a percentage, i.e. 2.00 means 2% annual growth. The first two rows show the mean and standard deviation for each series. The next two rows show the standard deviation for the growth rate of the permanent component and the transitory component. The next two rows show the volatility of the permanent and transitory components relative to each series. The final two rows show the first autocorrelation of each transitory component and the contemporaneous correlation of the transitory component with the permanent component. The data for non-residential investment, TFP, business debt, and business assets are sampled quarterly from 1952-2016. There are a total of 260 observations.

	NRI	TFP	Debt	Assets
Mean(Δy_t)	3.1628	1.2062	3.3393	1.7926
Sd(Δy_t)	9.0565	3.1613	3.9726	5.2242
Mean($\Delta \mu_t$)	3.7056	1.0966	3.5262	2.1917
Sd($\Delta \mu_t$)	12.4128	2.6673	9.2937	9.1242
Mean($\Delta \psi_t$)	-0.3912	0.0994	-0.2014	-0.3685
Sd($\Delta \psi_t$)	7.2107	3.4988	7.7331	7.3944
$\frac{\text{Mean}(\Delta \mu_t)}{\text{Mean}(\Delta y_t)}$	1.1693	0.9095	1.0553	1.2209
$\frac{\text{Mean}(\Delta \psi_t)}{\text{Mean}(\Delta y_t)}$	-0.1253	0.0828	-0.0611	-0.2072
$\frac{\text{Sd}(\Delta \mu_t)}{\text{Sd}(\Delta y_t)}$	1.3706	0.8437	2.3395	1.7465
$\frac{\text{Sd}(\Delta \psi_t)}{\text{Sd}(\Delta y_t)}$	0.7962	1.1068	1.9466	1.4154
AR1(ψ_t)	0.9180	0.9150	0.9020	0.9330
Corr($\Delta \mu, \Delta \psi$)	-0.7205	-0.5111	-0.9067	-0.8192

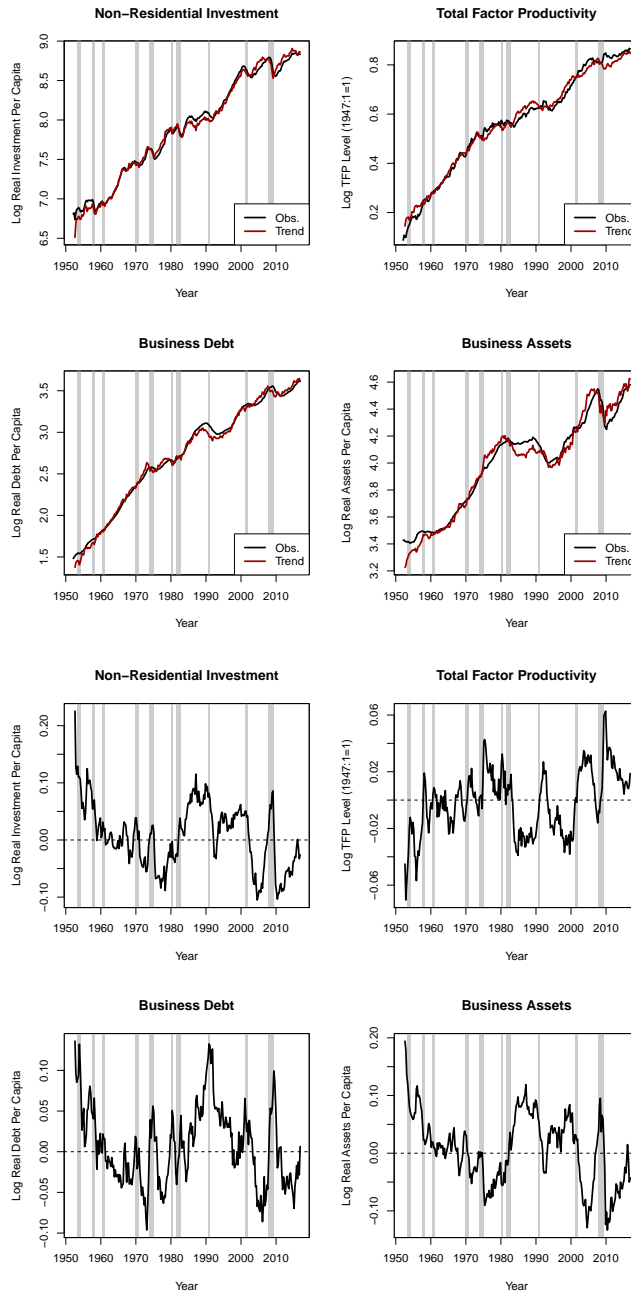


Figure 2.7 Permanent-Transitory Decomposition, Business Model. These plots show the permanent-transitory decomposition for the firm VECM. The top four panels plot the observed data and the calculated permanent component. The bottom four panels plot the transitory components. Shaded areas represent NBER recession periods.

