AN ENERGY METHOD FOR THE TRI-AXIAL ANALYSIS OF A RECTANGULAR SLAB WITH A CENTRAL HOLE AND ARBITRARY BOUNDARY CONDITIONS

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SUMMARY

The term "energy method", in case of complicated analysis, in most cases means the finite element method. Other energy methods, such as the Rayleigh-Ritz method, are more often used for simpler analysis, an important field being verification for the complicated methods.

One main difficulty in connection with the Rayleigh-Ritz method is to find admissible functions for expansion of the solutions. Hence the application of this and related methods have been restricted to cases with very simple kinematic boundary conditions.

The present method overcomes these difficulties by spring suspensions on surfaces, where kinematic boundary conditions are prescribed:

The rectangular slab is supported by springs on the vertical edges and loaded by uniform pressure on the upper surface. The hole is elliptic, running vertical in the center of the slab.

The displacement field is approximated by a truncated series in powers of the independent variables, so that arbitrary displacements can be represented on the boundaries. By pre-stressing of the springs, displacements or stresses (depending of the spring stiffnesses) can be prescribed.

The coefficients in the expansion are determined by the Rayleigh-Ritz method, the elastic energy of the springs being added to the energy of the slab. The prestressing of the springs enters the computations as coefficients in an expansion similar to that of the displacements. Examples on solutions will be shown in the lecture.

The present study is sponsored by the Danish AEC as a part of the investigation of the thick, perforated bottom slab in a PCRV. The slab considered here forms an element of the bottom slab.

The author considers the following as the most significant in the investigation:

(a) The familiar spring concept is used in connection with the Rayleigh-Ritz method to develop a method of computing solutions to a complicated, classical problem under arbitrary boundary conditions, using one and the same set of coordinate functions.
(b) The method is specially suited for slabs with a large hole diameter/side length ratio and a hole diameter/plate thickness ratio of the order of magnitude 1.
(c) The method yields smooth solutions.

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1. Introduction

The calculation of the stress concentration around one or more holes in a plate is one of the classical problems of the theory of elasticity. Exact solutions to problems of this kind have been found only for rather few, simple cases.

Due to the fact, that perforated plates have been of increasing importance in the design of heat exchangers, boilers and reactor pressure vessels, an ever increasing number of authors have worked with this kind of problems. Most of the published works have used the theory of thin plates (see references in Meijers [4], [5], Chen Pang Tan [2] and Neubert & Hahn [8]) but in recent years especially the development of safe power reactors has accellerated the triaxial calculation of stress concentrations in perforated plates. Except for the more fundamental considerations (Youngdale, Sternberg [11], Dvorak [3] e.a.) energy methods are commonly used, especially the finite element method.

The present analysis has been initiated by the need for more detailed investigation of the perforated bottom slab in the PCRV for the BWR dealt with in the paper H 3/9 [7] of the present conference. It is attempted in the analysis to combine the advantages of a series solution with a main advantage of the finite element method, namely the ability to cope with arbitrary boundary conditions.

2. The boundary value problem

Since the actual bottom slab is loaded on the upper surface and supported on the cylindrical side surface, shear forces should be taken into account. A solution in doubly-periodic functions for an infinite plate will fail in this respect, and therefore the investigation is restricted to one hole, the plate of fig. 1 being investigated.

The plate is a rectangular slab with sidelengths $2L_1$, $2L_2$ and $2L_3$ in the x_1- , x_2- and x_3-directions, respectively. It is penetrated by an elliptic hole with semiaxes R_1 and R_2 in the x_1- and x_2-directions and is loaded on the upper surface by uniform pressure P in the positive x_3-direction. The displacements U_i are arbitrarely prescribed on the 4 edges $x = \pm L_1$, $x_2 = \pm L_2$.

In order to secure compatibility the problem is solved in displacements. Hence we have a boundary value problem, where the differential equations are the full Naviers equations with body forces set to zero, and the boundary conditions are prescribed arbitrarely:

$$\sum_{j=1}^{3} \left[U_{i,jj} + \frac{1}{1-2\nu} U_{j,ji}\right] = 0 \qquad (1)$$

$$U_i \quad \text{on} \quad S_u$$

S_u are the surfaces, where displacements are prescribed.

