DYNAMIC INTERACTION OF COMPONENTS, STRUCTURE, AND FOUNDATION OF NUCLEAR POWER FACILITIES

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SUMMARY

A solution is formulated for the dynamic analysis of structures and components with different stiffness and damping characteristics, including the consideration of soil-structure interaction effects. Composite structures are often analysed approximately, in particular with regards to damping. For example, the reactor and other equipment in nuclear power plant structures are often analysed by assuming them uncoupled from the supporting structures. To achieve a better accuracy, the coupled system is hereby analysed as a composite component-structure-soil system. Although derivation of mass and stiffness matrices for the component-structure-soil system is a simple problem, the determination of the damping characteristics of such a system is more complex. This emphasis on the proper evaluation of system damping is warranted on the grounds that, when resonance conditions occur, the response amplitude is governed to a significant degree by the system damping.

The damping information is usually available for each sub-structure separately with its based fixed or devoid of rigid-body modes of motion. The rigid-body motions are often free of damping resistance but sometimes, such as in the case of soil-structure interaction, or in the case of aerodynamic resistance, are uniquely defined. The composite damping matrix for the complete structure is hereby derived from the above-mentioned information. Thus, the damping matrix is first obtained for the free-free model of each sub-structure (the model containing the structural degrees of freedom together with rigid-body modes of motion), and then the submatrices for the free-free models are assembled to form the composite damping matrix in accordance with an assembly technique relating the sub-structure coordinates to the global coordinates of the composite structure.

To demonstrate the assembly technique, two examples are considered: (a) a steel structure sitting on a concrete stem and linked by a steel bridge to another concrete structure, and (b) an actual model of a nuclear power plant containment structure. In the latter example, the model consists of the containment structure, the internal structure, the pedestal, the shield wall, and the complete model of a boiling water reactor, including rod elements and hydrodynamic effects. All the subsystems are interconnected with each other and with the foundation. The stiffness matrix is also set up by the proposed method as well as an independent standard procedure to verify the validity of the proposed assembly technique.

1. Introduction

Nuclear power plant structures are usually made up of several sub-structures with different dynamic characteristics. Thus, for example, a reactor building, including the foundation and the reactor components is a highly complex system. The evaluation of the interaction of components, structure and foundation during earthquakes of such complex systems can be made only when the total system is correctly modeled. The modeling of the inertia and stiffness characteristics of such systems can, at the present time, be easily accomplished through the use of general purpose finite element computer programs. The proper modeling of the damping characteristics however has not been adequately treated. Several approximate methods have been described in the technical literature to approximate the damping characteristics of the total system. The validity of these methods can be verified by either in-situ tests or by a comparison of results using the correct formulation of the problem. The former, although highly desirable, is not a simple task. Thus, there is the need for a rigorous fomulation of the damping matrix of complex systems. The damping characteristics of sub-structures are, in general, adequately defined based on test data of similar structures. Thus, for structures, damping is usually specified as a characteristic of the modes of vibration [1]; and for soils, as the imaginary part of the foundation impedances [2].

The emphasis in this paper is on the formulation of a rigorous composite damping matrix for complex systems. This damping matrix is derived by first obtaining the damping sub-matrices of sub-structures as free-free systems, and then assembling these into a global matrix that represents the damping characteristics of the total system. The damping matrix thus formulated is, in general, not diagonalizable in the classical sense. However, having obtained the correct damping matrix, several alternatives are available to determine directly the response of the system. Alternatively, based on this total system damping matrix, approximate equivalent modal damping values for the total system can be calculated. Reference [3] describes two such methods.

2. Damping Matrix for Free-Free Model

Considering the fixed-base degrees of freedom x_f together with the rigid-body degrees of freedom x_r, the degrees of freedom x of the free-free model can be expressed as:

$$\left\{x\right\} = \begin{bmatrix} T \end{bmatrix} \begin{Bmatrix} \{x_f\} \\ \{x_r\} \end{Bmatrix} = \begin{bmatrix} I & T_r \\ 0 & I \end{bmatrix} \begin{Bmatrix} \{x_f\} \\ \{x_r\} \end{Bmatrix} \tag{1}$$

where the displacement transformation T_r with the elements

$$(T_r)_{ij} = \frac{\partial (x)_i}{\partial (x_r)_j} \tag{2}$$

gives the displacements induced by unit rigid-body motions.