BIBLIOGRAPHY

- Anderson, Richard G. and Charles S. Gascon (2009). “The commercial paper market, the Fed, and the 2007-2009 financial crisis”. *Review* Nov, pp. 589–612.
- Beveridge, Stephen and Charles R. Nelson (1981). “A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the ‘business cycle’”. *Journal of Monetary Economics* 7.2, pp. 151–174.
- Bigio, Saki (2012). “Liquidity shocks and the business cycle: What next”.
- (2015). “Endogenous liquidity and the business cycle”. *The American Economic Review* 105.6, pp. 1883–1927.
- Bordo, Michael D. and Joseph G. Haubrich (2010). “Credit crises, money and contractions: An historical view”. *Journal of Monetary Economics* 57.1, pp. 1–18.
- Bussiere, Matthieu and Marcel Fratzscher (2006). “Towards a new early warning system of financial crises”. *Journal of International Money and Finance* 25.6, pp. 953–973.
- Christiano, Lawrence, Massimo Rostagno, and Roberto Motto (2010). *Financial factors in economic fluctuations*. Working Paper Series 1192. European Central Bank.
- Claessens, Stijn, M. Ayhan Kose, and Marco E. Terrones (2011). “Financial Cycles: What? How? When?” *NBER International Seminar on Macroeconomics* 7.1, pp. 303–344.
- Cui, Wei and Sren Radde (2016). “Money and Asset Liquidity in Frictional Capital Markets”. *American Economic Review* 106.5, pp. 496–502.
- Doornik, Jurgen A (1998). “Approximations to the asymptotic distributions of cointegration tests”. *Journal of Economic Surveys* 12.5, pp. 573–593.
- Eisfeldt, Andrea L. (2004). “Endogenous Liquidity in Asset Markets”. *The Journal of Finance* 59.1, pp. 1–30.
- Engle, Robert F and Clive WJ Granger (1987). “Co-integration and error correction: representation, estimation, and testing”. *Econometrica: journal of the Econometric Society*, pp. 251–276.
- Epstein, Larry G. and Stanley E. Zin (1989). “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework”. English. *Econometrica* 57.4, pp. 937–969.
- Epstein, Larry G and Stanley E Zin (1991). “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: An Empirical Analysis”. *Journal of Political Economy* 99.2, pp. 263–86.

- Fernald, John (2012). *A quarterly, utilization-adjusted series on total factor productivity*. Tech. rep.
- Gallant, A., Peter Rossi, and George Tauchen (1993). “Nonlinear Dynamic Structures”. *Econometrica* 61.4, pp. 871–907.
- Gertler, Mark and Peter Karadi (2011). “A model of unconventional monetary policy”. *Journal of Monetary Economics* 58.1, pp. 17–34.
- Gorton, Gary (2009). “Information, Liquidity, and the (Ongoing) Panic of 2007”. English. *The American Economic Review* 99.2, pp. 567–572.
- Gorton, Gary and Andrew Metrick (2010). “Haircuts”. *Review* Nov, pp. 507–520.
- Gourinchas, Pierre-Olivier and Maurice Obstfeld (2012). “Stories of the Twentieth Century for the Twenty-First”. *American Economic Journal: Macroeconomics* 4.1, pp. 226–265.
- Hart, Oliver and J. Moore (1994). “A Theory of Debt Based on the Inalienability of Human Capital”. *Quarterly Journal of Economics* 109.4, pp. 841–879.
- Hecq, Alain, Franz C Palm, and Jean-Pierre Urbain (2000). “Permanent-transitory Decomposition in Var Models With Cointegration and Common Cycles”. *Oxford Bulletin of Economics and Statistics* 62.4, pp. 511–532.
- Heer, Burkhard and Alfred Maussner (2009). *Dynamic General Equilibrium Modeling*. Second Edition. Springer.
- Hördahl, Peter and Michael R King (2008). “Developments in repo markets during the financial turmoil”. *BIS Quarterly Review*.
- Issler, Joo and Farshid Vahid (2001). “Common cycles and the importance of transitory shocks to macroeconomic aggregates”. *Journal of Monetary Economics* 47.3, pp. 449–475.
- Jermann, Urban and Vincenzo Quadrini (2012). “Macroeconomic Effects of Financial Shocks”. *American Economic Review* 102.1, pp. 238–71.
- Johansen, Søren (1988). “Statistical analysis of cointegration vectors”. *Journal of economic dynamics and control* 12.2-3, pp. 231–254.
- (1991). “Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models”. *Econometrica: Journal of the Econometric Society*, pp. 1551–1580.
- (1994). “The role of the constant and linear terms in cointegration analysis of nonstationary variables”. *Econometric reviews* 13.2, pp. 205–229.