The boundary value problem eq. (1) is solved approximately by Rayleigh-Ritz' method, the displacement field being expressed as series expansions in powers of the independent variables.

It is well known (Mikhlin [6] e.a.) that solving eq. (1) is identical to finding among a set of admissible functions ϕ_i those functions ϕ_i^* that gives minimum value to the total energy E_{tot} of the system so that

$$E_{tot}[\phi_i^*] = \min E_{tot}[\phi_i] \ . \qquad (2)$$

If the displacements are represented by truncated series of the form

$$U_i = \sum_{n=1}^{N} C_n^i \, \phi_n(x_1,x_2,x_3) \qquad (3)$$

the constants C_n^i can be determined as solutions of the system of equations

$$\frac{\partial E_{tot}}{\partial C_n} = 0 \qquad n=1,2,\ldots,N \ . \qquad (4)$$

In this case it is wanted, that the kinematic boundary conditions can be arbitrarily prescribed in each calculation. Normally this demands that every case be computed with special coordinate functions, satisfying the kinematic boundary conditions, which in fact means that a new EDP-program must be developed for each case.

In order to avoid this, the problem is formulated in such a way, that only stresses are prescribed in the boundaries. In such case all configurations can be computed with one single set of coordinate functions if only these functions can produce arbitrary displacements on the boundaries. The stresses are prescribed by means of imaginary springs: On each of the 4 edges acts an elastic continuum (see fig. 2). By stiffness $F_s\ (m,n)$ is denoted the stiffness on edge No. m in direction n , that is, in a given point on edge No. m is the stress σ_n in direction n given by

$$\sigma_n = F_s\ (m,n)\ [F\ (m,n) - U_n\ (m)] \qquad (5)$$

where $U_n\ (m)$ is the displacement in a point on edge No. m in direction n and $F\ (m,n)$ is a corresponding prestressing of the elastic continuum. If these continua is incorporated in the system, eq. (4) will minimize the total energy of the slab and the springs, and we are thus able to prescribe stresses or displacements, dependent of the value of F_s . Two limit cases are obtained:

a) F_s large: $U_n\ (m) \simeq F\ (m,n)$
b) F_s small: $\sigma_n \simeq F_s(m,n) \cdot F\ (m,n)$

In the calculations the springs are incorporated by adding their elastic energy to that of the slab, and so the total energy consists of

$$E_{tot} = E_p + E_f - V \tag{6}$$

where E_p is the elastic energy of the plate, E_f is the elastic energy of the springs and V the loss of potential energy of the loading. Thus the energy functional E_{tot} can be written as

$$E_{tot} = \int_R W_p dR + \int_{S_u} W_f dS_u - \int_{S_t} \sum_{i=1}^{3} P_i \cdot U_i dS_t \ . \tag{7}$$

In this equation, W_p and W_f are the energy densities for the plate and the springs, respectively, and P_i is the loading on the surface S_t . With this functional eq. (4) can be written

$$\frac{\partial E_{tot}}{\partial C_n} = \frac{\partial}{\partial C_n} \left[E_p + E_f - V\right] \qquad n=0,1,2,\ldots,N \ . \tag{8}$$

In cartesian coordinates the energy density W_p in the linear elastic slab is given by

$$\begin{aligned} W_p = \frac{E}{2(1+\nu)} \{ & \left[U_{1,1}^2 + U_{2,2}^2 + U_{3,3}^2\right] + \tfrac{1}{2}\left[(U_{1,2} + U_{2,1})^2 \right. \\ & \left. + (U_{1,3} + U_{3,1})^2 + (U_{2,3} + U_{3,2})^2\right] \\ & + \frac{\nu}{1-2\nu}\left[U_{1,1} + U_{2,2} + U_{3,3}\right]^2 \} \ . \end{aligned} \tag{9}$$

For the springs it is most convenient to express the energy density W_f as energy per unit slab edge area. In a point on edge No. m we have:

$$W_f\ (m) = \frac{1}{2} \sum_{n=1}^{3} F_s\ (m,n) \left[F\ (m,n) - U_n\ (m)\right]^2 \tag{10}$$

3. The system of equations

The coordinate functions should be chosen so that the expansion of arbitrary displacement fields is possible, but an extension of Naviers thin-plate solution to this purpose leads to quite complicated expressions (Woinowski-Krieger [10]). A simpler set of complete functions will be products of the kind

$$T_p\ (\xi_1) \cdot T_q\ (\xi_2) \cdot T_r\ (\xi_3)$$

where $T_m\ (\xi_n)$ is the Chebycheff polynomium of the order m in the dimensionless coordinate $\xi_n = x_n/L_n$.