Denoting by C_f and C_r the damping matrices for the fixed-base model and the rigid-body motion respectively, the damping matrix for the free-free model is found to be:

$$C = [T^{-1}]^T \begin{bmatrix} C_f & 0 \\ 0 & C_r \end{bmatrix} [T^{-1}] = \left[\begin{array}{c|c} C_f & -C_f T_r \\ \hline -T_r^T C_f & C_r + T_r^T C_f T_r \end{array}\right] \tag{3}$$

where C_r could represent a damping impedance function for rigid-body motion against foundation soil or the damping related to aerodynamic resistance against rigid-body motion. If such effects are not present, C_r is set equal to zero. The proof for the above relation lies in Bettie's law and its consequence that the force and displacement transformation matrices are the transpose of each other.

Given the fixed base modal damping values, C_f is obtained from eq. 4

$$C_f = [\phi^{-1}]^T \begin{bmatrix} \ddots & & \\ & 2\lambda_n\omega_n & \\ & & \ddots \end{bmatrix} [\phi^{-1}] \tag{4}$$

where ω_n and $\{\phi\}_n$ are the eigenvalues and eigenvectors of the undamped fixed-base structure obtained from the solution of the eigenvalue problem:

$$K_f\{\phi\}_n - \omega_n^2 M_f\{\phi\}_n = 0 \tag{5}$$

It is usually more convenient to relate x not to x_f but to the modal coordinates $q_f = \phi^{-1}x_f$

Thus,

$$\left\{ x \right\} = \begin{bmatrix} T \end{bmatrix} \begin{Bmatrix} \{x_f\} \\ \{x_r\} \end{Bmatrix} = \begin{bmatrix} T_\phi \end{bmatrix} \begin{Bmatrix} \{q_f\} \\ \{x_r\} \end{Bmatrix} = \begin{bmatrix} \phi & T_r \\ 0 & I \end{bmatrix} \begin{Bmatrix} \{q_f\} \\ \{x_r\} \end{Bmatrix} \tag{6}$$

If C_r lacks off-diagonal terms, the transformation T_ϕ would diagonalize the damping matrix:

$$T_\phi^T C T_\phi = \begin{bmatrix} \phi^T & 0 \\ T_r^T & I \end{bmatrix} \left[\begin{array}{c|c} C_f & -C_f T_r \\ \hline -T_r^T C_f & C_r + T_r^T C_f T_r \end{array}\right] \begin{bmatrix} \phi & T_r \\ 0 & I \end{bmatrix}$$

$$= \left[\begin{array}{c|c} \phi^T C_f \phi & 0 \\ \hline 0 & C_r \end{array}\right] = \left[\begin{array}{c|c} \diagdown 2\lambda_n\omega_n \diagdown & 0 \\ \hline 0 & C_r \end{array}\right] \tag{7}$$

Few examples of the transformation matrix T_ϕ can be found in References [4] and [5].

Occasionally, for sub-structures linking two other sub-structures, the damping information is available for a simply-supported model rather than the fixed-base model. For such a case, the same formulation presented above applies except that C_f and ϕ are related to the simply-supported model, the rigid-body motions are composed of relations inducing unit displacement at one end and zero at the other, and the transformation T_r, has a different form as shown below for a stick model with translational and rotational inertias.

$$[T_r]_{f.b.} = \begin{bmatrix} 1 & h_1 \\ 0 & 1 \\ 1 & h_2 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & h_n \\ 0 & 1 \end{bmatrix} \; ; \; [T_r]_{s.s.} = \begin{bmatrix} 1 & 1 \\ (h_2/h_1) & (h_2/h_1 - 1) \\ 1 & 1 \\ (h_3/h_1) & (h_3/h_1 - 1) \\ \vdots & \vdots \\ (h_n/h_1) & (h_n/h_1 - 1) \\ 1 & 1 \end{bmatrix} \tag{8}$$

3. Composite Damping Matrix by Sub-Structure Assembly

In the case of models composed of sub-structures with given damping properties, the free-free damping matrix is formulated for each sub-structure as described in Section 2. These sub-matrices are then assembled to form the composite damping matrix of the total system in accordance with the displacement transformation matrix relating the sub-structure coordinates to the global coordinates of the composite structure. These global coordinates are partitioned into the coordinates shared by various sub-structures and the coordinates belonging to one sub-structure alone. Accordingly, each set of sub-structure coordinates is partitioned into the coordinates shared with other sub-structures and the coordinates belonging to the particular sub-structure alone. Finally, each sub-matrix of each sub-structure is superposed to the corresponding location of the composite damping matrix. In this fashion, at each set of coordinates shared by one or more sub-structures, the contributions of the sub-structures are summed up to form that particular sub-matrix of the composite damping matrix.