- Judd, Kenneth L. (1998). *Numerical Methods in Economics*. Vol. 1. MIT Press Books 0262100711. The MIT Press.
- Kiyotaki, Nobuhiro and John Moore (1997). “Credit Cycles”. *Journal of Political Economy* 105.2, pp. 211–48.
- (2012). *Liquidity, Business Cycles, and Monetary Policy*. NBER Working Papers 17934. National Bureau of Economic Research, Inc.
- Kreps, David M and Evan L Porteus (1978). “Temporal resolution of uncertainty and dynamic choice theory”. *Econometrica: journal of the Econometric Society*, pp. 185–200.
- Kurlat, Pablo (2013). “Lemons Markets and the Transmission of Aggregate Shocks”. *American Economic Review* 103.4, pp. 1463–89.
- MacKinnon, James G. (2010). *Critical Values for Cointegration Tests*. Working Papers 1227. Queen’s University, Department of Economics.
- Maliar, Lilia and Serguei Maliar (2015). “Merging simulation and projection approaches to solve high-dimensional problems with an application to a new Keynesian model”. *Quantitative Economics* 6.1, pp. 1–47.
- Mehra, Rajnish and Edward C. Prescott (1985). “The equity premium: A puzzle”. *Journal of Monetary Economics* 15.2, pp. 145–161.
- Osterwald-Lenum, Michael (1992). “A note with quantiles of the asymptotic distribution of the maximum likelihood cointegration rank test statistics”. *Oxford Bulletin of Economics and statistics* 54.3, pp. 461–472.
- Proietti, Tommaso (1997). “Short-Run Dynamics in Cointegrated Systems”. *Oxford Bulletin of Economics and Statistics* 59.3, pp. 405–422.
- Schmitt-Grohe, Stephanie and Martin Uribe (2004). “Solving dynamic general equilibrium models using a second-order approximation to the policy function”. *Journal of Economic Dynamics and Control* 28.4, pp. 755–775.
- Schularick, Moritz and Alan M. Taylor (2012). “Credit Booms Gone Bust: Monetary Policy, Leverage Cycles, and Financial Crises, 1870-2008”. *American Economic Review* 102.2, pp. 1029–1061.
- Shi, Shouyong (2015). “Liquidity, assets and business cycles”. *Journal of Monetary Economics* 70.0, pp. 116–132.
- Shiller, Robert (2015). *U.S. Stock Markets 1871-Present and CAPE Ratio*.

Stock, James H and Mark W Watson (1988). “Testing for common trends”. *Journal of the American statistical Association* 83.404, pp. 1097–1107.

Terrones, Marco and Enrique G. Mendoza (2008). *An Anatomy of Credit Booms; Evidence From Macro Aggregates and Micro Data*. IMF Working Papers 08/226. International Monetary Fund.

Vahid, F and Robert F Engle (1993). “Common Trends and Common Cycles”. *Journal of Applied Econometrics* 8.4, pp. 341–360.

Whelan, Karl (2000). *A guide to the use of chain aggregated NIPA data*. Tech. rep.

APPENDICES

Appendix A

Comparison of Collateral Constraints

Jermann and Quadrini (2012) motivate their collateral constraint by allowing borrowing firms to default on intra-period loans. Firms can post capital worth k_t as collateral. They assume that the liquidation value of the collateral is determined after the decision to default. With some exogenous probability, ξ , the lender will be able to liquidate the capital for the full value, k_{t+1} , and with probability $(1 - \xi)$ will recover nothing. They further assume that the borrowing firm has all the bargaining power and that the lender collects the lowest amount that the borrower can threaten to repay.

If the liquidation value of the collateral is k_{t+1} , then the value of the decision to default to the firm is

$$l_t + E_t m_{t+1} V_{t+1} - k_{t+1} + \frac{b_{t+1}}{1 + r_t},$$

where l_t is the intra-period loan to the firm, V_{t+1} is the continuation value of the firm, m_{t+1} is the stochastic discount factor applied to the value of the firm, b_{t+1} is the inter-period debt due at $t + 1$, and r_t is the interest rate on the inter-period loan.

If the liquidation value of the collateral is 0, then the value of the decision to default to the

firm is

$$l_t + E_t m_{t+1} V_{t+1}.$$

When the contract is agreed to, the liquidation value of the collateral is unknown. The expected value of the decision to default to the firm is

$$l_t + E_t m_{t+1} V_{t+1} - \xi \left(k_{t+1} - \frac{b_{t+1}}{1 + r_t} \right).$$

The enforcement constraint requires that the expected value of deciding to repay the loan is at least as large as the expected value of deciding to default, or

$$E_t m_{t+1} V_{t+1} \geq l_t + E_t m_{t+1} V_{t+1} - \xi \left(k_{t+1} - \frac{b_{t+1}}{1 + r_t} \right).$$

Rearranging the above equation produces the enforcement constraint reported in their text,

$$\underbrace{l_t}_{\text{Size of loan}} \leq \underbrace{\xi}_{\text{Liquidity}} \underbrace{\left(k_{t+1} - \frac{b_{t+1}}{1 + r_t} \right)}_{\text{Net value of value of collateral}}.$$

Compare the above constraint to the collateral constraint used in this paper,

$$\underbrace{-b_{e,t}}_{\text{Size of loan}} \leq \underbrace{\theta}_{\text{Liquidity}} \underbrace{p_t^k k_{e,t}}_{\text{Net value of collateral}}$$

The algebraic differences between the Jermann and Quadrini constraint and this constraint are that debt is denoted with negative values in this model, the price of capital is 1 in their model and p_t^k in this model, and there is only one market for debt in this model.

This model relaxes some assumptions that Jermann and Quadrini make to simplify the determination of the constraint. They assume that there are only two potential states for the liquidation value of the capital. This model allows for the liquidation value of the capital to be determined by the market price. Lenders can always possess the land, but the corn value of that

land could be any fraction of the anticipated value of the land.