An expansion in T_m is in fact an expansion in powers of ξ_n , and the grouping of the powers of ξ_n in Chebycheff polynomiums is advantageous in cases, where the coefficients are determined by use of the orthogonality properties of these polynomiums. However, using Rayleigh-Ritz' method, all coefficients are determined at a time by solving the linear system of eq. (4), and therefore it gives simpler calculations to expand directly in powers of ξ_m . The following expansion, nondimensionalized by division by the plate dimensions, is used:

$$u_i = \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} P^i_{pqr} \, \xi_1^p \, \xi_2^q \, \xi_3^r \tag{11}$$

where u_i denotes the dimensionless displacement U_i/L_i .

The notation is simplified by the introduction of

$$\begin{aligned} G_{pqr} &= \xi_1^p \, \xi_2^q \, \xi_3^r \\ F^1_{pqr} &= \frac{\partial}{\partial \xi_1} G_{pqr} = p \cdot \xi_1^{p-1} \, \xi_2^q \, \xi_3^r \\ F^2_{pqr} &= \frac{\partial}{\partial \xi_2} G_{pqr} = q \cdot \xi_1^p \, \xi_2^{q-1} \, \xi_3^r \\ F^3_{pqr} &= \frac{\partial}{\partial \xi_3} G_{pqr} = r \cdot \xi_1^p \, \xi_2^q \, \xi_3^{r-1} \end{aligned} \tag{12}$$

which gives

$$\begin{aligned} u_i &= \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} P^i_{pqr} \, G_{pqr} \\ \frac{\partial u_i}{\partial \xi_j} &= \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} P^i_{pqr} \, F^j_{pqr} \\ \frac{\partial}{\partial P^k_{\ell mn}} (u_i) &= \delta_{ik} \cdot G_{\ell mn} \\ \frac{\partial}{\partial P^k_{\ell mn}} \left(\frac{\partial u_i}{\partial \xi_j}\right) &= \delta_{ik} \cdot F^j_{\ell mn} \end{aligned} \tag{13}$$

By introducing eqs. (12) and (13) into eq. (9), we obtain after rearrangement the following expression for the first term in eq. (8):

$$\frac{\partial E_p}{\partial P^{\ell}_{ijk}} = L_1 \cdot L_2 \cdot L_3 \int_R \frac{E}{1+\nu} \sum_{pqr}\sum\sum \{ [\frac{1-\nu}{1-2\nu}\delta_{1\ell}F^1_{pqr}F^1_{ijk} + \frac{\nu}{1-2\nu}[\delta_{2\ell}F^1_{pqr}F^2_{ijk}+\delta_{3\ell}F^1_{pqr}F^3_{ijk}]$$

$$+ \frac{1}{2}(f^2_{12}\delta_{1\ell}F^2_{pqr}F^2_{ijk} + f_{21}f_{12}\delta_{2\ell}F^2_{pqr}F^1_{ijk}$$

$$+ f^2_{13}\delta_{1\ell}F^3_{pqr}F^3_{ijk} + f_{13}f_{31}\delta_{3\ell}F^3_{pqr}F^1_{ijk})] \cdot P^1_{pqr}$$

$$+ [\frac{1-\nu}{1-2\nu}\delta_{2\ell}F^2_{pqr}F^2_{ijk} + \frac{\nu}{1-2\nu}[\delta_{1\ell}F^2_{pqr}F^1_{ijk}+\delta_{3\ell}F^2_{pqr}F^3_{ijk}]$$