The assembly technique will be demonstrated by the aid of an example. The structure shown in Fig. 1 with four distinct sub-structures is considered. The global coordinates (deformations relative to free-field) are represented as:

$$\bar{x}^T = (\bar{x}_i^T, \bar{x}_j^T, \bar{x}_k^T, \bar{x}_m^T, \bar{x}_n^T, \bar{x}_p^T, \bar{x}_q^T, \bar{x}_r^T, \bar{x}_s^T, \bar{x}_t^T, \bar{x}_u^T) \tag{9}$$

where

$\bar{x}_j$ = degrees of freedom shared by sub-structures 1 and 2

$\bar{x}_m$ = degrees of freedom shared by sub-structures 2 and 3

$\bar{x}_s$ = degrees of freedom shared by sub-structures 3 and 4

$\bar{x}_p$ = base degrees of freedom for sub-structure 2

$\bar{x}_u$ = base degrees of freedom for sub-structure 4

and the free-free coordinates for each sub-structure are denoted by $\bar{y}$, where

$$\bar{y}_1 = \begin{Bmatrix} \bar{x}_i \\ \bar{x}_j \end{Bmatrix} \qquad \bar{y}_2 = \begin{Bmatrix} \bar{x}_j \\ \bar{x}_k \\ \bar{x}_m \\ \bar{x}_n \\ \bar{x}_p \end{Bmatrix} \qquad \bar{y}_3 = \begin{Bmatrix} \bar{x}_q \\ \bar{x}_m \\ \bar{x}_n \end{Bmatrix} \qquad \bar{y}_4 = \begin{Bmatrix} \bar{x}_r \\ \bar{x}_s \\ \bar{x}_t \\ \bar{x}_u \end{Bmatrix} \tag{10}$$

The damping sub-matrix is set up for each free-free sub-structure, as discussed in Section 2, and appropriately partitioned as indicated:

$$c_1 = \begin{bmatrix} c_{ii}^{(1)} & c_{ij}^{(1)} \\ c_{ji}^{(1)} & c_{jj}^{(1)} \end{bmatrix}$$

$$c_2 = \begin{bmatrix} c_{jj}^{(2)} & c_{jk}^{(2)} & c_{jm}^{(2)} & c_{jn}^{(2)} & c_{jp}^{(2)} \\ c_{kj}^{(2)} & c_{kk}^{(2)} & c_{km}^{(2)} & c_{kn}^{(2)} & c_{kp}^{(2)} \\ c_{mj}^{(2)} & c_{mk}^{(2)} & c_{mm}^{(2)} & c_{mn}^{(2)} & c_{mp}^{(2)} \\ c_{nj}^{(2)} & c_{nk}^{(2)} & c_{nm}^{(2)} & c_{nn}^{(2)} & c_{np}^{(2)} \\ c_{pj}^{(2)} & c_{pk}^{(2)} & c_{pm}^{(2)} & c_{pn}^{(2)} & c_{pp}^{(2)} \end{bmatrix} \tag{11}$$

$$c_3 = \begin{bmatrix} c_{qq}^{(3)} & c_{qm}^{(3)} & c_{qs}^{(3)} \\ c_{mq}^{(3)} & c_{mm}^{(3)} & c_{ms}^{(3)} \\ c_{sq}^{(3)} & c_{sm}^{(3)} & c_{ss}^{(3)} \end{bmatrix}$$

$$c_4 = \begin{bmatrix} c_{rr}^{(4)} & c_{rs}^{(4)} & c_{rt}^{(4)} & c_{ru}^{(4)} \\ c_{sr}^{(4)} & c_{ss}^{(4)} & c_{st}^{(4)} & c_{su}^{(4)} \\ c_{tr}^{(4)} & c_{ts}^{(4)} & c_{tt}^{(4)} & c_{tu}^{(4)} \\ c_{ur}^{(4)} & c_{us}^{(4)} & c_{ut}^{(4)} & c_{uu}^{(4)} \end{bmatrix}$$

It should be noted that $\bar{x}_m$ and $\bar{x}_s$ are those degrees of freedom only that are shared by the two sub-structures, and if the connections of sub-structure 3 to sub-structures 2 and 4 are hinged, the rotational degrees of freedom should be included elsewhere and not in

$\overline{x}_m$ and $\overline{x}_s$. Furthermore, it should be noted that each set of sub-structure coordinates must possess all rigid-body motions for the assembly technique to be valid.