Appendix B

Deriving Optimality Conditions

The entrepreneurs' problem is represented with the Lagrangean

$$L_{e,t} = E_t \left[\frac{C_{e,t}^{1-\eta_e}}{1-\eta_e} + \lambda_{e,t} \left(A_{e,t} k_{e,t-1}^{\alpha_e} + P_t^k (k_{e,t-1} + f_{e,t-1}) + \max \{ P_t^k - P_{t-1}^x, 0 \} o_{e,t-1} \right. \right. \\ \left. \left. - C_{e,t} - P_t^k k_{e,t} - P_t^f f_{e,t} - P_t^o o_{e,t} \right) + \nu_{e,t} A_{s,t}^{1-\eta_e} (k_{e,t} + f_{e,t}) \right],$$

and the savers' problem is represented with the Lagrangean

$$L_{s,t} = E_t \left[\frac{C_{s,t}^{1-\eta_s}}{1-\eta_s} + \lambda_{s,t} \left(A_{s,t} k_{s,t-1}^{\alpha_s} + P_t^k (k_{s,t-1} + f_{s,t-1}) + \max \{ P_t^k - P_{t-1}^x, 0 \} o_{s,t-1} \right. \right. \\ \left. \left. - C_{s,t} - P_t^k k_{s,t} - P_t^f f_{s,t} - P_t^o o_{s,t} \right) \right].$$

Optimality conditions for entrepreneurs are

$$\begin{aligned} \frac{\partial L}{\partial C_{e,t}} &= C_{e,t}^{-\eta_e} - \lambda_{e,t} = 0, \\ \frac{\partial L}{\partial k_{e,t}} &= E_t \left[\beta_e \lambda_{e,t+1} (P_{t+1}^k + \alpha_e k_{e,t+1}^{\alpha_e - 1}) \right] - \lambda_{e,t} P_t^k + \nu_{e,t} = 0, \\ \frac{\partial L}{\partial f_{e,t}} &= E_t \left[\beta_e \lambda_{e,t+1} P_{t+1}^k \right] - \lambda_{e,t} P_t^f + \nu_{e,t} = 0, \\ \frac{\partial L}{\partial o_{e,t}} &= E_t \left[\beta_e \lambda_{e,t+1} \max \{ P_{t+1}^k - P_t^x, 0 \} \right] - \lambda_{e,t} P_t^o = 0, \end{aligned}$$

Similarly for savers, optimality conditions are

$$\begin{aligned}
\frac{\partial L}{\partial C_{s,t}} &= C_{s,t}^{-\eta_s} - \lambda_{s,t} = 0, \\
\frac{\partial L}{\partial k_{s,t}} &= E_t \left[\beta \lambda_{s,t+1} (P_{t+1}^k + \alpha_s A_{s,t+1} k_{s,t+1}^{\alpha_s-1}) \right] - \lambda_{s,t} P_t^k = 0, \\
\frac{\partial L}{\partial f_{s,t}} &= E_t \left[\beta_s \lambda_{s,t+1} P_{t+1}^k \right] - \lambda_{s,t} P_t^f = 0, \\
\frac{\partial L}{\partial o_{s,t}} &= E_t \left[\beta_s \lambda_{s,t+1} \max \left\{ P_{t+1}^k - P_t^x, 0 \right\} \right] - \lambda_{s,t} P_t^o = 0,
\end{aligned}$$

Eliminating the budget Lagrange multipliers gives the six Euler equations,

$$\begin{aligned}
E_t \left[\beta_s C_{s,t+1}^{-\eta_s} (P_{t+1}^k + \alpha_s A_{s,t+1} k_{s,t+1}^{\alpha_s-1}) \right] &= C_{s,t}^{-\eta} P_t^k \\
E_t \left[\beta_s C_{s,t+1}^{-\eta_s} P_{t+1}^k \right] &= C_{s,t}^{-\eta_s} P_t^f, \\
E_t \left[\beta_s C_{s,t+1}^{-\eta_s} \max \left\{ P_{t+1}^k - P_t^x, 0 \right\} \right] &= C_{s,t}^{-\eta_s} P_t^o, \\
E_t \left[\beta_e C_{e,t+1}^{-\eta_e} (P_{t+1}^k + \alpha_e A_{e,t+1} k_{e,t+1}^{\alpha_e-1}) \right] + A_{s,t}^{1-\eta_e} \nu_{e,t} &= C_{e,t}^{-\eta_e} P_t^k, \\
E_t \left[\beta_e C_{e,t+1}^{-\eta_e} P_{t+1}^k \right] + A_{s,t}^{1-\eta_e} \nu_{e,t} &= C_{e,t}^{-\eta_e} P_t^f, \\
E_t \left[\beta_e C_{e,t+1}^{-\eta_e} \max \left\{ P_{t+1}^k - P_t^x, 0 \right\} \right] &= C_{e,t}^{-\eta_e} P_t^o
\end{aligned}$$

to go along with the market-clearing conditions, budget constraint, collateral constraint, and

TFP processes,

$$\xi C_{e,t} + (1 - \xi)C_{s,t} = \xi A_{e,t} k_{e,t-1}^{\alpha_e} + (1 - \xi) A_{s,t} k_{s,t-1}^{\alpha_s},$$

$$\xi k_{e,t} + (1 - \xi)k_{s,t} = \bar{k},$$

$$\xi f_{e,t} + (1 - \xi)f_{s,t} = 0,$$

$$\xi o_{e,t} + (1 - \xi)o_{s,t} = 0,$$

$$f_{e,t} + o_{e,t} = 0,$$

$$C_{e,t} + P^k k_{e,t} + P^f f_{e,t} + P^o o_{e,t} = A_{e,t} k_{e,t-1}^{\alpha_e} + P_t^k (k_{e,t-1} + f_{e,t-1}) + \max \left\{ P_t^k - P_{t-1}^x, 0 \right\} o_{e,t},$$