$$+ \frac{1}{2}(f_{21}f_{12}\delta_{1\ell}F^1_{pqr}F^2_{ijk} + f^2_{21}\delta_{2\ell}F^1_{pqr}F^1_{ijk} \qquad (14)$$

$$+ f^2_{23}\delta_{2\ell}F^3_{pqr}F^3_{ijk} + f_{23}f_{32}\delta_{3\ell}F^3_{pqr}F^2_{ijk})] \cdot P^2_{pqr}$$

$$+ [\frac{1-\nu}{1-2\nu}\delta_{3\ell}F^3_{pqr}F^3_{ijk} + \frac{\nu}{1-2\nu}[\delta_{1\ell}F^3_{pqr}F^1_{ijk}+\delta_{2\ell}F^3_{pqr}F^2_{ijk}]$$

$$+ \frac{1}{2}(f_{31}f_{13}\delta_{1\ell}F^1_{pqr}F^3_{ijk} + f^2_{31}\delta_{3\ell}F^1_{pqr}F^1_{ijk}$$

$$+ f_{32}f_{23}\delta_{2\ell}F^2_{pqr}F^3_{ijk}+F^2_{32}\delta_{3\ell}F^2_{pqr}F^2_{ijk})] \cdot P^3_{pqr}\} d\xi_1 d\xi_2 d\xi_3$$

where $f_{ij} = L_i/L_j$ and R is the whole volume of the slab. This equation is written

$$\frac{\partial E_p}{\partial P^{\ell}_{ijk}} = \sum_{n=1}^{3} \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} A^{\ell ijk}_{npqr} \cdot P^n_{pqr} \qquad (15)$$

and contains only straightforward integrals.

From eq. (10) we see that the total elastic energy of the springs is:

$$E_f = \frac{1}{2} \sum_{m=1}^{4} \sum_{n=1}^{3} \int_{S_u} [F_s\ (m,n) - U_n\ (m)]^2\ dS_u \qquad (16)$$

By introducing eqs. (12) and (13) together with the notation $f\ (m,n) = F\ (m,n)/L_n$ into eq. (16) we obtain as the second term in eq. (8):

$$\frac{\partial E_f}{\partial P^{\ell}_{ijk}} = \sum_{n=1}^{3} \left\{ \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} \delta_{n\ell} \Big[F_s(1,n) \cdot L_n^2 \cdot L_2 \cdot L_3 \int_{S_1} G_{pqr}(1) \cdot G_{ijk}(1)\, d\xi_2 d\xi_3 \right.$$

$$+ F_s(3,n) \cdot L_n^2 \cdot L_2 \cdot L_3 \int_{S_3} G_{pqr}(3) \cdot G_{ijk}(3)\, d\xi_2 d\xi_3$$

$$+ F_s(2,n) \cdot L_n^2 \cdot L_1 \cdot L_3 \int_{S_2} G_{pqr}(2) \cdot G_{ijk}(2)\, d\xi_1 d\xi_3$$

$$+ F_s(4,n) \cdot L_n^2 \cdot L_1 \cdot L_3 \int_{S_4} G_{pqr}(4) \cdot G_{ijk}(4)\, d\xi_1 d\xi_3 \Big] \cdot P^n_{pqr}$$

$$- \delta_{n\ell} \Big[F_s(1,n) \cdot L_n^2 \cdot L_2 \cdot L_3 \int_{S_1} f(1,n) \cdot G_{ijk}(1)\, d\xi_2 d\xi_3 \qquad (17)$$

$$+ F_s(3,n) \cdot L_n \cdot L_2 \cdot L_3 \int_{S_3} f(3,n) \cdot G_{ijk}(3)\, d\xi_2 d\xi_3$$

$$+ F_s(2,n) \cdot L_n^2 \cdot L_1 \cdot L_3 \int_{S_2} f(2,n) \cdot G_{ijk}(2)\, d\xi_1 d\xi_3$$

$$\left. + F_s(4,n) \cdot L_n^2 \cdot L_1 \cdot L_3 \int_{S_4} f(4,n) \cdot G_{ijk}(4)\, d\xi_1 d\xi_3 \Big] \right\}$$

where G_{pqr} (m) stands for the term G_{pqr} evaluated on surface S_m , the edge number m .