The displacement transformation matrix relates the global coordinates to the sub-structure coordinates:

$$\begin{Bmatrix} \overline{y}_1 \\ \overline{y}_2 \\ \overline{y}_3 \\ \overline{y}_4 \end{Bmatrix} = A\overline{x} \tag{12}$$

where

$$A = \left[\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
I & & & & & & & & & & \\ \hline
 & I & & & & & & & & & \\ \hline
 & I & & & & & & & & & \\ \hline
 & & I & & & & & & & & \\ \hline
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 & & & & & & & & & & I
\end{array}\right]$$

Therefore, the composite damping matrix for the complete structure is given as:

$$C = A^T \begin{bmatrix} c_1 & & & \\ & c_2 & 0 & \\ & 0 & c_3 & \\ & & & c_4 \end{bmatrix} A \tag{13}$$

The expanded form of the damping matrix **C** is given below:

$$
C = \begin{bmatrix}
c_{ii}^{(1)} & c_{ij}^{(1)} & & & & & & & & & \\
c_{ji}^{(1)} & c_{jj}^{(1)} + c_{jj}^{(2)} & c_{jk}^{(2)} & c_{jm}^{(2)} & c_{jn}^{(2)} & c_{jp}^{(2)} & & & & & \\
 & c_{kj}^{(2)} & c_{kk}^{(2)} & c_{km}^{(2)} & c_{kn}^{(2)} & c_{kp}^{(2)} & & & & & \\
 & c_{mj}^{(2)} & c_{mk}^{(2)} & c_{mm}^{(2)} + c_{mm}^{(3)} & c_{mn}^{(2)} & c_{mp}^{(2)} & c_{mq}^{(3)} & & c_{ms}^{(3)} & & \\
 & c_{nj}^{(2)} & c_{nk}^{(2)} & c_{nm}^{(2)} & c_{nn}^{(2)} & c_{np}^{(2)} & & & & & \\
 & c_{pj}^{(2)} & c_{pk}^{(2)} & c_{pm}^{(2)} & c_{pn}^{(2)} & c_{pp}^{(2)} & & & & & \\
 & & & c_{qm}^{(3)} & & & c_{qq}^{(3)} & & c_{qs}^{(3)} & & \\
 & & & & & & & c_{rr}^{(4)} & c_{rs}^{(4)} & c_{rt}^{(4)} & c_{ru}^{(4)} \\
 & & & c_{sm}^{(3)} & & & c_{sq}^{(3)} & c_{sr}^{(4)} & c_{ss}^{(3)} + c_{ss}^{(4)} & c_{st}^{(4)} & c_{su}^{(4)} \\
 & & & & & & & c_{tr}^{(4)} & c_{ts}^{(4)} & c_{tt}^{(4)} & c_{tu}^{(4)} \\
 & & & & & & & c_{ur}^{(4)} & c_{us}^{(4)} & c_{ut}^{(4)} & c_{uu}^{(4)}
\end{bmatrix}
$$

4. Discussion and Comments

The methodology described in this paper is easily verified by applying it to the assembly of the stiffness matrix of a complicated structure. The choice of the stiffness matrix is based on the fact that it can also be derived directly through the use of general purpose finite element computer codes. Thus, a direct comparison of the stiffness matrices of the same structure derived by the two methods would conclusively validate the proposed assembly technique. An example of this type of comparison is given in Reference [5]. Although a similar successful check was done for the present approach the results will not be shown as the example used was extremely large. It is recommended that this check be routinely done specially for the more complicated structures in order to assure that the transformation matrices have been properly developed. Response comparisons between the coupled and uncoupled models have not been made considering the prohibitive size of the model used. However, parametric response studies of a similar model would be the subject of another paper.

The objective of this study was the rigorous formulation of the damping matrix of composite structures composed of many sub-structures with different damping characteristics. This formulation provides a more appropriate response calculation of coupled systems. Its usefulness lies also in the fact that it can be the basis by which to judge the adequacy of the often used approximate techniques for the calculation of equivalent modal damping values of complex systems.

References

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[5] Ibrahim, A. M., and Hadjian, A. H., "The Composite Damping Matrix for Three Dimensional Soil-Structure Systems," 2nd ASCE Specialty Conference on Structural Design of Nuclear Plant Facilities, New Orleans, December 1975.

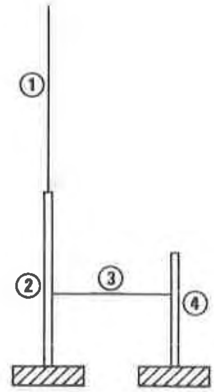

Figure 1. EXAMPLE STRUCTURE TO DEMONSTRATE DAMPING MATRIX ASSEMBLY TECHNIQUE