$$-f_{e,t} = k_{e,t},$$

$$\ln A_{e,t} = \ln A_{e,t-1} + \ln \mu_s + (\rho_a - 1)(\ln A_{e,t-1} - \ln A_{s,t-1} - \ln \bar{a}) + \sigma_a v_{a,t},$$

$$\text{and } \ln A_{s,t} = \ln A_{s,t-1} + \ln \mu_s$$

Appendix C

Stationary Equations

To produce equations in stationary variables, I normalize the non-stationary variables relative to savers' TFP, $A_{s,t}$. A non-stationary variable X_t has a stationary equivalent $x_t = \frac{X_t}{A_{s,t}}$. The exception is $p_t^x = \frac{P_t^x}{A_{s,t+1}}$, where the shift in time index is because repurchase prices set at time t are paid at time $t + 1$.

Replacing variables stationary equivalents using the definition above, the Euler equations become

$$\begin{aligned}
 E_t \left[\beta_s A_{s,t+1}^{-\eta_s} c_{s,t+1}^{-\eta_s} (A_{s,t+1} p_{t+1}^k + \alpha_s A_{s,t+1} k_{s,t}^{\alpha_s-1}) \right] &= A_{s,t}^{-\eta_s} c_{s,t}^{-\eta_s} A_{s,t} p_t^k, \\
 E_t \left[\beta_s A_{s,t+1} c_{s,t+1}^{-\eta_s} A_{s,t+1} p_{t+1}^k \right] &= A_{s,t}^{-\eta_s} c_{s,t}^{-\eta_s} A_{s,t} p_t^d, \\
 E_t \left[\beta_s A_{s,t+1}^{-\eta_s} c_{s,t+1}^{-\eta_s} \max \left\{ A_{s,t+1} p_{t+1}^k - A_{s,t+1} p_t^x, 0 \right\} \right] &= A_{s,t}^{-\eta_s} c_{s,t}^{-\eta_s} A_{s,t} p_t^o, \\
 E_t \left[\beta_e A_{s,t+1}^{-\eta_e} c_{e,t+1}^{-\eta_e} (A_{s,t+1} p_{t+1}^k + \alpha_e A_{e,t+1} k_{e,t}^{\alpha_e-1}) \right] + A_{s,t}^{1-\eta_e} \nu_{e,t} &= A_{s,t}^{-\eta_e} c_{e,t}^{-\eta_e} A_{s,t} p_t^k, \\
 E_t \left[\beta_e A_{s,t+1}^{-\eta_e} c_{e,t+1}^{-\eta_e} A_{s,t+1} p_{t+1}^k \right] + A_{s,t}^{1-\eta_e} \nu_{e,t} &= A_{s,t}^{-\eta_e} c_{e,t}^{-\eta_e} A_{s,t} p_t^d, \\
 \text{and } E_t \left[\beta_e A_{s,t+1}^{-\eta_e} c_{e,t+1}^{-\eta_e} \max \left\{ A_{s,t+1} p_{t+1}^k - A_{s,t+1} p_t^x, 0 \right\} \right] &= A_{s,t}^{-\eta_e} c_{e,t}^{-\eta_e} A_{s,t} p_t^o.
 \end{aligned}$$

Combine the $A_{s,t}$ terms to be written in terms of the growth rate of savers' TFP, μ .

$$\begin{aligned}
E_t \left[\mu_s^{1-\eta_s} \beta_s c_{s,t+1}^{-\eta_s} (p_{t+1}^k + \alpha_s k_{s,t}^{\alpha_s-1}) \right] &= c_{s,t}^{-\eta_s} p_t^k, \\
E_t \left[\mu_s^{1-\eta_s} \beta_s c_{s,t+1}^{-\eta_s} p_{t+1}^k \right] &= c_{s,t}^{-\eta_s} p_t^f, \\
E_t \left[\mu_s^{1-\eta_s} \beta_s c_{s,t+1}^{-\eta_s} \max \left\{ p_{t+1}^k - p_t^x, 0 \right\} \right] &= c_{s,t}^{-\eta_s} p_t^o, \\
E_t \left[\mu_s^{1-\eta_s} \beta_e c_{e,t+1}^{-\eta_e} (p_{t+1}^k + \alpha_e a_{t+1} k_{e,t}^{\alpha_e-1}) \right] + \nu_{e,t} &= c_{e,t}^{-\eta_e} p_t^k, \\
E_t \left[\mu_s^{1-\eta_s} \beta_e c_{e,t+1}^{-\eta_e} p_{t+1}^k \right] + \nu_{e,t} &= c_{e,t}^{-\eta_e} p_t^f, \\
\text{and } E_t \left[\mu_s^{1-\eta_s} \beta_e c_{e,t+1}^{-\eta_e} \max \left\{ p_{t+1}^k - p_t^x, 0 \right\} \right] &= c_{e,t}^{-\eta_e} p_t^o.
\end{aligned}$$