In abbreviated notation this is written

$$\frac{\partial E_f}{\partial P^{\ell}_{ijk}} = \sum_{n=1}^{3} \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} B^{\ell ijk}_{npqr} \cdot P^n_{pqr} - C_{\ell ijk} \qquad (18)$$

The integrals containing G_{pqr} only are straightforward which the integrals containing f (m,n) are not. It is important that it should be possible to prescribe f arbitrarely, since the variable boundary conditions are introduced by prestressing the springs. Therefore the evaluation of the latter type of integrals is performed in a special way as shown in appendix A.

The change in potential energy of the load (0,0,P) on the upper slab surface is

$$V = P \cdot L_3 \int_{S_t} u_3\ (S_t)\ dS_t \qquad (19)$$

where u_3 (S_t) denotes u_3 evaluated on the upper surface. By means of eq. (13) we obtain

$$\frac{\partial V}{\partial P^{\ell}_{ijk}} = \delta_{3\ell} \cdot P \cdot L_3 \int_{S_t} G_{ijk}(S_t)\, dS_t \equiv D_{\ell ijk} \tag{20}$$

The linear system of equations eq. (8) can now be written

$$\frac{\partial E_{tot}}{\partial P^{\ell}_{ijk}} = \sum_{n=1}^{3} \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} \left\{A^{\ell ijk}_{npqr} + B^{\ell ijk}_{npqr}\right\} \cdot P^{n}_{pqr} - C_{\ell ijk} - D_{\ell ijk} = 0 \tag{21}$$

$\ell=1,2,3$
$i=0,1,2,\ldots,N$
$j=0,1,2,\ldots,N$
$k=0,1,2,\ldots,N$

Eqs. (21) are solved approximately by successive overrelaxation. To this purpose a FORTRAN IV program has been written for use on an IBM 370/165 computer. The program makes it possible to operate with separate limits for each of the indices p, q and r.

4. Stresses and strains

When eqs. (21) have been solved, we have approximate values for the coefficients P^{i}_{pqr} in eq. (11). The true displacements U_i are determined from

$$U_i = L_i \cdot u_i = L_i \cdot \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} P^{i}_{pqr} \cdot G_{pqr} \tag{22}$$

and the derivatives from

$$\frac{\partial U_i}{\partial x_j} = \frac{L_i}{L_j} \cdot \frac{\partial u_j}{\partial \xi_j} = f_{ij} \sum_{p=0}^{N} \sum_{q=0}^{N} \sum_{r=0}^{N} P^{i}_{pqr} \cdot F^{j}_{pqr} \tag{23}$$

From this the elements in the linear strain tensor are given by

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) \tag{24}$$

and Hookes law now yields the elements in the stress tensor:

$$\sigma_{ij} = \frac{E}{1+\nu} \{\varepsilon_{ij} + \frac{\nu}{1-2\nu} \delta_{ij} \sum_{k=1}^{3} \varepsilon_{kk}\} \tag{25}$$

Now let in a given point P in a plane α three orthogonal vectors a_i , b_i and c_i be given and let a_i be the normal to α . Then the normal and shear stresses on α in P according to the directions a , b and c (see fig. 3) are computed as

$$\sigma_a = \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma_{ij}\, a_i\, a_j$$
$$\tau_{ab} = \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma_{ij}\, a_i\, b_j \qquad (26)$$
$$\tau_{ac} = \sum_{i=1}^{3} \sum_{j=1}^{3} \sigma_{ij}\, a_i\, c_j$$

The actual strains are computed in analogous way.

5. Results

Computations for a number of cases have shown, that the convergence of the iterative solution method is reliable. It has been checked, that the spring concept is able to represent all reasonable equilibrium boundary conditions, in most cases with a very high degree of accuracy.

The case of a quadratic slab under uniaxial tension with a circular hole having a diameter of half the sidelength of the slab, has been computed with powers of ξ_1 , ξ_2 and ξ_3 up to 17 , 17 and 1 , respectively. This corresponds to plane stress, and the result after 50 iterations is compared in fig. 4 with the solutions of O'Connell [9] and Ando e.a. [1].

Further examples including triaxial solutions will be shown at the lecture.

Appendix A . Evaluation of integrals containing f(m,n).
The "prestressing surface" of fig. 2 is approximated by an expansion similar to eq. (11). For f(m,n) the expansion is

$$f(m,n) = \sum_{p=1}^{N1} \sum_{q=1}^{N2} F_{pq}\, t_1^{p-1}\, t_2^{q-1} \qquad (27)$$

where t_1 and t_2 are dimensionless coordinates on the actual surface. By prescribing f(m,n) in N1 · N2 nodal points on the surface, the coefficients F_{pq} are determined by a linear system of equations

$$\sum_{p=1}^{N1} \sum_{q=1}^{N2} F_{pq} \cdot (t_1(i))^{p-1}(t_2(j))^{q-1} = f(m,n,i,j) \qquad (28)$$

i = 1,N1
j = 1,N2

where $t_1(i)$ and $t_2(j)$ are coordinates to nodal point i,j and $f(m,n,i,j)$ is the prestressing in the point. The evaluation of the integrals are now straigthforward.

Notation

a_i, b_i, c_i	Orthogonal unit vectors
$A^{\ell ijk}_{npqr}$	Terms in eq. (21)
$B^{\ell ijk}_{npqr}$	Terms in eq. (21)
$C_{\ell ijk}$	Terms in eq. (21)
$D_{\ell ijk}$	Terms in eq. (21)
c^i_n	Coefficients in the expansion of u_i
E	The modulus of elasticity
E_p	The elastic energy of the slab
E_f	The elastic energy of the springs
E_{tot}	The total energy of the system
F(m,n)	The prestressing in direction x_n of the spring on S_m
f(m,n)	$F(m,n)/L_n$
$F_s(m,n)$	The stiffness of the spring on S_m for displacements in direction x_n
F_{pq}	Coefficients in expansion of f(m,n)
F^i_{pqr}	The derivative of G_{pqr} with respect to ξ_i
f_{ij}	The ratio L_i/L_j
G_{pqr}	The product $\xi_1^p \xi_2^q \xi_3^r$ in the expansion of u_i
$G_{pqr}(m)$	G_{pqr} evaluated on S_m
L_1, L_2, L_3	Half sidelengths of the slab in directions x_1, x_2, x_3, respectively
P_i	The loading on the upper surface of the slab
P	The loading component in the x_3-direction
P^ℓ_{ijk}	Coefficient in the expansion of u_i
R	The volume of the slab
R_1, R_2	Semiaxes of the hole in directions x_1, x_2, respectively
S_u	The part of the surface, where displacements are prescribed
S_t	The part of the surface, where stresses are prescribed
S_m	The surface (edge) no m
$T_p(\xi_i)$	The Chebycheff polynomium of the order p in ξ_i
t_1, t_2	Dimensionless coordinates on S_m

U_i	True displacements in the x_i-direction
$U_{i,j}$	$\frac{\partial U_i}{\partial x_j}$
u_i	Dimensionless displacement , $u_i = U_i/L_i$
$U_n(m)$	U_n evaluated on S_m
V	The loss of potential energy for the loading
W_p	Energy density for the slab
W_f	Energy density for the springs
$W_f(m)$	W_f evaluated on S_m
x_1, x_2, x_3	Coordinates
δ_{ij}	The Kronecker's symbol
ϕ_i	Admissible functions for expansion of U_i
ϕ_i^*	The solution to Naviers equations
ξ_i	Dimensionless coordinates $\xi_i = x_i/L_i$
σ_n	Stress in direction x_n
σ_a	Normal stress
τ_{ab}	Shear stress
ν	Poissons ratio