The stationary Euler equations, along with the stationary market-clearing conditions, budget constraint, and collateral constraint, describe the equilibrium conditions for the economy. The the stationary market-clearing conditions, budget constraint, and collateral constraint are

$$\begin{aligned}
\xi c_{e,t} + (1 - \xi) c_{s,t} &= \xi a_t k_{e,t-1}^{\alpha_e} + (1 - \xi) k_{s,t-1}^{\alpha_s}, \\
\xi k_{e,t} + (1 - \xi) k_{s,t} &= \bar{k}, \\
\xi f_{e,t} + (1 - \xi) f_{s,t} &= 0, \\
\xi o_{e,t} + (1 - \xi) o_{s,t} &= 0, \\
f_e + o_e &= 0, \\
c_{e,t} + p^k k_{e,t} + p^f f_{e,t} + p^o o_{e,t} &= a_t k_{e,t-1}^{\alpha_e} + p_t^k (k_{e,t-1} + f_{e,t-1}) + \max \left\{ p_t^k - p_{t-1}^x, 0 \right\} o_{e,t-1}, \\
\text{and } -f_{e,t} &= k_{e,t}.
\end{aligned}$$

This is a system of thirteen equations in the thirteen variables $c_{e,t}$, $c_{s,t}$, $k_{e,t}$, $k_{s,t}$, $f_{e,t}$, $f_{s,t}$, $o_{e,t}$, $o_{s,t}$, p_t^k , p_t^f , p_t^o , p_t^x , and $\nu_{e,t}$.

Appendix D

Consolidated Optimality Conditions

The optimality conditions for land, collateral, and interest can be combined to produce a consolidated asset pricing equation.

The optimality conditions for entrepreneurs and savers can be combined to eliminate p_t^k , p_t^f , and p_t^o ,

$$\begin{aligned} E_t \left[m_{s,t+1}(p_{t+1}^k + \alpha_s k_{s,t}^{\alpha_s - 1}) \right] &= E_t \left[m_{e,t+1}(p_{t+1}^k + \alpha_e a_{t+1} k_{e,t}^{\alpha_e - 1}) \right] + \frac{\nu_{e,t}}{c_{e,t}^{-\eta_e}}, \\ \frac{\nu_{e,t}}{c_{e,t}^{-\eta_e}} &= E_t \left[(m_{s,t+1} - m_{e,t+1}) p_{t+1}^k \right], \\ 0 &= E_t \left[(m_{s,t+1} - m_{e,t+1}) \max \left\{ p_{t+1}^k - p_t^x, 0 \right\} \right]. \end{aligned}$$

The collateral and interest conditions can be combined,

$$\frac{\nu_{e,t}}{c_{e,t}^{-\eta_e}} = E_t \left[(m_{s,t+1} - m_{e,t+1}) \min \left\{ p_t^x, p_{t+1}^k \right\} \right].$$

Insert into the land condition to eliminate the liquidity premium,

$$\begin{aligned} E_t \left[m_{s,t+1}(p_{t+1}^k + \alpha_s k_{s,t}^{\alpha_s - 1}) \right] &= E_t \left[m_{e,t+1}(p_{t+1}^k + \alpha_e a_{t+1} k_{e,t}^{\alpha_e - 1}) \right] \\ &+ E_t \left[(m_{s,t+1} - m_{e,t+1}) \min \left\{ p_t^x, p_{t+1}^k \right\} \right], \end{aligned}$$

Collect terms by discount factors

$$E_t \left[m_{s,t+1} \left(\alpha_s k_{s,t}^{\alpha_s - 1} + \min \{ p_{t+1}^k - p_t^x, 0 \} \right) \right] = E_t \left[m_{e,t+1} \left(\alpha_e a_{t+1} k_{e,t}^{\alpha_e - 1} + \min \{ p_{t+1}^k - p_t^x, 0 \} \right) \right],$$

Appendix E

Cointegration Tests

This section provides detail on the Engle-Granger tests for cointegration between consumption, total factor productivity, and liquidity.

To conduct the Engle-Granger tests, I run the first stage regressions as log liquidity on a constant, time trend, time trend squared, and log of either TFP or consumption. These expressions correspond to equation 4 in MacKinnon (2010). Plots of the levels, fitted values, and residuals are shown in Figure (E.1)

I run the second stage regressions as the residual from the two first stage regressions on the lagged residual and four lags of residual differences. The four lags are chosen by starting with 12 lags and reducing the number of lags until the last t-statistic is greater than 1.96.

The results of the second-stage regressions on the residuals are shown in Table E.1.

If liquidity is not cointegrated with either TFP or consumption, then the residuals should still be $I(1)$ and the coefficients on u_1 should still approximately 0. Because the test statistics follow a non-standard distribution, I use the critical values as calculated by MacKinnon (2010). Specifically, I use the values for the quadratic time trend case and $N = 2$ or $N = 3$ in Table 4 on page 15. Applying the small-sample correction for 247 observations I get the following critical values:

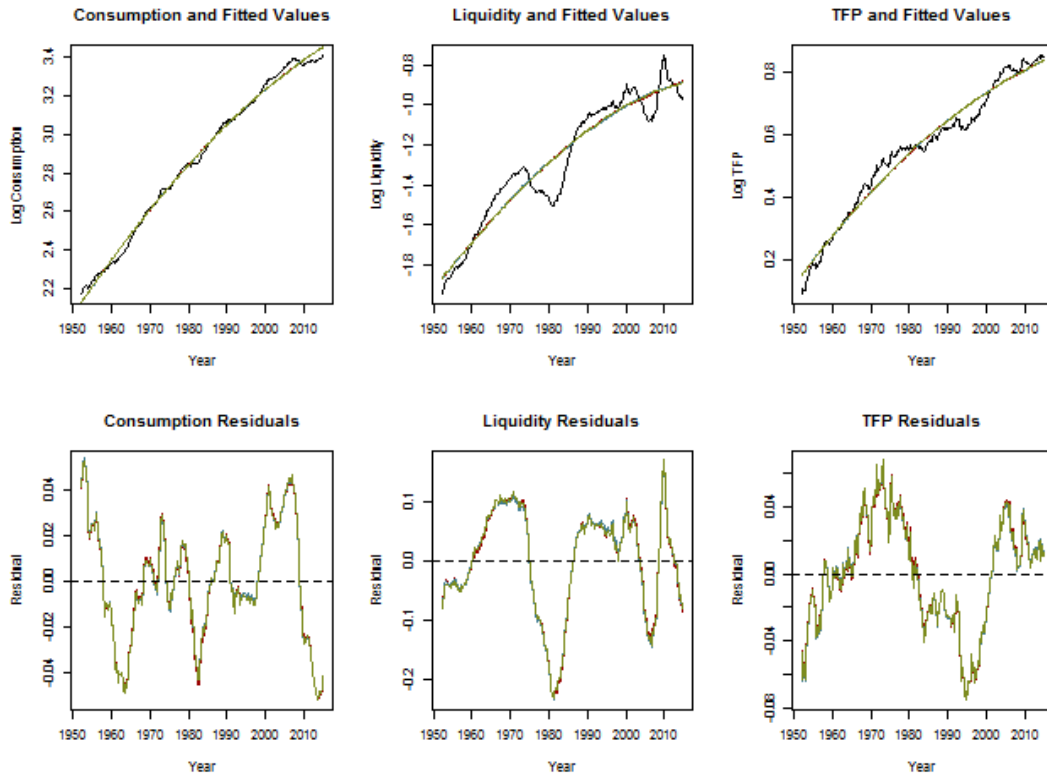


Figure E.1 Engle-Granger Tests, Stage 1. Levels for each series are in black. In the top row, the red, blue, and green lines are the fitted values. Changing the independent variables in the regression has little effect on the fitted values, as the time trend dominates the fit. In the bottom row, the red, blue, and green lines are the residuals from the separate first stage regressions.

Confidence Level	$N = 2$	$N = 3$
1%	-4.775715	-5.087454
5%	-4.208223	-4.517491
10%	-3.916104	-4.223659

The observed t-statistics fall short of the critical values, which suggests we cannot reject the null hypothesis that the residuals are $I(1)$. That is, the data are consistent with the hypothesis that the three series are not cointegrated.

Table E.1 Engle-Granger Tests, Second Stage. Estimates are the coefficient on the lagged residual term in an augmented Engle-Granger test. Under the null hypothesis that there is no cointegration between the two series, the coefficient is 0.

Regression	Estimate	Std. Err.	t-Stat.
Consumption on TFP	-0.03084	0.01108	-2.783
Consumption on Liquidity	-0.03035	0.01104	-2.749
Liquidity on TFP	-0.02554	0.00839	-3.043
Liquidity on Consumption	-0.02544	0.00829	-3.067
TFP on Consumption	-0.03667	0.01519	-2.413
TFP on Liquidity	-0.03733	0.01522	-2.453
Consumption on TFP and Liquidity	-0.03080	0.01106	-2.786
Liquidity on Consumption and TFP	-0.02564	0.00838	-3.059
TFP on Consumption and Liquidity	-0.03712	0.01523	-2.438

Appendix F

Solution Algorithm

The solution involves using polynomial approximations to solve for some of the unknown equilibrium functions, then using the remaining equilibrium equations to solve for the remaining variables. The exact algorithm is

1. Initialization

- (a) Choose a set of points ω_n for $n = 1, \dots, N$ that are the Gauss-Hermite nodes for integrating shocks $v_{a,t+1}$ and $v_{\theta,t+1}$. For the parameterized-liquidity and endogenous-liquidity models, I use 51 nodes. For the stochastic-liquidity model, I use 7 nodes for each shock, for a total of 49 nodes.
- (b) Use a perturbation solution to initialize the vector of coefficients for approximating the policy functions. I calculate a second-order approximation using code from Schmitt-Grohe and Uribe (2004).

2. Create an EDS of points in the state space from the simulation.

- (a) Choose a target size M_{eds} for the EDS. I choose $M_{eds} = 200$.
- (b) Sample every κ th point from the simulation to get an approximately independent sample from the distribution of state space variables. I choose $\kappa = 10$.