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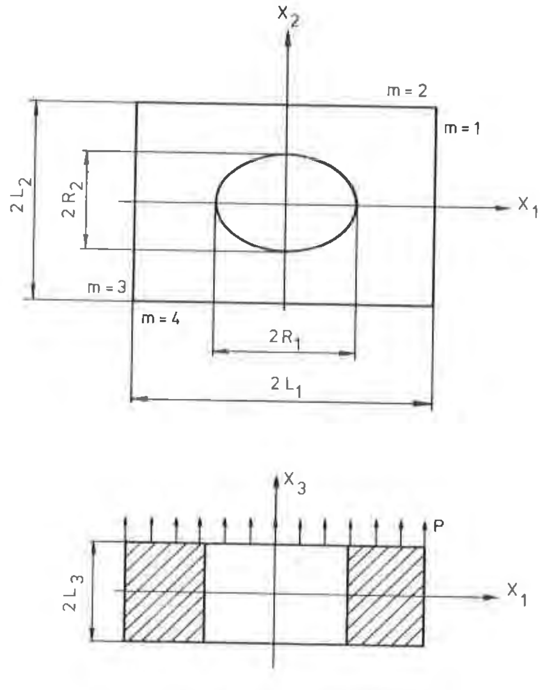

Fig. 1

Fig. 1: The slab. Note, that the vertical sides have numbers $m = 1$ to $m = 4$

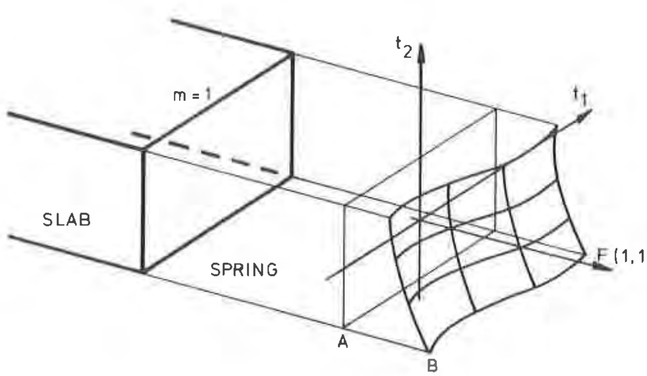

Fig. 2

Fig. 2: The prestressing in the direction x_1 for the spring on side $m = 1$. During the prestressing, the surface of the spring is moved from position A to position B. The amount $F(1,1)$ of prestressing is given by its volves in the nodal points indicated by the mesh.

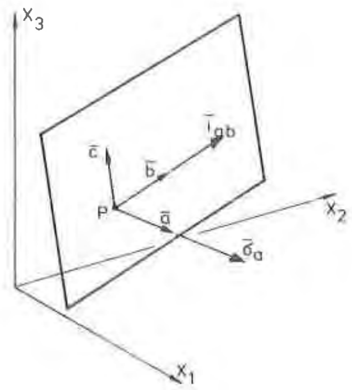

Fig. 3

Fig. 3: Visualization of the stress vectors σ_a of τ_{ab} .

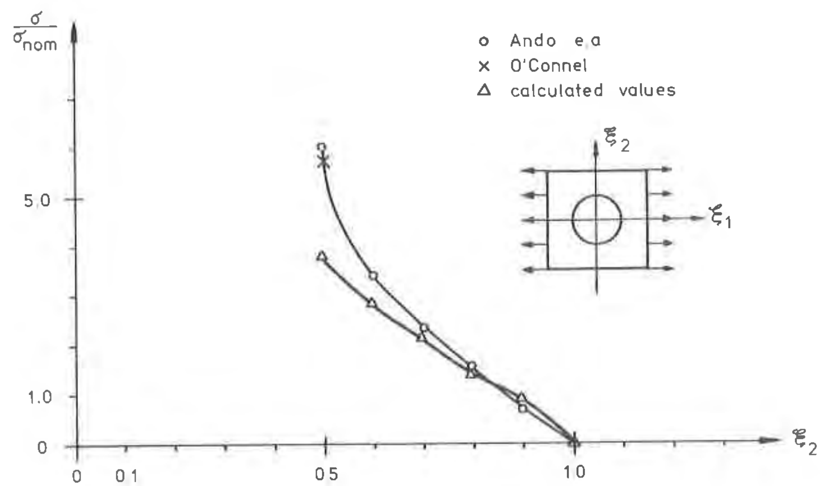

Fig. 4: The distribution along the ξ_2-axis of the tensile stress in the ξ_1 direction for a square slab under uniaxial tension.