- (c) Normalize the sample to zero mean and unit variance.
 - (d) Orthogonalize the sample.
 - (e) Choose a range $[\epsilon_{min}, \epsilon_{max}]$ and set $\epsilon = 0.5(\epsilon_{min}, \epsilon_{max})$. Pick a point from the sample and place it in the EDS. Remove every other point in the sample that is within ϵ of the chosen point. Repeat until all points from the sample have been selected or eliminated.
 - (f) If the EDS has the target size, then move to the next step. Otherwise, if the EDS is too small, set $\epsilon = 0.5(\epsilon_{min} + \epsilon)$, and if the EDS is too large, set $\epsilon = 0.5(\epsilon + \epsilon_{max})$, then repeat.
3. For $\omega_n, n = 1, \dots, N$
- (a) Set the one-step-ahead productivity level as

$$\ln \hat{a}_{t+1} = (1 - \rho_a) \ln \bar{a} + \rho_a \ln a_t + \sigma \omega_n.$$

- (b) Apply the policy functions with $\hat{z}_{m,t+1} = (\hat{k}_{e,t}, \hat{p}_t^s, \hat{a}_{t+1})$. Compute the following functions, which correspond to the Euler equations.

$$g_1(z_m, \omega_n) = \mu^{1-\eta_e} \beta_e \hat{c}_{e,t+1}^{-\eta} \frac{\hat{p}_{t+1}^k + \hat{a}_{t+1} \alpha \hat{k}_{e,t}^{\alpha-1}}{\hat{p}_t^k} + \frac{\hat{v}_{e,t}}{\hat{p}_t^k},$$

$$g_2(z_m, \omega_n) = \mu^{1-\eta_s} \beta_s \left(\frac{\hat{c}_{s,t+1}}{\hat{c}_{s,t}} \right)^{-\eta} (\hat{p}_{t+1}^k + \alpha_s \hat{k}_{s,t}^{\alpha_s-1}),$$

$$g_3(z_m, \omega_n) = \mu^{1-\eta_s} \beta_s \left(\frac{c_{s,t+1}}{c_{s,t}} \right)^{-\eta} \hat{r}_t^i \min\{\hat{p}_m^x, \hat{p}_{m,t+1}^k\},$$

$$g_4(z_m, \omega_n) = \mu^{1-\eta_s} \beta_s \left(\frac{c_{s,t+1}}{c_{s,t}} \right)^{-\eta} \hat{p}_{m,t+1}^k,$$

and $g_5(z_m, \omega_n) = \mu^{1-\eta_e} \beta_e \left(\frac{\hat{c}_{e,t+1}}{\hat{c}_{e,t}} \right)^{-\eta} \max\{p_{t+1}^k - p_t^s, 0\}.$

4. The conditional expectations are calculated as the following integrals where m quadrature nodes are used and ω_i is the weight at each node.

5. The residual function is

$$R_j(z_m) = \phi_j(z_m) - \psi_j(\gamma, z_m), \quad j = 1, \dots, 6.$$

6. Find parameters γ that minimize Euclidian norm $\|R(\gamma, z_t)\|$,

$$\hat{\gamma} = \arg \min \sum_{z \in Z} R(z)^2 dz$$

Appendix G

Euler Errors

Table G.1 displays the average and maximum Euler equation errors during a simulation of 8,000 quarters for each of the three models.

Table G.1 Euler Errors. Euler errors are calculated as the absolute difference between the value implied by the Euler equation and the value set by the polynomial approximation as a percentage of the value set by the polynomial approximation.

Model	Error	$c_{e,t}^{-\eta_e}$	p_t^k	p_t^x	p_t^f	p_t^o
Param.	Avg.	0.30%	0.30%	0.30%	–	–
	Max	0.35%	0.66%	0.64%	–	–
Stoch.	Avg.	0.30%	0.30%	0.30%	–	–
	Max	0.50%	1.16%	1.17%	–	–
Endog.	Avg.	0.48%	0.59%	0.20%	0.19%	0.18%
	Max	1.17%	3.35%	2.06%	2.08%	2.12%

Average errors are on the order of one-half of one percent, suggesting good fit for most of the sample. However, the maximum errors for the endogenous-liquidity model range from one to three percent. Errors for all models increase as the share of land held by entrepreneurs increases and the share held by savers decreases, and entrepreneurs' share is typically higher in the endogenous-liquidity model. This suggests that higher-order approximations of the policy

functions would be helpful in capturing the behavior of prices as savers' share of the land falls, and thus their marginal product of land rises.

Appendix H

Data Description

Consumption is measured by Personal Consumption Expenditures as reported by the Bureau of Economic Analysis (NIPA Table 1.1.5). Real PCE is calculated by dividing the nominal value by the corresponding price index (NIPA 1.1.4) to get a measure of real PCE.

Investment is measured by gross private domestic investment as reported by the Bureau of Economic Analysis (NIPA Table 1.1.5). Real PCE is calculated by dividing the nominal value by the corresponding price index (NIPA 1.1.4) to get a measure of real PCE.

The TFP series used in this paper is the utilization-adjusted TFP series taken from Fernald (2012). This data is available at the website of the Federal Reserve Bank of San Francisco.

Data on debt and assets comes from the Financial Accounts of the United States, published by the Federal Reserve. Nominal data are deflated using the PCE price index. The following series are used:

Series	Code
Nonfinancial corporate business; debt securities and loans; liability	LA104104005.Q
Nonfinancial business; debt securities and loans; liability	LA144104005.Q
Households and nonprofit organizations; debt securities and loans; liability	LA154104005.Q
Nonfinancial corporate business; nonfinancial assets	LM102010005.Q
Nonfinancial noncorporate business; nonfinancial assets	LM112010005.Q
Households and nonprofit organizations; nonfinancial assets	LM152010005.Q

Non-financial non-corporate debt is calculated by subtracting non-financial corporate debt from the total nonfinancial debt.

All quantity variables are converted to a per capita basis by dividing by the U.S. population. The population data is taken from NIPA Table 2.1.